Graph Piecewise Functions

by Sophia

WHAT'S COVERED

In this lesson, you will graph piecewise functions. Specifically, this lesson will cover:

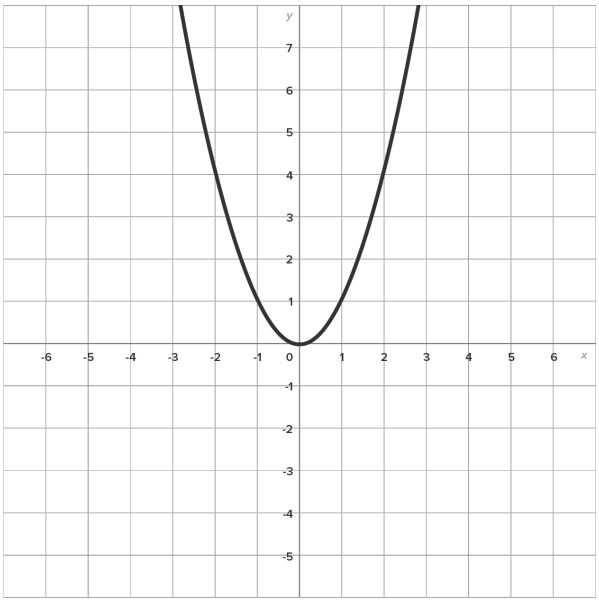
1. Graphing a Function on a Restricted Domain
2. Graphing Piecewise Functions

1. Graphing a Function on a Restricted Domain

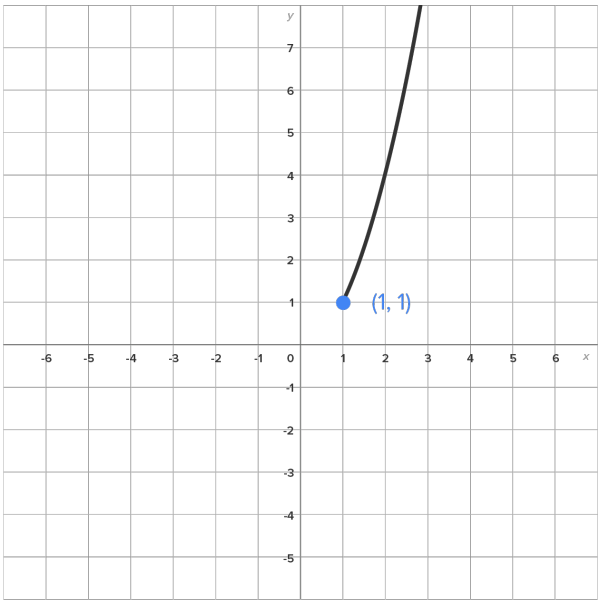
When we graph a function, we are considering the entire function. What if we only wanted part of the graph?

↱ EXAMPLE For example, consider the function $f(x) = x^2$, and several "pieces" of the graph, as shown below:

Graph 1: $f(x) = x^2$ (entire graph)

