TD 1.

Reduced density matrix of states shared between two parties.

$$\rho_A = tr_B(|\psi_{AB}\rangle\langle\psi_{AB}|), \quad \rho_B = tr_A(|\psi_{AB}\rangle\langle\psi_{AB}|)$$

$$|\phi\rangle_{AB} = \sum_{a,b} \alpha_{ab} \underbrace{|\varepsilon_a\rangle}_{\text{orthonormal basis.}} |e_b\rangle$$

$$tr_A(|\phi_{AB}\rangle\langle\phi_{AB}|) = \sum_{a,b} |\alpha_{ab}|^2 |e_b\rangle\langle e_b|_B.$$

$$|\phi'\rangle_{AB} = \sum_{ab} \mu_{ab} |e_a\rangle \underbrace{|\varepsilon_b\rangle}_{\text{orthonormal basis}}$$

$$tr_B(|\phi'_{AB}\rangle\langle\phi'_{AB}|) = \sum_{ab} |\mu_{ab}|^2 |e_a\rangle\langle e_a|$$

- Norm of density matrix is one.

- Vector normalization.
  - it is a process of make a vector into a unit vector which has norm equal to one

$$u = \frac{v}{\|v\|}$$

u : unit vector

v : original vector.

TD 2.

Trace distance.

$$\Delta(\rho,\sigma) = \frac{1}{2}\mathrm{Tr}\left(\sqrt{(\rho-\sigma)^\dagger(\rho-\sigma)}\right)$$

→ if ρ and σ are two pure state : $\rho = |\psi\rangle\langle\psi|$, $\sigma = |\phi\rangle\langle\phi|$.

$$\Delta(\rho,\sigma) = \sqrt{1 - |\langle\psi|\phi\rangle|^2}$$

→ if ρ and σ are diagonalizable in the same basis.

$\rho = \sum_i p_i |e_i\rangle\langle e_i|$, $\sigma = \sum_i q_i |e_i\rangle\langle e_i|$,

$$\Delta(\rho,\sigma) = \frac{1}{2}\sum_i |p_i - q_i| .$$

$$\Rightarrow \max\left(\Pr[\text{Bob guesses } b]\right) = \frac{1}{2} + \frac{\Delta(\rho,\sigma)}{2}.$$

note :)

$$\frac{1}{2} + \frac{1}{2\sqrt{2}} = \cos^2(\pi/8)$$

TD5

The angle is a distance.

- $\text{Angle}(\rho,\sigma) = 0 \iff \rho = \sigma$
- $0 \leq \text{Angle}(\rho,\sigma) \leq \pi/2$
- $\text{Angle}(\rho,\sigma) = \text{Angle}(\sigma,\rho)$
- $\text{Angle}(\rho,\tau) \leq \text{Angle}(\rho,\sigma) + \text{Angle}(\sigma,\tau)$

Probability of Bob distinguishing between $|0\rangle$ & $|+\rangle$,

* it appears multiple times over several TDs.

- This is something we did on lecture.

- measure in $\{|v\rangle, |v^\perp\rangle\}$ basis with

$$|v\rangle = \cos(\pi/8)|0\rangle - \sin(\pi/8)|1\rangle$$
$$|v^\perp\rangle = \sin(\pi/8)|0\rangle + \cos(\pi/8)|1\rangle$$

* in $|0\rangle$ and $|+\rangle$ probabilities are 1 and $1/2$. But with $\{|v\rangle, |v^\perp\rangle\}$? Probabilities are $\cos^2(\pi/8) \approx 0.85$ each.

Success w.p $\cos^2(\pi/8)$.