Linear Algebra
for Quant Finance

Green Book
Section 3.6 - Linear Algebra

Vectors - an $m \times 1$ one dimensional array, m rows, 1 column $a = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix}$ $b = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix}$

Vector Addition: $a + b = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \dots \\ a_n + b_n \end{bmatrix}$

Scalar Multiplication: $ca = \begin{bmatrix} c \cdot a_1 \\ c \cdot a_2 \\ \dots \\ c \cdot a_n \end{bmatrix}$

Inner/Dot Product: $a \cdot b = a_1 \cdot b_1 + a_2 \cdot b_2 + \dots + a_n \cdot b_n$

Matrix - an $m \times n$ two-dimensional array, m rows, n colums

Matrix Addition: given two matrices, A and B, both of dimension $m \times n$,
$C_{ij} = A_{ij} + B_{ij}$ for all i in m and j in n
$A = \begin{bmatrix} a_{11} & a_{1j} \\ a_{i1} & a_{ij} \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{1j} \\ b_{i1} & b_{ij} \end{bmatrix}$ $C = \begin{bmatrix} a_{11} + b_{11} & a_{1j} + b_{1j} \\ a_{i1} + b_{i1} & a_{ij} + b_{ij} \end{bmatrix}$

Matrix Subtraction: $C_{ij} = A_{ij} - B_{ij}$

Matrix Multiplication: Given two matrices A and B of dimensions $m \times n$ and $n \times p$, matrix multiplication calculates the inner/dot product of A's rows and B's column. Thus, A's and B's dimensions are inverse. Returned is $C\ (m \times p)$
$C_{ij} = \sum_{k=1}^{n} A_{ik} \cdot B_{kj}$
$A\ (m \times n) = \begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$ $B\ (n \times p) = \begin{bmatrix} b_{11} & b_{1p} \\ b_{21} & b_{2p} \\ b_{n1} & b_{np} \end{bmatrix}$ $m = 2$, $n = 3$, $p = 3$

$C\ (n \times p) = \begin{bmatrix} (a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{1n} \cdot b_{n1}) & (a_{m1} \cdot b_{1p} + a_{m2} \cdot b_{2p} + a_{mn} \cdot b_{np}) \\ (a_{m1} \cdot b_{1p} + a_{m2} \cdot b_{1p} + a_{mn} \cdot b_{np}) & (a_{m1} \cdot b_{1p} + a_{m2} \cdot b_{2p} + a_{mn} \cdot b_{np}) \end{bmatrix}$

Matrix Scalar Multiplication: $cA_{ij} = c \cdot a_{ij}$ for all i in m and j in n

Identity Matrix: matrix with 1's in main diagonal and all 0 elsewhere, I

Matrix Inverse: for matrix A, denoted A^{-1}, is the matrix that results in the identity matrix after matrix multiplication,
$A \cdot A^{-1} = I$