Other Asymptotes As x Approaches ∞ and -∞

by Sophia

WHAT'S COVERED

In this lesson, you will investigate other types of asymptotes that are neither horizontal nor vertical. Specifically, this lesson will cover:

1. Slant (Oblique) Asymptotes
2. Other Nonlinear Asymptotes

1. Slant (Oblique) Asymptotes

When a rational function $f(x)$ doesn't have a horizontal asymptote, it could have a**slant asymptote**, which is a slanted line that the graph of $f(x)$ approaches as $x \to \pm\infty$.

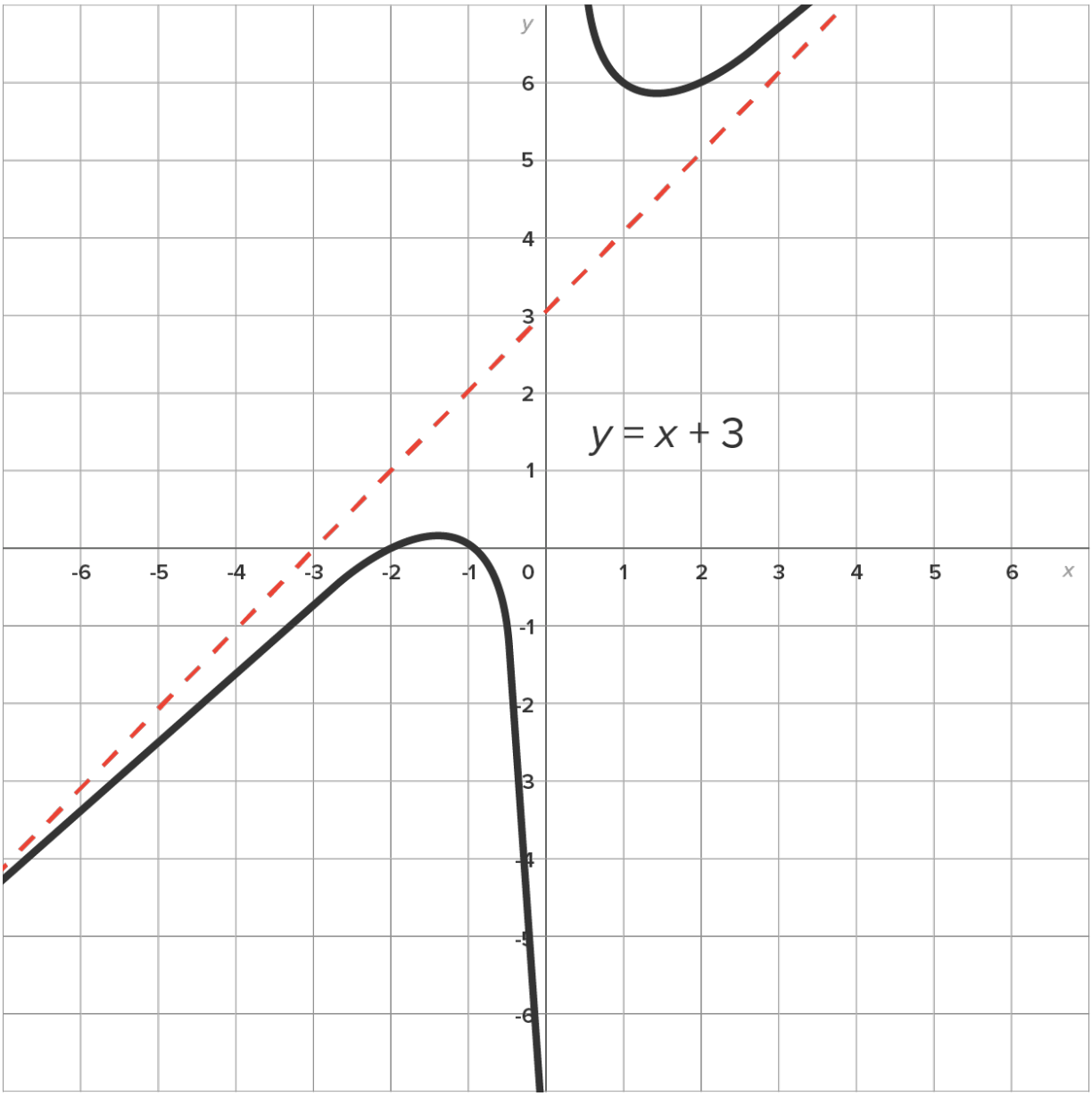
To see how to recognize a slant asymptote, let's look at this first example.

EXAMPLE Consider the function $f(x) = \frac{x^2+3x+2}{x}$.

Performing the division, we have $f(x) = \frac{x^2}{x} + \frac{3x}{x} + \frac{2}{x} = x+3+\frac{2}{x}$.

As $x \to \pm\infty$, $\frac{2}{x} \to 0$, which means the graph of $f(x)$ gets closer to the graph of $y = x+3$. Thus, the slant asymptote is $y = x+3$.

The graph of $f(x)$ along with its slant asymptote (dashed) is shown in the figure. Note how the graph approaches its slant asymptote as $x \to \pm\infty$.



TRY IT

Consider the function $f(x)=\frac{3x^2+5x+2}{x+2}=3x-1+\frac{4}{x+2}$.

Identify the slant asymptote of the graph of this function. +

The graph of $f(x)$ has the slant asymptote $y=3x-1$ since $\frac{4}{x+2}\to 0$ as $x\to\pm\infty$.

