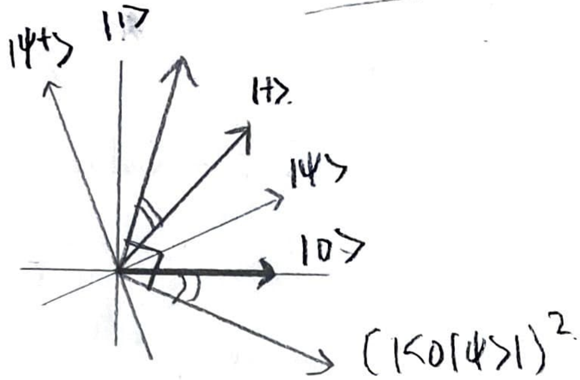
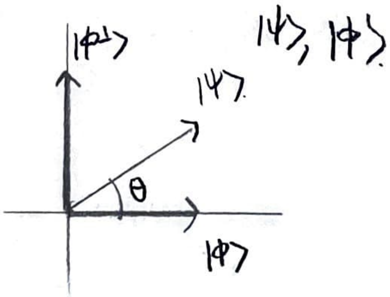
QINTRO - TD5 22. Feb. 23





$|\langle\phi|\psi\rangle| = \cos\theta$

$\text{Angle} = \arctan(|\langle\phi|\psi\rangle|)$

$$\text{Angle}(\rho, \sigma) = \text{Arccos}(F(\rho,\sigma))$$

- $0 \leq \text{Angle}(\rho,\sigma)$ with equality if $\rho = \sigma$
- $\text{Angle}(\rho,\sigma) = \text{Angle}(\sigma,\rho)$
- $\text{Angle}(\rho,\tau) \leq \text{Angle}(\rho,\sigma) + \text{Angle}(\sigma,\tau)$

$0 \leq \text{Angle}(\rho,\sigma) \leq \pi/2$

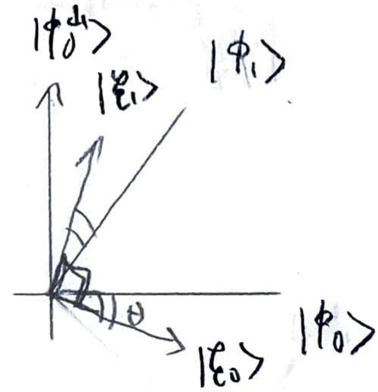
Ex. 2

1. $\langle \varphi_0 | \varphi_1 \rangle = \langle ++ | \frac{1}{\sqrt{2}} (|+\rangle|1\rangle - |-\rangle|0\rangle)$

$= \frac{1}{2}$. $\quad = \frac{1}{\sqrt{2}} \left(1 + \frac{1}{\sqrt{2}} + 0 + \frac{1}{\sqrt{2}} \right)$

$= \frac{1}{\sqrt{2}} \left(\frac{2+\sqrt{2}}{\sqrt{2}} \right) = \frac{2}{2} + \frac{\sqrt{2}}{2}$

1



$\theta = \left(\frac{\pi}{2} - \text{Angle}(|\varphi_0\rangle |\varphi_1\rangle) \right) / 2.$

$= \left(\frac{\pi}{2} - \frac{\pi}{3} \right) / 2 = \frac{\pi}{12}$

$|\xi_0\rangle = \cos\left(\frac{\pi}{12}\right) |\varphi_0\rangle - \sin\left(\frac{\pi}{12}\right) |\varphi_0^{\perp}\rangle$

$|\xi_1\rangle = \sin\left(\frac{\pi}{12}\right) |\varphi_0\rangle + \cos\left(\frac{\pi}{12}\right) |\varphi_0^{\perp}\rangle$

