Chapter 8: Quantitative Concepts

Time Value of Money

1. Introduction
    a. Time Value of Money: issues relate to the idea that the timing of cash flows affect their values.

2. Interest
    a. Principal: the amount of money that was originally borrowed by the borrower and from the lender.
    b. Interest: the amount of money that the borrower pays back to the lender on top of the principal. This is the payment / cost of borrowing money.
    c. The use of interest arises from two components:
        i. The borrower pays a price (interest) for not being able to wait until they have money and pays the lender for the opportunity cost of other investments by the lender. Opportunity cost in this circumstance can also be thought of as delaying consumption.
        ii. Additionally, the lender bears risk, in terms of both the potential for borrower default, and the risk of inflation causing the principal amount and the interest amount to devalue over time. The higher the levels of risk of default and the higher the level of inflation,the higher the asking interest rate will be.

3. Simple Interest
    a. Simple Interest Rate: the cost to the borrow or the rate of return to the lender, per period, on the original principle. Conventionally, these are expressed as annual rates, so the period is assumed to be a year unless stated otherwise.
        i. Simple Interest = (SIR) * (Principal) * (Number of Periods)
    b. If the Simple Interest gained over a period is added to the principal, the new formula os:
        i. Future Value = (Original Principal) * [1 + (SIR * Number of Periods)]

4. Compound Interest
    a. Compound Interest: known as “interest on interest” occurs when interest is added on to the original principal.
    b. Future Value = (Original Principal) * [1+(SIR)^*(Number of Periods)]

5. Comparing Simple Interest and Compound Interest
    a. Compound Interest leads to greater returns over time than Simple Interest, and the difference between the two is significant when considered over longer periods of time and when the quantities involved are greater.

6. Annual Percentage Rate and Effective Annual Return
    a. Unless specified otherwise, interest rates are stated as annual rates.
    b. Annual Percentage Rate (APR): is the typically quoted simple interest rate that does not involve compound interest.
    c. Effective Annual Rate (EAR): a quoted compound interest rate that is usually compounded monthly, quarterly, or semi-annually.
        i. EAR = [(1 + (APR / Periods per Year)^(Number of Periods per Year))] - 1
    d. Whenever an interest rate compounds more frequently than annually, the EAR is greater than the APR.

7. Present Value and Future Value
    a. Time Value of Money problems typically involve two kinds of calculations for cash flows: their present value and their future value. The premise is that a dollar in the present is worth more than a dollar in the future, as receiving a dollar in the future includes the opportunity cost of missed investment and consumption, as well as risk. The rate (which is an interest) at which a dollar in the present is worth more than a dollar in the future is called the Discount Rate.
        i. FV = PV * (1 + (Interest Rate)^(Number of Periods))
        ii. PV = FV / (1 + (Discount Rate)^ (Number of Periods))
    b. Useful considerations to be made while assessing the value of investments are:
        i. The cash flows each investment will generate in the future
        ii. The timing of these cash flows
        iii. The risk associated with each investment, which is reflected in the discount rate
    c. Net Present Value
        i. NPV: of an investment is the present value of future cash flows or returns mimus the present value of the cost of the investment (which often, but not always, occurs only in the initial period.)
        ii. If costs were to occur at times other than zero, then they would also be discounted back to time zero for the purposes of calculation and comparison of NPV.
        iii. If NPV > 0, the investment is earning at least the discount rate.
        iv. If NPV < 0, the investment should not be made
        v. Individual discounts should be made for investments and they should accuracy reflect the risks associated with the investment they are applied to.
    d. Present Value and the Valuation of Financial Instruments
        i. Similar to all investments, the NPV should be positive

    e. Time Value of Money and Regular Payments
        i. Annuity: involved the initial payment of an amount, usually to an insurance company, in exchange for a fixed number of future payments of a certain amount. Each period, the insurance company makes payments to the annuity holder, and at the end of the term the balance is zero.
        ii. Mortgage: of the amortizing variety involves a loan and a series of fixes payments. While the payments are fixed, the interest rates for each payment are

based on the remaining principal. This means that the amount of principal paid off in each payment increases as time goes on.

