Shifting and Stretching Graphs

by Sophia

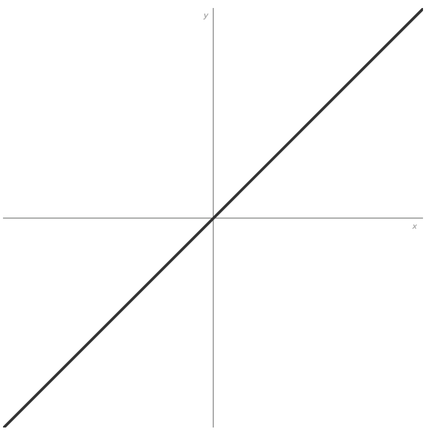
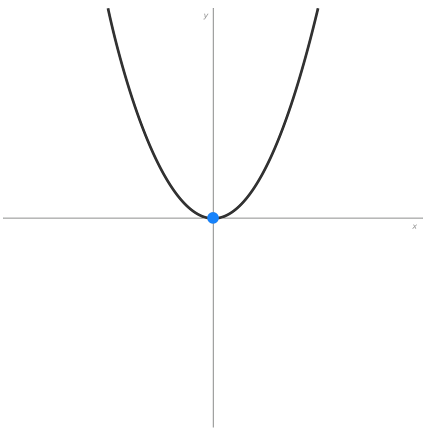
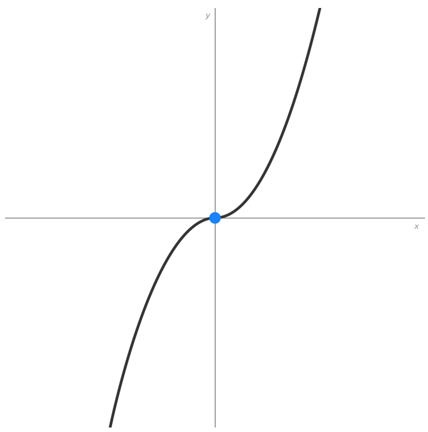
WHAT'S COVERED

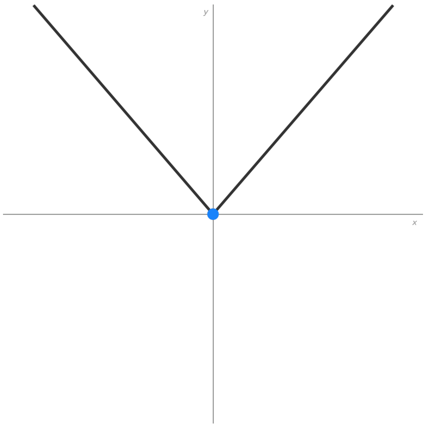
In this lesson, you will investigate how translations affect the graph of a function. Specifically, this lesson will cover:

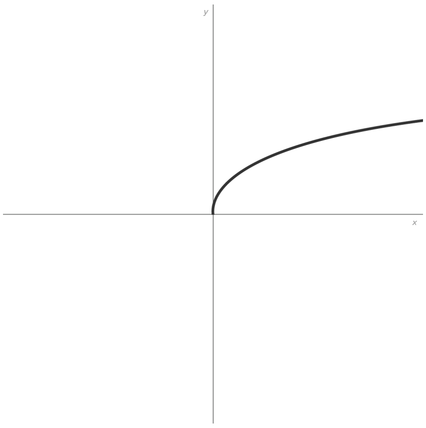
1. Commonly Used Basic Functions and Their Graphs
2. Applying Basic Translations to $y=f(x)$
3. Applying Several Translations to $y=f(x)$

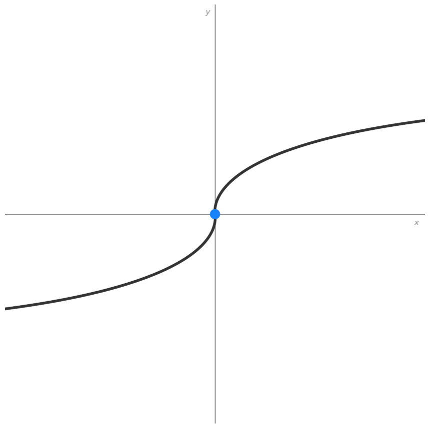
1. Commonly Used Basic Functions and Their Graphs

