The Graph Method

by Sophia

WHAT'S COVERED

In this lesson, you will evaluate limits by using the graph of a function. Specifically, this lesson will cover:

1. Defining Limit Notation
2. Using Graphs to Evaluate Limits

1. Defining Limit Notation

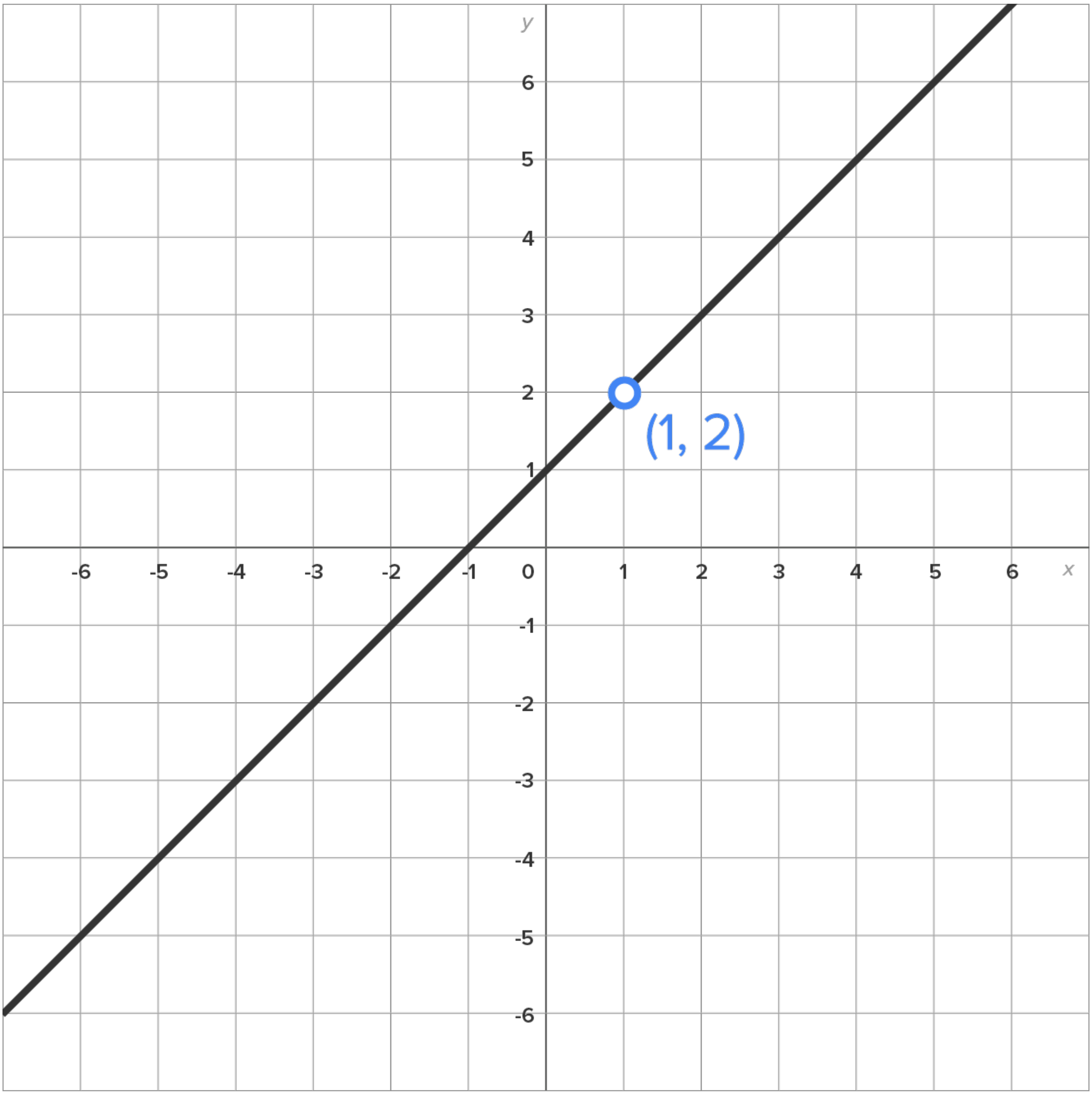
Consider the function $f(x) = \frac{x^2 - 1}{x - 1}$.