TERM TO KNOW

Slant (Oblique) Asymptote

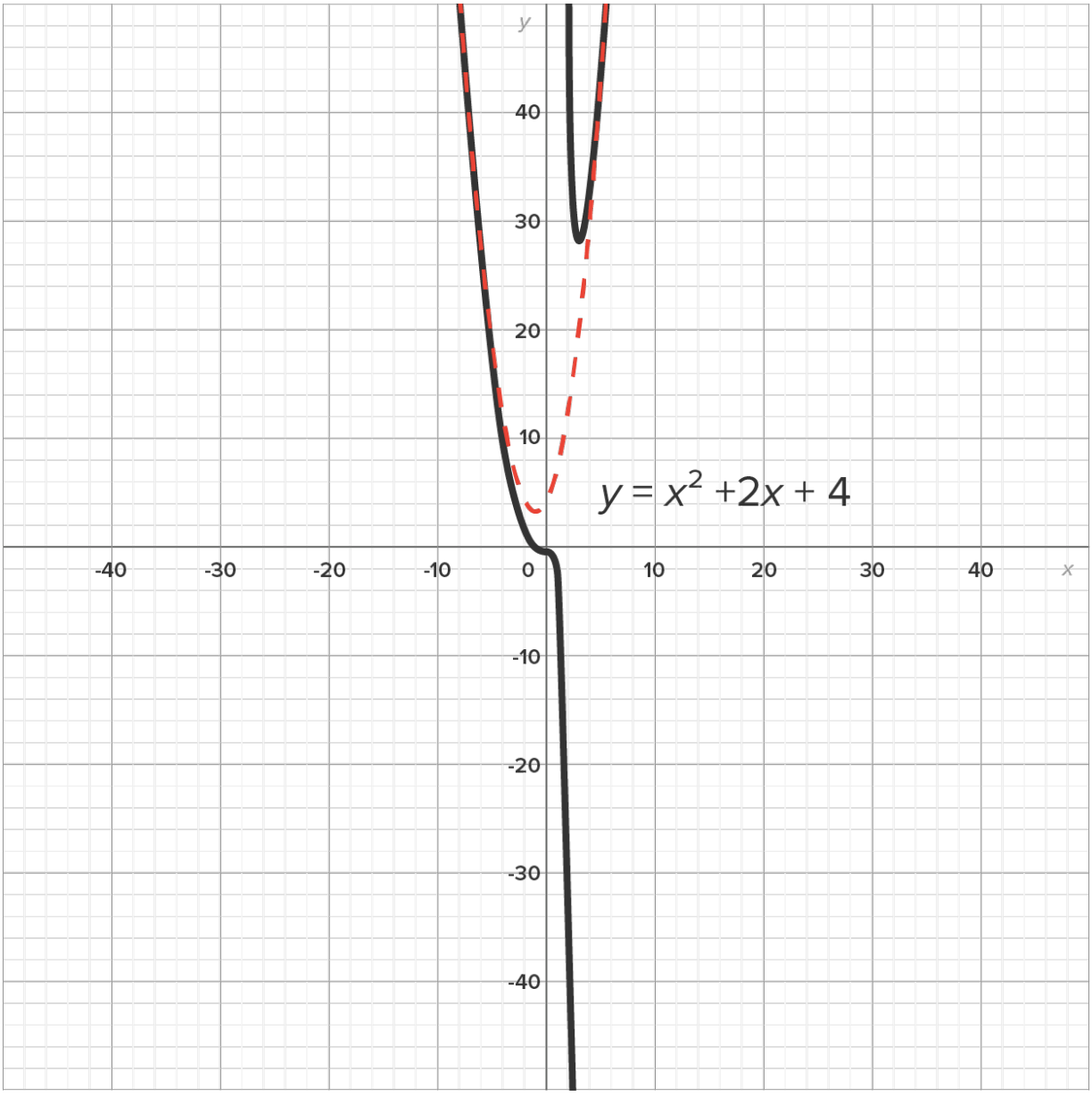
The slanted line that a graph approaches as $x\to\pm\infty$.

2. Other Nonlinear Asymptotes

A **nonlinear asymptote** is the curve that a graph approaches as $x \to \pm\infty$.

EXAMPLE Consider the function $f(x) = \dfrac{x^3+1}{x-2} = x^2+2x+4+\dfrac{9}{x-2}$.

Since $\dfrac{9}{x-2} \to 0$ as $x \to \pm\infty$, the graph of $f(x)$ has a nonlinear asymptote $y = x^2+2x+4$. The graph of $f(x)$ along with the nonlinear asymptote (dashed) is shown in the figure.



TRY IT

Consider the function $f(x) = x^2 + \dfrac{x}{x^2+1}$.

Write the equation of the nonlinear asymptote of the function. +

The nonlinear asymptote is $y = x^2$.

TERM TO KNOW

Nonlinear Asymptote

The curve that a graph approaches as $x \to \pm\infty$.

SUMMARY

In this lesson, you learned that when a rational function doesn't have a horizontal asymptote, it could have either a **slant (oblique) asymptote**, which is a slanted line that the graph of $f(x)$ approaches as $x \to \pm\infty$, or a **nonlinear asymptote**, which is the curve that a graph approaches as $x \to \pm\infty$.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 3 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Nonlinear Asymptote

The curve that a graph approaches as $x \to \pm\infty$.

Slant (Oblique) Asymptote

The slanted line that a graph approaches as $x \to \pm\infty$.