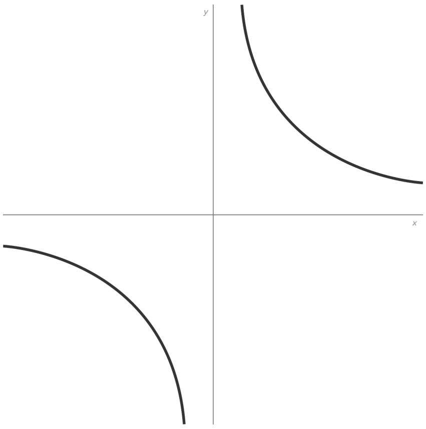
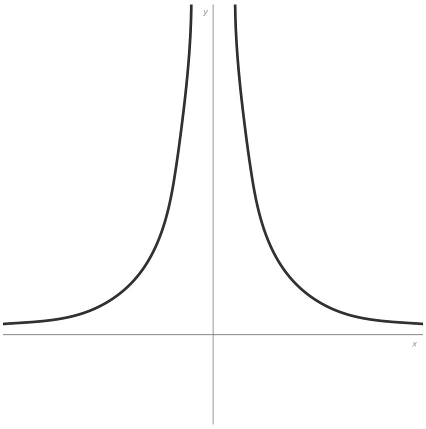
Here are the most commonly used graphs that are encountered in a typical algebra course. There are others as well, which will be investigated in future challenges.

Linear Function $f(x)=x$	Quadratic Function $f(x)=x^2$	Cubic Function $f(x)=x^3$
		
Absolute Value Function $f(x)=\lvert x\rvert$	**Square Root Function** $f(x)=\sqrt{x}$	**Cube Root Function** $f(x)=\sqrt[3]{x}$







Reciprocal Function $f(x)=\frac{1}{x}$	Reciprocal Square Function $f(x)=\frac{1}{x^2}$
	

In this challenge, we will investigate translations to an equation and how they affect the graph of said equation.

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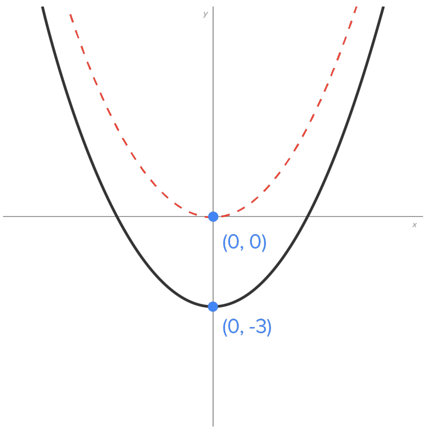
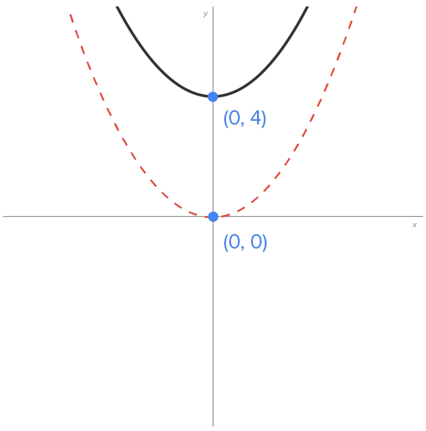
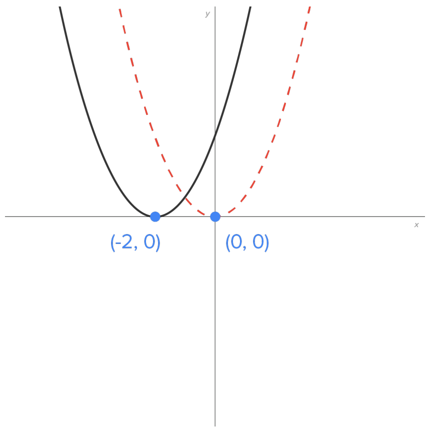
2. Applying Basic Translations to $y = f(x)$

Shown below are several functions which are translations of $f(x)=x^2$. In each graph, the graph of $f(x)=x^2$ is shown with a dashed line.

