Equations of Lines

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WHAT'S COVERED

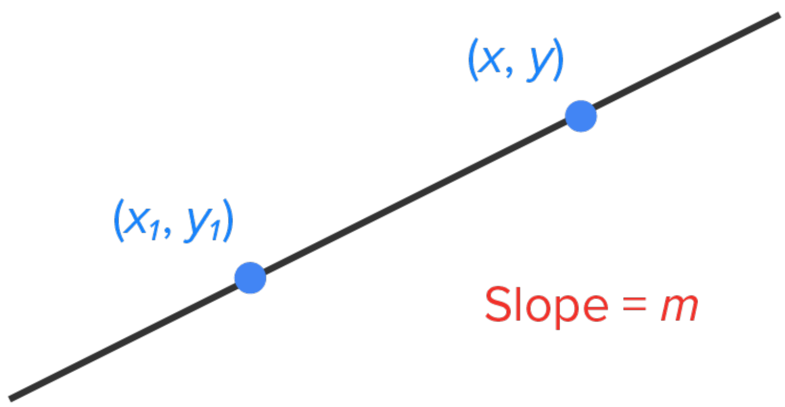
In this lesson, you will be able to write the equation of a line when given the appropriate information. Specifically, this lesson will cover:

1. Point-Slope Form
2. Slope-Intercept Form

1. Point-Slope Form

A line has the property that the slope between any two points on that line is always the same (we call it m).

Let (x, y) represent any point on a line and (x_1, y_1) a specific point on the line.



Using the slope formula, we know $m = \frac{y - y_1}{x - x_1}$. If we multiply both sides by $x - x_1$ this gives us the point-slope form of a linear equation:

FORMULA

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

HINT

Typically, the point (x_1, y_1) and the slope m are substituted into this equation, then the final answer is solved for y.

EXAMPLE Use point-slope form to write the equation of the line that contains the point (-1, 4) and has slope 3.

$y - y_1 = m(x - x_1)$	Point-Slope Form
$y - 4 = 3(x - (-1))$	Substitute the value for m and the known point for x_1 and y_1.
$y - 4 = 3(x + 1)$	Simplify the subtraction inside the parentheses.
$y - 4 = 3x + 3$	Use the distributive property to simplify the right-hand side.
$y = 3x + 7$	Add 4 to both sides.

The equation of the line is $y = 3x + 7$.

2. Slope-Intercept Form

Another form of a line you may be familiar with is $y = mx + b$, which is the slope-intercept form of a line. The variable m is the slope, where the variable b is the y-coordinate of the y-intercept. Thus, another way to think about the line in the previous section is that it has slope 3 and y-intercept (0, 7).

FORMULA

Slope-Intercept Form

$$y = mx + b$$

EXAMPLE Write the equation of the line that contains the points (1, 5) and (4, 7) in slope-intercept form.

First, label the variables: $x_1 = 1$, $y_1 = 5$, $x_2 = 4$, $y_2 = 7$.

Then, the slope of the line is $m = \frac{7-5}{4-1} = \frac{2}{3}$.

You can then use point-slope form, along with either given point and the slope you just found. In this example, the point (1, 5) is used.

$y - y_1 = m(x - x_1)$	Point-Slope Form
$y - 5 = \frac{2}{3}(x - 1)$	Substitute the value for m and the known point for x_1 and y_1.

$y-5=\frac{2}{3}x-\frac{2}{3}$ Distribute the right-hand side.

$y=\frac{2}{3}x+\frac{13}{3}$ Add 5 to both sides.

Thus, the equation of the line is $y=\frac{2}{3}x+\frac{13}{3}$. This tells us that the line has a slope of $\frac{2}{3}$ and a y-intercept $\left(0,\frac{13}{3}\right)$.

WATCH

The following video illustrates how to write the equation of a line.

Video Transcription

[MUSIC PLAYING] Hi, and welcome back. What we're going to do in this video is take an actual, real life situation and form a linear model that describes the situation. So what we have here, the profit after selling 20 units is $100, while the profit after selling 50 units is $850. So profit is increasing as we sell more units. That actually makes sense. And what we're going to do is write the equation, assuming that it's a linear equation, that gives the profit after x units are sold.

So we have x being the number of units, and we're going to call y the profit. And the information was actually given to us as ordered pairs, without explicitly saying that. We know that when x is 20, the prophet is $100, and when x is 50, the prophet is $850. So we have two ordered pairs. I'm just going to pretend like there's a grid here. We have the point 20 comma 100, and we have the point 50 comma 850. And we want to write the equation of the line that connects those two points.

So remember, when we're writing the equation of a line, we use the point slope form when we're not given the y-intercept. So this is just to remind us that we're going to need this equation right here, y minus y1 equals m times x minus x1. So that's going to be where we actually do the work of writing the equation of the line.

First thing we need is the slope. Now, we know that the slope is the difference between the y-coordinates. So I'm going to say 850 minus 100 divided by the difference in the x-coordinates. And remember, 50 goes with 850, and 20 goes with 100, and then we simplify. So it's 750 divided by 30, which is 25. So 25 is the slope, and that's going to get substituted into our equation here.

Now, remember, it doesn't matter which point, you use when you're going to write the equation of your line, because the slope is what pulls them together. So we're going to use the easier numbers. I like 20 and 100 for x1 and y1. So we're going to say y minus 100 equals 25 times x minus 20, which means y minus 100 is equal to 25x minus 500. And remember, we usually like these solved for y, so I'm going to add 100 to both sides. So it's going to give us, as a final equation, y equals 25x minus 400. And that is the equation of the profit after x units are sold.

And just to kind of double-check reality, here, if x is 0-- so if you sell no units-- this tells us that the profit is negative $400, which seems a little weird. But remember, when you're in the business of selling items, you usually start off at a deficit, and your goal is to at least make up for that deficit. That's called break-even. So this equation actually does model reality pretty well. So there is writing the equation of a linear model, given the information.

[MUSIC PLAYING]

SUMMARY

In this lesson, you learned that a line has the property that the slope between any two points on that line is always the same (m). You learned that given the slope and a point on the line (or two points contained on the line), you can use the **point-slope form** to write its equation. You also learned how to write the equation of a line using the **slope-intercept form**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

FORMULAS TO KNOW

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Slope-Intercept Form

$$y = mx + b$$

