TS2

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Northwestern University, Time-Series Analysis & Forecasting

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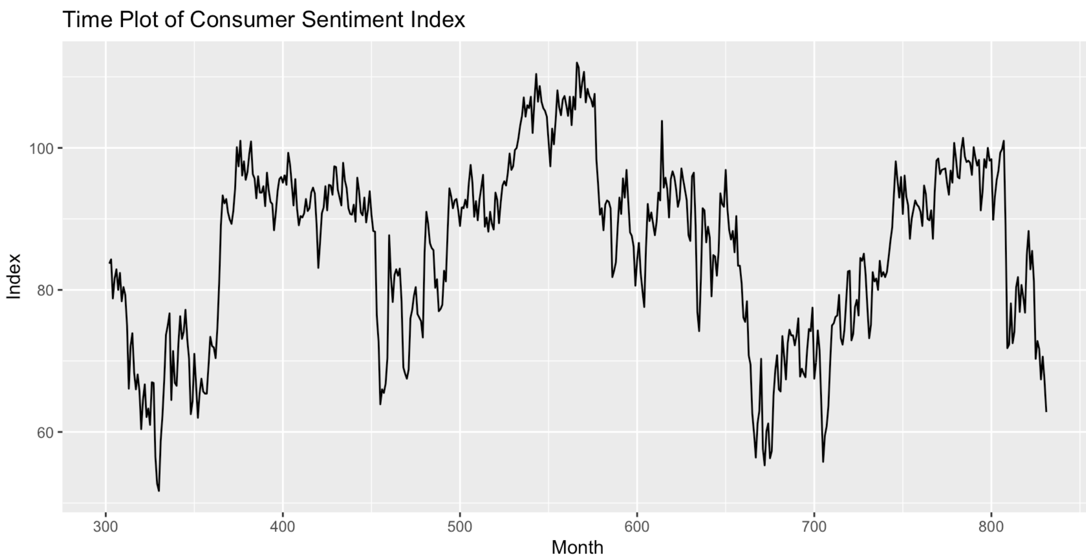
1. EDA

1.1 Missing Data

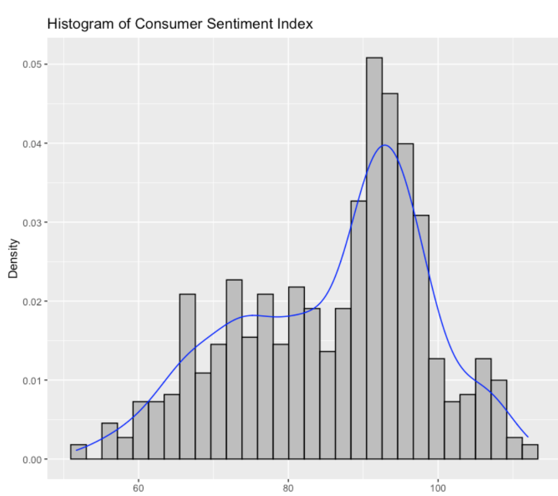
Data before 1978 has quartlerty or other non-monthly time intervals between observations. Removing them leaves only monthly data from 1978 forward.

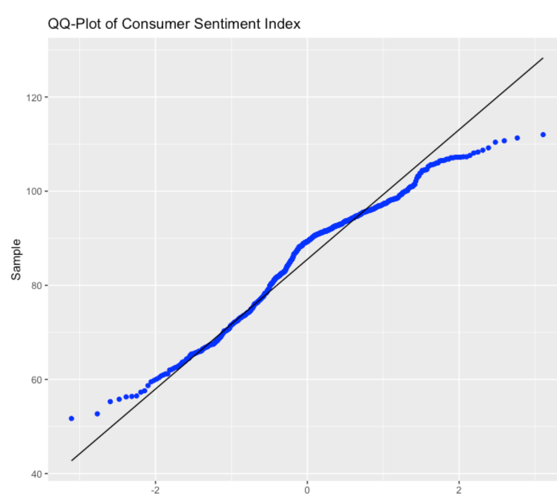
1.2 EDA



The time plot shows that this data is not stationary; it has numerous level changes and a potential seasonal component. However, there is no obvious overall linear trend in the data. Given that there is a time-dependence of some form, I will difference the data once.