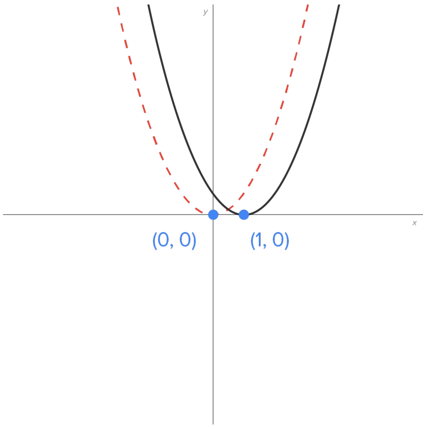
$y=x^2-3$ Shifted Down 3 Units	$y=x^2+4$ Shifted Up 4 Units	$y=(x+2)^2$ Shifted Left 2 Units

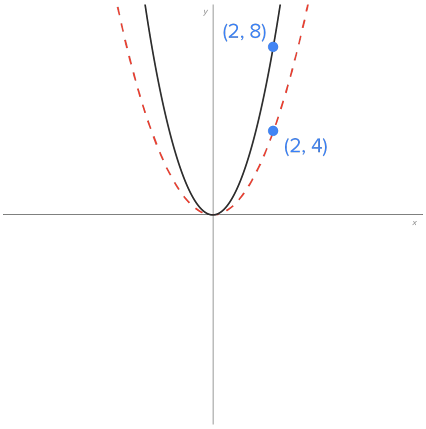


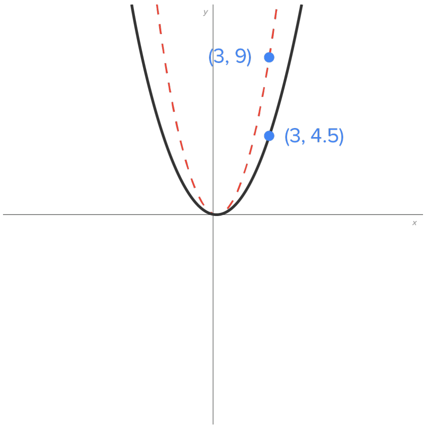




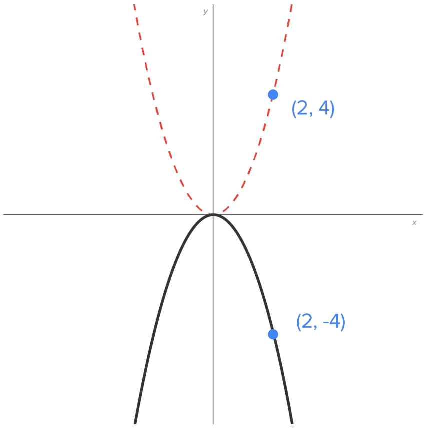
$y = (x-1)^2$ Shifted 1 Unit Right	$y = 2x^2$ Vertical Stretch	$y = 0.5x^2$ Vertical Compression