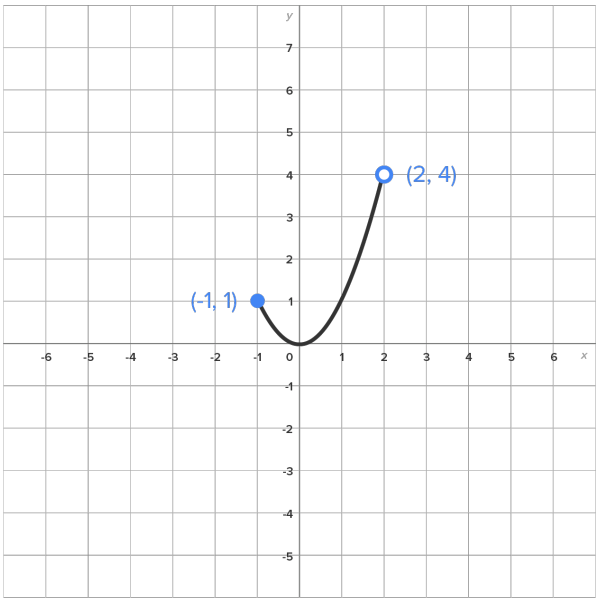


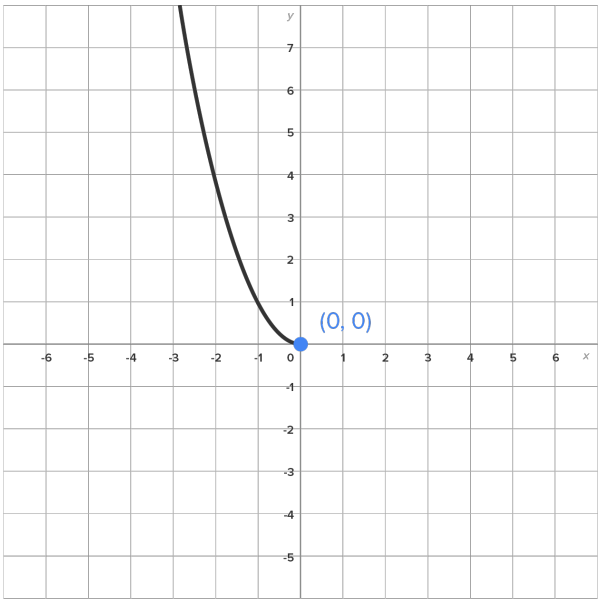
Graph 2: $f(x) = x^2, x \geq 1$



Graph 3: $f(x) = x^2, -1 \leq x < 2$

Graph 4: $f(x) = x^2, x \leq 0$





To sketch a portion of the graph, a **restricted domain** is used. Recall that the domain of a function is the set of all possible inputs for a function.

For example, in Graph 3 above, the "$-1 \leq x < 2$" is the domain restriction since it is not the entire domain of $f(x) = x^2$ (which is all real numbers).

HINT

When an endpoint is included, we represent it by using a closed circle. See Graphs 2, 3, and 4.

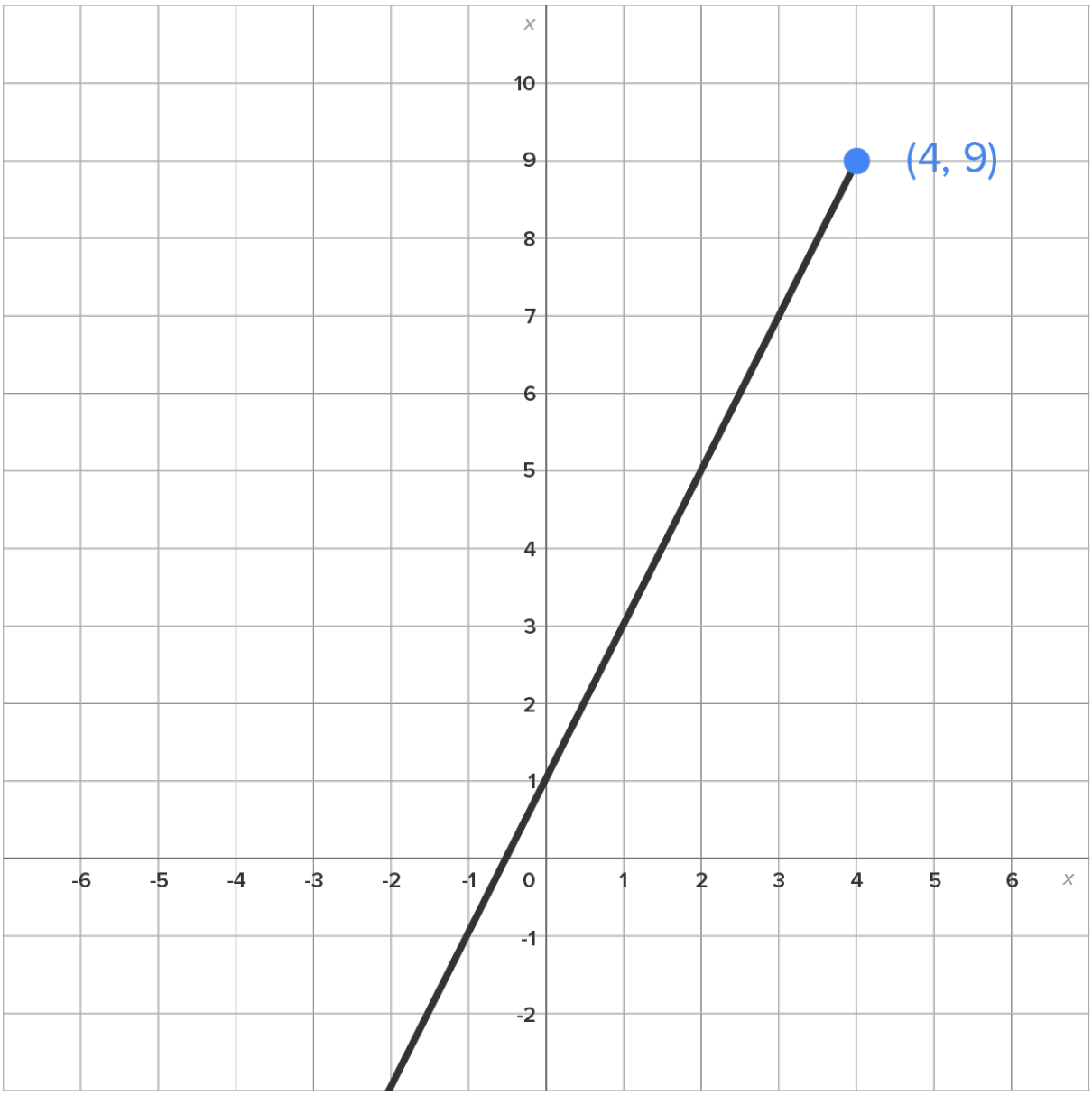
When an endpoint is not included, we represent it by using an open circle. See Graph 3.

