The Mean Value Theorem for Integrals

by Sophia

WHAT'S COVERED

In this lesson, you will connect the mean value theorem to integrals. Specifically, this lesson will cover:

1. The Mean Value Theorem for Integrals
2. Finding the Value of c Guaranteed by the Mean Value Theorem for Integrals

1. The Mean Value Theorem for Integrals

Similar to the mean value theorem for derivatives, we can establish a theorem for integrals. If $f(x)$ is continuous on $[a, b]$, then at some point c in $[a, b]$:

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

In other words, there is at least one value of c in the interval $[a, b]$ such that $f(c) =$ the average value of $f(x)$ on $[a, b]$.

TERM TO KNOW

The Mean Value Theorem for Integrals

If $f(x)$ is continuous on $[a, b]$, then at some point c in $[a, b]$:

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

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2. Finding the Value of *c* Guaranteed by the Mean Value Theorem for Integrals

Let's look at a few examples to help illustrate the mean value theorem for integrals.

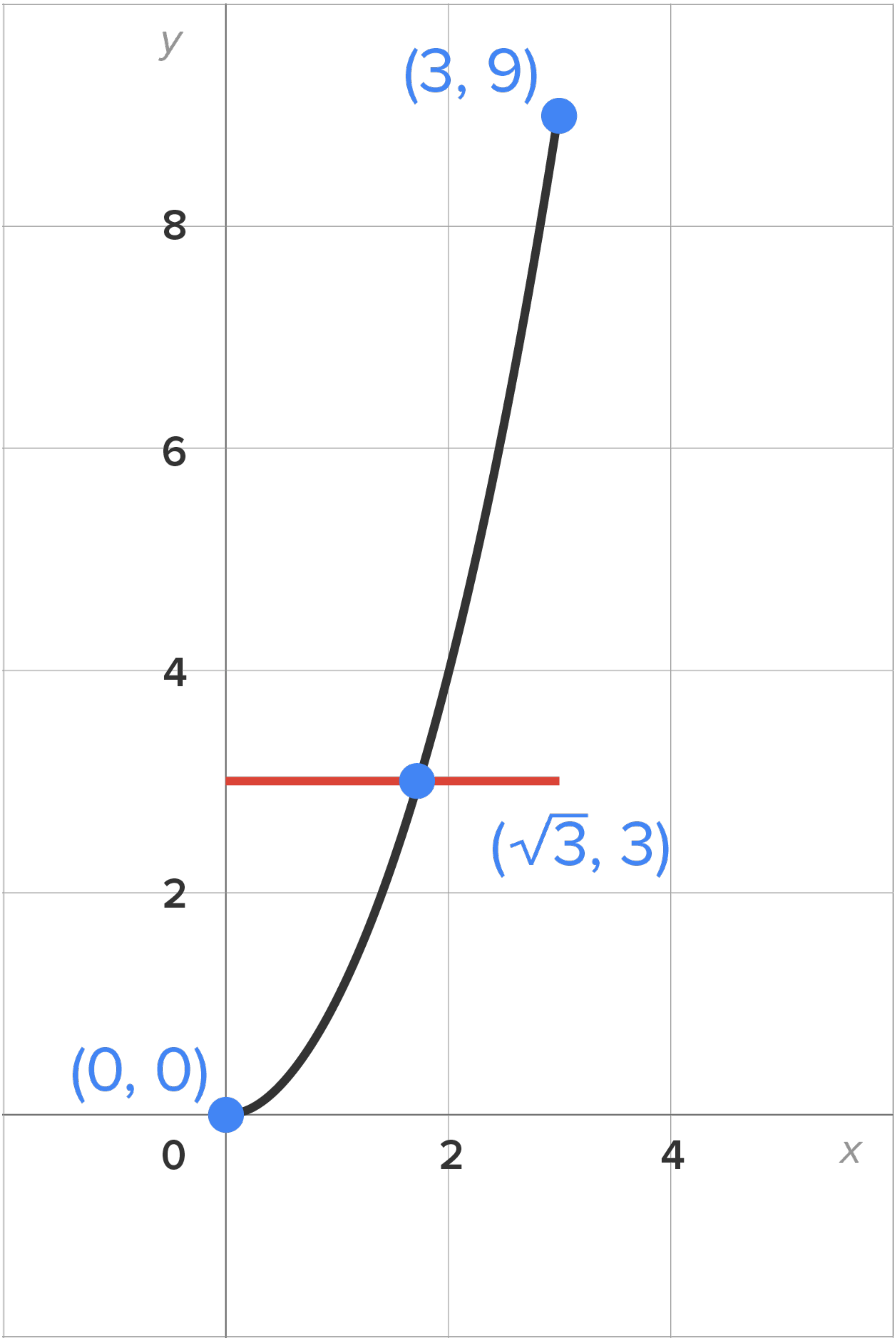
EXAMPLE Consider the function $f(x) = x^2$ on the interval $[0, 3]$.

The average value of $f(x)$ on $[0, 3]$ is $\frac{1}{3}\int_0^3 x^2 dx$. Evaluating, we have:

$$\frac{1}{3}\int_0^3 x^2 dx = \frac{1}{3} \cdot \frac{1}{3}x^3\Big|_0^3 = \frac{1}{9}(3)^3 - \frac{1}{9}(0)^3 = 3$$

To find the value of *c*, set $f(c) = 3$. This means $c^2 = 3$, which means $c = \pm\sqrt{3}$. Since $-\sqrt{3}$ is not in the interval $[0, 3]$, the value of *c* guaranteed by the theorem is $c = \sqrt{3}$.

Here is the graph of $f(x) = x^2$ on the interval $[0, 3]$ along with the line $y = 3$ (the average value). Note that they intersect at the point $(\sqrt{3}, 3)$.



WATCH

Check out this video to see the example to find the average value and the value of c guaranteed by the mean value theorem for $f(x) = 2x^2 - x$ on $[-1, 3]$.

TRY IT

Consider the function $f(x) = \frac{16}{x^3}$ on the interval $[1, 2]$.

Find the average value and the value of c guaranteed by the mean value theorem for integrals. +

Average value = 6, $c = \sqrt[3]{\frac{8}{3}} \approx 1.39$

SUMMARY

In this lesson, you learned that through **the mean value theorem for integrals**, you are able to guarantee that there is some input value (c) of a function $f(x)$ on $[a, b]$ in which $f(x)$ is equal to its average value on $[a, b]$. Next, you practiced **finding the value of c guaranteed by the mean value theorem for integrals**.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

The Mean Value Theorem for Integrals
If $f(x)$ is continuous on $[a, b]$, then at some point c in $[a, b]$:

$$f(c) = \frac{1}{b-a} \int_a^b f(x)dx$$