

# Equations of Circles

by Sophia



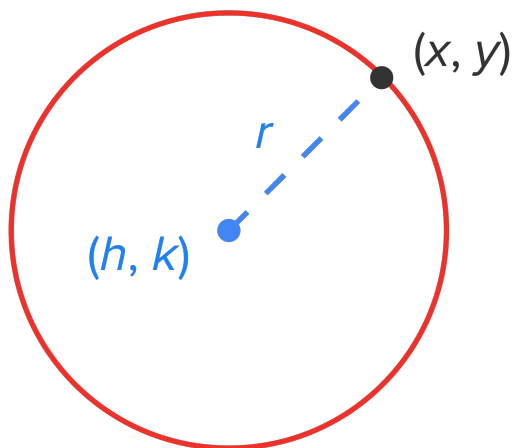
## WHAT'S COVERED

In this lesson, you will learn how to write the equation of a circle. Specifically, this lesson will cover:

1. Parts of a Circle
2. Standard Form Equation of a Circle

## 1. Parts of a Circle

Consider the circle shown below.



In the image above:

- The center of the circle is labeled  $(h, k)$ .
- $(x, y)$  represents any point on the circle.
- The radius of the circle is  $r$ , which is the distance from  $(h, k)$  to  $(x, y)$ .

So, how can we calculate this distance?

Using the distance formula from the previous lesson, we can set up a relationship. The distance between  $(h, k)$  and  $(x, y)$  is the radius,  $r$ , and can be found with the following formula:



## FORMULA

### Radius of a Circle

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

Where:

$(h, k)$  is the center and  $(x, y)$  is a point on the circle.

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## 2. Standard Form Equation of a Circle

If we take the radius formula from the above section and square it, we get the following equation:

$$(x-h)^2 + (y-k)^2 = r^2$$

This actually is the standard form of the equation of a circle. So, if a circle has center  $(h, k)$  and radius  $r$ , the equation for all points on the circle is  $(x-h)^2 + (y-k)^2 = r^2$ .



## FORMULA

### Standard Form Equation of a Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Where:

$(h, k)$  is the center and  $r$  is the radius.

