# Indefinite Integrals of Exponential Functions

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WHAT'S COVERED

In this lesson, you will find antiderivatives of exponential functions and incorporate them into the antiderivatives we already know (powers and trigonometric functions). Specifically, this lesson will cover:

1. Antiderivatives of Exponential Functions
2. Antiderivatives of Functions Containing Exponential Functions

## 1. Antiderivatives of Exponential Functions

Recall that $D[e^x]=e^x$ and $D[a^x]=a^x\cdot\ln a$, assuming $a>0$. This leads to the following antiderivative formulas:

FORMULA

**Antiderivatives of Exponential Functions**

$$\int e^x dx=e^x+C$$

$$\int a^x dx=\frac{a^x}{\ln a}+C$$

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## 2. Antiderivatives of Functions Containing Exponential Functions

Let’s get into some examples.

EXAMPLE Find the indefinite integral: $\int (4e^x-2x+1)dx$

Note that the same properties can be used.

$\int (4e^x-2x+1)dx$ Start with the original expression.

 

$$= 4\int e^x dx - 2\int x dx + \int 1 dx$$ Use the sum/difference properties and the constant multiple rule.

$$= 4(e^x) - 2\left(\frac{x^2}{2}\right) + x + C$$ Apply exponential and power rules.

$$= 4e^x - x^2 + x + C$$ Simplify.

Thus, $\int (4e^x - 2x + 1)dx = 4e^x - x^2 + x + C$.

EXAMPLE Find the indefinite integral: $\int \left(3^x - \frac{2}{3}\sin x\right)dx$

$$\int \left(3^x - \frac{2}{3}\sin x\right)dx$$ Start with the original expression.

$$= \int 3^x dx - \frac{2}{3}\int \sin x dx$$ Use the sum/difference properties and the constant multiple rule.

$$= \frac{3^x}{\ln 3} - \frac{2}{3}(-\cos x) + C$$ Apply formulas for $\int a^x dx$ and $\int \sin x dx$.

$$= \frac{3^x}{\ln 3} + \frac{2}{3}\cos x + C$$ Simplify.

Thus, $\int \left(3^x - \frac{2}{3}\sin x\right)dx = \frac{3^x}{\ln 3} + \frac{2}{3}\cos x + C$.

TRY IT

Consider $\int (x^2 - 5e^x)dx$.

Find the indefinite integral. +

$$\frac{1}{3}x^3 - 5e^x + C$$

TRY IT

Consider $\int \left(10^x - \frac{6}{\sqrt{x}}\right)dx$.

Find the indefinite integral. +

$$\frac{10^x}{\ln 10} - 12\sqrt{x} + C$$

## SUMMARY

 

In this lesson, you learned the formula for **antiderivatives of exponential functions**, expanding your abilities to find derivatives to include finding **antiderivatives of functions containing exponential functions**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 4 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

## FORMULAS TO KNOW

**Antiderivatives of Exponential Functions**

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

 