Notice that $f(x)$ is undefined when $x = 1$. However, we still may want to analyze the behavior of $f(x)$ around $x = 1$. The mathematical tool used to do this sort of analysis is called a limit.

BIG IDEA

$\lim_{x \to a} f(x) = L$ means "the limit of $f(x)$ as x gets closer to a is equal to L". In other words, as x gets closer to a, the value of $f(x)$ gets closer to L. We call L the limit of the function $f(x)$.

To see how this works graphically, shown below is the graph of $f(x) = \frac{x^2 - 1}{x - 1}$.



Notice that there is a hole in the graph at the point (1, 2), indicating that the graph of $f(x)$ is a line, but excludes the point (1, 2).

Since $f(x)$ is undefined when $x = 1$, we analyze the behavior of $f(x)$ by using limits.

That is, we want to evaluate $\lim_{x \to 1} f(x)$ or more specifically, $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$.

By examining the graph, it appears that as x gets closer and closer to 1, $f(x)$ gets closer and closer to 2. Thus, we can write $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$.

TERM TO KNOW

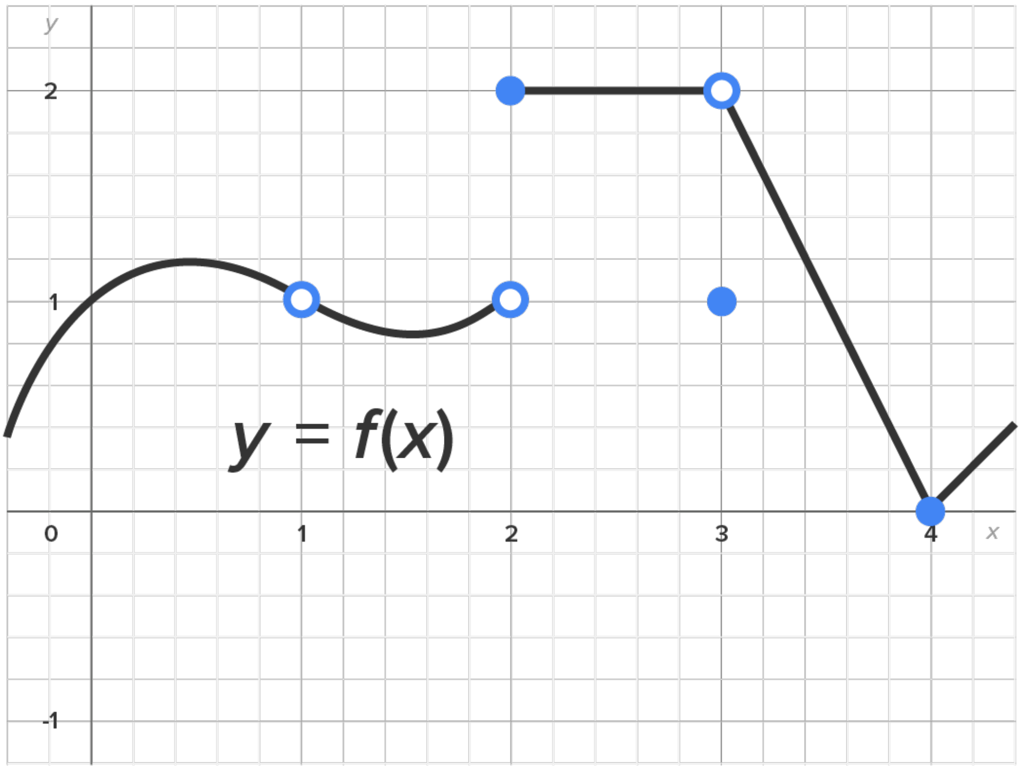
Limit

The value that a function $f(x)$ approaches as x gets closer to a specified number.

2. Using Graphs to Evaluate Limits

We can use the information from a graph to evaluate a limit.

EXAMPLE Consider the graph of some function $y = f(x)$.