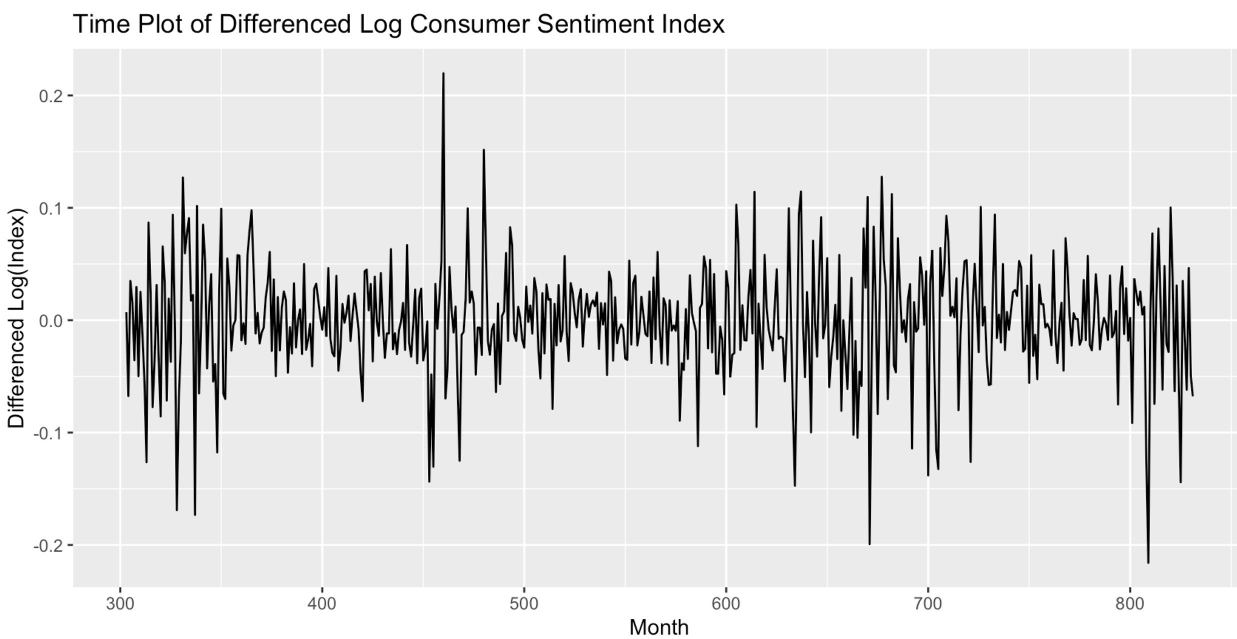
A histogram and QQ plot of the index both show deviation from normality. The histogram demonstrates that the distribution is bimodal and skewed left, and the QQ Plot shows how the tail ends of the distributions stray from normal.





The actual skew of the index was -0.415 and its excess kurtosis was -0.577. Testing for the skew provided a 95% confidence interval of (-0.417,-0.41). Testing for excess kurtosis provided a 95% confidence interval or (-0.578,-0.573). Neither of these confidence intervals contain 0, and given that the plots above suggest non-normality, we can conclude that the index values are not normally distributed. Thus, I will make a log transformation.