TRY IT

Consider the following function: $f(x) = 2x + 1, x \leq 4$.

Graph this function. +

Remembering that $y = 2x + 1$ is a line with slope 2 and y-intercept 1, we graph the line but only for values of x up to and including 4.



TERM TO KNOW

Restricted Domain

Part of, but not the entire, domain of a function.

2. Graphing Piecewise Functions

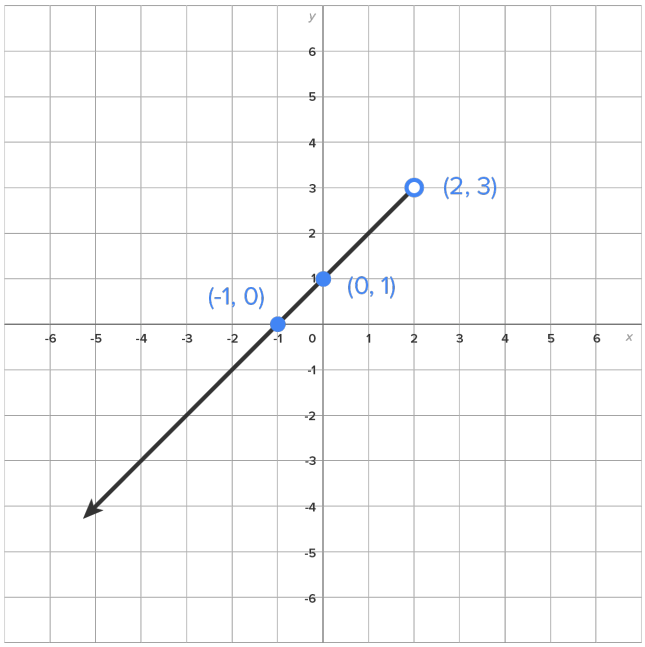
A piecewise function is made up of other functions that are on restricted domains. For example, consider the function:

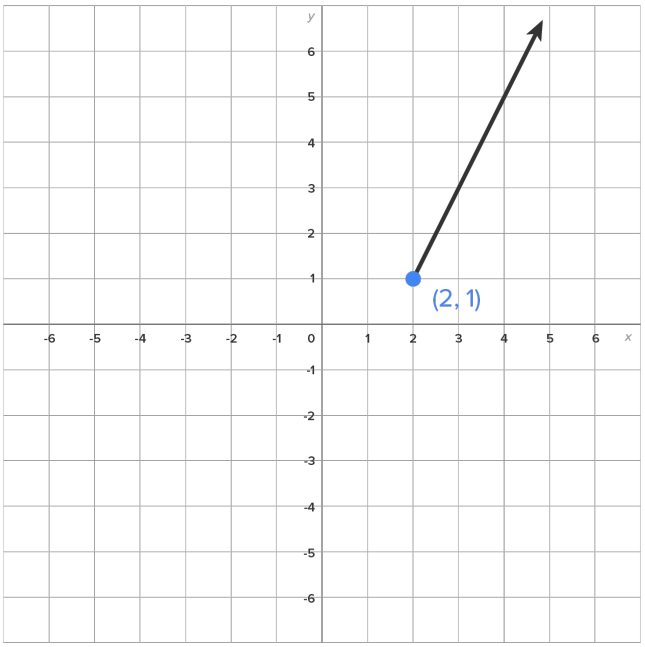
$$f(x)=\begin{cases} x+1 & \text{if } x<2 \\ 2x-3 & \text{if } x\geq 2 \end{cases}$$

The function tells us to use "$x+1$", but only if the input is less than 2; and to use "$2x-3$" if the input is at least 2.

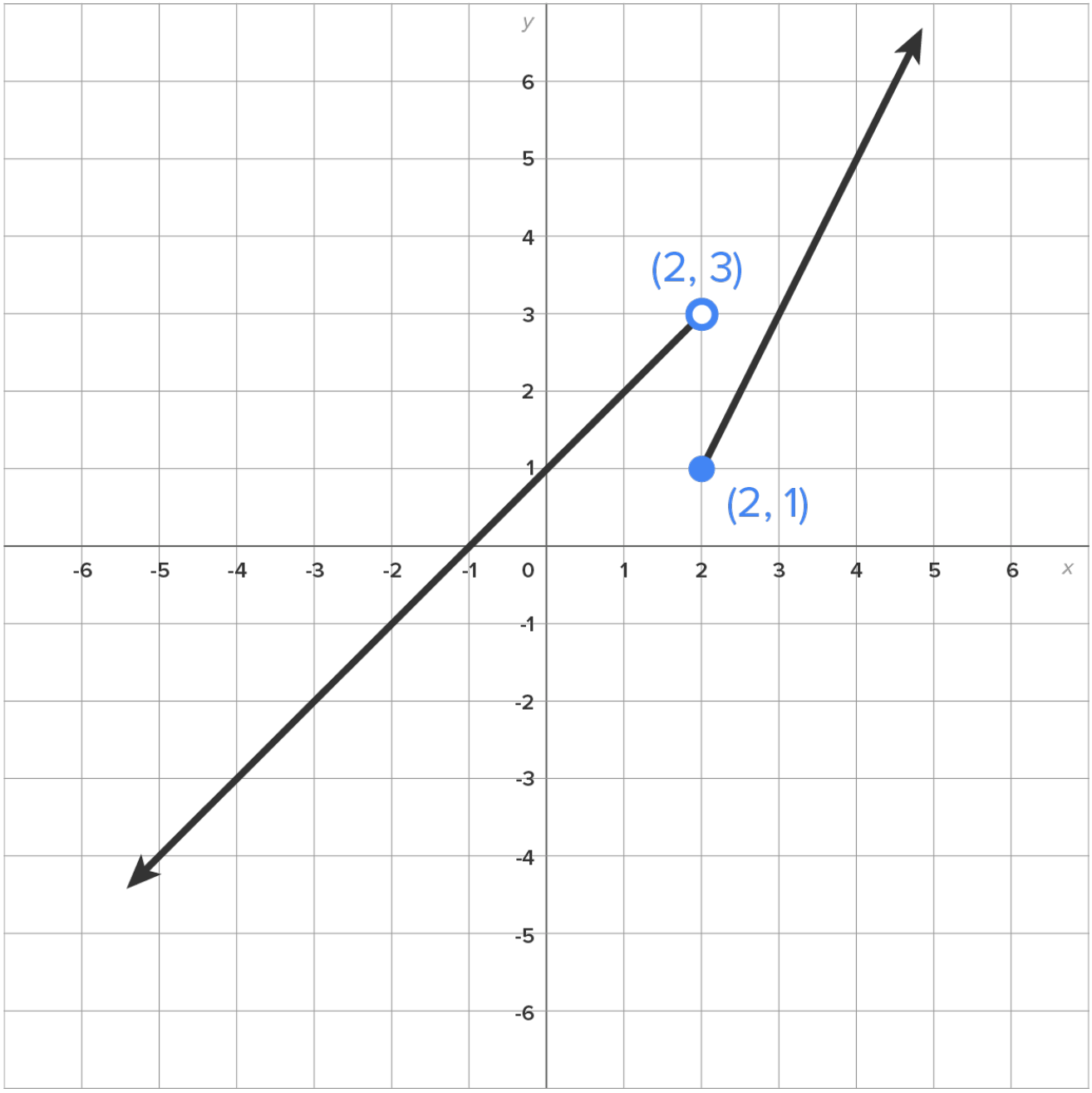
This means that the graph of the function will be "part of" the graph of $y=x+1$ along with "part of" the graph of $2x-3$. Here is how we put this together:

