Evaluate Functions

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WHAT'S COVERED

In this lesson, you will learn function notation and use it to evaluate functions. Specifically, this lesson will cover:

1. Evaluate Functions Using Function Notation
2. Why Use Function Notation?

1. Evaluate Functions Using Function Notation

When a relationship meets the requirements of being a function, it can be written using function notation. For instance, $y = x^2$ is a function. In function notation, this is written $f(x) = x^2$.

When written this way, x is considered the *input variable* and $f(x)$ is the *output variable*. In the example above, x^2 is the rule for computing the output.

Note, the letter f is the name of the function. This could have been called $g(x) = x^2$ or $k(x) = x^2$, for instance.

EXAMPLE Consider the function $f(x) = x^2$. Use it to find $f(2)$, $f(-4)$, and $f(a+3)$.

$f(x) = x^2$	Solution
$f(2) = (2)^2$ $= 4$	$f(2) = 4$
$f(-4) = (-4)^2$ $= 16$	$f(-4) = 16$
$f(a+3) = (a+3)^2$ $= (a+3)(a+3)$ $= a^2 + 3a + 3a + 9$ $= a^2 + 6a + 9$	$f(a+3) = a^2 + 6a + 9$

HINT

For this function, remember to square the whole input! When you square (a + 3), you multiply it by itself.

TRY IT

Use the function $f(x) = x^2 - 4x + 2$ to answer the following problems.

Find *f*(1). +

$$\begin{aligned} f(1) &= (1)^2 - 4(1) + 2 \\ &= 1 - 4 + 2 \\ &= -1 \end{aligned}$$

Find *f*(-3). +

$$\begin{aligned} f(-3) &= (-3)^2 - 4(-3) + 2 \\ &= 9 + 12 + 2 \\ &= 23 \end{aligned}$$

Find *f*(a + 1). +

$$\begin{aligned} f(a+1) &= (a+1)^2 - 4(a+1) + 2 \\ &= a^2 + 2a + 1 - 4a - 4 + 2 \\ &= a^2 - 2a - 1 \end{aligned}$$

WATCH

Depending on which function is used, applying function notation can get very technical. Here is a video that helps guide you through a more complicated function.

Video Transcription

[MUSIC PLAYING] Hello, and thank you for visiting. What we're going to do is a little check in with function notation. We're given the function f of x equals 2x squared minus 5x plus 8, and we're going to find various values of f of x. So remember that with a function, the x is the input. So if we take a look here, everywhere we have an x, it's going to be replaced by a number.

So f of 3, for example, we have 2 times something squared minus 5 times something plus 8. So since it's 3 we are substituting, 3 goes here and here. And then we are left to perform the order of operations. So remember, order of operation sets.

We do the exponents first. We don't have anything inside parentheses, so we jump to the exponents. 2 times 3 squared would be 2 times 9 minus 5 times 3 plus 8, and then we just go from left to right. 2 times 9 is 18. 5 times 3 is 15. Plus 8. 18 -15 is 3. Plus 8. And 3 plus 8 is 11. So there we have our function evaluated at 3. So in summary, we would say f of 3 is equal to 11.

So we do the same thing for f of negative 2. So we have 2 times negative 2 squared minus 5 times negative 2 plus 8. Now here's where we have to be really careful, and this is why the parentheses are crucial. We have negative 2 quantity squared here. Many people think that that's negative 4. It might be tempting to write negative 4. Remember, negative 2 squared is negative 2 times negative 2, which is positive 4. And I'm just going to go ahead and simplify down the line here.

So we have minus 5 times negative 2, which ends up being a plus 10. And then plus 8, which is then 8 plus 10 plus 8. And if we add all three of those numbers together, 8 plus 10 is 18, plus 8 is 26. So there again just to pull that together, f of negative 2 is 26. When you substitute negative 2, you get 26 as an output. So now to substitute a more complicated expression. This is where, again, the grouping symbols and whatnot are very crucial.

So again, our function was 2 times something squared minus 5 times the something plus 8. And inside the parentheses, we're going to put the x plus 4. So now we have to perform x plus 4 the quantity squared. So let's just take a little side step for that. X plus 4 the quantity squared is x plus 4 times x plus 4.

So then if we FOIL that, first times the first is x squared. Outers give us 4x. The inners also give us 4x. And the last times the last gives us 16. And the simplest form of that is x squared plus 8x plus 16. So pulling this into our function, we have 2 times x squared plus 8x plus 16. I'll just leave the rest of it as it was.

And now we can go ahead and do the algebra. So here, the two is going to get distributed to everything. So we have 2x squared plus 16x plus 32. We can go ahead and distribute the minus 5, because that other term is behind us now. So you have minus 5x -20, and then plus 8. And then we'll combine like terms. There is no other squared term, so we have 2x squared.

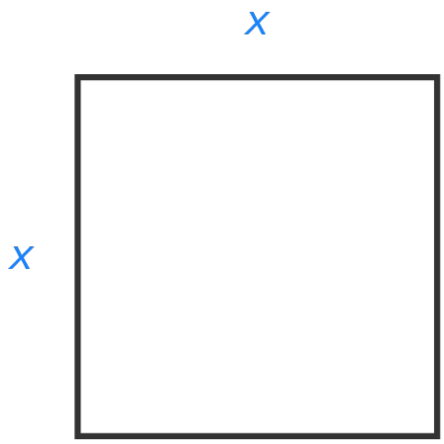
It looks like we have a plus 16x and a minus 5x. So that's going to be plus 11x. And then for our numbers, we have plus 32 minus 20 plus 8. 32 plus 8 is 40. Minus 20 is 20. And there we have it. So f of x plus 4 is 2x squared plus 11x plus 20. And there we have some examples of function notation.

[MUSIC PLAYING]

2. Why Use Function Notation?

There are situations in which we want to compute two (or more) different values given a single input. This is where naming functions can be convenient.

→ EXAMPLE Suppose a square has sides with length x.



- The area of the square is x^2. Using function notation, we could write $A(x) = x^2$.
- The perimeter of the square is $4x$. Using function notation, we could write $P(x) = 4x$.
- The length of the diagonal is $x\sqrt{2}$. Using function notation, we could write $D(x) = x\sqrt{2}$.

Let's now say that the length of the square is 5 inches.

- The area is $A(5) = 5^2 = 25$ square inches.
- The perimeter of the square is $P(5) = 4(5) = 20$ inches.
- The length of the diagonal is $D(5) = (5)\sqrt{2} = 5\sqrt{2}$ inches.

The function names (*A, P,* and *D*) are important here since *A* is used for area, *P* is used for perimeter, and *D* is used for the length of the diagonal. Therefore, we can provide meaningful names for the output of a function rather than using *y* all the time.

HINT

The input variable doesn't necessarily have to be *x*. For example, let's say you are tracking the height of a projectile after *t* seconds. You might name the function $h(t)$.

SUMMARY

In this lesson, you learned that when a relationship is a function, function notation is used to emphasize the roles of the input and output values, and their relationship to each other. In this context, you can **evaluate functions using function notation**. You also learned **why we use function notation**, noting that naming functions can be convenient in situations in which we want to compute the outputs of two (or more) different functions given input values, such as determining the area, perimeter, and length of the diagonal of a square.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.