First Shape Theorem

by Sophia

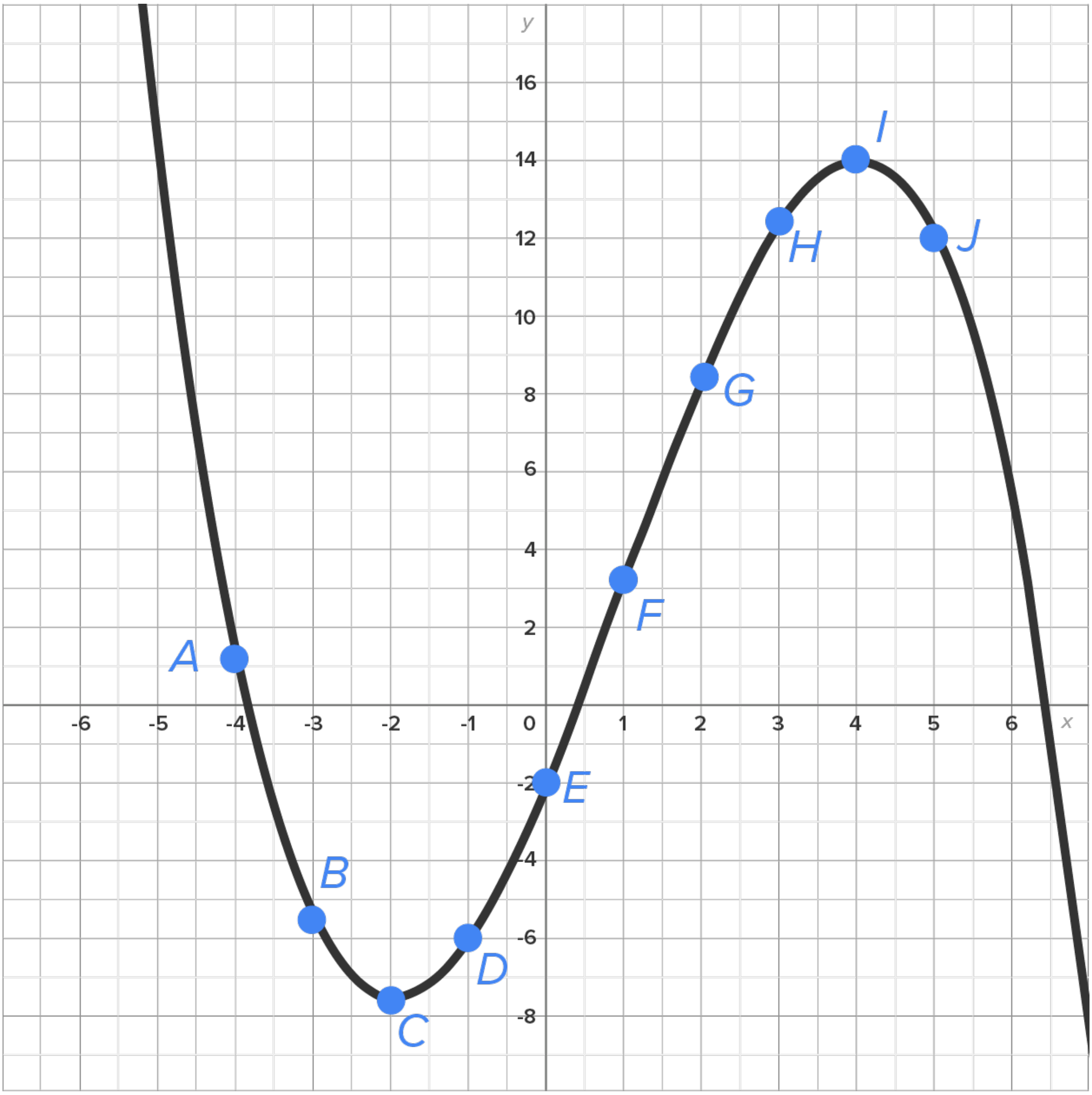
WHAT'S COVERED

In this lesson, you will use properties of a function $f(x)$ to sketch the graph of its derivative, $f'(x)$. Specifically, this lesson will cover:

1. What $f'(x)$ Tells Us About the Graph of $y = f(x)$
2. Using Slope to Graph $y = f'(x)$ Given $y = f(x)$

1. What $f'(x)$ Tells Us About the Graph of $y = f(x)$

Consider the graph of a function $y = f(x)$, shown below.



Note that the graph is decreasing at points *A*, *B*, and *J*. Notice also that the slopes of the tangent lines at each of these points are negative.

Note that the graph increases at points *D*, *E*, *F*, *G*, and *H*. Notice also that the slopes of the tangent lines at each of these points are positive.

Finally, points *C* and *I* are local maximum/minimum points. Notice also that the slope of the tangent line at each of these points is zero.

This leads to a very useful link between the behavior of $f(x)$ and the value of $f'(x)$.

