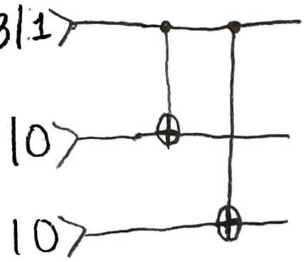
QINTRO - TD10

Ex 1.

Extending circuit for the 3 qubits - bit phlip.

bit phlip.

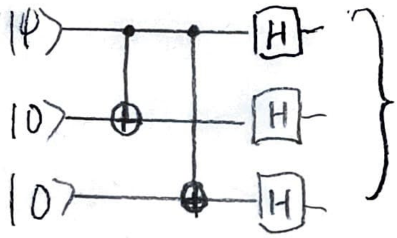
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



$\Rightarrow \alpha|000\rangle + \beta|111\rangle$

then.

phase - phlip.



$\Rightarrow \alpha|{+}{+}{+}\rangle + \beta|{-}{-}{-}\rangle$

remember

$|\tilde{0}\rangle = |\tilde{+}\tilde{+}\tilde{+}\rangle$

$|\tilde{1}\rangle = |\tilde{-}\tilde{-}\tilde{-}\rangle$

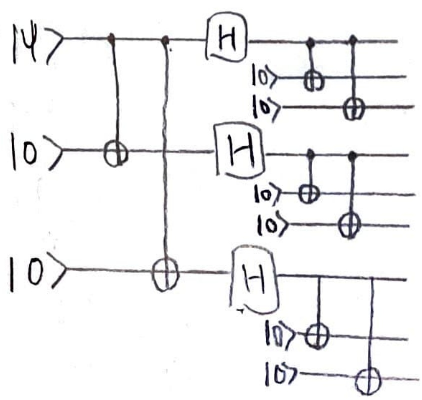
when $|\tilde{+}\rangle = \frac{1}{\sqrt{2}}(|\hat{0}\rangle + |\hat{1}\rangle)$

$|\tilde{-}\rangle = \frac{1}{\sqrt{2}}(|\hat{0}\rangle - |\hat{1}\rangle)$

$|\hat{0}\rangle = |000\rangle$

$|\hat{1}\rangle = |111\rangle$

$\Leftarrow$



Ex. 2

$$|\bar{0}\rangle = \frac{1}{2}(|00\rangle + |11\rangle) \otimes (|00\rangle + |11\rangle) = |\phi_+\rangle|\phi_+\rangle$$

$$|\hat{1}\rangle = \frac{1}{2}(|00\rangle - |11\rangle) \otimes (|00\rangle - |11\rangle) = |\phi_-\rangle|\phi_-\rangle$$

where $|\phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, $|\phi_-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

a) $X_1|\bar{0}\rangle = \frac{1}{2}(|10\rangle + |01\rangle) \otimes (|00\rangle + |11\rangle)$

$X_1|\hat{1}\rangle = \frac{1}{2}(|10\rangle - |01\rangle) \otimes (|00\rangle - |11\rangle)$

To detect for bit flip we need to find a way to compare the first 2 qubit togethere (to detect X_1 or X_2) and the last two qubits together (to detect X_3 or X_4)

b) $Z_1|\hat{0}\rangle = \frac{1}{2}(|00\rangle - |11\rangle) \otimes (|00\rangle + |11\rangle) = \overset{(1)}{|\phi_-\rangle}\overset{(2)}{|\phi_+\rangle}$

$Z_1|\hat{1}\rangle = \frac{1}{2}(|00\rangle + |11\rangle) \otimes (|00\rangle + |11\rangle = \overset{(3)}{|\phi_+\rangle}\overset{(4)}{|\phi_-\rangle}$

To detect for phase phlip we need to compare the (1) and (2), state on the first two qubits and the state of the last two states

$$Z_1 \text{ or } Z_2 \to \begin{cases} |\hat{0}\rangle \longmapsto |\phi_-\rangle|\phi_+\rangle \\ |\hat{1}\rangle \longmapsto |\phi_+\rangle|\phi_-\rangle \end{cases}$$

$$Z_3 \text{ or } Z_4 \to \begin{cases} |\hat{0}\rangle \longmapsto |\phi_+\rangle|\phi_-\rangle \\ |\hat{1}\rangle \longmapsto |\phi_-\rangle|\phi_+\rangle \end{cases}$$

vete a la porta!

QINTRO - TD10 vete a la porra.

d) The code is $|\tilde{0}\rangle = |\hat{+}\hat{+}\rangle$ where $|\hat{+}\rangle = \frac{1}{\sqrt{2}}(|\hat{0}\rangle + |\hat{1}\rangle)$

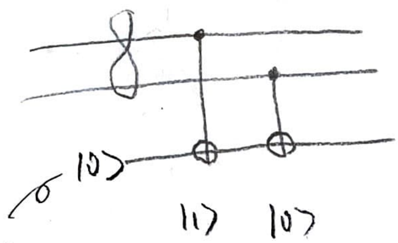
$|\tilde{1}\rangle = |\hat{-}\hat{-}\rangle$ $\qquad |\hat{-}\rangle = \frac{1}{\sqrt{2}}(|\hat{0}\rangle - |\hat{1}\rangle)$

and $|\hat{0}\rangle = |00\rangle$

$|\hat{1}\rangle = |11\rangle$.