2. Accounting for a Linear Trend in Time Series

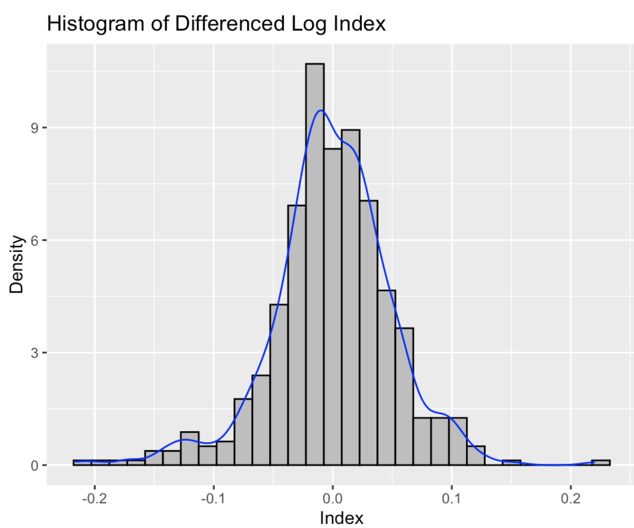


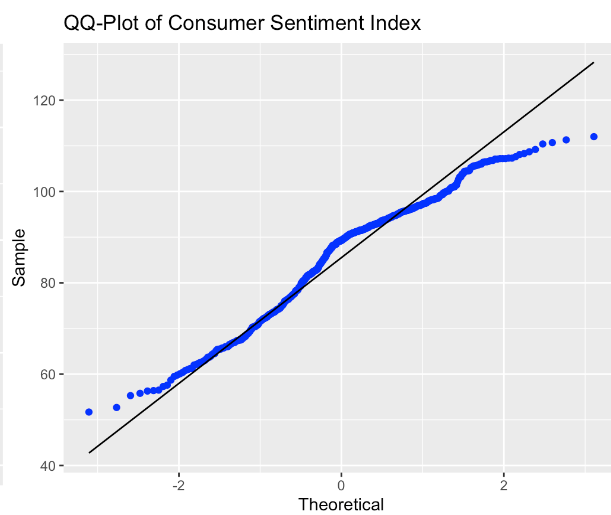
2.1 - Expected Change in Consumer Sentiment

Applying a log transformation to the data and differencing it once resulted in the time plot shown above. As one can see, these transformations have gotten rid of the level changes and it is plausible that the expected change in consumer sentiment is 0. A t-test of the null hypothesis that the mean change is equal to zero showed that the sample mean was -0.005 and produced a p-value of 0.805 with a confidence interval of (-0.0049,0.0038), which contains 0. Thus, we cannot reject the null hypothesis that the mean change is equal to 0.

2.2 - Gaussian Analysis

These transformations resulted in a distribution that is more closely approximated by normality. This is most obviously demonstrated in the histogram below on the left, which is now unimodal and has better symmetry. It does appear taller than the standard normal distribution, which is evidenced by its excess kurtosis of 1.438 and a 95% confidence interval for excess kurtosis of (1.441,1.509). This is a notable increase from the undifferenced and non-log transformed data, which had an excess kurtosis of -0.577. Unlike kurtosis, skew improved. From a previous value of -0.415, the skew of the differenced log data was -0.239 and had a 95% confidence interval of (-0.256, -0.23). Assessing change in the QQ Plot is less easy than with other visualization and descriptive methods. The two plots are similar enough that I cannot definitively say I see a difference.





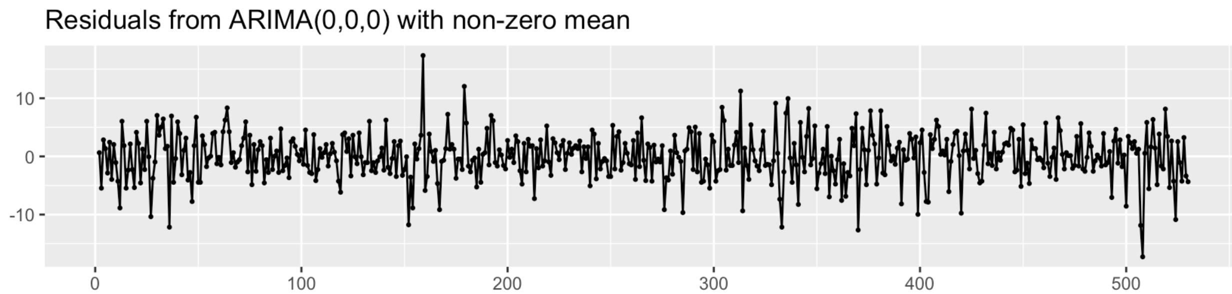
3. Identifying Autocorrelation, Stationarity, and White Noise

3.1. Mean Model

Running a mean model of the form ARIMA(0,0,0) applied to the differenced log index data produced a coefficient of -0.395 with a standard error of 1.74. This coefficient is the mean of the data, as the name suggests. With 529 observations and d, q, and p values of 0, this model has 529 degrees of freedom.

