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Elliptic Curves Ove a Field

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Abstract: Basic Theory of Weierstrass equations over a DVR (including semistable reduction theorem, Neron-Ogg-Shafarevich criterion)

Main Results

Let E/K be an elliptic curve.

Proposition (Torsion injects I): Suppose $m \in \mathbb{Z}_{\geq 1}$ w/ $\gcd(m, \mathrm{char}(k)) = 1$. If E/K is nonsingular, then the map $E(K)[m] \hookrightarrow E[k]$ is injective.

Proposition (Torsion injects II): Assume E has good reduction. Suppose $K/\mathbb{Q}_p$ is an extension with ramification index $e < p-1$. Then, $\forall m \in \mathbb{Z}_{\geq 1}$, the map $E(K)[m] = \bar{E}(k)[m]$ is injective.

Theorem (Semistable Reduction): there exists a finite extension L/K s.t. E_L has a good reduction or multiplicative reduction. Moreover, can choose L w/ $[L:K] = 6$ if $\mathrm{char}(k) \neq 2, 3$

Theorem (Neron-Ogg-Shafarevich): TFAE

1) E has good reduction

b) $T_\ell E$ is unramified at v for all primes $\ell \neq \mathrm{char}(k)$

Neron Models