# Indefinite Integrals and Antiderivatives of Polynomial Functions

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WHAT'S COVERED

In this lesson, you will begin your journey in finding antiderivatives (or indefinite integrals) by using properties and formulas, similar to how you learned derivatives. Finding an antiderivative is the reverse of the process used to find a derivative, so this will be used to our advantage. We'll start with polynomial functions, then work our way through trigonometric and exponential functions. Specifically, this lesson will cover:

1. The Definition of Indefinite Integrals
2. The Power Rule
3. Properties of Antiderivatives and Antiderivatives of Polynomials

## 1. The Definition of Indefinite Integrals

In the last tutorial, we defined the antiderivative of a function. Another name for an antiderivative is an **indefinite integral**. The indefinite integral of a function $f(x)$, written $\int f(x)dx$, is the collection of functions whose derivatives are equal to $f(x)$. In other words, the indefinite integral of $f(x)$ is the antiderivative of $f(x)$.

If $F'(x)=f(x)$, then we write $\int f(x)dx=F(x)+C$, where $C$ is an arbitrary constant. $C$ is also known as the constant of integration.

TERM TO KNOW

**Indefinite Integral of *f* (*x*)**

The collection of functions whose derivatives are equal to $f(x)$. In other words, the indefinite integral of $f(x)$ is the antiderivative of $f(x)$.

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## 2. The Power Rule

To get a better understanding of the antiderivative of $f(x)=x^n$, watch this video.

 

WATCH

In this video, we'll develop the ideas needed to establish a rule to find $\int x^n dx$.

Here is a summary of the antiderivative formulas for $f(x) = x^n$.

FORMULA

**Power Rule for Antiderivatives**

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C,\ n \neq -1$$

**Antiderivative of a Constant**

$$\int k dx = kx + C$$

**Natural Logarithm Rule**

$$\int \frac{1}{x} dx = \ln|x| + C$$

EXAMPLE Find $\int x^4 dx$.

| | |
|---|---|
| $\int x^4 dx$ | Start with the original expression. |
| $= \frac{1}{5} x^5 + C$ | Use the power rule with $n = 4$. |
| $= \frac{x^5}{5} + C$ | Rewrite. |

Thus, $\int x^4 dx = \frac{x^5}{5} + C$.

EXAMPLE Find $\int t^{1/2} dt$.

| | |
|---|---|
| $\int t^{1/2} dt$ | Start with the integral. |
| $= \frac{1}{3/2} t^{3/2} + C$ | Use the power rule with $n = \frac{1}{2}$: $\frac{1}{2} + 1 = \frac{3}{2}$ |
| $= \frac{2}{3} t^{3/2} + C$ | Rewrite: $\frac{1}{3/2} = \frac{2}{3}$<br>Fractions should always be written in the simplest form. |

Thus, $\int t^{1/2} dt = \frac{2}{3} t^{3/2} + C$.

TRY IT

Consider $\int x^{-4} dx$.

Find the indefinite integral.                                                                                                   +

$\frac{-1}{3}x^{-3}+C$

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# 3. Properties of Antiderivatives and Antiderivatives of Polynomials

Recall the rules for derivatives of combinations of functions (sums, differences, and constant multiples). For example, recall that $D[\sin x+\cos x]=D[\sin x]+D[\cos x]$.

Below are the formulas for the antiderivative of a sum, difference, and constant multiple of a function:

FORMULA

**Antiderivative of a Constant Multiple of a Function**

$$\int k\cdot f(x)dx=k\cdot\int f(x)dx$$

**Antiderivative of a Sum of Functions**

$$\int[f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx$$

**Antiderivative of a Difference of Functions**

$$\int[f(x)-g(x)]dx=\int f(x)dx-\int g(x)dx$$

BIG IDEA

In general:

- The antiderivative of a constant times $f(x)$ is the constant times the antiderivative of $f(x)$.
- The antiderivative of a sum is the sum of the antiderivatives.
- The antiderivative of a difference is the difference of the antiderivatives.

Recall that a polynomial function is a function in which all the terms are nonnegative integer powers of some variable. For example, $f(x)=15x^4-3x^2+20x-41$. This means we have all the properties necessary to find the antiderivative of any polynomial function.

EXAMPLE Find the indefinite integral: $\int(15x^4+2x^3-12x^2)dx$

| | |
|---|---|
| $\int(15x^4+2x^3-12x^2)dx$ | Start with the original expression. |
| $=\int 15x^4dx+\int 2x^3dx-\int 12x^2dx$ | Use the sum/difference properties. |
| $=15\int x^4dx+2\int x^3dx-12\int x^2dx$ | Apply the constant multiple property. |
| $=15\left(\frac{1}{5}x^5\right)+2\left(\frac{1}{4}x^4\right)-12\left(\frac{1}{3}x^3\right)+C$ | Use the power rule. Note: there is only one "+$C$" needed. If a constant were added to each indefinite integral, they could be merged and written as one |

constant.

$= 3x^5 + \frac{1}{2}x^4 - 4x^3 + C$ Simplify.

Thus, $\int (15x^4 + 2x^3 - 12x^2)dx = 3x^5 + \frac{1}{2}x^4 - 4x^3 + C$.

TRY IT

Consider $\int (6x^8 - 14x^6 + 22x + 17)dx$.

Find the indefinite integral. +

$\frac{2}{3}x^9 - 2x^7 + 11x^2 + 17x + C$

WATCH

In this video, we'll find $\int \left(2x - \frac{8}{x}\right)dx$.

## SUMMARY

In this lesson, you learned that another name for an antiderivative (covered in the last tutorial) is **indefinite integral**. The indefinite integral of $f(x)$ is the antiderivative of $f(x)$. You also learned that by applying **the power rule** for antiderivatives, as well as **properties of antiderivatives and antiderivatives of polynomials**, you can find antiderivatives of sums, differences, and constant multiples of powers of a variable.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 4 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

## TERMS TO KNOW

**Indefinite Integral of f(x)**

The collection of functions whose derivatives are equal to $f(x)$. In other words, the indefinite integral of $f(x)$ is the antiderivative of $f(x)$.

## FORMULAS TO KNOW

**Antiderivative of a Constant**

$\int k dx = kx + C$

**Antiderivative of a Constant Multiple of a Function**

 

$$\int k \cdot f(x)dx = k \cdot \int f(x)dx$$

**Antiderivative of a Difference of Functions**

$$\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$$

**Antiderivative of a Sum of Functions**

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

**Natural Logarithm Rule**

$$\int \frac{1}{x}dx = \ln|x| + C$$

**Power Rule for Antiderivatives**

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C,\ n \neq -1$$