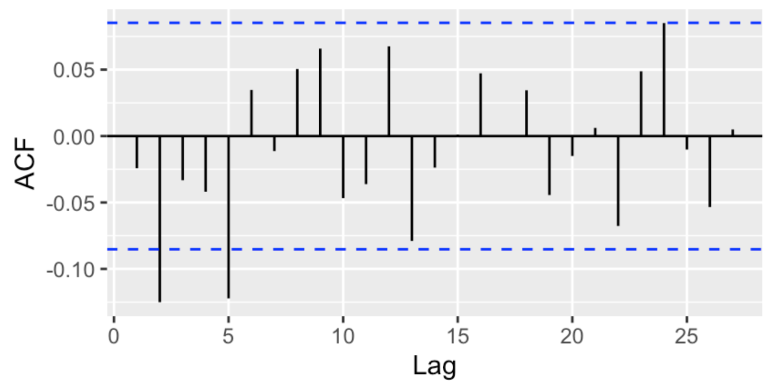
3.2. Model Assessment

To assess the fit of the model, the residuals (shown as a time plot below) can be assessed for stationarity, autocorrelation, independence of lags, and normality.



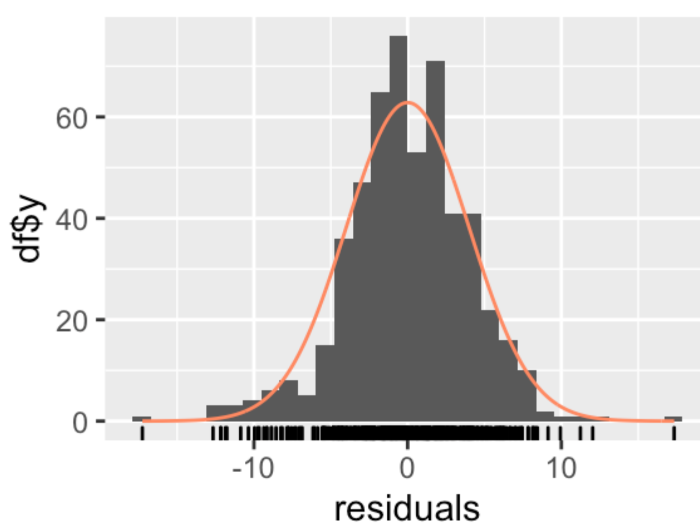
The mean of the model's residuals (the difference between each observation and the mean) was within 15 decimal places of 0. A t-test of the null hypothesis that the mean was zero produced a p-value of 1 and a confidence interval of (-0.342,0.342). Thus, we fail to reject the null hypothesis that the residuals' mean is zero.

Stationarity of the residuals is suggested by ADF and KPSS tests. The p-value of the ADF test was 0.01, indicating random walk startionarity at the 95% confidence level. The test-statistic of the KPSS test, 0.0356, was well below the critical value of the 95% confidence level, 0.176, indicating linear trend stationarity.



The ACF plot of the residuals on the right shows the p-values for auto-correlations of multiple lags have p-values above 0.05, indicating that the residuals do have some auto-correlation.

The residuals appear to be approximately normally distributed, as shown in the histogram on the bottom right. They are unimodal and have good symmetry. However, neither the confidence interval for skew (-0.233,-0.204) nor excess kurtosis (1.467,1.528) contain zero.



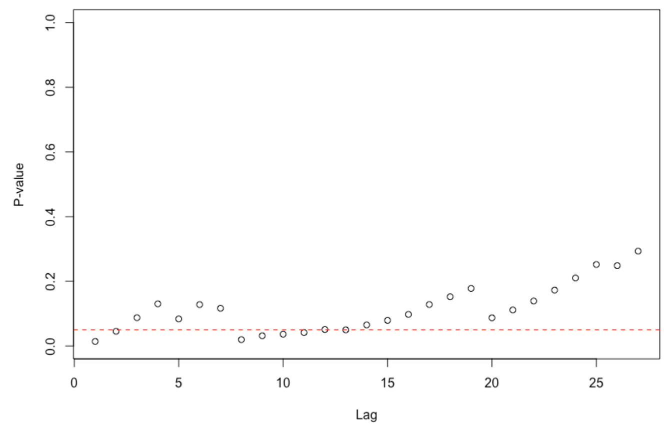
Overall, the residuals of the model look good. While they do feature some auto-correlation, they are stationary according to the ADF and KPSS tests and approximately normally distributed. We can conclude that the model fits well.

3.3 Business Cycles in Model Residuals

The function provided in the R code that identifies imaginary roots returned no results when analyzing the model's residuals. Thus, we can conclude that they contain no detectable business cycles. This indicates that the model fits well, as detectable business cycles in the residuals would indicate that the model does not capture the cycles.