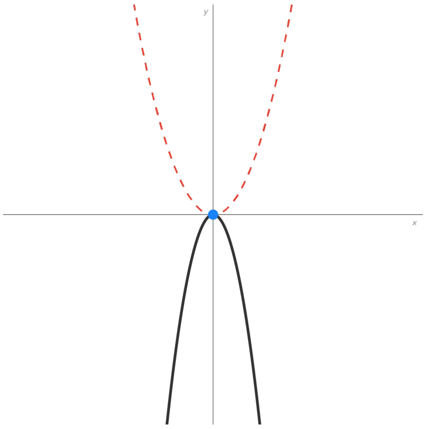


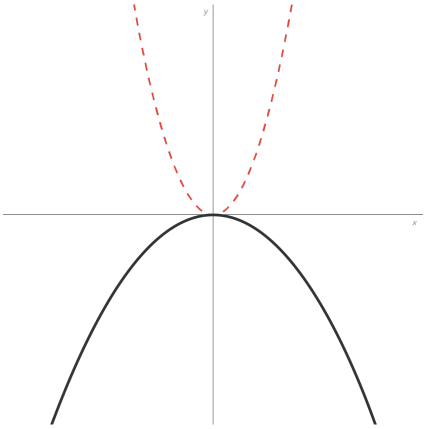




$y = -x^2$ Reflection Across the x-axis	$y = -3x^2$ Vertical Stretch and Reflection Across the x-axis	$y = -0.25x^2$ Vertical Compression and Reflection Across the x-axis







BIG IDEA

Given the graph of $y = f(x)$ and a positive constant *k*:

- The graph of $y = f(x) + k$ shifts the graph up *k* units.
- The graph of $y = f(x) - k$ shifts the graph down *k* units.
- The graph of $y = f(x - k)$ shifts the graph right *k* units.
- The graph of $y = f(x + k)$ shifts the graph left *k* units.
