Trigonometric Functions

by Sophia

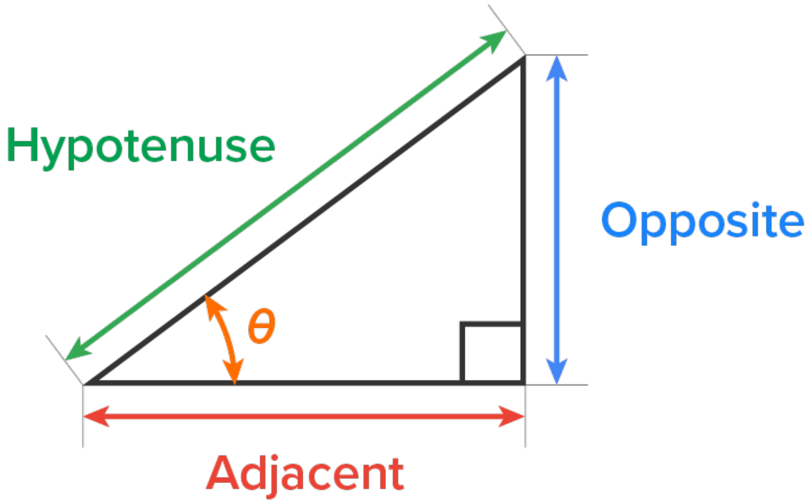
WHAT'S COVERED

In this lesson, you will get a brief overview of the trigonometric functions, some properties, and graphs. Specifically, this lesson will cover:

1. The Three Basic Trigonometric Functions
2. Evaluating Trigonometric Functions
   a. Using Right Triangles to Evaluate Trigonometric Functions
   b. Evaluating Trigonometric Functions for Any Acute Angle
   c. Evaluating Trigonometric Functions for 30°, 45°, and 60°
   d. Defining Non-Acute Angles
   e. The Unit Circle and Evaluating Trigonometric Functions for Any Angle
3. Radian Measure
   a. Converting Between Degrees and Radians
   b. Evaluating Trigonometric Functions Using Radian Measure
4. Finding an Input for a Known Output (Solving Trigonometric Equations)
5. Basic Trigonometric Graphs
   a. The Graph of $y = \sin x$
   b. The Graphs of $y = \cos x$ and $y = \tan x$
6. Frequently Used Trigonometric Identities

1. The Three Basic Trigonometric Functions

Consider the right triangle.



The sides “Opposite” and “Adjacent” are named this way relative to the angle θ (theta) marked in the triangle.

Going forward, “Opposite” means the length of the opposite side and “Adjacent” means the length of the adjacent side.

The hypotenuse is the side opposite the right angle, and is consequently the longest side of the right triangle.

Using this convention, there are six possible ratios that can be computed between two distinct sides of a right triangle. We will focus on three of these for now.

A **trigonometric function** assigns an angle to a ratio. That is, the input is the angle and the output is the ratio. The three basic trigonometric functions we will discuss for now are called the sine function (sin), the cosine function (cos), and the tangent function (tan).

To compute these functions, an angle must be used. Here are their definitions:

FORMULA

Definitions of the Sine, Cosine, and Tangent Functions

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

TERM TO KNOW

Trigonometric Function

Uses an angle as an input and returns a ratio as the output.

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2. Evaluating Trigonometric Functions

At first, we are going to only consider situations where θ is an **acute angle**, meaning its measure is more than $0°$ and less than $90°$. When using trigonometric functions, it is important to note that an angle must accompany the name of the function. For example, we write $\sin\theta = \frac{5}{7}$, not $\sin = \frac{5}{7}$. (Think about the square root function: You had to write $\sqrt{16} = 4$ as opposed to $\sqrt{} = 4$.)

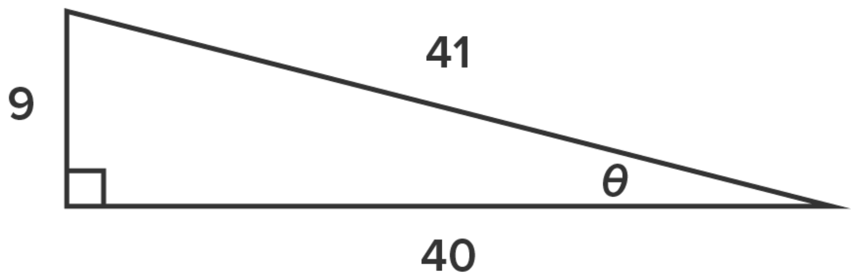
TERM TO KNOW

Acute Angle

An angle whose measure is more than $0°$ and less than $90°$.

2a. Using Right Triangles to Evaluate Trigonometric Functions

EXAMPLE Using the right triangle, find the sine, cosine, and tangent of angle θ.

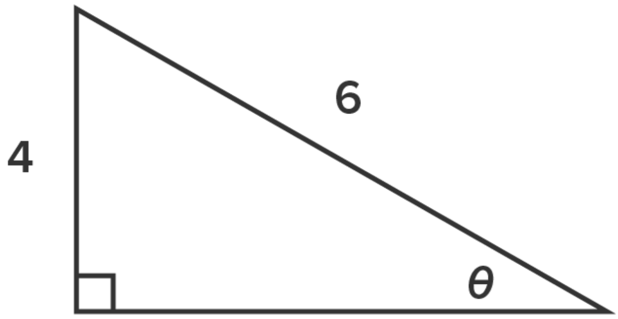


$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{41} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{40}{41} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{40}$$

HINT

All right triangles satisfy the Pythagorean theorem. Therefore, if two sides are given, the Pythagorean theorem can be used to find the unknown side.