3.4 White Noise Process

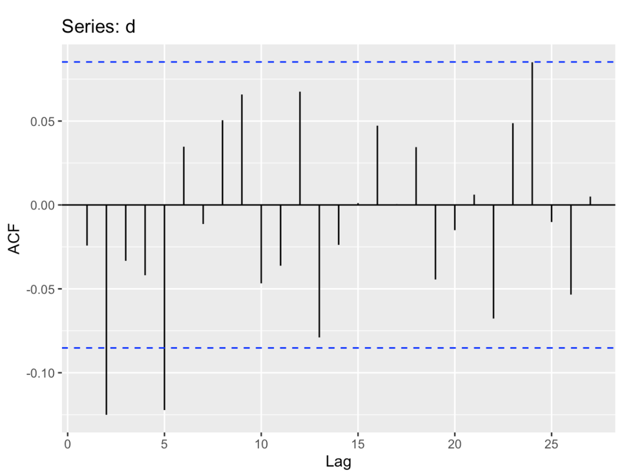
The two requirements for white noise are mean zero and constant variance. A McLeod test of this model produced the plot to the right. There are numerous lags that fall below the 0.05 threshold, indicating that we reject the null hypothesis of constant variance. Thus, we can conclude that this model is not a white noise process.

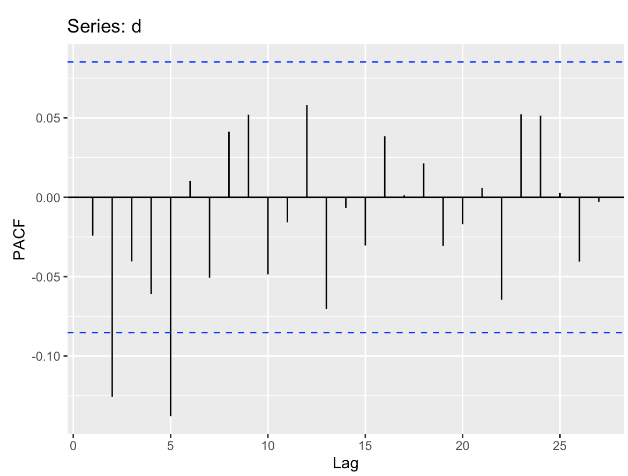


4. Autoregressive (AR) Models

4.1. Autocorrelation

A Box-Ljung Test of the null hypothesis that the differenced log data has no auto-correlation returned a p-value of 0.00775. We reject the null hypothesis, concluding that there the data does feature auto-correlation. The plots below confirm that there are multiple lags with ACF and PACF values exceeding critical values.





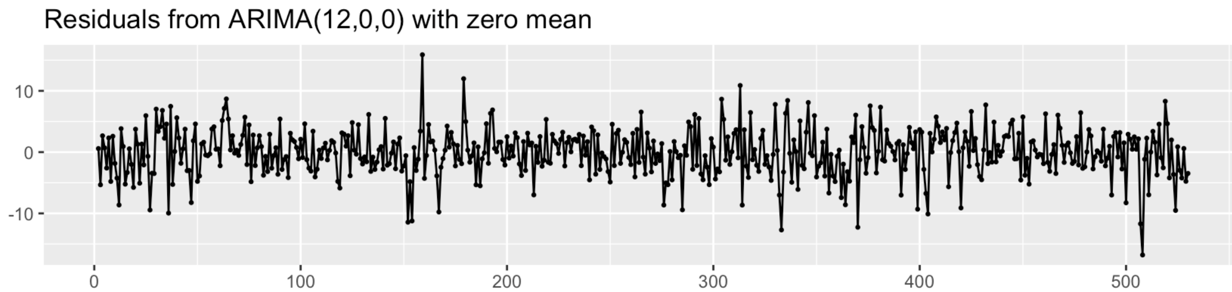
4.2. AR(12) Model Construction and Fit

Constructing an AR(12) model applied to the differenced log index data produced the following output. The model residuals are then plotted as a time plot below.

```
Series: d
ARIMA(12,0,0) with zero mean

Coefficients:
          ar1      ar2      ar3      ar4      ar5     ar6      ar7     ar8     ar9     ar10     ar11    ar12
      -0.0389  -0.1404  -0.0618  -0.0598  -0.1452  0.0131  -0.0404  0.0354  0.0524  -0.0455  -0.0152  0.0626
s.e.   0.0434   0.0435   0.0439   0.0440   0.0441  0.0444   0.0448  0.0444  0.0444   0.0444   0.0442  0.0442

sigma^2 = 15.51:  log likelihood = -1469.85
AIC=2965.7   AICc=2966.4   BIC=3021.22
```