- The graph of $y = a \cdot f(x)$ is a **vertical stretch** if $|a| > 1$ and a **vertical compression** if $|a| < 1$. Also, if a is negative, the graph also reflects over the x-axis.

TERMS TO KNOW

Vertical Compression

A translation that makes all y-values of a graph smaller in magnitude, pulling a graph toward the x-axis. This is represented by $y = a \cdot f(x)$, where $|a| < 1$.

Vertical Stretch

A translation that makes all y-values of a graph larger in magnitude, pulling a graph toward the y-axis. This is represented by $y = a \cdot f(x)$, where $|a| > 1$.

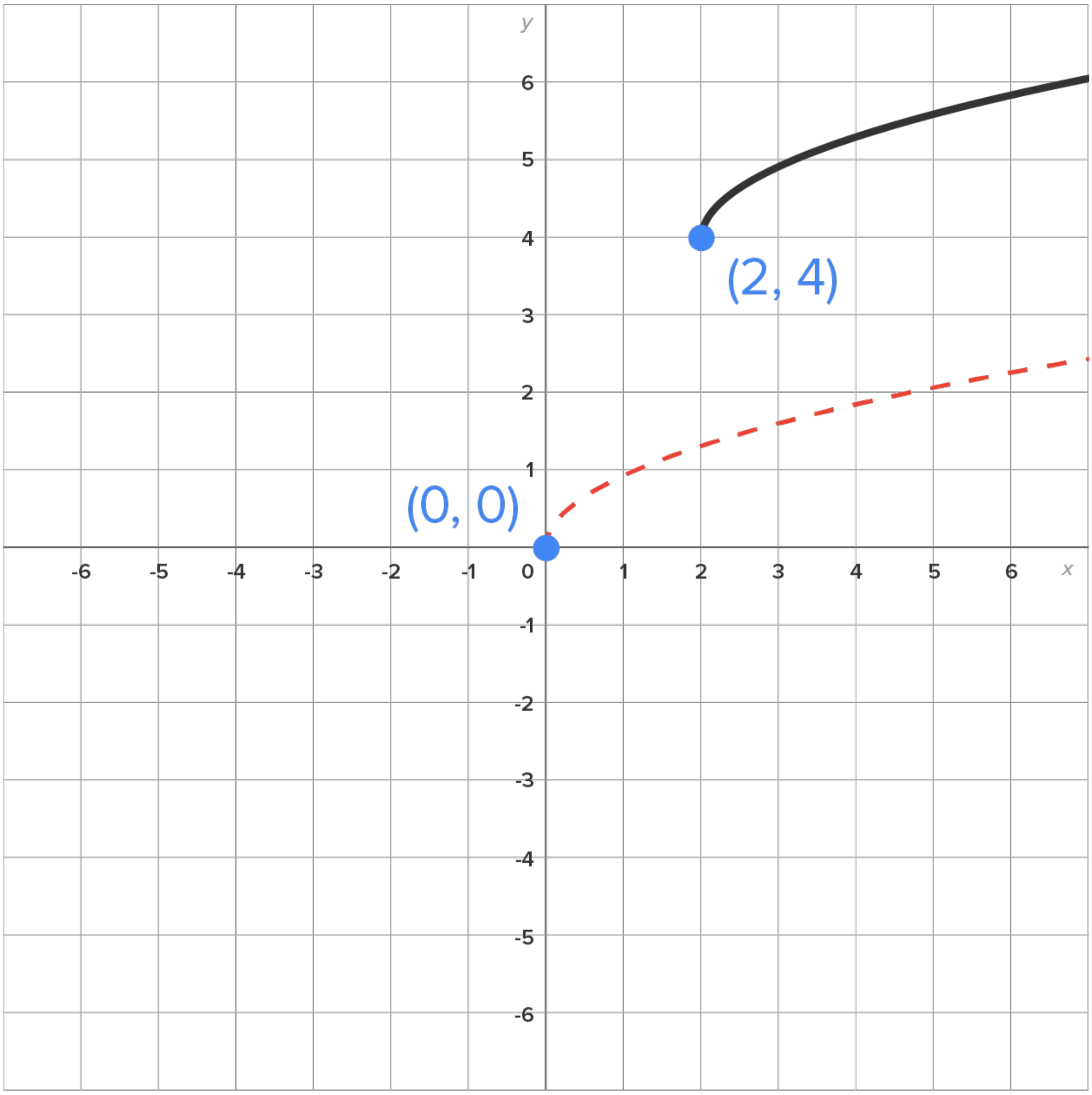
3. Applying Several Translations to $y = f(x)$