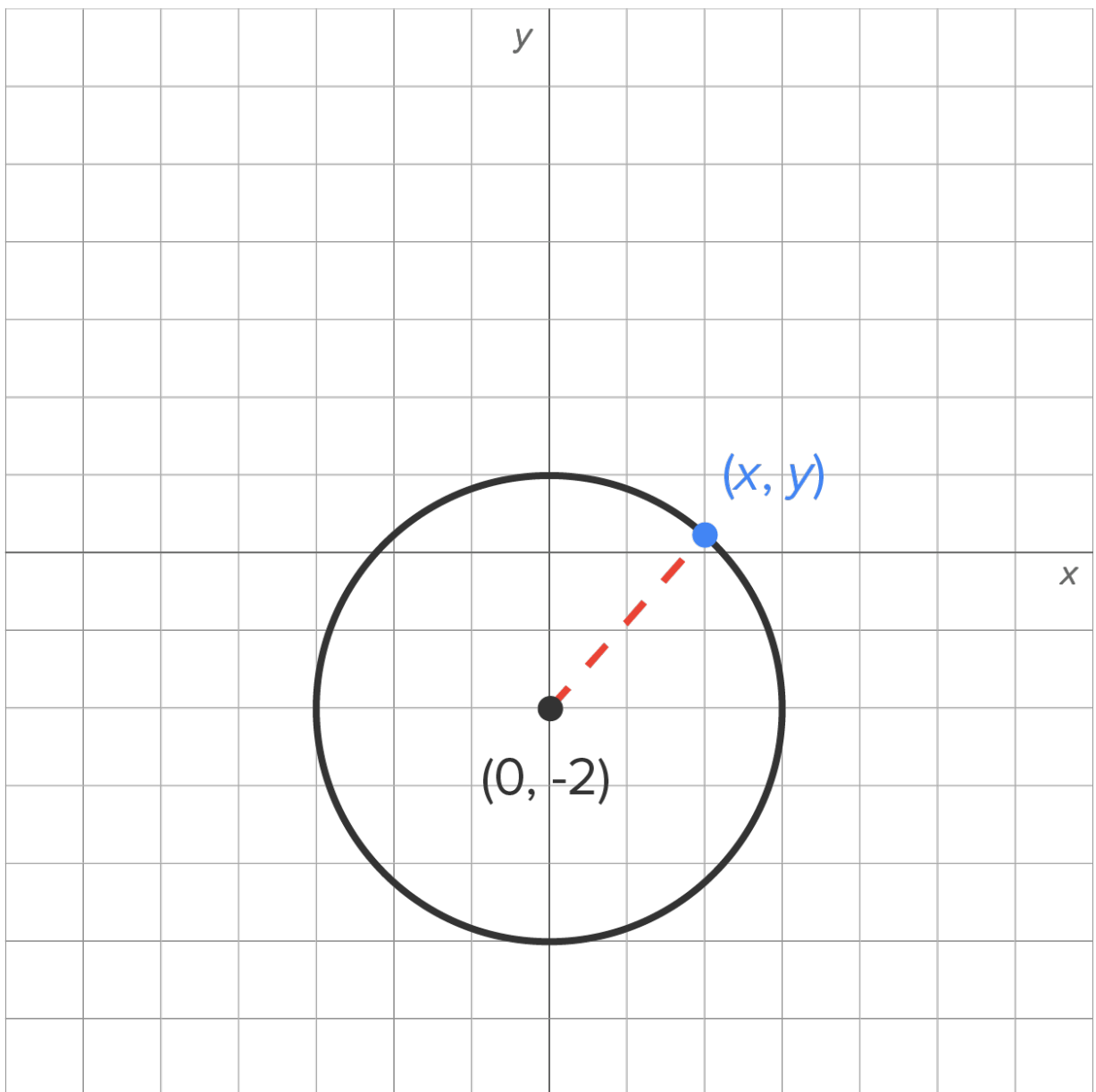
➞ **EXAMPLE** Write the equation of a circle whose center is  $(0, -2)$  and which has a radius of 3.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Standard Form Equation of a Circle}$$

$$(x-0)^2 + (y-(-2))^2 = 3^2 \quad \text{Substitute known values: } h=0, k=-2, \text{ and } r=3.$$

$$x^2 + (y+2)^2 = 9 \quad \text{Simplify the equation.}$$

The equation of the circle whose center is  $(0, -2)$  and which has a radius of 3 is  $x^2 + (y+2)^2 = 9$  and has the following graph:



TRY IT

A circle has the equation  $(x - 1)^2 + (y + 2)^2 = 16$ .

What is the center?

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Matching the equation to the standard form, we see that  $h = 1$  and  $k = -2$ , so the center of the circle is  $(1, -2)$ .

