Distance Between Two Points

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WHAT'S COVERED

In this lesson, you will learn how to find the distance between two points on a number line and also on the xy-plane. Specifically, this lesson will cover:

1. The Distance Between Two Numbers on a Number Line
2. The Distance Between Two Points in the xy-Plane

1. The Distance Between Two Numbers on a Number Line

Suppose you want to calculate the **distance** between two locations on a number line, as shown below.

The distance between these two points is $b - a$, but that is assuming that b is larger than a.

In general, just so we don't have to worry about which number is larger, the distance between two numbers a and b is $dist(a, b) = |b - a|$. The absolute value is used to ensure that the result is not negative.

FORMULA

Distance on a Number Line

$dist(a, b) = |b - a|$

TRY IT

Find the distance between a and b in each example below.

What is the distance when $a = 13$ and $b = 5$? +

The distance between 13 and 5 is 8.

$dist(13,5) = |5-13| = |-8| = 8$

What is the distance when $a = -21$ and $b = 9$? +

The distance between -21 and 9 is 30.

$dist(-21,9) = |9-(-21)| = |9+21| = |30| = 30$

TERM TO KNOW

Distance
The length of a line segment between two points.

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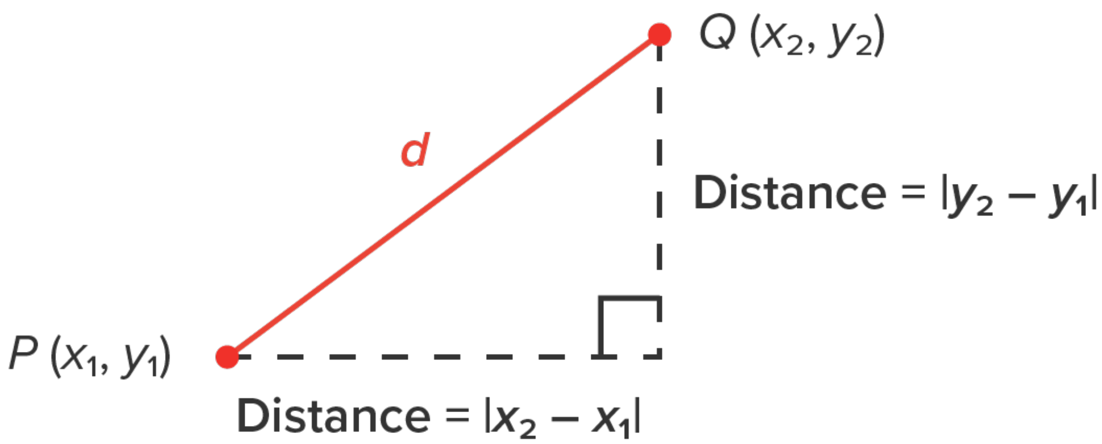
2. The Distance Between Two Points in the xy-Plane

The following image shows two points, P and Q, and the distance between them in the xy-plane, d. Let's find a formula for the distance between these two points.



In the image above:

- The vertical side is the distance between the y-coordinates, which is $|y_2 - y_1|$.
- The horizontal side is the distance between the x-coordinates, which is $|x_2 - x_1|$.
- The distance between the points is labeled as d.

Notice that we have three sides of a right triangle. This means that the Pythagorean theorem can be used to relate the sides to each other. Recall that the Pythagorean theorem states that $(leg_1)^2+(leg_2)^2=(hypotenuse)^2$, where a leg is defined as a side that makes up the right angle and the hypotenuse is the side opposite the right angle (the longest side).

Applying the Pythagorean theorem to our image, we have $|x_2-x_1|^2+|y_2-y_1|^2=d^2$.

HINT

Notice that the first two terms are squares of absolute values. Since squaring also guarantees a nonnegative result, there is no need to include the absolute value. Thus, the relationship actually can be rewritten as $(x_2-x_1)^2+(y_2-y_1)^2=d^2$.

To write an expression for the distance, *d*, take the square root of both sides to get the following formula:

FORMULA

Distance in the xy-Plane

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

HINT

You might remember from algebra that taking the square root of both sides results in a positive solution and a negative solution. Since distance is always nonnegative, only the positive square root is considered.

EXAMPLE Calculate the exact distance between the points (4, 5) and (8, 1).

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Distance Formula
$d=\sqrt{(8-4)^2+(1-5)^2}$	Substitute known quantities: $x_1=4$, $y_1=5$, $x_2=8$, $y_2=1$.
$d=\sqrt{4^2+(-4)^2}$	Evaluate subtraction inside parentheses.
$d=\sqrt{16+16}$	Square values.
$d=\sqrt{32}$	Add values under the square root.
$d=\sqrt{16\cdot 2}$	Rewrite the square root with any perfect square factors.
$d=\sqrt{16}\sqrt{2}$	Apply the product property of square roots.
$d=4\sqrt{2}$	Simplify the radical.

The distance between the points (4, 5) and (8, 1) is $4\sqrt{2}$, or about 5.66 units.

WATCH

The following video further illustrates the use of the distance formula.

Video Transcription

Welcome. And what we're going to look at right now is the distance formula and using it to find the distance between two points and the x-y plane.

So we have labeled here are two points, negative 2, 3 and 4, 5. And what we're curious about is how far apart are they. So you might recall the distance formula over to the right is d equals the square root of x2 minus x1 quantity squared plus y2 minus y1 one the quantity squared. Remember that really represents the difference in the x's squared, which we call delta x, and the difference between the y-coordinates squared.

So the first thing we should do whenever we use a formula is to label the inputs. And our first point I'm just going to call x sub 1, y sub 1. And 4, 5 I'm going to call x sub 2, y sub 2. And that makes it very clear what we are going to substitute into this formula.

So we have d is equal to the square root of-- so x2 is 4. x1 is negative 2. And we're squaring that, plus y2 is 5 and y1 is 3. So that's 5 minus 3, the quantity squared. And now we just use our order of operations to simplify the expression.

So we have the square root of 4 minus negative 2 is 6, and 6 squared is 36, plus 5 minus 3 is 2, and 2 squared is 4. And now we combine under the radical. So we have the square root of 40. And the exact form of the answer is what we're going to look for.

So remember that some square roots can be simplified, in that we remove the perfect square factors from the radicand, which is 40. So thinking about a perfect square that divides into 40 evenly, we could think of 4. So 40 can be rewritten as 4 times 10. Remember property of radicals, this is rewritten as the square root of 4 times the square root of 10, which is 2 square roots of 10.

And this is the exact form of the distance. If you want to approximate it, naturally you can take a calculator. And you can either type in the square root of 40 or 2 times the square root of 10, and they should both give you the same result. And we know that 2 times square root of 10 is simplified because there is no other perfect square factor that divides into 10 evenly. So there is a demonstration of the distance formula.

SUMMARY

In this lesson, you learned how to calculate **the distance between two numbers on a number line** by calculating the absolute value of their difference. Next, you applied this idea, along with the Pythagorean theorem, to arrive at the distance formula to calculate **the distance between two points in the xy-plane**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Distance
The length of a line segment between two points.

FORMULAS TO KNOW

Distance in the xy-Plane

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Distance on a Number Line

$$dist(a,b)=|b-a|$$