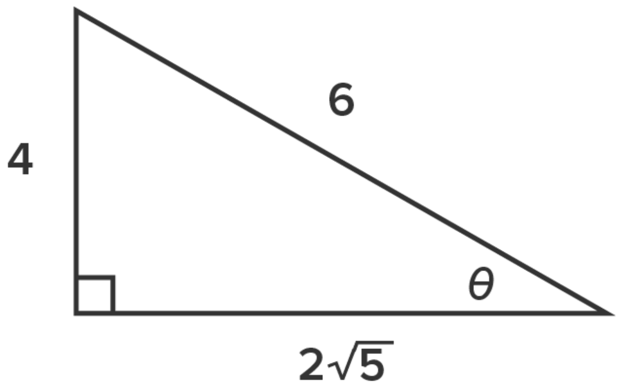
EXAMPLE Using the right triangle, find the sine, cosine, and tangent of angle θ.



Let x = the adjacent side. To find x, the Pythagorean theorem is needed:

$$4^2 + x^2 = 6^2$$
$$16 + x^2 = 36$$
$$x^2 = 20$$
$$x = \sqrt{20} = 2\sqrt{5}$$

Now, here is the triangle with the unknown side filled in:



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{4}{6} = \frac{2}{3}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

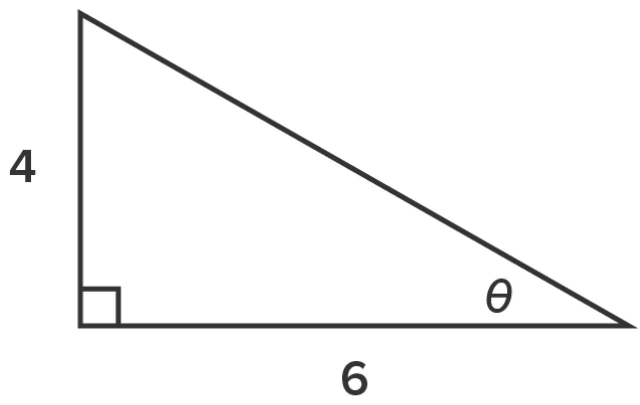
$$\tan\theta = \frac{opposite}{adjacent} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

HINT

In the result for tan θ in the above example, the expression $\frac{2}{\sqrt{5}}$ is not considered simplified since there is a radical in the denominator. The process by which an expression like this is simplified is called rationalizing the denominator. In short, to rationalize a fraction with $\sqrt{b}$ in the denominator, multiply by $\frac{\sqrt{b}}{\sqrt{b}}$.

TRY IT

Consider the right triangle below.



What is the length of the hypotenuse? +

The length of the hypotenuse is $\sqrt{52} = 2\sqrt{13}$.

What is the sine of angle θ? +

$$\sin\theta = \frac{4}{2\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

What is the cosine of angle θ? +

$$\cos\theta = \frac{6}{2\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

What is the tangent of angle θ? +

$$\tan\theta = \frac{4}{6} = \frac{2}{3}$$

2b. Evaluating Trigonometric Functions for Any Acute Angle

Most trigonometric functions require calculator use. On a typical scientific calculator, you will notice the “sin,” “cos,” and “tan” buttons.

Before practicing this, be sure that your calculator is in “degree” mode, which should be the default setting.

> ↪ EXAMPLE Suppose we want the sine of the angle 40°. This is written $\sin(40^\circ)$.
>
> Using your calculator, press the “sin” key, then type in the number 40, then close the parenthesis. The result is a long decimal. Rounded to 4 places, we can say $\sin(40^\circ) \approx 0.6428$.

TRY IT

Use your calculator to answer the following questions.

What is the value of cos40°? +

$\cos 40^\circ \approx 0.7660$

What is the value of tan40°? +

$\tan 40^\circ \approx 0.8391$

2c. Evaluating Trigonometric Functions for 30°, 45°, and 60°

Most trigonometric functions produce values that are long decimals, which are irrational numbers. It turns out that the ratios corresponding to 30°, 45°, and 60° are concise enough that they are worth noting (and remembering!). This is why we sometimes refer to these angles as special angles.

WATCH

This video reviews how these values are derived.

Video Transcription

[MUSIC PLAYING] Greetings, and welcome back. What we're going to look at in this video is the trigonometric functions, but applied to specific angles, because they have such nice ratios that they're worth remembering and noting. So if you take a look at the equilateral triangle on the left, we notice that all sides are length 2. And remember that with an equilateral triangle, each angle is also the same. So all three of those angles are 60 even though the one at the top is not marked.

Because here's what we're going to do. Remember that we can only find trig ratios when a right triangle is involved. So I'm going to drop a height here in the middle, and what that's going to produce at the bottom is two right angles. And that means that each of these sides is 1, and that also means that 60-degree angle at the top is now two 30-degree angles.

And focusing on the triangle on the left-- I'm going to highlight that for you here-- if we look at the height, the hypotenuse, and the side whose length is 1, that height, by the Pythagorean theorem, is the square root of 3. So now we have three sides of a right triangle, and we can establish the ratios that are involved with 30 degrees and 60 degrees.

So starting with 30, the sign of 30 is the opposite, which is 1, over the hypotenuse, which is 2. So there we have that. Cosine of 30 is the adjacent side, which is the square root of 3, and hypotenuse is 2. Tangent of 30, that one's a little bit more tricky. Tangent of 30 is opposite over adjacent. So that's going to be 1 over square root of 3, which, in rationalized form, is square root of 3 divided by 3.

Now we could do the same thing for 60 degrees. Opposite the 60 is the square root of 3, hypotenuse is still 2. Cosine of 60, the adjacent side to the 60 degree angle, is 1, and the hypotenuse is still 2. And tangent of 60 is the opposite, which is the square root of 3 divided by the adjacent, which is 1. So tangent of 60 actually simplifies to the square root of 3.