So it is the concatination of a 2-qubit phase shift repetition code with a 2-qubit bit philip repetition code.

e)

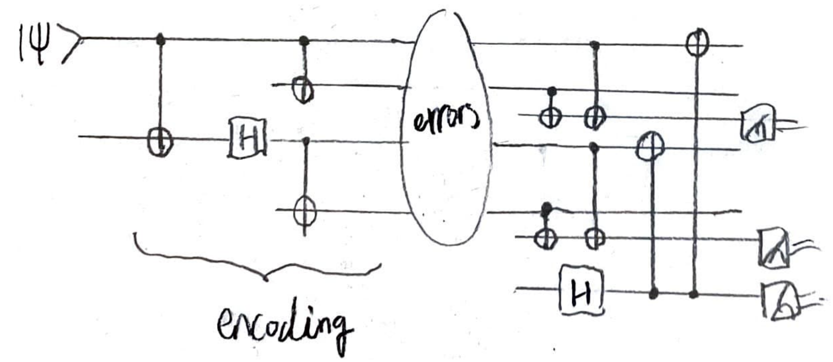


for $|+\rangle$ you flip CNOT gates !

$00 \to |0\rangle$
$01 \to |1\rangle$
$10 \to |1\rangle$
$11 \to |0\rangle$

CNOT.

$|++\rangle \to |+\rangle|+\rangle$
$|+-\rangle \to |-\rangle|-\rangle$
$|-+\rangle \to |-\rangle|+\rangle$
$|--\rangle \to |+\rangle|-\rangle$

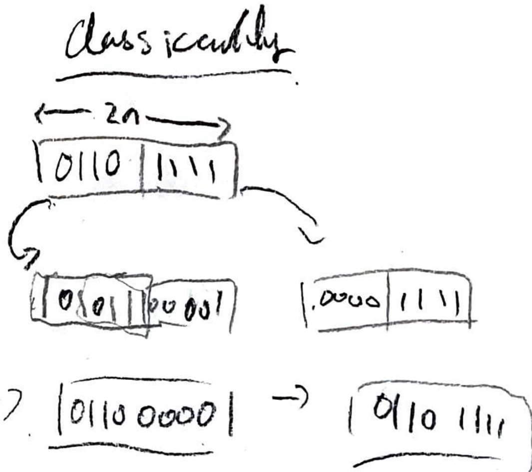


Ex. 3 idea: Show that if such a code exist, we can use it to copy a quantum state
contradiction with the no-cloning theorem.

$$|\psi\rangle = \sum \alpha_{k_0 \dots k_{2n-1}} \underbrace{|k_0\rangle \dots\dots |k_{2n-1}\rangle}_{2n \text{ physical cubit}} \qquad \sum |\alpha|_{k_0 \dots} = 1$$

$\rightarrow$ n-qubits $|\psi_0\rangle \dots$,

n-qubits $|\psi_1\rangle \dots\dots$



let $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ be a quantum state. We first encode it into the 2n qubit code.

Assume, A keeps n qubits of the 2n qubits state.
B keeps the other n qubits. Each of them can the append $|0\rangle^{\otimes n}$ to their state and they each detain a "copy" of $|\psi\rangle$ with erros on at most n qubits.

By assumption they can correct those error as they each obtain a copy of the encoded $|\psi\rangle$,
contradiction with the non-clone theorem.

Ex. 4

a) $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\frac{1}{\sqrt{2}} = (|0\rangle + e^{i\pi/4}|1\rangle)$

$CNOT(|\phi\rangle, \frac{1}{\sqrt{2}}|0\rangle + e^{i\pi/4}|1\rangle)$

$CNOT(\alpha|0\rangle + \beta|1\rangle, \frac{1}{\sqrt{2}}|0\rangle + e^{i\pi/4}|1\rangle)$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes (\frac{1}{\sqrt{2}}|0\rangle + e^{i\pi/4}|1\rangle)$$

$$= \frac{1}{\sqrt{2}}(\alpha|00\rangle + \alpha e^{i\pi/4}|01\rangle + \beta|10\rangle + \beta e^{i\pi/4}|11\rangle$$

$$\xrightarrow{CNOT} \frac{1}{\sqrt{2}}(\alpha|00\rangle + \alpha e^{i\pi/4}|01\rangle + \beta|11\rangle + \beta e^{i\pi/4}|10\rangle).$$

b) $P_0 = I \otimes |0\rangle\langle 0|$, $P_1 = I \otimes |1\rangle\langle 1|$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\alpha|00\rangle + \alpha e^{i\pi/4}|01\rangle + \beta|11\rangle + \beta e^{i\pi/4}|10\rangle$$

$$= \frac{1}{\sqrt{2}}\left[(\alpha|0\rangle + \beta e^{i\pi/4}|1\rangle) \otimes |0\rangle + (\alpha e^{i\pi/4}|0\rangle + \beta|1\rangle) \otimes |1\rangle\right]$$

$$P_0 = |\langle 0|\psi\rangle|^2 = \frac{1}{2}(|\alpha|^2 + |\beta \cancel{e^{i\pi/4}}|^2) = 1/2$$

$$P_1 = |\langle 1|\psi\rangle|^2 = \frac{1}{2}(|\alpha \cancel{e^{i\pi/4}}|^2 + |\beta|^2) = 1/2.$$

or

$|\langle\psi|P_0|\psi\rangle|$

$|\langle\psi|P_1|\psi\rangle|$

c) If measurement 0, then we have $|\phi\rangle = (\alpha|0\rangle + \beta e^{i\pi/4}|1\rangle)$

$$T|\phi\rangle = \alpha|0\rangle + \beta e^{i\pi/4}|1\rangle$$

we can apply Identity and obtain $T|\phi\rangle$.

d) If measurement yield 1, $|\phi\rangle = \alpha e^{i\pi/4}|0\rangle + \beta|1\rangle$

To get $T|\phi\rangle$, $= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta e^{i\pi/4} \end{pmatrix}$

Apply S.

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} \alpha e^{i\pi/4} \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha e^{i\pi/4} \\ \beta i \end{pmatrix} = e^{i\pi/4} \times (\alpha|0\rangle + \beta e^{i\pi/4}|1\rangle)$$