

Harris-Venkatesh plus Stark, Robin Zhang

**Abstract:** The class number formula describes the behavior of the Dedekind zeta function at  $s = 0$  and  $s = 1$ . The Stark and Gross conjectures extend the class number formula, describing the behavior of Artin  $L$ -functions and  $p$ -adic  $L$ -functions at  $s = 0$  and  $s = 1$  in terms of units and class numbers. The Harris-Venkatesh conjecture describes the residue of Stark units modulo  $p$ , giving a modular analogue to the Stark and Gross conjectures while also serving as the first verifiable part of the broader Prasanna-Venkatesh conjectures. In this talk, I will draw a picture, formulate a unified conjecture combining Harris-Venkatesh and Stark for weight one modular forms, and describe the proof of this in the imaginary dihedral case.

Modular Forms

Zeta Function

Dedekind Zeta Function

Stark and Gross Conjecture

Title: Modular Gelfand pairs multiplicity-free triplets (maybe gamma factor)

Question: When is an induced representation multiplicity free?  $\text{Ind}_H^G(\rho)$   
Multiplicity free triple: if each component is multiplicity free  $(G, H, \rho)$   
Gelfand Pair:  $(G, H)$  if  $(G, H, \text{triv}_H)$  is multiplicity free

- $\S_1$ : Multiplicity free  $\text{triv}_H$  in finite groups
- $\S_2$ : " finite groups with characteristic  $\neq 0$
- $\S_3$ : Gamma factors in  $\text{char}(0)$  or  $\text{char} = p$

General criterion over  $\mathbb{Q}$   
Consider Hecke algebra  $\mathcal{H}(G, H, p, \mathbb{Q}) = \text{space of all continuous functions.}$   
If commutative, multiplicity free.

Schur's lemma, Gelfand's Trick

Example: Whittaker Models

- Representation Theory
- Symmetry of groups
- Classical branching rules
- Adding blocks
- Familiarity with common groups
- Subrepresentation
- Induction
  - $\text{Triv}_H \rightarrow \text{trivial}$
- Reducability
- $p$ -adic groups
- Endomorphism
- Anti-involution
- Surjective
- Functor
- Projective
- Injective
- Local Langlands's correspondence
- Field
- Whittaker Model
- Ramanujan-Selberg Gamma Factor
- Cover
- Colloqure