An Mereological and Algebraic Formalization of MegaL's Prelude

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Disclaimer:
These notes are at draft level and may be very imperfect.

Abstract. ...

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1 Introduction

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2 A Mereological Formalization

2.1 Parthood

A mereology focuses on the **parthood** predicate $\mathcal{P}$. We read the $\mathcal{P}xy$ as *"x is part of y"* for two arbitrary objects x and y. [**?**]

Definition 1 (Parthood). *Parthood is a binary predicate $\mathcal{P}$ between two entities x and y. We write*

$$\mathcal{P}xy \tag{1}$$

and read "x is part of y".

Axiom 1 (Parthood is Reflexive)

$$\mathcal{P}xx \tag{2}$$

An object x is always part of itself.

Note that this opens the door for problems similar to Russel's antinomy for naive sets.

Axiom 2 (Parthood is Antisymmetric)

$$\mathcal{P}xy \wedge \mathcal{P}yx \Rightarrow x = y \tag{3}$$

If two objects are part of one another, we assume both objects to be identical.

Axiom 3 (Parthood is Transitive)

$$\mathcal{P}xy \wedge \mathcal{P}yz \Rightarrow \mathcal{P}xz \tag{4}$$

We assume parthood to be transitive in the sense of object composition. If one object is part of a second object, which in turn is part of a third object, the first is considered to be also a part of the latter one.

Specializations of Parthood We define the following specializations of parthood:

Definition 2 (Proper Part).

$$\mathcal{PP}xy := \mathcal{P}xy \wedge \neg\mathcal{P}yx \tag{5}$$

Definition 3 (Overlap).

$$\mathcal{O}xy := \exists z(\mathcal{P}zx \wedge \mathcal{P}zy) \tag{6}$$

Definition 4 (Underlap).

$$\mathcal{U}xy := \exists z(\mathcal{P}xz \wedge \mathcal{P}yz) \tag{7}$$

2.2 Correspondence

Correspondence in the sense that two objects are similar to one another.

Definition 5 (Correspondence). *Correspondence is a binary predicate $\mathcal{K}$ between two entities x and y. We write*

$$\mathcal{K}xy \tag{8}$$

and read "x corresponds to y".

Axiom 4 (Correspondence is Reflexive)

$$\mathcal{K}xx \tag{9}$$

Axiom 5 (Correspondence is Symmetric)

$$\mathcal{K}xy \Rightarrow \mathcal{K}yx \tag{10}$$

Axiom 6 (Correspondence and Parthood)

$$\mathcal{K}e_1e_2 \Rightarrow \exists p_1, p_2(\mathcal{P}p_1e_1 \wedge \mathcal{P}p_2e_2 \wedge \mathcal{K}p_1p_2) \tag{11}$$

2.3 Conformance

Definition 6 (Conformance). *Conformance is a binary predicate $\mathcal{C}$ between two entities x and y. We write*

$$\mathcal{C}xy \tag{12}$$

and read "x conforms to y".

Axiom 7 (Conformance and Parthood)

$$\mathcal{C}xy \Rightarrow \forall u \exists v (\mathcal{P}uy \wedge \mathcal{P}vx \Rightarrow \mathcal{C}uv) \tag{13}$$

2.4 Representation

Definition 7 (Representation). *Representation is a binary predicate $\mathcal{K}$ between two entities x and y. We write*

$$\mathcal{R}xy \tag{14}$$

and read "x is a representation of y".

3 An Algebraic Formalization

Definition 8 (Entities).

$$\mathsf{ENTITY} := \textit{"Set of all entities"} \tag{15}$$

Definition 9 (Relationships).

$$\mathsf{RELATIONSHIP} := \mathsf{ENTITY} \times \mathsf{ENTITY} \tag{16}$$

Set of all binary relationships between entities

Definition 10 (Megamodel). *A megamodel is a four-tuple*

$$M := (ET, E, RT, R) \tag{17}$$

with:

– *A set of **Entity-Types***

$$ET \subseteq \{X \subseteq \mathsf{ENTITY}\} \tag{18}$$

– *A set of **Entities** (also called instances)*

$$E \subseteq \mathsf{ENTITY} \tag{19}$$

– *A set of **Relationship-Types***

$$RT \subseteq \{X \subseteq \mathsf{RELATIONSHIP}\} \tag{20}$$

– *A set of **Relationships***

$$R \subseteq \mathsf{RELATIONSHIP} \tag{21}$$

Definition 11 (The Empty Megamodel).

$$0 := (\emptyset, \emptyset, \emptyset, \emptyset) \tag{22}$$

Definition 12 (The Set of All Megamodels).

$$\mathsf{MEGA} := \{(ET, E, RT, R)\} \tag{23}$$

Definition 13 (Megamodel Inclusion).

$$\begin{aligned} &+ : \mathsf{MEGA} \times \mathsf{MEGA} \to \mathsf{MEGA} \\ &M_1 + M_2 := (ET_1 \cup ET_2, E_1 \cup E_2, RT_1 \cup RT_2, R_1 \cup R_2) \end{aligned} \tag{24}$$