BIG IDEA

If $f(x)$ is increasing at $x = a$, then $f'(a) > 0$.
If $f(x)$ is decreasing at $x = a$, then $f'(a) < 0$.

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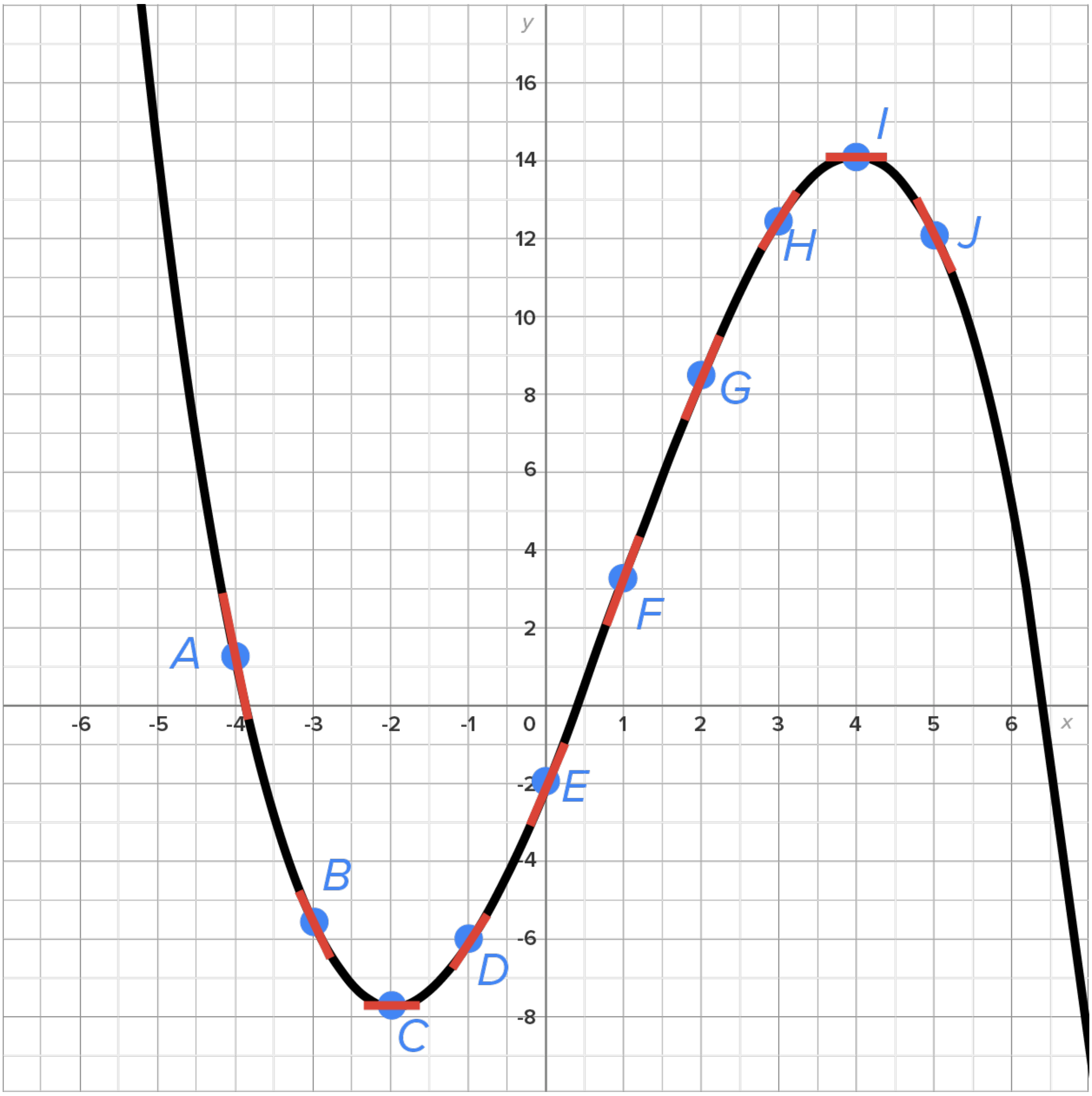
2. Using Slope to Graph $y = f'(x)$ Given $y = f(x)$

Given what we know about $f'(x)$ when $f(x)$ is increasing or decreasing, we can get a rough sketch of the graph of $f'(x)$ when given the graph of $f(x)$.

EXAMPLE Consider the graph of $y = f(x)$ shown below with tangent line segments at points *A* through *J*. Notice also the local minimum at point *C* and the local maximum at point *I*.