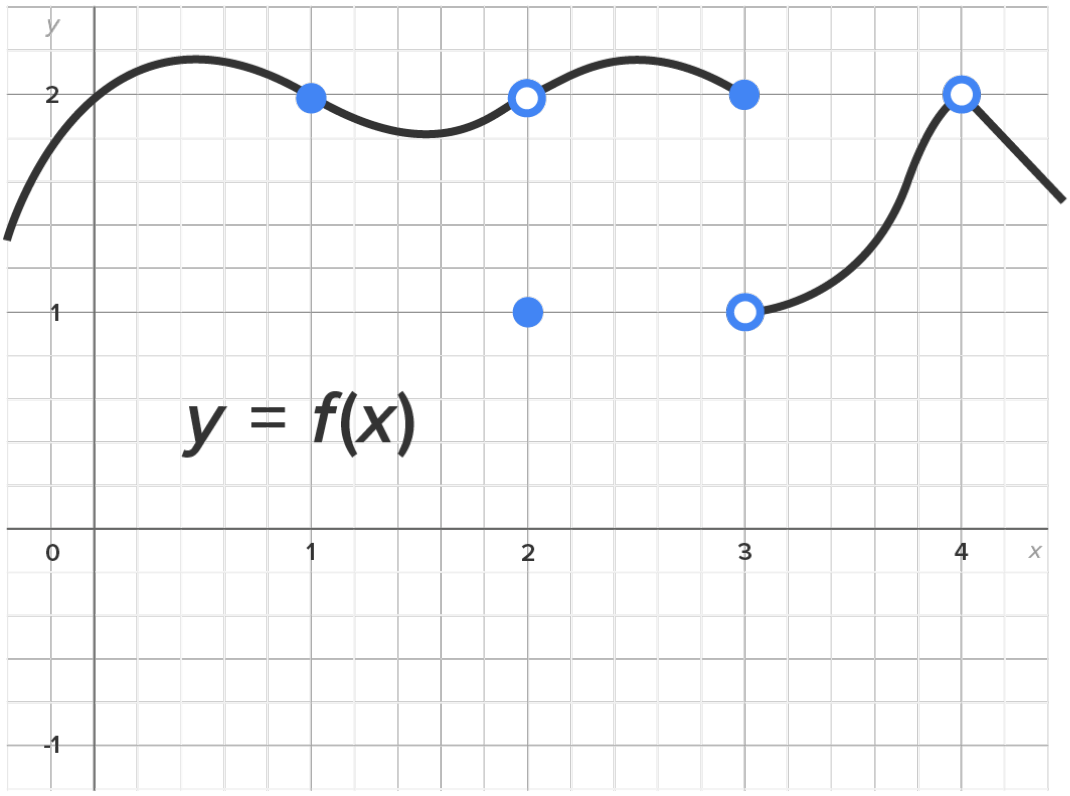


We can say the following:

Statement	Description
$\lim_{x \to 0} f(x) = 1$	As x gets closer to 0, $f(x)$ gets closer to 1.
$\lim_{x \to 1} f(x) = 1$	As x gets closer to 1, $f(x)$ gets closer to 1.
$\lim_{x \to 2} f(x)$ does not exist.	As x gets closer to 2 from the left (values smaller than 2), $f(x)$ gets closer to 1. However, as x gets closer to 2 from the right (values larger than 2), $f(x)$ gets closer to 2. Since $f(x)$ approaches two different values, as x approaches 2, we say the limit does not exist.
$\lim_{x \to 3} f(x) = 2$	As x gets closer to 3, $f(x)$ gets closer to 2. Note that the actual value of $f(3)$ is 1 (closed dot at $x = 3$), but the limit tells us what is happening as we get closer and closer to 3, not what is happening right at 3.
$\lim_{x \to 4} f(x) = 0$	As x gets closer to 4, $f(x)$ gets closer to 0.

TRY IT

Consider the graph pictured below.



Evaluate the function as x approaches 1. +

$$\lim_{x \to 1} f(x) = 2$$

Evaluate the function as x approaches 2. +

$$\lim_{x \to 2} f(x) = 2$$

Evaluate the function as x approaches 3. +

$\lim_{x \to 3} f(x)$ does not exist

Evaluate the function as x approaches 4. +

$$\lim_{x \to 4} f(x) = 2$$

SUMMARY

In this lesson, you learned about **defining limit notation**, or how the limit of a function is used to determine the behavior (or value) a function $f(x)$ approaches as x gets closer to some value. You also learned that you can **use the information from a graph to evaluate a limit**

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 1 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Limit
The value that a function $f(x)$ approaches as x gets closer to a specified number.