$|\langle \varphi_0 | \xi_0 \rangle|^2 = \cos^2\left(\frac{\pi}{12}\right)$

$|\langle \varphi_1 | \xi_1 \rangle| = \cos^2\left(\frac{\pi}{12}\right)$

$P_{success} = \cos^2\left(\frac{\pi}{12}\right)$

Ex. 3

if $|\psi_{0000}\rangle = |+\rangle|+\rangle$, and $|\psi_{1111}\rangle = -|+\rangle|+\rangle$

if $|\psi_{0011}\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(|0\rangle|+\rangle - |1\rangle|+\rangle\right)$

$= |-\rangle|+\rangle$

1, $x_1 = x_2 = 0$	$x_3 = x_4 = 1$	$x_1 = x_2 = 1$	$x_3 = x_4 = 0$
2, $x_1 = x_3 = 0$	$x_2 = x_4 = 1$	$x_1 = x_3 = 1$	$x_2 = x_4 = 0$
3 $x_1 = x_4 = 0$	$x_2 = x_3 = 1$	$x_1 = x_4 = 1$	$x_2 = x_3 = 0$

$x_1 = x_2$

$x_3 = x_4$ $\quad |\psi_{x_1 x_2 x_3 x_4}\rangle$

$= \frac{1}{2}(-1)^{x_1}|00\rangle + (-1)^{x_1}|01\rangle + (-1)^{x_3}|10\rangle + (-1)^{x_3}|11\rangle$

$= \frac{1}{2}\left[|0\rangle\left((-1)^{x_1}(|0\rangle + |1\rangle)\right) + |1\rangle\left((-1)^{x_3}(|0\rangle + |1\rangle)\right)\right]$

$= \frac{1}{\sqrt{2}}\left[(-1)^{x_1}|0\rangle|+\rangle + (-1)^{x_3}|1\rangle|+\rangle\right]$

$= \frac{1}{\sqrt{2}}\left[\left((-1)^{x_1}|0\rangle + (-1)^{x_3}|1\rangle\right)|+\rangle\right]$

$= (-1)^{x_1}\underbrace{\frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]}_{\Rightarrow \sqrt{2}|-\rangle}|+\rangle = (-1)^{x_1}|-\rangle|+\rangle = \boxed{\pm |-\rangle|+\rangle}$

$$\left|\begin{array}{l} x_1 = x_3 \\ x_2 = x_4 \end{array}\right. \quad \pm |+\rangle|-\rangle$$

$$\left|\begin{array}{l} x_1 = x_4 \\ x_2 = x_3 \end{array}\right. \quad \pm |-\rangle|-\rangle$$

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Ex. 4

1.

$$\cos(\alpha+\beta) \geq \frac{1}{2}\left(2\cos^2(\alpha) - 1 + 2\cos^2(\beta) - 1\right)$$

$\Longleftrightarrow$ let $x = 2\alpha$, $y = 2\beta$

$$\cos\left(\frac{2\alpha+2\beta}{2}\right) \geq \frac{1}{2}\left(\cos(2\alpha) + \cos(2\beta)\right)$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$$

2. let ξ be any dencity matrix.

$$\frac{1}{2}\left(F^2(\rho,\xi) + \frac{1}{2}F^2(\xi,\sigma)\right) = \frac{1}{2}\left(\cos^2(\alpha) + \cos^2(\beta)\right)$$

$$\leq \frac{1}{2}\cos(\alpha+\beta) + 1$$

$$\text{Angle}(\rho,\xi) + \text{Angle}(\xi,\sigma) \geq \text{Angle}(\rho,\sigma)$$

$$\boxed{\begin{aligned} F(\rho,\xi) &= \cos(\underbrace{\text{Angle}(\rho,\xi)}_{\alpha}) \\ F(\xi,\rho) &= \cos(\underbrace{\text{Angle}(\xi,\sigma)}_{\beta}) \end{aligned}}$$

$$\alpha + \beta \geq \gamma$$

$$\cos(\alpha+\beta) \leq \cos(\gamma)$$

$$F(\rho,\sigma) = \cos(\underbrace{\text{Angle}(\rho,\sigma)}_{\gamma})$$

$\Rightarrow \cos(\gamma)$