How to normalize a vector

→ It is a process of make a vector into a unit vector which has norm equal to one.

→ $u = \frac{v}{\|v\|}$ u: unit vector, v: original vector.

Example 1.

- Normalize $\left(\frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle\right)$.

$v = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$.

$\|v\| = \sqrt{\left|\frac{1}{\sqrt{3}}\right|^2 + \left|\frac{1}{\sqrt{3}}\right|^2} = \sqrt{\frac{2}{3}}$

$u = \frac{\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle}{\sqrt{\frac{2}{3}}} = \sqrt{\frac{3}{2}} \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$ ($\frac{1}{\|v\|}$, v)

$= \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

So, $\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$.

Example 2

- Normalize $\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|1\rangle\right)$

$$\frac{\left\langle \frac{1}{\sqrt{2}}, \frac{1}{2} \right\rangle}{\frac{\sqrt{3}}{2}}$$

$$\frac{2}{\sqrt{3}}\left\langle \frac{1}{\sqrt{2}}, \frac{1}{2} \right\rangle$$

$$\Rightarrow \left\langle \frac{\sqrt{2}}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

$$\sqrt{\left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{2}\right|^2}$$

$$= \sqrt{\frac{1}{2} + \frac{1}{4}} = \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

$$\therefore \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle\right)$$