Using Properties of Limits

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WHAT'S COVERED

In this lesson, you will utilize limit properties to evaluate more complex limits. Specifically, this lesson will cover:

1. Limit Properties
2. Evaluating Limits Using Limit Properties

1. Limit Properties

Suppose we know that $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$. Then, we can establish the following properties of limits:

Property of Limits	Formula
Limit of a Constant	$\lim_{x \to a} k = k$
Limit of a Sum or Difference	$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm M$
Limit of a Product	$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = LM$
Limit of a Quotient	$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$ (as long as $M \neq 0$)
Constant Multiple	$\lim_{x \to a} k \cdot f(x) = k \cdot \lim_{x \to a} f(x) = kL$
Limit of a Power	$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n = L^n$
Limit of an nth Root	$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L}$ (If n is even, then $L > 0$.)

2. Evaluating Limits Using Limit Properties

Here are a few examples of using the limit properties to evaluate limits.

EXAMPLE Given $\lim_{x \to 3} f(x) = 20$ and $\lim_{x \to 3} g(x) = 4$, evaluate $\lim_{x \to 3} [2f(x) + 3]$, $\lim_{x \to 3} \frac{f(x)}{1 + g(x)}$, and $\lim_{x \to 3} \sqrt{f(x) - g(x)}$.

$\lim_{x\to 3}[2f(x)+3]$	Start with the original limit.
$=\lim_{x\to 3}2f(x)+\lim_{x\to 3}3$	Apply the sum/difference property.
$=2\cdot\lim_{x\to 3}f(x)+\lim_{x\to 3}3$	Apply the constant multiple property.
$=2(20)+3$	Apply the limit of a constant property: $\lim_{x\to 3}f(x)=20$
$=43$	Simplify the expression.

$\lim_{x\to 3}\frac{f(x)}{1+g(x)}$	Start with the original limit.
$=\frac{\lim_{x\to 3}f(x)}{\lim_{x\to 3}[1+g(x)]}$	Apply the quotient property.
$=\frac{\lim_{x\to 3}f(x)}{\lim_{x\to 3}1+\lim_{x\to 3}g(x)}$	Apply the sum/difference property.
$=\frac{20}{1+4}$	Apply the limit of a constant property: $\lim_{x\to 3}f(x)=20$, $\lim_{x\to 3}g(x)=4$
$=4$	Simplify the expression.

$\lim_{x\to 3}\sqrt{f(x)-g(x)}$	Start with the original limit.
$=\sqrt{\lim_{x\to 3}[f(x)-g(x)]}$	Apply the nth root property.
$=\sqrt{\lim_{x\to 3}f(x)-\lim_{x\to 3}g(x)}$	Apply the sum/difference property.
$=\sqrt{20-4}$	$\lim_{x\to 3}f(x)=20$, $\lim_{x\to 3}g(x)=4$
$=\sqrt{16}$	Simplify the expression.
$=4$	Simplify the expression.

TRY IT

Suppose $\lim_{x\to a}f(x)=-4$ and $\lim_{x\to a}g(x)=12$ and you want to find $\lim_{x\to a}\frac{[f(x)]^2}{g(x)}$.

Evaluate this limit. +

$$\lim_{x\to a}\frac{[f(x)]^2}{g(x)}=\frac{4}{3}$$

SUMMARY

In this lesson, you learned that **limit properties** can be helpful in evaluating limits where other related limits are known. You also explored several examples of **evaluating limits using limit properties**. Future topics will illustrate the need to use these properties.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 1 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.