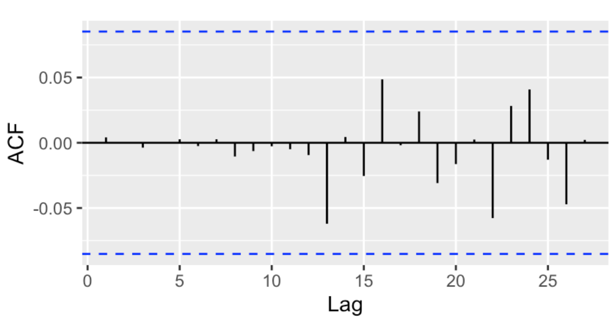
The residuals mean was 0.0487. A t-test of the null hypothesis that the residuals mean is 0 produced a p-value of 0.7738 with a 95% confidence interval of (-0.382,0.284). Given the magnitude of the p-value and the inclusion of 0 in the confidence interval, we do not reject the null hypothesis that the mean of the residuals is 0.

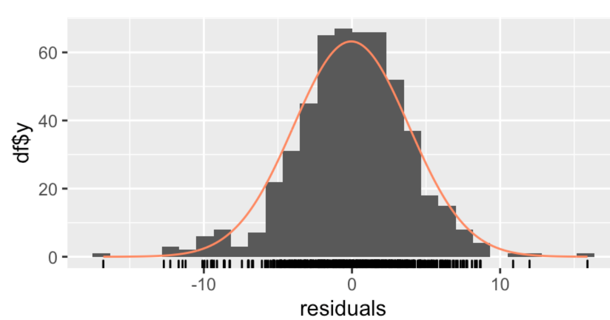
Stationarity is suggested by both KPSS and ADF tests. A KPSS test of the null hypothesis that the residuals have linear-trend stationarity returned a test-statistic of 0.0424, which is below the critical value of the 95% confidence level, 0.176. Thus, we fail to reject the null hypothesis. The ADF, which tests random-walk stationarity instead of linear-trend stationarity, returned a p-value < 0.01, indicating we also fail to reject the null hypothesis and conclude random-walk stationarity.

As shown by the ACF chart below on the left, the residuals also feature auto-correlation. There are multiple spikes between beyond 10+ lags that have p-values above 0.05.

The residuals of this model appear to be normally distributed according to the histogram below on the right. They are unimodal and have good symmetry. However, neither of 95% confidence intervals for skew (-0.297,-0.278) nor excess kurtosis (1.35,1.41) contain 0.

Overall, the residuals indicate that model fit is good. They have mean zero, stationarity, and are approximately normally distributed.





4.3. AR(12) and Mean Model Comparison

Assessment of the residuals of the Mean and AR(12) models produced similar results. Based on the statistical tests run, it is suggested that the residuals of both models have mean zero, random walk and linear trend stationarity, auto-correlation, and appear to be approximately normal. Many of the confidence intervals and test statistics were very close as well. The largest difference was in the auto-correlation. The ACF plot of the residuals of the AR(12) model had considerably less auto-correlation and lags with p-values above 0.05. For this reason, I would recommend the AR(12) model.