$y = x + 1, x < 2$	$y = 2x - 3, x \geq 2$





The graph of $f(x)$ is these pieces put together on one graph as follows:



WATCH

The following video walks you through the process of graphing a piecewise function.

Video Transcription

[MUSIC PLAYING] Hello. I hope you're learning experiences going well so far. What we're going to look at here is how to graph a piecewise function, and we are going to focus on the function f of x equals 3x minus 4 if x is less than 2 and x squared if x is greater than or equal to 2.

So you'll notice that there's two graphs written right below the function, and these are the complete graphs of each piece. This is the graph of y equals 3x minus 4, and this is the graph of y equals x squared. So the thing to understand about a piecewise function is that we just want a piece of each of those graphs, and those two pieces will be put together to form the graph of the piecewise function.

So we're looking at y equals 3x minus 4, but only if x is less than 2. So I'm going to find the point where x

is equal to 2, and that's right here. And you notice I'm going to put an open circle there, because we're not including 2. So we want the piece where x is less than 2, which means we are going to only take this piece of the graph.

Now, when we go to put our graphs together, it's going to be important to have some key points labeled. This point right here is 2 comma 2. And we've also managed to include the y-intercept, which is 0, negative 4. So that's going to help us when we go to graph the piecewise function.

Now, for y equals x squared, we're only taking the piece where x is greater than or equal to 2. So if we look where x is equal to 2, this point right here is 2 comma 4, because 2 squared is 4. And we're only taking the piece where x is greater than or equal to 2. So we're only taking that piece that's drawn in black there.

So to put these pieces together, we're just going to make a rough sketch here. I'm just going to zoom out just a little. So we have the point 2, 2 as a key point, because that's where the first graph ends. And it's also passing through 0, negative 4. So our first piece looks like that. And then our second piece starts at 2, 4, so just two units above the open circle. And it curves upward from there, just like y equals x squared. And that is the graph of our piecewise function.

[MUSIC PLAYING]

SUMMARY

In this lesson, you recalled that when you graph a function, you consider the entire function. However, if you only want part of the graph, you learned how to **graph a function on a restricted domain**, which is part of, but not the entire, domain of a function. You learned how to apply this knowledge to **graphing piecewise functions**—which are made up of other functions that are on restricted domains—which requires you to graph each piece on their respective restricted domains of the function.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Restricted Domain
Part of, but not the entire, domain of a function.