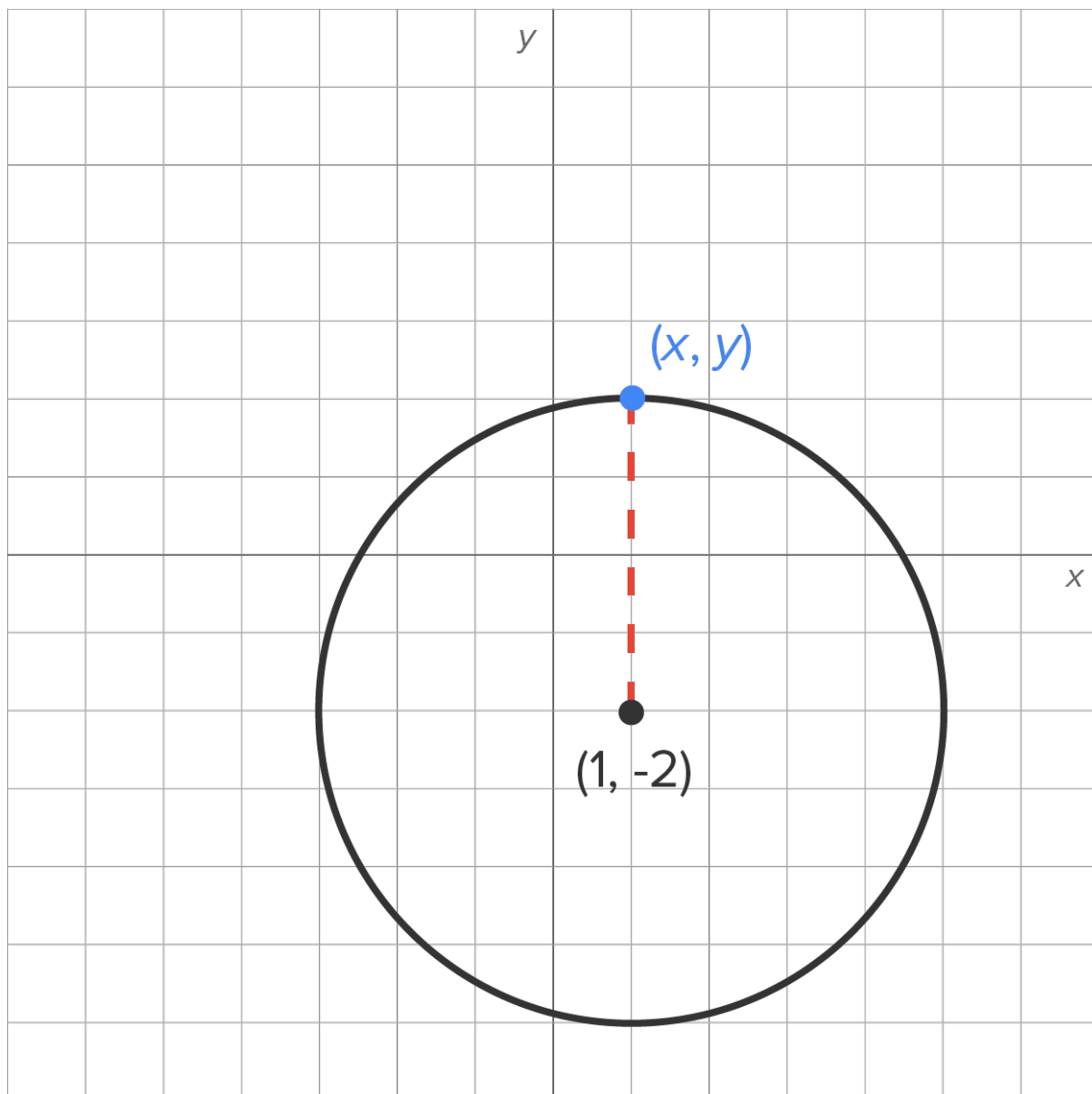
What is the radius?

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Also from the equation, we see that the right side of the equation, 16, corresponds with  $r^2$ , which means the radius,  $r$ , is 4.

What does this graph look like?

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## SUMMARY

In this lesson, you learned about the **parts of a circle**, including the center of the circle, the representation of any point on the circle, and the radius, which is the distance between the center and a given point on the circle. You learned how to calculate this distance using the formula for the radius of a circle. You also learned how to write the equation of a circle using the **standard form equation of a circle**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.



## FORMULAS TO KNOW

### Radius of a Circle

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

Where:  $(h, k)$  is the center and  $(x, y)$  is a point on the circle.

**Standard Form Equation of a Circle**

$$(x-h)^2 + (y-k)^2 = r^2$$

Where:  $(h, k)$  is the center and  $r$  is the radius.