4.4. Simplified Model Construction and Fit

Applying the parameterTest() function to the AR(12) model indicated that only two of the 12 ar variables had significance levels below 0.05. They were ar2 and ar5. Creating an AR model that used only those two ar variables produced the following output. The model residuals are then plotted as a time plot below.

```
Call:
arima(x = d, order = c(12, 0, 0), include.mean = F, fixed = c1)

Coefficients:
      ar1      ar2  ar3  ar4      ar5  ar6  ar7  ar8  ar9  ar10  ar11  ar12
        0  -0.1303    0    0  -0.1275    0    0    0    0     0     0     0
s.e.    0   0.0429    0    0   0.0429    0    0    0    0     0     0     0

sigma^2 estimated as 15.52:  log likelihood = -1475.92,  aic = 2955.85
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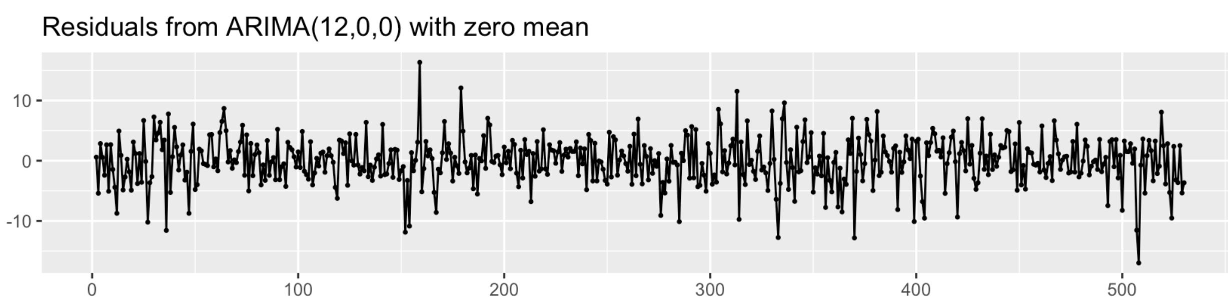


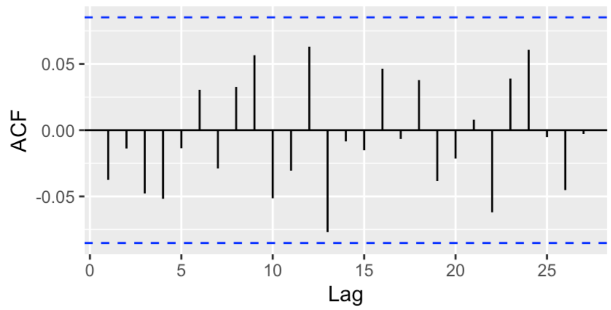
The residuals mean was 0.045. A t-test of the null hypothesis that the residuals mean is zero returned a p-value of 0.7926 with a 95% confidence interval of (-0.381,0.292). This p-value is greater than 0.05 and the confidence interval includes zero, so we can conclude that the residuals are mean zero at the 95% confidence level.

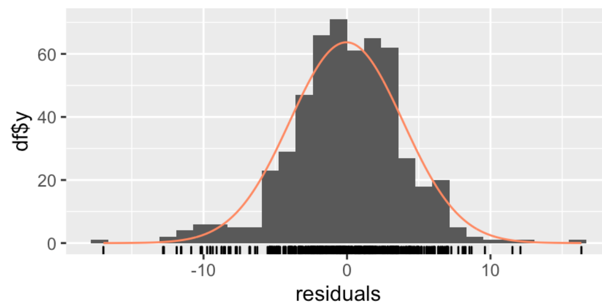
The results of KPSS and ADF tests of the residuals indicate stationarity. The KPSS test, for which the null hypothesis was that the residuals are linear-trend stationary, returned a p-value of 0.0424, below the 0.176 critical value to reject at the 95% confidence level. Thus, accept the null hypothesis. The ADF test, for which the null hypothesis was that the residuals are random-walk stationarity, returned a p-value < 0.01. So, the null hypothesis is accepted.

The residuals do have auto-correlation, as shown by the ACF plot on the left. There are a number of lags that feature p-values above 0.05.

The residuals are approximately normally distributed according to the histogram below on the right. They are unimodal and have good symmetry. However, neither of 95% confidence intervals for skew (-0.296,-0.285) nor excess kurtosis (1.54,1.595) contain zero.

The model fit is good. Its residuals have mean zero, stationarity, and are approximately normally distributed.





4.5. Business Cycles

The residuals of the AR(12) and Simplified models both had detectable business cycles. Their respective lengths were (6.57, 2.998, 10.618, 2.054, 2.346) and (9.188, 3.399). The difference in number of cycles and duration indicate that the hypothetical business cycles in these residuals are different.

4.6. Simplified and Mean Model Comparison