Now, if we come over to the 45, 45, 90 triangle, the isosceles-- and just full point of reference, you've probably heard this word before. Isosceles means that two angles are the same, which, in turn, means that two sides are the same. And since this is a right triangle, that means the other two angles have to split the difference between 180 and 90. So that means they're each 45 degrees.

Now, if we apply our trig ratios to this triangle, if I take the sine of 45 degrees-- which, I'll use this one, the incredible disappearing triangle there. I'll use this one. So sine of 45 is 1 over root 2, opposite over hypotenuse, which, again, can be written in rationalized form. So that's the square root of 2 over 2. Cosine of 45, same thing. And then tangent of 45, this is probably the most surprising of them all. But tangent of 45 is opposite over adjacent, which is 1 over 1, which is 1. And that is how we establish the trig ratios for 30 degrees, 60 degrees, and 45 degrees.

[MUSIC PLAYING]

The exact values for all the special angles are as follows:

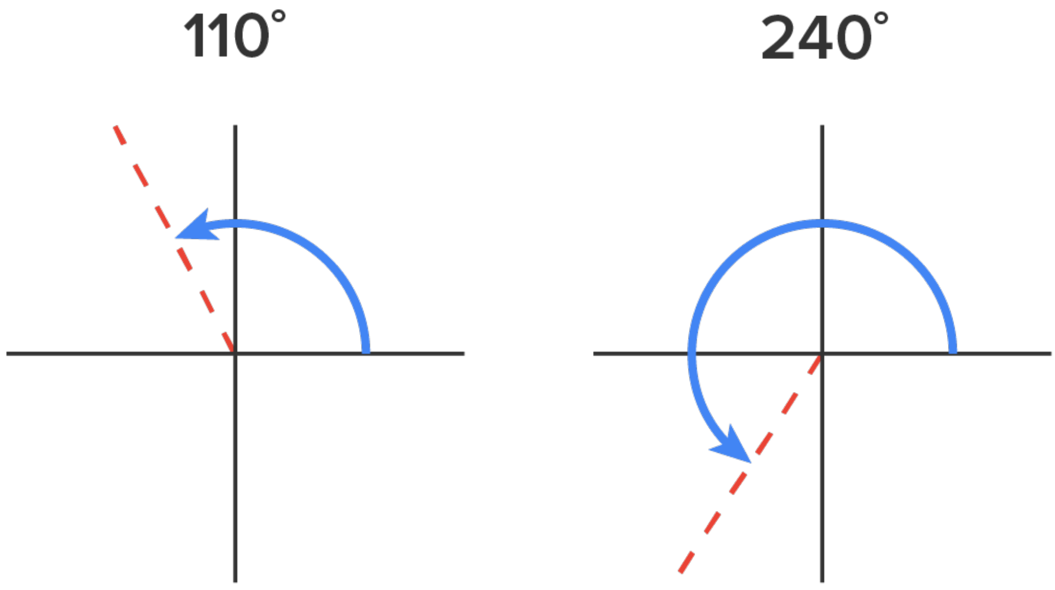
$\sin 30^\circ = \frac{1}{2}$	$\sin 45^\circ = \frac{\sqrt{2}}{2}$	$\sin 60^\circ = \frac{\sqrt{3}}{2}$
$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\cos 45^\circ = \frac{\sqrt{2}}{2}$	$\cos 60^\circ = \frac{1}{2}$
$\tan 30^\circ = \frac{\sqrt{3}}{3}$	$\tan 45^\circ = 1$	$\tan 60^\circ = \sqrt{3}$

2d. Defining Non-Acute Angles

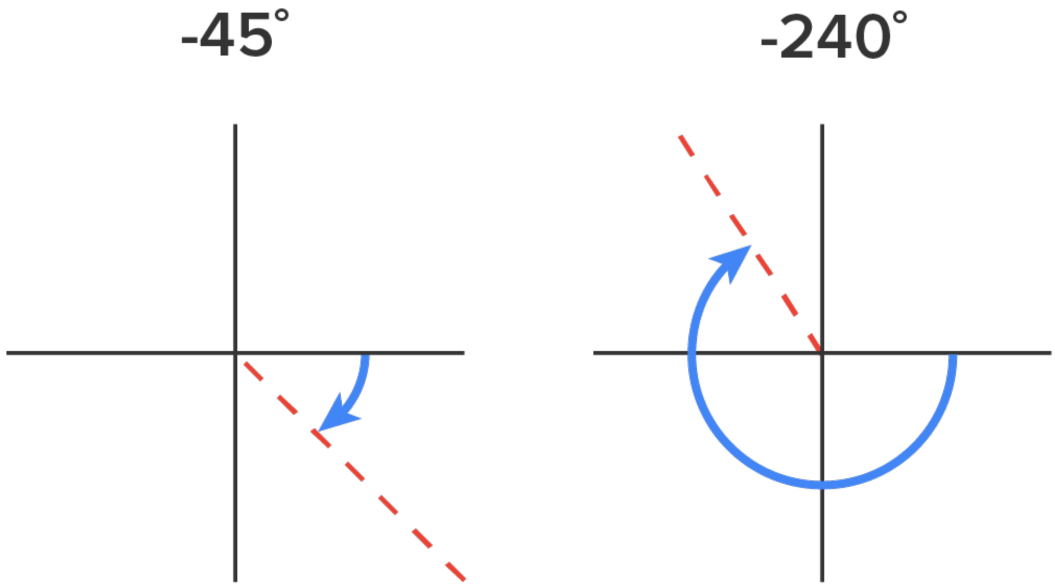
As it turns out, the work we have done with the trigonometric functions can be applied to any angle, not just acute angles. But how are non-acute angles handled?