Descriptive Statistics

1. Introduction
    a. Descriptive statistics are numbers that summarise essential features of a data set. Individual data points are called observations.
    b. Distribution: of a variable is the values that a variable can take and the number of observations associated with each of these values.
    c. Descriptive statistics included in this section are measures of central tendency and dispersion.

2. Measures of Frequency and Average
    a. Arithmetic Mean
        i. Simple arithmetic mean assumes that each observation is equally likely to occur. If this is not true, then a proper weight can be assigned to each observation and the arithmetic mean can be calculated by summing the product of each observation and its associated probability.
        ii. The main disadvantage of arithmetic means is that they are highly sensitive to outliers. The presence of outliers will cause an arithmetic mean to be skewed in the direction of the outlier.
    b. Geometric Mean
        i. The Geometric Mean is the average return assuming that returns are compounding.
        ii. Formula for Geometric Mean:
            1. GM = (x1 * x2 *...* xn)^(1/n) - 1
            2. GM = [(1+r1)*...*(1+rn)]^(1/n) - 1
    c. Median
        i. Median is the observation that lies in the middle of a dataset when all observations are ordered by ascending value.
    d. Mode
        i. Mode is the most frequently occurring value in a data set. The mode is a useful measurement of central tendency for data that has been sorted into categories.
        ii. The mode of a dataset does not exist if there are multiple values that share an identical frequency and are the most frequent.
        iii. The mode is also not useful when working with continuous data, which is data that can take on an infinite number of values between whole numbers.
        iv. Discrete data takes on distinct values. Mode is also typically not a useful measure here as the likelihood of observations taking the exact same values is much lower, and therefore less significant.
        v. Also, the mode may actually be far away from other measurements such as the mean or median.

3. Measures of Dispersion
    a. Introduction:
        i. Measures of dispersion are used to describe the spread of data / its variation around a central value. A good description of data should include both measures of central tendency and measures of dispersion.
        ii. Investment risk is often measured in a way that includes variability. Investors are interested in both the expected return as well as the degree of risk that is caused by its potential variance. Risk averse investors choose to use financial instruments that have lower degrees of variance.
    b. Range
        i. Range is the difference between the highest and lowest values of observations in a data set. This measurement is extremely sensitive to outliers, and in their presence, will say little about the real distribution of the data set.
        ii. The range can be further divided into percentiles. The 50th percentile marks the median, the 25th and 75th percentiles mark the quarters. A given percentile is the value at which that percent of the observations fall under.
    c. Standard Deviation
        i. Standard deviation measures the variability of a data set around the arithmetic mean. Specifically, it measures the average squared deviation from the mean. The deviations from the mean are squared to account for negative deviations.
        ii. Variance: the value taken before the square root, which is another measure of dispersion, this time measuring the entire dispersion of a data set. The variance is measured in units squared.
    d. Normal Distribution
        i. Distribution: the set of values that a variable can take and their associated frequency of observation.
        ii. This data can be visualized by creating a histogram, a bar chart that plots the values of the variable on the x-axis and the frequency on the y-axis. Values of the variable can be grouped as well.
        iii. Symmetrical Distribution: has observations falling fairly evenly on both sides of the center of the range. Distributions that are not symmetrical are skewed. In a perfectly symmetrical distribution, mean, median, and mode are identical. The median will fall on whichever side a distribution is skewed to.
        iv. Normal Distribution: a distribution that is symmetrical about the mean.
            1. The exact shape of the normal distribution is dependent upon the mean and standard deviation. The mean determines the center of the distribution and the standard deviation determines the height and width.
            2. A large standard deviation causes a normal distribution to be tall and thin, while a small standard deviation causes a normal distribution to be short and wide.
            3. Approximately two-thirds of the observations lie within one standard deviation of the mean, and 95% of observations lie within two standard deviations of the mean.

4. Distributions with “fat tails” include values for which the observational probability is higher than would be prescribed by a perfectly normal distribution.