The comparison of the Simplified and Mean models is quite close to the comparison of the AR(12) and Mean models. Both the Simplified and Mean models feature residuals that are mean zero, random-walk and linear-trend stationary, feature auto-correlation, and are approximately normal. And like the previous comparison, the Simplified model has less auto-correlation, and for this reason, I find it preferable to the Mean model.

4.7. Simplified and AR(12) Model Comparison

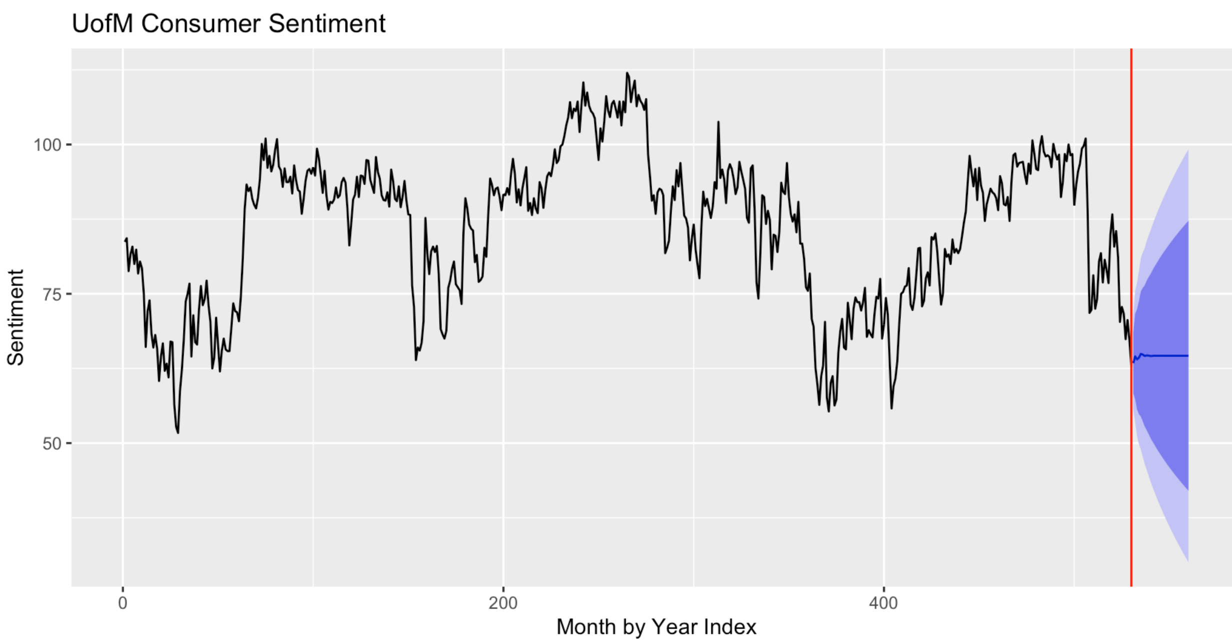
Previous comparisons have established that the AR(12) and Simplified models both have residuals that are mean zero, stationary, approximately normal, and do have auto-correlation. I chose both of these models over the mean model because of the improvements in auto-correlation. The improvement in auto-correlation was significantly larger in the AR(12) model than in the Simplified model, and this is demonstrated by both fewer and smaller spikes in their respective ACF plots. However, I actually find the Simplified model preferable to the AR(12) model because of its simplicity. The model has only two variables that were both statistically significant, while the AR(12) model has an additional 10 that are not significant.

4.8. Backtesting

The results of backtesting using the AT(12) and Simplified models were identical. As shown below, they had the same RMSEs, MAEs, and Bias for out of sample forecasts. Given that the performance of these models were identical, I would still argue that the Simplified model is preferable given its fewer variables.

5. Reports

The following diagram represents predictions of the University of Michigan's Consumer Sentiment Index. The red line represents where current data ends and prediction begins. The blue areas represent the possible future values and their respective likelihoods. The darker the color, the more likely the outcome. The dark line represents the average forecasted value (which can be thought of as the expected value), the darker of the two shaded areas represent where the Index should fall within 95% of the time, and the lighter shaded region represents the remaining 5% on the margins.



These predictions were made using a model that predicts the future value of the Index based on previous values of the Index. A variety of types of these models were tested, and the final selection was made based on how well each model fit the data. A model's 'goodness of fit' can be measured by things like how well it can predict past values or how well it captures certain types of economic or otherwise relevant patterns. The final prediction values and their probabilities are determined by how the model fits the data and understands its patterns to work.