For reference, we draw the x-axis and y-axis. The angle is represented by starting at the positive x-axis (which we call the initial side of an angle), then drawing counterclockwise. To show where the angle stops, a side is created (dashed). This is called the terminal side of the angle.

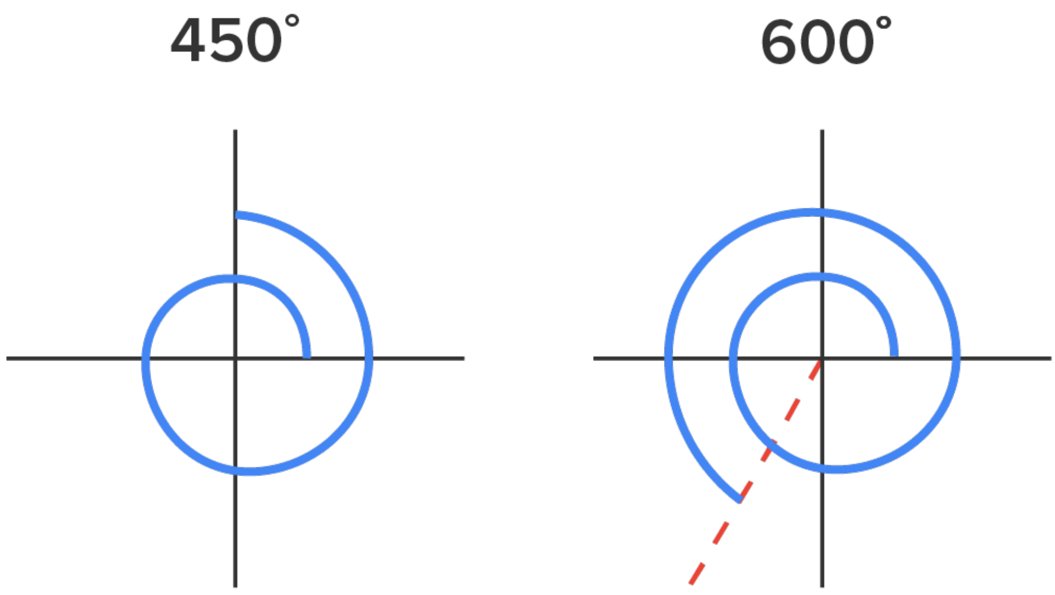
For example, here are what 110° and 240° look like:



It follows that angles can also be measured clockwise. These angles are negative. Some examples:



Lastly, it is also possible to talk about angles larger than $360°$. These angles go through more than one full revolution.



Notice that the 450° angle above terminates at the same place that a 90° angle terminates. We call these angles coterminal. In general, coterminal angles have measures that differ by some multiple of 360°.

TRY IT

Suppose you have the angle 110°.

Give measures of two angles that are coterminal to this angle. +

There are many answers, but the most straightforward are 470° and 830°.

2e. The Unit Circle and Evaluating Trigonometric Functions for Any Angle

WATCH

This video shows how a unit circle, which is a circle with radius 1, can be connected with the trigonometric functions.

Video Transcription

[MUSIC PLAYING] Hi, and welcome back. What we're going to look at right now in what we're talking about, trigonometric functions, is its relationship to this thing called the unit circle. And the unit circle is just a circle with a radius of 1. So everything coming out from the center to any point that's on the circle has a length of 1.

So the big question is, what is the relationship between all of these points that are labeled on the circle and the angles that they correspond to? So just for example, we see that the point root 2 over 2, root 2 over 2 corresponds to 45 degrees. Why is that? So let's see. Just to get a snapshot, let's say we're in the first quadrant. So I'm going to take the point 0, 0 and go out to a point that's on the unit circle and call it x comma y. And I'm going to make a triangle out of it.

Now, the major thing to notice about these angles is they seem to be measured from the positive x-axis and going counterclockwise. So that's how we measure angles. We start at the positive x-axis. That's the initial side, and the terminal side is where we end up moving counterclockwise. And that's how we measure the angle. So this angle is theta.

Now, that means, in this case, that this side has length x and this side has length y. So looking at the three basic trigonometric functions, we know that the sine of theta is y over-- remember, this side is 1-- 1. So that's just y. So the y-coordinate of the point is the sine of the angle that corresponds to it.

Cosine of theta is the adjacent side divided by the hypotenuse, so that's just x. So that means, right there, that the actual coordinates tell us the cosine of the corresponding angle and the sine of the corresponding angle. Cool, OK, now let's look at tangent of theta. Tangent is opposite over adjacent, which is y over x. So that means that we could also easily find the tangent of the corresponding angle by just finding the ratio of the coordinates.

