# Derivative of Elementary Combinations of Functions

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WHAT'S COVERED

In this lesson, you will use rules of differentiation to find derivatives of combinations of functions. One reason this is important to learn is to be able to analyze situations in which the function is more complex. For example, suppose that the height of a projectile $t$ seconds after being launched is given by $h(t) = -16t^2 + 80t + 5$. If we learn some more derivative rules, we can analyze this function more efficiently without having to use the limit definition. Specifically, this lesson will cover:

1. Derivatives of Constant Multiples of Functions
2. Derivatives of Sums and Differences of Functions

# 1. Derivatives of Constant Multiples of Functions

To introduce this topic, let's say a jogger goes out for a run.

- Let $y(x) =$ the number of yards that a jogger runs in $x$ minutes.
- Let $f(x) =$ the number of feet that a jogger runs in $x$ minutes.

Since there are 3 feet in every yard, we also know that $f(x) = 3 \cdot y(x)$.
Now, let's think about the rates of change, namely $f'(x)$ and $y'(x)$.

If the jogger's rate of change at a certain instant is 40 yards per minute, then their rate is also 120 feet per minute (3 feet in every yard). Therefore, if we know the rate of change in the number of yards per minute, we just need to multiply by 3 to get the rate of change in the number of feet per minute.

Using symbols, here is the summary: If $f(x) = 3y(x)$, then $f'(x) = 3y'(x)$.

This leads us to the constant multiple rule for derivatives. If $k$ is a constant, then we have the following rule: $D[k \cdot f(x)] = k \cdot D[f(x)]$. In other words, find the derivative of the variable part first, then multiply it by the constant.

FORMULA

**Derivative of a Constant Multiple**

$D[k \cdot f(x)] = k \cdot D[f(x)]$

EXAMPLE Find the derivative of $f(x) = 3x^4$.

| | |
|---|---|
| $f'(x) = 3D[x^4]$ | Use the constant multiple rule. |
| $f'(x) = 3(4x^3)$ | Use the power rule. |
| $f'(x) = 12x^3$ | Simplify. |

Thus, $f'(x) = 12x^3$.

EXAMPLE Find the derivative of $f(x) = 12\sqrt{x}$.

First, rewrite as $\sqrt{x} = x^{1/2}$. Then:

| | |
|---|---|
| $f'(x) = 12D[x^{1/2}]$ | Use the constant multiple rule. |
| $f'(x) = 12\left(\frac{1}{2}x^{-1/2}\right)$ | Use the power rule. |
| $f'(x) = 6x^{-1/2}$ | Simplify. |
| $f'(x) = \frac{6}{x^{1/2}}$ | Write with positive exponents. |

Thus, $f'(x) = \frac{6}{x^{1/2}}$. Using radicals, this could also be written $f'(x) = \frac{6}{\sqrt{x}}$.

TRY IT

Consider the functions $f(x) = -10x^2$, $g(x) = 6\sqrt[3]{x}$, and $h(x) = \frac{5}{x^4}$.

Find the derivatives of the formulas above. +

$f'(x) = -20x$, $g'(x) = \frac{2}{x^{2/3}}$, and $h'(x) = \frac{-20}{x^5}$

## 2. Derivatives of Sums and Differences of Functions

Let’s say that Fred and Gabby are baking cookies, where Fred makes 100 cookies per hour and Gabby makes 80 cookies per hour.

Then, the total number of cookies made per hour is 100 + 80 = 180 cookies per hour. Thus, the rate of change of the sum is the sum of the individual rates of change.

We can also say that Fred's rate of change is 20 cookies more per hour than Gabby's. Thus, the rate of change of their difference is the difference between the individual rates of change.

This leads to two more derivative rules:

FORMULA

**Derivative of a Sum**
$D[f(x)+g(x)]=D[f(x)]+D[g(x)]$

**Derivative of a Difference**
$D[f(x)-g(x)]=D[f(x)]-D[g(x)]$

EXAMPLE We know that $D[x^3]=3x^2$ and $D[\sin x]=\cos x$.

Then, $D[x^3+\sin x]=3x^2+\cos x$.

EXAMPLE Find the derivative of $f(x)=\frac{4}{x^6}-7\cos x$.

Since this is a difference of functions, use the difference rule. To find $D\left[\frac{4}{x^6}\right]$:

| | |
|---|---|
| $D\left[\frac{4}{x^6}\right]=D[4x^{-6}]$ | Rewrite using negative exponents. |
| $=4D[x^{-6}]$ | Use the constant multiple rule. |
| $=4(-6)x^{-7}$ | Use the power rule. |
| $=-24x^{-7}$ | Simplify. |
| $=-\frac{24}{x^7}$ | Write using positive exponents. |

To find $D[7\cos x]$:

| | |
|---|---|
| $D[7\cos x]=7D[\cos x]$ | Use the constant multiple rule. |
| $=7(-\sin x)$ | $D[\cos x]=-\sin x$ |
| $=-7\sin x$ | Simplify. |

Then, $D\left[\frac{4}{x^6}-7\cos x\right]=-\frac{24}{x^7}-(-7\sin x)=-\frac{24}{x^7}+7\sin x$.

Thus, $f'(x)=-\frac{24}{x^7}+7\sin x$.

TRY IT

In the introduction, the function $h(t) = -16t^2 + 80t + 5$ was mentioned.

Find $h'(t)$. +

$h'(t) = -32t + 80$

TRY IT

Consider the function $f(x) = 6\sqrt{x} + 8x - 3\cos x$.

Find the derivative. +

$$f'(x) = \frac{3}{x^{1/2}} + 8 + 3\sin x$$

## SUMMARY

In this lesson, you learned that the sum/difference and constant multiple rules for derivatives allow us to expand on the types of functions that can be differentiated by using rules rather than the limit definition. For example, you learned that to find the **derivative of constant multiples of functions**, you find the derivative of the variable part first, then multiply it by the constant. You also explored an example involving finding **derivatives of sums and differences of functions**, noting that the rate of change of the sum is the sum of the individual rates of change, and the rate of change of the difference is the difference between the individual rates of change. We're only getting started since there are several rules to discuss yet!

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 2 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

## FORMULAS TO KNOW

**Derivative of a Constant Multiple**
$D[k \cdot f(x)] = k \cdot D[f(x)]$

**Derivative of a Difference**
$D[f(x) - g(x)] = D[f(x)] - D[g(x)]$

**Derivative of a Sum**
$D[f(x) + g(x)] = D[f(x)] + D[g(x)]$