Proposition 1 (Megamodel Inclusion is Commutative).

$$\forall M_1, M_2 \in \mathsf{MEGA} : M_1 + M_2 = M_2 + M_1 \tag{25}$$

Proof:

$$\begin{aligned} & M_1 + M_2 \\ & = (ET_1 \cup ET_2, E_1 \cup E_2, RT_1 \cup RT_2, R_1 \cup R_2) \\ & = (ET_2 \cup ET_1, E_2 \cup E_1, RT_2 \cup RT_1, R_2 \cup R_1) \\ & = M_2 + M_1 \end{aligned} \tag{26}$$

Proposition 2 (Megamodel Inclusion is Associative).

$$\forall M_1, M_2, M_3 \in \mathsf{MEGA} : M_1 + (M_2 + M_3) = (M_1 + M_2) + M_3 \tag{27}$$

Proof:

$$\begin{aligned} & M_1 + (M_2 + M_3) \\ & = (ET_1, E_1, RT_1, R_1) + ((ET_2, E_2, RT_2, R_2) + (ET_3, E_3, RT_3, R_3)) \\ & = (ET_1, E_1, RT_1, R_1) + (ET_2 \cup ET_3, E_2 \cup E_3, RT_2 \cup RT_3, R_2 \cup R_3) \\ & = (ET_1 \cup (ET_2 \cup ET_3), E_1 \cup (E_2 \cup E_3), RT_1 \cup (RT_2 \cup RT_3), R_1 \cup (R_2 \cup R_3)) \\ & = ((ET_1 \cup ET_2) \cup ET_3, (E_1 \cup E_2) \cup E_3, (RT_1 \cup RT_2) \cup RT_3, (R_1 \cup R_2) \cup R_3) \\ & = (ET_1 \cup ET_2, E_1 \cup E_2, RT_1 \cup RT_2, R_1 \cup R_2) + (ET_3, E_3, RT_3, R_3) \\ & = ((ET_1, E_1, RT_1, R_1) + (ET_2, E_2, RT_2, R_2)) + (ET_3, E_3, RT_3, R_3) \\ & = (M_1 + M_2) + M_3 \end{aligned} \tag{28}$$

Proposition 3 (Megamodel Inclusion Identity).

$$\forall M \in \mathsf{MEGA} : M + 0 = 0 + M = M \tag{29}$$

Proof:

$$\begin{aligned} M + 0 &= (ET, E, RT, R) + (\emptyset, \emptyset, \emptyset, \emptyset) \\ &= (ET \cup \emptyset, E \cup \emptyset, RT \cup \emptyset, R \cup \emptyset) \\ &= (ET, E, RT, R) = M \end{aligned} \tag{30}$$

$M + 0 = 0 + M$ *follows directly from proposition 1.*

Proposition 4 (Megamodel Inclusion Semigroup).

$$(\mathsf{MEGA}, +) \textit{ is a Semigroup} \tag{31}$$

Proof: *Follows directly from proposition 2.*

Proposition 5 (Megamodel Inclusion Abelian Semigroup).

$$(\mathsf{MEGA}, +) \textit{ is an Abelian Semigroup} \tag{32}$$

Proof: *Follows directly from propositions 4 and 1.*

Proposition 6 (Megamodel Inclusion Abelian Monoid).

$$(\mathsf{MEGA}, +, 0) \textit{ is an Abelian Monoid} \tag{33}$$

Proof: *Follows directly from propositions 5 and 3.*

Definition 14 (Megamodel). *A megamodel is a labeled directed multigraph*

$$M := (\Sigma_V, \Sigma_A, V, A, s, t, l_V, l_A) \tag{34}$$

with

- *A set of vertices* $V = \mathscr{E} \cup \mathscr{M}$
- *A set of arcs* $A = V \times V$
- *Two alphabets* Σ_V *and* Σ_A

4 Other Stuff

A Mereology for Naive Sets Let A, B be sets.

Axiom 8 (Element Parthood)

$$a \in A \Rightarrow \mathcal{P}aA \tag{35}$$

Axiom 9 (Subset Parthood)

$$A \subseteq B \Rightarrow \mathcal{P}AB \tag{36}$$

Axiom 10 (Proper Subset Parthood)

$$A \subset B \Rightarrow \mathcal{P}\mathcal{P}AB \tag{37}$$

A Mereology for Tuples Let $p = (a, b) \in A \times B$

A Mereology for Formal Languages

4.1 The Parthood Relationship

$$\begin{aligned} &(.) \subset \mathsf{RELATIONSHIP} \\ &(.) := \{(e_1, e_2) \in \mathsf{RELATIONSHIP} \mid \mathcal{P}e_1e_2\} \end{aligned} \tag{38}$$

$$\forall e \in \mathsf{ENTITY} : e.e \tag{39}$$

$$\forall x, y \in \mathsf{ENTITY} : x.y \wedge y.x \Rightarrow x = y \tag{40}$$

$$\forall x, y, z \in \mathsf{ENTITY} : x.y \wedge y.z \Rightarrow x.z \tag{41}$$