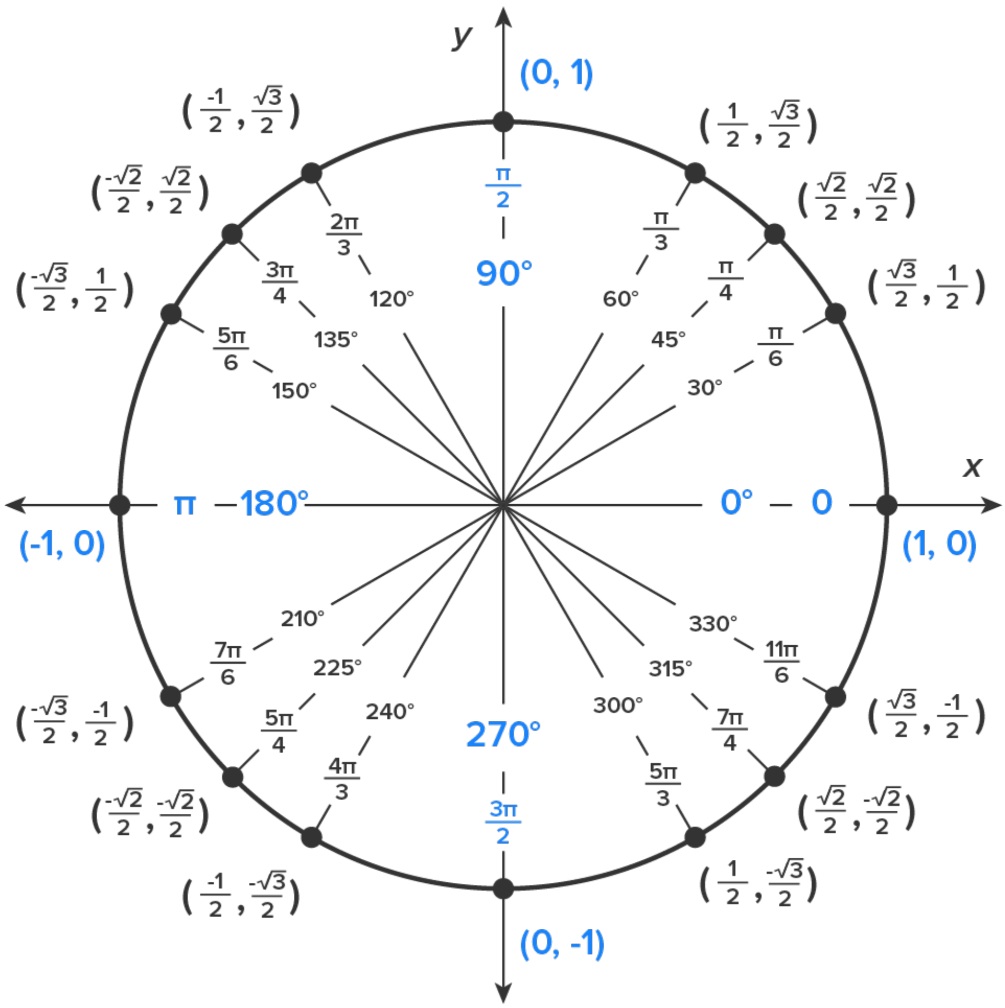
So let's just take a look at how we can apply this. So sine of 120 degrees. 120 degrees terminates at the point 1/2 comma root 3 over 2. Sine of 120 would be the y-coordinate of the 120-degree angle. So that is square root 3 over 2. And notice that when we get into bigger angles, that opens the door for the ratios to end up being negative. This is our way of defining sine, cosine, and tangent moving outside of a right triangle, OK?

So cosine of 135, the 135-degree angle terminates at negative root 2 over 2 comma root 2 over 2. Cosine is the x-coordinate, so that means that cosine of 135 is negative square root of 2 divided by 2. Tangent of 240 terminates here, at the point negative 1/2, negative square root 3 over 2, so we're going to have y over x. So negative square root 3 over 2 divided by negative 1 over 2 is, when we simplify it, the square root of 3. And sine of 90, remember, sine is the y-coordinate of the corresponding point. That is the point 0, 1. So sine of 90 degrees is equal to 1.

So this unit circle can be used to find any trig function, sine, cosine, tangent, or the other three that we haven't really spoken too much about so far, of any of those special angles. And how do you know it's a special angle? Basically, it's any multiple of 30 or any multiple of 45. That's when you know it's going to have a special ratio. And now with this added [INAUDIBLE] with this unit circle, we can also say it's either going to be positive or negative. So there is a complete picture, with sine cosine, and tangent of the special angles.

[MUSIC PLAYING]

For your reference, here is the unit circle. It will also be accessible on the course dashboard.



HINT

From the unit circle, you can see that $\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$. This is a very useful identity that can be used to compute θ.

EXAMPLE Find the exact value of sin $180°$ by using the unit circle.

On the unit circle, $180°$ corresponds to the point (-1, 0). Since the y-coordinate of the point is 0, it follows that sin $180° = 0$.

EXAMPLE Find the exact value of tan $315°$ by using the unit circle.

On the unit circle, $315°$ corresponds to the point $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$. Since $\tan\theta = \frac{y}{x}$, it follows that

$\tan 315° = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$.

For negative angles and angles larger than $360°$, use a coterminal angle between $0°$ and $360°$ to evaluate the trigonometric function.

> EXAMPLE Find the exact value of $\cos 420°$.
>
> On the unit circle, $420°$ has the same terminal side as $60°$. This means that $\cos 420° = \cos 60°$. Since $60°$ corresponds to the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $\cos 420° = \cos 60° = \frac{1}{2}$.

> EXAMPLE Find the exact value of $\tan(-150°)$.
>
> First, realize that $-150°$ is coterminal with $-150° + 360° = 210°$. Thus, $\tan(-150°) = \tan 210°$. Since $210°$ corresponds to the point $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$, it follows that $\tan 210° = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$.

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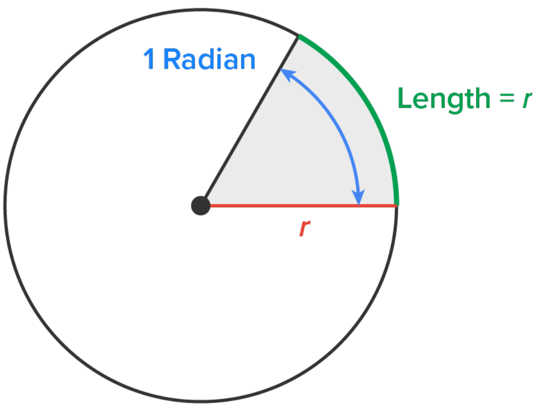
3. Radian Measure

Another way to measure angles is to use radians. One **radian** is defined as the central angle in the circle (see figure) so that the length of the circular arc is equal to the radius of the circle. Radians are used as a way of measuring angles because they represent a quantity, while degrees represent a scale.