Given the translations discussed in the previous section, it is possible to apply several to one function.

EXAMPLE Consider the function $g(x) = \sqrt{x-2} + 4$, which is related to the function $f(x) = \sqrt{x}$.

- There is an "$x - 2$" under the radical where the "x" is in the base function, indicating that the graph is moved to the right two units.
- The "+ 4" outside of the radical indicates that the graph is shifted up 4 units.

The graphs of $f(x)$ and $g(x)$ are shown on the same axes below. The graph of $f(x)$ is dashed to show its relationship to $g(x)$.

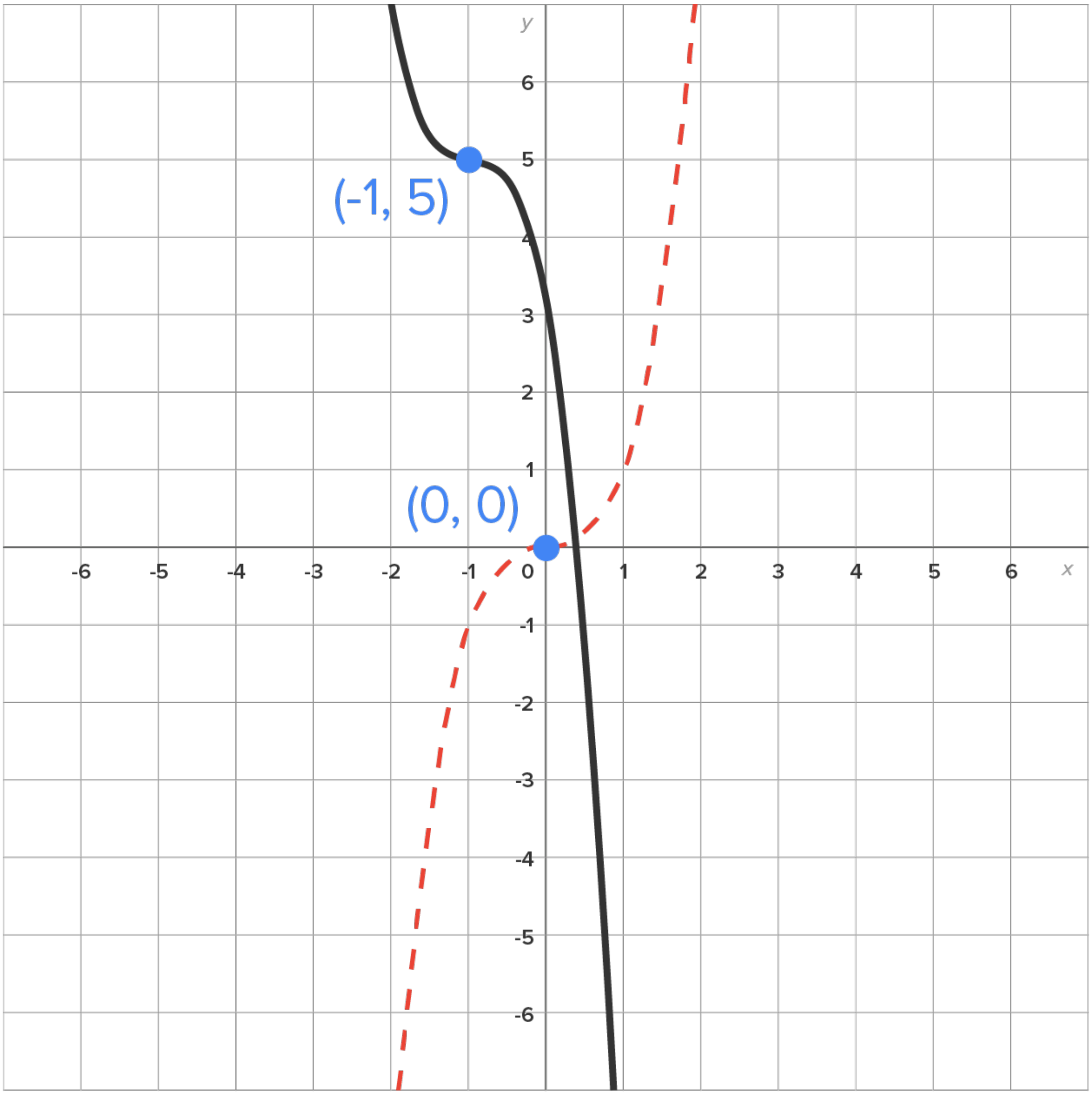


The graph of $g(x)$ is obtained by moving the graph of $f(x)$ to the right 2 units and up 4 units.

EXAMPLE Describe the sequence of transformations that are required to graph $g(x) = -2(x+1)^3+5$ based on $f(x) = x^3$.

- The "$x+1$" tells us that the graph is shifted to the left by 1 unit.
- The "-2" multiplied to the cubed term tells us that the graph is reflected around the x-axis and stretched vertically (since 2 > 1).
- The "+ 5" tells us that the graph is then shifted up five units.

The graph is shown here, with $f(x) = x^3$ dashed.



BIG IDEA

The order in which translations are applied only matters when there is a vertical shift. Here is the order in which translations should be applied:

1. Horizontal Translations
2. Vertical Compressions/Stretches/Reflections
3. Vertical Translations

TRY IT

Consider the equation $g(x) = 0.75(x+4)^2 - 3$.

Identify the basic function. +

The basic function is $f(x) = x^2$.

List the sequence of translations required to graph $g(x)$ based on the basic function. +

The graph is shifted to the left by 4 units, vertically compressed by a factor of 0.75, and shifted down 3 units.

SUMMARY

In this lesson, you began by exploring **commonly used basic functions and their graphs**. You investigated several types of translations to an equation and how they affect the graph of said equation, **applying basic translations to $y = f(x)$** to shift, stretch, compress, and reflect its graph. You also learned how to graph a function $g(x)$ by **applying several translations to $y = f(x)$**, noting the order in which translations should be applied.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Vertical Compression
A translation that makes all y-values of a graph smaller in magnitude, pulling a graph toward the x-axis. This is represented by $y = a \cdot f(x)$, where $|a| < 1$.

Vertical Stretch
A translation that makes all y-values of a graph larger in magnitude, pulling a graph toward the y-axis. This is represented by $y = a \cdot f(x)$, where $|a| > 1$.