September 11^{th}, 2023 - Robert Burklund

There is a natural dichotomy between telescopic (T(n)-local) and chromatic (K(n)-local) homotopy theory. Telescopic homotopy theory is more closely tied to the stable homotopy groups of spheres and through them to geometric questions, but is generally computationally intractable. Chromatic homotopy theory is more closely tied to arithmetic geometry and powerful computational tools exist in this setting. Ravenel's telescope conjecture asserted that these two sides coincide. I will present a family of counterexamples to this conjecture based on using trace methods to analyze the algebraic K-theory of a family of K(n)-local ring spectra beginning with the K(1)-local sphere. As a consequence of this we obtain a new lower bound on the average rank of the stable homotopy groups of spheres. Time permitting, I will then describe the galois group of the T(n)-local sphere and how this informs our understanding of telescopic homotopy theory. This talk is based on projects joint with Carmeli, Clausen, Hahn, Levy, Schlank and Yanovski.

Algebraic Topology
Hatcher, Allen

Chapter 0 - Some Underlying Geometric Notions

Morphism: a structure preserving map between two mathematical structures.

Isomorphism: a bijective morphism (homeomorphism)

Homeomorphism: continuous transformation, an equivalence relation and one-to-one correspondence between points in two geometric figures or topological spaces that is continuous in both directions.

△ A bijective continuous function $f\ X \to Y$ between topological spaces X and Y for which $f^{-1} X \to Y$ is also continuous, in case the first definition isn't 'it' △

Homotopy: two continuous functions $f, g: X \to Y$ are homotopic if there exists a continuous function $H: X\ [0,1]\ Y$ such that $H(x,0) = f(x)$ for $x \in X$ and $G(0,y) = g(x)$ for $x \in X$.
H is the homotopy. This is a continuous deformation.

Loop: a continuous function f in a topological space X over interval $[0,1]$ where $f(0) = f(1)$, i.e is starting point is its end point.

Ring:

Abelian Group
Fundamental Group
Homology
Cohomology
Stability
Spheres
Base Point
Loop
Suspension
Smash Product