TERM TO KNOW

Radian

The angle required to produce a circular arc whose length is equal to the radius. One radian is

$\frac{180}{\pi}$ degrees.

3a. Converting Between Degrees and Radians

Thinking about the previous figure, consider making one trip around the entire circle, which has length $2\pi r$ (circumference). Remembering that each radian contributes a length of *r* to the circle, it follows that one full trip around the circle is $\frac{2\pi r}{r} = 2\pi$ radians.

Also recall that one full trip around the circle is $360°$.

As a result, we see that $360° = 2\pi$ radians. Dividing both sides by 2, we can also say that $180° = \pi$ radians. Dividing both sides by π and 180 respectively, we have two rules to use when converting between radians and degrees:

	Formula	When To Use It
Divide by π:	$\left(\frac{180}{\pi}\right)° = 1\ radian$	When given an angle measured in radians, multiply by $\left(\frac{180}{\pi}\right)°$ to get the angle measurement in degrees.
Divide by 180:	$1° = \frac{\pi}{180}\ radians$	When given an angle measured in degrees, multiply by $\frac{\pi}{180}$ to get the angle measurement in radians.

FORMULA

Conversions Between Degrees and Radians

$$1\ degree = \frac{\pi}{180}\ radians$$

$$1\ radian = \frac{180}{\pi}\ degrees$$

EXAMPLE Convert $45°$ to radians.

$$45 \cdot \frac{\pi}{180} = \frac{\pi}{4}\ \text{radians}$$

EXAMPLE Convert 2.1 radians to degrees.

$$2.1 \cdot \frac{180}{\pi} \approx 120.32°$$

TRY IT

Convert the following from degrees to radians or vice versa.

Convert 240 degrees to radians. Leave your answer in terms of pi. +

$$240 \cdot \frac{\pi}{180} = \frac{240\pi}{180} = \frac{4\pi}{3}\ \text{radians}$$

Convert 5.7 radians to degrees. Round your final answer to the nearest tenth. +

$$5.7 \cdot \frac{180}{\pi} \approx 326.6°$$

HINT

When expressing angles in radians, it is customary to leave the angle in terms of π rather than approximate it. These angles are usually multiples of $15°$.

Here is a list of some common angles you will encounter.

Degrees	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$270°$	$360°$
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

3b. Evaluating Trigonometric Functions Using Radian Measure

Remember that radian measure is another way to represent an angle, so it may be helpful to convert the angle to degrees first before evaluating. (Recall the special values you were given earlier and the values from the unit circle).

↪ EXAMPLE

$$\cos\left(\frac{\pi}{4}\right) = \cos(45°) = \frac{\sqrt{2}}{2} \qquad \sin\left(\frac{2\pi}{3}\right) = \sin(120°) = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{5\pi}{6}\right) = \tan(150°) = -\frac{\sqrt{3}}{3} \qquad \cos\left(\frac{\pi}{2}\right) = \cos(90°) = 0$$

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4. Finding an Input for a Known Output (Solving Trigonometric Equations)

An equation where the angle is unknown and the ratio is known is called a**trigonometric equation**.

Examples:

$$\cos\theta = \frac{1}{2} \qquad \tan x = 3 \qquad \sin A = 0.4$$

Based on what we know from the unit circle, there are infinite solutions to a trigonometric equation if we include all coterminal angles. In most situations, we want to find all solutions in the first revolution of the circle, namely the interval $[0, 360°)$ or in radians, $[0, 2\pi)$.

For now, we will focus on known ratios that correspond to special angles. Other angles/ratios will be investigated in Unit 3.

↪ EXAMPLE Find all solutions to $\sin x = 0$ on the interval $[0, 2\pi)$.

From the unit circle, the y-coordinate is the sine of the angle. The two points with a y-coordinate of 0

are $0°$ and $180°$, or in radians, 0 and π.

Thus, the solutions to the equation are $x = 0$ and $x = \pi$.

↪ EXAMPLE Solve the equation $\tan x + 1 = 0$ on the interval $[0, 2\pi)$.

First, isolate the tan *x* term on one side by subtracting 1 from both sides: $\tan x = -1$.

We seek all angles *x* such that the ratio is -1. Consulting the unit circle, this means that $x = 135°$ and $315°$. In radians, that means $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$.

TRY IT

Solve $2\sin x + 1 = 0$ on the interval $[0, 2\pi)$ by completing the following steps.

First, isolate sin *x* on one side. +

$\sin x = -\frac{1}{2}$

Now consult the unit circle. +

$x = 210°, x = 330°$

Convert to radians. +

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

TERM TO KNOW

Trigonometric Equation

An equation in which trigonometric functions are involved and the angle is unknown.

