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Evaluate Piecewise Functions

by Sophia

WHAT'S COVERED

In this lesson, you will learn how a piecewise function is defined and how it is evaluated. Specifically, this lesson will cover:

1. Uses of Piecewise Functions in the Real World
2. Defining Piecewise Functions
3. Evaluating Piecewise Functions

1. Uses of Piecewise Functions in the Real World

A job pays $24 per hour as long as an employee works at most 40 hours in a week. If an employee works more than 40 hours, they still get $24 for each of the first 40 hours, but they also get $36 for every extra hour beyond 40.

How would we calculate an employee's earnings?

There are two rules, depending on the number of hours worked:

40 hours or less	Earnings = 24 $\cdot$ (Hours)
More than 40 hours	Earnings = 24 $\cdot$ 40 + 36 $\cdot$ (Extra Hours)

If the goal is to calculate several employees' earnings, a piecewise function is needed since there are two rules to calculate the same output (earnings).

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2. Defining Piecewise Functions

Let's take this situation and try to represent it mathematically. Let x = the number of hours an employee works in a week.

Each entry in the table above can be translated into a mathematical statement involving the variable x.

Mathematical Statement	Statement in Words
$x \leq 40$	"40 hours or less"

$x > 40$	"More than 40 hours"
Earnings = 24x	Earnings = 24 · (Hours)
Earnings = 24(40) + 36(x – 40) Note: "x – 40" is the number of extra hours. If someone works more than 40 hours, then you subtract 40 from the hours they worked to get the number of extra hours.	Earnings = 24*40 + 36 · (Extra Hours)

In this situation, notice also that the weekly earnings depend on the number of hours worked. That is, the earnings is a function of the number of hours worked. Using function notation, let $E(x)$ represent the earnings after x hours.

Since there are two rules to find $E(x)$, we can express $E(x)$ as a **piecewise function**. Here is how it would be written:

$$E(x) = \begin{cases} 24x & \text{if } x \le 40 \\ 24(40) + 36(x - 40) & \text{if } x > 40 \end{cases}$$

This function isn't quite accurate for the following reasons:

- x cannot be negative (the number of hours worked cannot be negative).
- x cannot be more than 168 (# hours in a week).

The expression 24(40) + 36(x – 40) is not in its final form because it can be simplified.
After addressing these things, the function could be written as:

$$E(x) = \begin{cases} 24x & \text{if } 0 \le x \le 40 \\ 36x - 480 & \text{if } 40 < x \le 168 \end{cases}$$

TERM TO KNOW

Piecewise Function

Assigns an input to an output, but the rule used to determine the output depends on the value of the input.

3. Evaluating Piecewise Functions

The purpose of the function we built in the previous section is to calculate earnings for employees. In order to do so, remember that the input determines which rule is used:

If $0 \le x \le 40$, then use 24x to compute $E(x)$.
If $40 < x \le 168$, then use 36x - 480 to compute $E(x)$.

EXAMPLE Let's use the function to compute earnings for several employees:

Employee, Hours	Hours written in terms of x	Which Rule Should We Use?	Calculate $E(x)$

Holly, 42 hours	$x=42$	Since 42 satisfies $40<x\leq 168$, use $36x-480$.	$E(42)=36(42)-480=1032$
George, 36 hours	$x=36$	Since 36 satisfies $0\leq x\leq 40$, use $24x$.	$E(36)=24(36)=864$
Israel, 40 hours	$x=40$	Since 40 satisfies $0\leq x\leq 40$, use $24x$.	$E(40)=24(40)=960$
Savannah, 50 hours	$x=50$	Since 50 satisfies $40<x\leq 168$, use $36x-480$.	$E(50)=36(50)-480=1320$

Conclusion: Holly earned $1032, George earned $864, Israel earned $960, and Savannah earned $1320.

TRY IT

Let $f(x)=\begin{cases} 4x+5 & \text{if } x<2 \\ x^2+x+3 & \text{if } x\geq 2 \end{cases}$

Evaluate $f(-3)$, $f(5)$, and $f(2)$.

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$f(-3)=4(-3)+5=-7$	Use the first rule since $-3<2$.
$f(5)=5^2+5+3=33$	Use the second rule since $5\geq 2$.
$f(2)=2^2+2+3=9$	Use the second rule since $2\geq 2$.

SUMMARY

In this lesson, you learned that whenever a situation involves more than one rule for computing an output, it can be represented mathematically by a piecewise function. A piecewise function assigns an input to an output, where the rule used to determine the output depends on the value of the input. You began the lesson by exploring **uses of piecewise functions in the real world**, then learned how to **define piecewise functions** by taking entries in a table and translating them into a mathematical statement involving the variable *x*. Lastly, you learned how to **evaluate piecewise functions** by calculating earnings for employees, applying the appropriate rule, determined by the input, to calculate the output (earnings).

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Piecewise Function

Assigns an input to an output, but the rule used to determine the output depends on the value of the input.