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Analysis & PDEs

Global Well-Posedness and Scattering for the Conformal Nonlinear Wave Equations in Higher Dimensions with Radial Data

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Abstract: In this talk, we prove global well-posedness and scattering for the conformally invariant, radially symmetric nonlinear wave equation in the defocusing case. This result is sharp in the radial case. We spend the first half of the talk discussing the broader context of the nonlinear wave equation. We also explain why the result is sharp.

§1 Introduction

$u_{tt} + \Delta u \pm |u|^{p-1} u = 0$ (nonlinear wave equation)

$u_{tt} + \Delta u = 0$ (linear wave equation)

can also drop leplace component: $u_{tt} - |t|^{p-1} u$, then ODE → $u(t) = C_p t^{\frac{-2}{p-1}}$

when (time invariant?) time derivative is zero: $\Delta u - |u|^{p-1} u = 0$ (stationary)

Well-Posedness: a problem is said to be well posed if the following criteria hold:

1) Solution exists
2) Solution is unique
3) Solution depends continuously on input data

Scattering Theory: studies how waves and particles propogate and interact with obstacles or potential fields.

- Incident wave: wave that gets scattered
- Scattered wave: the wave produced by scattering
- Scattering operator: maps the incident state to the scattered state

Linear/Nonlinear Waves

- Linear waves are described by linear equations in which each dependent variable and its derivative are at most first order. The superposition principle applies to linear waves. They are more easily tractable and share analytic techniques.
- Nonlinear waves are described by nonlinear equations, allowing their dependent variables and derivatives to be of order greater than one. The superposition principle does not apply to nonlinear waves. They are less tractable and treated uniquely.

Radial Symmetry

- Waves are radially symmetric if their propogation in all directions from a central point are identical. Thus, there is spatial symmetry and behavior is dependent on distance from point.
- This seems to mean that, in 2 dimensions, a circular wavefront, and in 3, a spherical wavefront.

Conformal Invariance

- Conformal Map: a function that preserves angles, but not necessarily length
- Conformal Invariance means that the wave equation is maintained under conformal transformation (via conformal map)

Defocusing: occurs when waves disperse as they propogate

Sharp: an inequality is said to be sharp if it is an equality for some cases.