5. Basic Trigonometric Graphs

5a. The Graph of *y* = sin *x*

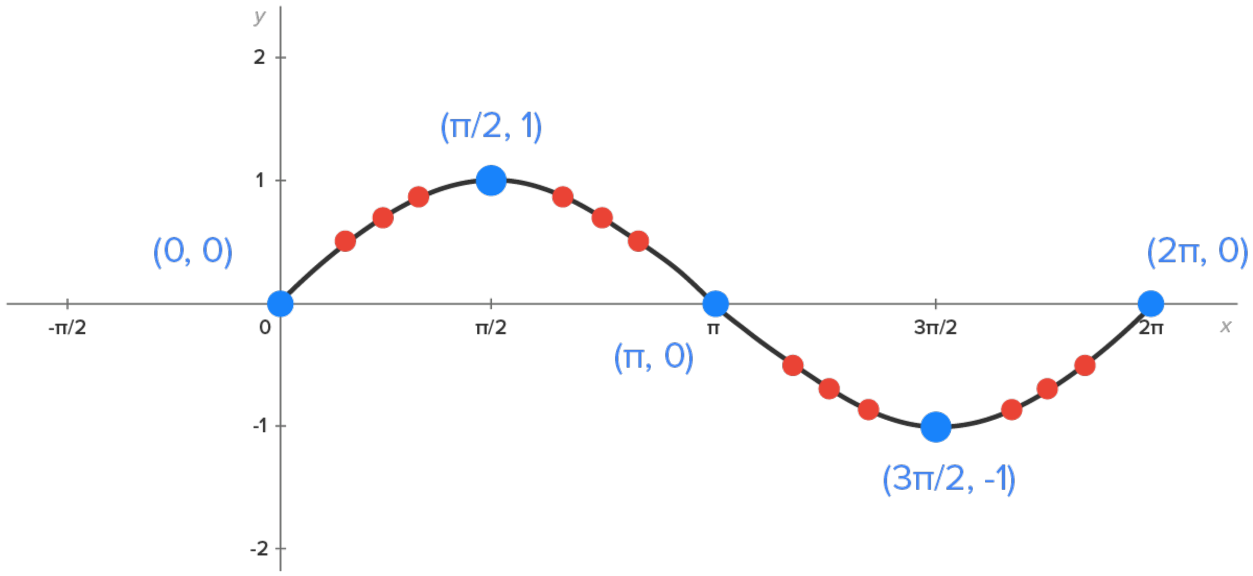
Consider the function *y* = sin *x*.

From the unit circle, here is a table of values that shows how the angles and the ratios are related.

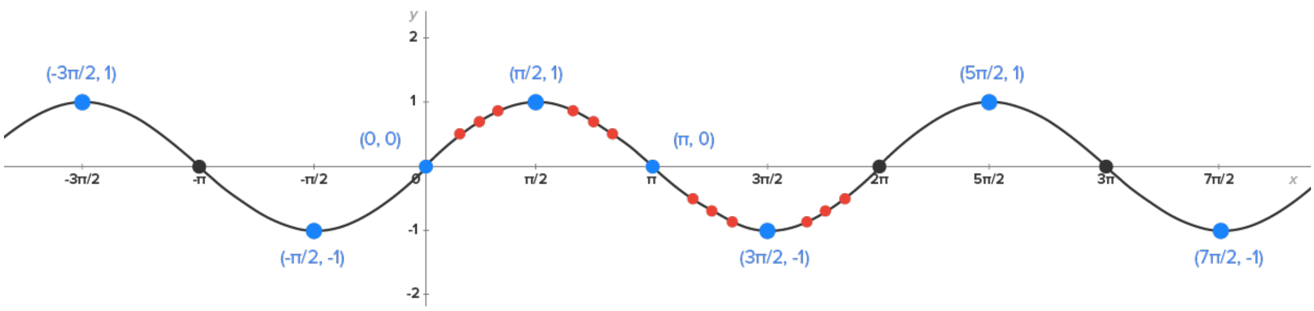
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y = sin*x*	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

x	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

Here is the graph, limited to the points that are listed above.



Remember also that **coterminal angles** produce the same trigonometric values. Thus, we can say that $\sin(x \pm 2\pi) = \sin x$. Because of this relationship, we say that the sine function has a period of 2π, meaning that the graph repeats itself every 2π units. As a result, the complete graph of $y = \sin x$ is as follows:



The graph continues in this pattern indefinitely. Since there are no breaks or holes in the graph, the domain of $f(x) = \sin x$ is the set of all real numbers, also written in interval notation as $(-\infty, \infty)$. The range of this function is $[-1, 1]$ since the graph goes no higher than $y = 1$ and no lower than $y = -1$.

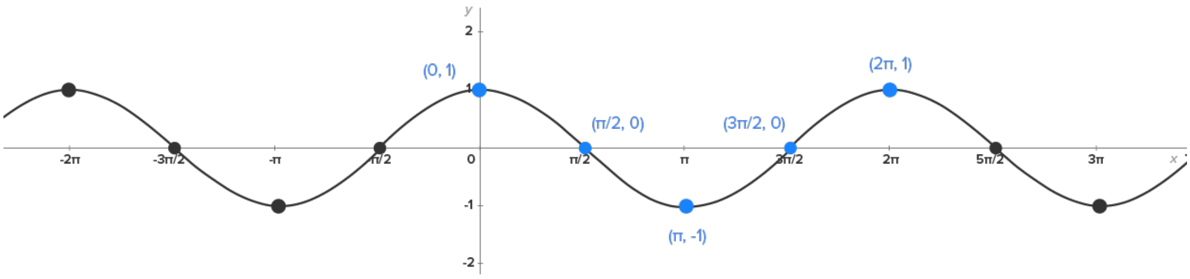
TERM TO KNOW

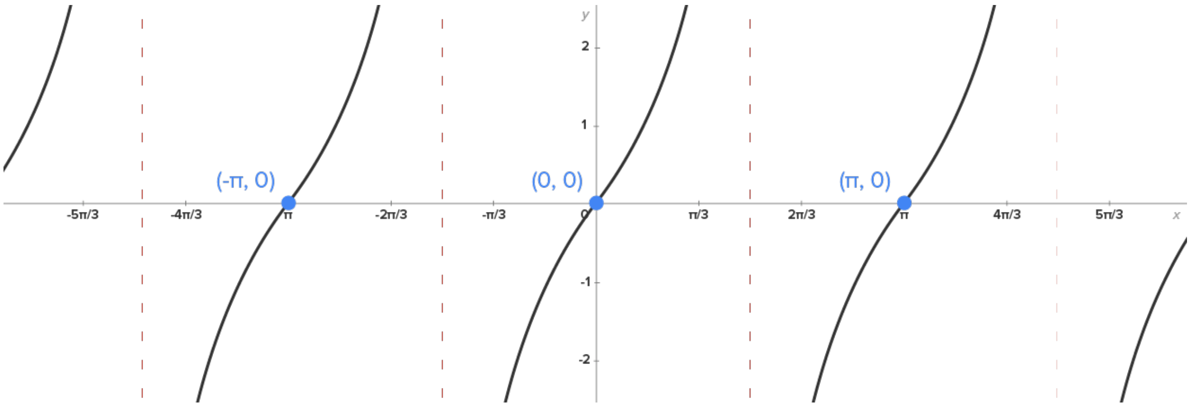
Coterminal Angles

Angles that have the same terminal side.

