QINTRO - TD 4 — 17. Feb. 23

$$|\psi_{x_1x_2x_3x_4}\rangle = \frac{1}{2}\left((-1)^{x_1}|00\rangle + (-1)^{x_2}|01\rangle + (-1)^{x_3}|10\rangle + (-1)^{x_4}|11\rangle\right)$$

Ex. 1

Assume $x_2, x_3, x_4 = 0$,

$$|\varphi_b\rangle = |\psi_{b000}\rangle = \frac{1}{2}\left((-1)^b|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

1. $|\varphi_0\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) = |+\rangle|+\rangle$

$|\varphi_1\rangle = \frac{1}{2}\left(-|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) = \frac{1}{2}\left(-\sqrt{2}|-\rangle|0\rangle + \sqrt{2}|+\rangle|1\rangle\right)$

2. $\rho_b^A = tr_B |\varphi_0\rangle\langle\varphi_0|$ $\quad |\varphi_b\rangle = \sum_{m,n} a_{m,n} |f_n\rangle|e_m\rangle$

$= \sum_{mn} |a_{mn}|^2 |f_n\rangle\langle f_n|$

$|\varphi_0\rangle = |+\rangle|+\rangle$. $\quad tr_A(|\varphi_0\rangle\langle\varphi_0|) = |+\rangle\langle+|$

$tr_B(|\varphi_0\rangle\langle\varphi_0|) = |+\rangle\langle+|$

$|\varphi_1\rangle = \frac{1}{2}\left(\sqrt{2}|+\rangle|1\rangle - \sqrt{2}|-\rangle|0\rangle\right)$

$\rho_1^A = tr_B\left(|\varphi_1\rangle\langle\varphi_1|\right) = \frac{1}{2} Id.$

$= \frac{1}{\sqrt{2}}|+\rangle\langle+| + \frac{1}{\sqrt{2}}|-\rangle\langle-|$

$= \frac{1}{2}\left(|+\rangle\langle+| + |-\rangle\langle-|\right)$

3. $\Delta(\rho_0^A, \rho_1^A) = \frac{1}{2} tr \sqrt{(\rho_0 - \rho_1)(\rho_0 - \rho_1)} = \frac{1}{2}\sum_i |d_i|$.

$$(\rho_0^A - \rho_1^A) = \frac{1}{2}(|+X+| - |-X-|)$$

$$d_0 = 1/2$$

$$d_1 = -1/2$$

$$\Delta(\rho_0^A, \rho_1^A) = \frac{1}{2}\left(\frac{1}{2} + \left|-\frac{1}{2}\right|\right) = \frac{1}{2}$$

$$P_{max} = \frac{1}{2} + \frac{\Delta(\rho_0^A, \rho_1^A)}{2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

To distinguish between ρ_0, ρ_1 as recieve R_b.
Measure it in $|+\rangle, |-\rangle$ basis

$\rightarrow$ if you get $|+\rangle$, conclude that $\rho_b = \rho_0$

$$\Rightarrow tr(\langle+|\rho_0|+\rangle) = 1$$

$\rightarrow$ if you get $|-\rangle$, conclude that $\rho_b = \rho_1$.

$$\Rightarrow tr(\langle-|\rho_1|-\rangle) = 1/2$$

$$\Downarrow$$

$$\frac{1}{2}\left(1 + \frac{1}{2}\right) = 3/4$$