QINTRO - TD7

Ex.4

- $a \oplus b \oplus c = OR(x, y, z)$
- $x, y, z \in \{0,1\}$, containing an even number of 1.
- They each output a bit, noted respectively a, b, and c.

x	y	z	$OR(x,y,z)$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	1

→ win if $a \oplus b \oplus c = 0$

→ win if $a \oplus b \oplus c = 1$

(1) $f_A \quad x \longmapsto f_A(x) = a$

$f_B \quad y \longmapsto f_B(y) = b$

$f_C \quad z \longmapsto f_C(z) = c$

$$f_A(0) \oplus f_B(0) \oplus f_C(0) = 0$$
$$f_A(0) \oplus f_B(1) \oplus f_C(1) = 1$$
$$f_A(1) \oplus f_B(0) \oplus f_C(1) = 1$$
$$f_A(1) \oplus f_B(1) \oplus f_C(0) = 1$$

Those 4 equations can't be simultaneously satisfied so summing all of them gives $0 = 1$.
Since the strategy is detemistic, it means one of the equations will never be satisfied so, A, B, C can only win with prob $\leq 3/4$.
It is possible to achive a proba. of $3/4$. For instance by taking $f_A(x) = 1$, $f_B(x) = f_C(x) = 0$ for all $x \in \{0,1\}$

So a deterministic clacial strategy success w.p. at most $3/4$

(2) $|\phi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right)$

In case of $x = y = z = 0$.

$$|\phi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right)$$

In case of $x = z = 1$ $y = 0$

$$H^{\otimes 1,3} \otimes I \otimes |\phi\rangle = \frac{1}{4}\left(|{+}0{+}\rangle - |{+}1{-}\rangle - |{-}0{-}\rangle - |{-}1{+}\rangle\right)$$

$$= \frac{1}{4}\Big(\big((|0\rangle+|1\rangle)|0\rangle(|0\rangle+|1\rangle)\big) - \big((|0\rangle+|1\rangle)|1\rangle(|0\rangle-|1\rangle)\big)$$
$$- \big((|0\rangle-|1\rangle)|0\rangle(|0\rangle-|1\rangle)\big) - \big((|0\rangle-|1\rangle)|1\rangle(|0\rangle+|1\rangle)\big)\Big)$$

$$= \frac{1}{4}\Big(\big(|000\rangle + |001\rangle + |100\rangle + |101\rangle\big)$$
$$-\big(|010\rangle - |011\rangle + |110\rangle - |111\rangle\big)$$
$$-\big(|000\rangle - |001\rangle - |100\rangle + |101\rangle\big)$$
$$-\big(|010\rangle + |011\rangle - |110\rangle - |111\rangle\big)\Big)$$

$$= \frac{1}{4}\Big(\cancel{|000\rangle} + |001\rangle + |100\rangle + \cancel{|101\rangle} - |010\rangle + \cancel{|011\rangle} - \cancel{|110\rangle}$$
$$+ |111\rangle - \cancel{|000\rangle} + |001\rangle + |100\rangle - \cancel{|101\rangle}$$
$$- |010\rangle - \cancel{|011\rangle} + \cancel{|110\rangle} + |111\rangle\Big)$$

$$= \frac{1}{4}\left(2|001\rangle + 2|100\rangle + 2|111\rangle - 2|010\rangle\right)$$

$$= \frac{1}{2}\left(|001\rangle + |100\rangle + |111\rangle - |010\rangle\right)$$

QINTRO - TD4

In case of $x, y, z = 1, 1, 0$

$$H \otimes I |\phi\rangle = \tfrac{1}{2}\left(|{+}{+}0\rangle - |{+}{-}1\rangle - |{-}{+}1\rangle - |{-}{-}0\rangle\right)$$

$$= \tfrac{1}{2}\left(|010\rangle + |100\rangle - |001\rangle + |111\rangle\right)$$

In case of $x, y, z = 0, 1, 1$

$$H \otimes I |\phi\rangle = \tfrac{1}{2}\left(|0{+}{+}\rangle - |0{-}{-}\rangle - |1{+}{-}\rangle - |1{-}{+}\rangle\right)$$

$$= \tfrac{1}{2}\left(|001\rangle + |010\rangle - |100\rangle + |111\rangle\right)$$