5b. The Graphs of $y = \cos\ x$ and $y = \tan\ x$

By following a similar process as above, we can obtain the graphs of $y = \cos x$ and $y = \tan x$:

$y = \cos x$	
Graph	
Domain	$(-\infty, \infty)$
Range	$[-1, 1]$
Period	2π

$y = \tan x$	
Graph	
Domain	All reals except $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$
Range	$(-\infty, \infty)$
Period	π

HINT

Remember that $\tan x = \frac{\sin x}{\cos x}$. Therefore, $\tan x$ is undefined whenever $\cos x = 0$, which is when *x* is an odd multiple of $\frac{\pi}{2}$.

6. Frequently Used Trigonometric Identities

Recall that an identity is an equation that is true for all possible values of the variable.

> EXAMPLE $2(x+3)=2x+6$ is an identity. No matter what is substituted for x, both sides of the equation will have the same value.

The following are the most commonly used trigonometric identities.

Reciprocal Identities	Tangent/Cotangent Identities
Secant: $\sec\theta=\dfrac{1}{\cos\theta}$ Cosecant: $\csc\theta=\dfrac{1}{\sin\theta}$ Cotangent: $\cot\theta=\dfrac{1}{\tan\theta}$	$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$
Cofunction Identities	**Pythagorean Identities**
$\sin\left(\dfrac{\pi}{2}-\theta\right)=\cos\theta$ $\cos\left(\dfrac{\pi}{2}-\theta\right)=\sin\theta$ $\tan\left(\dfrac{\pi}{2}-\theta\right)=\cot\theta$ $\cot\left(\dfrac{\pi}{2}-\theta\right)=\tan\theta$ $\sec\left(\dfrac{\pi}{2}-\theta\right)=\csc\theta$ $\csc\left(\dfrac{\pi}{2}-\theta\right)=\sec\theta$ (Note: If θ is measured in degrees, replace $\dfrac{\pi}{2}$ with $90°$.)	$\sin^2\theta+\cos^2\theta=1$ $1+\tan^2\theta=\sec^2\theta$ $1+\cot^2\theta=\csc^2\theta$ (Note: The notation $\sin^2\theta$ means $(\sin\theta)^2$.)
Sum of Angles Identities	**Double-Angle Identities**
$\sin(A\pm B)=\sin A\cos B\pm\sin B\cos A$ $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$ $\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$	$\sin 2x=2\sin x\cos x$ $\cos 2x=\cos^2x-\sin^2x$ $\cos 2x=2\cos^2x-1$ $\cos 2x=1-2\sin^2x$ $\tan 2x=\dfrac{2\tan x}{1-\tan^2x}$
Power-Reducing Identities	
$\cos^2x=\dfrac{1+\cos 2x}{2}$ $\sin^2x=\dfrac{1-\cos 2x}{2}$	

SUMMARY

In this lesson, you learned about trigonometric functions, which assign an angle to a real number; in other words, they take an angle as input and return a real number as output. These real numbers stem from the ratio of sides of right triangles. You learned about **the three basic trigonometric functions**: the sine function (sinθ), the cosine function (cosθ), and the tangent function (tanθ).

You learned that there are many representations that are useful in **evaluating trigonometric functions**, such as **using right triangles**. You also explored **evaluating trigonometric functions for any acute angle** (an angle whose measure is more than 0° and less than 90°), **evaluating trigonometric functions for 30°, 45°, and 60°** (special angles with concise corresponding ratios),**defining non-acute angles**, **the unit circle**, and **evaluating trigonometric functions for any angle**.

You learned about another way to measure angles by using **radian measure**, noting that one radian is defined as the central angle in the circle so that the length of the circular arc is equal to the radius of the circle. Radians are used as a way of measuring angles because they represent a quantity, while degrees represent a scale. Using this knowledge, you explored **converting between degrees and radians** and **evaluating trigonometric functions using radian measure**.

You investigated **finding an input for a known output, or solving trigonometric equations**, which are equations where the angle is unknown and the ratio is known, and explored **basic trigonometric graphs**, including **the graph of $y = \sin x$, $y = \cos x$ and $y = \tan x$**. Lastly, you covered a selection of **frequently used trigonometric identities** that will be utilized later in this course.

TERMS TO KNOW

Acute Angle

An angle whose measure is more than 0° and less than 90°.

Coterminal Angles

Angles that have the same terminal side.

Radian

The angle required to produce a circular arc whose length is equal to the radius. One radian is $\frac{180}{\pi}$ degrees.

Trigonometric Equation

An equation in which trigonometric functions are involved and the angle is unknown.

Trigonometric Function

Uses an angle as an input and returns a ratio as the output.

FORMULAS TO KNOW

Conversions Between Degrees and Radians

$$1 \textit{ degree} = \frac{\pi}{180} \textit{ radians}$$

$$1 \textit{ radian} = \frac{180}{\pi} \textit{ degrees}$$

Definitions of the Sine, Cosine, and Tangent Functions

$$\sin\theta = \frac{opposite}{hypotenuse}$$
$$\cos\theta = \frac{adjacent}{hypotenuse}$$
$$\tan\theta = \frac{opposite}{adjacent}$$