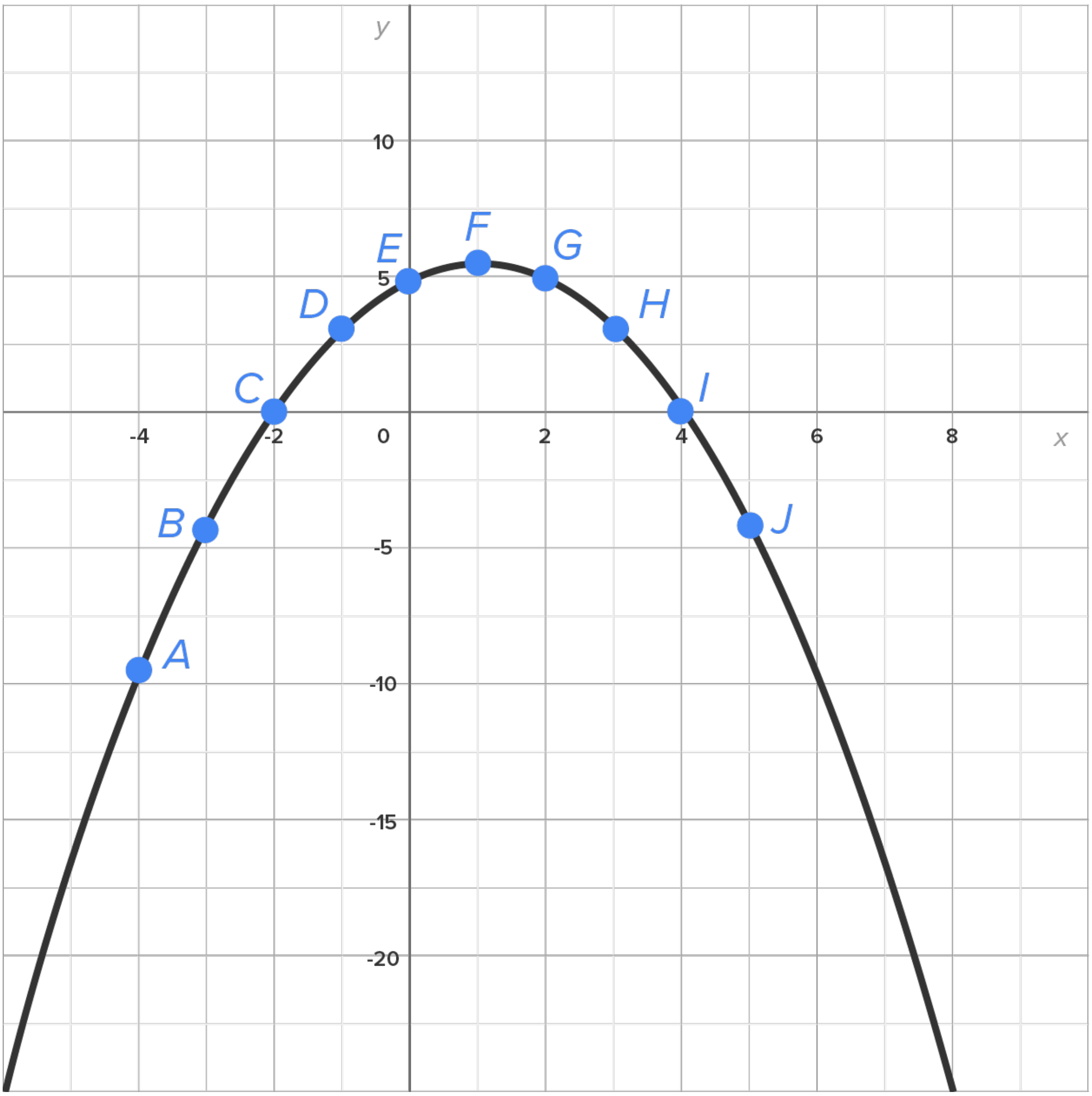


The behavior of $f'(x)$ can be summarized in the following table at each point. Remember that $m_{\tan}$ is the value of $f'(x)$ at any point.

Point	Value of $f'(x)$
A	$f'(x) < 0$
B	$f'(x) < 0$, but the value of $f'(x)$ is larger than its value at *A*
C	$f'(x) = 0$ (horizontal tangent line)

D	$f'(x) > 0$
E	$f'(x) > 0$, but its value is noticeably greater than the slope at point *D*
F	$f'(x) > 0$, but its value is slightly greater than the slope at point *E*
G	$f'(x) > 0$, but its value is slightly less than the slope at point *F*
H	$f'(x) > 0$, but its value is noticeably less than the slope at point *G*
I	$f'(x) = 0$ (horizontal tangent line)
J	$f'(x) < 0$

The graph of the derivative is shown here. Note that the points *A* through *J* have the same x-coordinates as those marked on the graph of $f(x)$.



WATCH

In this video, we'll sketch the derivative of a function given its graph.

WATCH

In this next video, we'll sketch the derivative of a function given its graph.

SUMMARY

In this lesson, you learned about a useful link between the behavior of $f(x)$ and the value of $f'(x)$. Specifically, **given the graph of** $y=f(x)$, it is possible to sketch **the graph of** $y=f'(x)$ by **using slopes** of the tangent lines at given points and their respective behavior.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 3 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.