Slope of a Line Between Two Points

by Sophia

WHAT'S COVERED

In this lesson, you will investigate the slope between two points. Specifically, this lesson will cover:

1. Slope of a Line
2. Slope of a Line Between Two Points on a Curve
3. Applications of Slope to Real-Life Situations
    a. Population
    b. Temperature

1. Slope of a Line

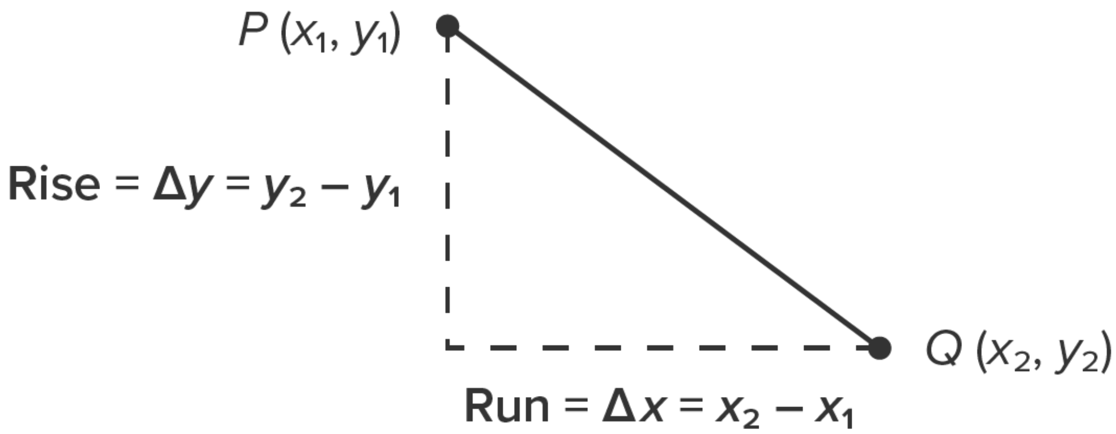
Slope is a very important concept in calculus, as it opens the door to explore rates of change.

When given two points, the slope between them is represented by the letter *m*, and is given by the following formula:

FORMULA

Slope

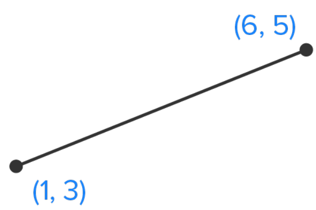
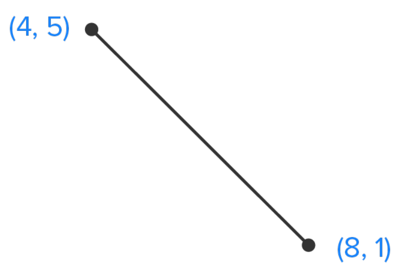
$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



HINT

The easiest way to look at slope is from left to right. When the line falls, or decreases, from left to right, the rise is negative. When the line increases from left to right, the rise is positive.

The table below shows examples of different types of slopes, along with how to compute and graph the slope.

Type of Slope	Points	Calculation	Graph
Positive Slope	$(1, 3)$ and $(6, 5)$	Slope = $m = \frac{5-3}{6-1} = \frac{2}{5}$	
Negative Slope	$(4, 5)$ and $(8, 1)$	Slope = $m = \frac{1-5}{8-4} = \frac{-4}{4} = -1$	
Zero Slope (Horizontal Line)	$(2, 5)$ and $(8, 5)$	Slope = $m = \frac{5-5}{8-2} = \frac{0}{6} = 0$	

Undefined Slope (Vertical Line)	$(1, 4)$ and $(1, 6)$	Slope = $m = \frac{6-4}{1-1} = \frac{2}{0}$ This is undefined.	

HINT

Notice that the slope for vertical lines is undefined. This is because you cannot divide by zero.

TERM TO KNOW

Slope

The ratio of the change in y to the change in x; measure of the steepness of a line.

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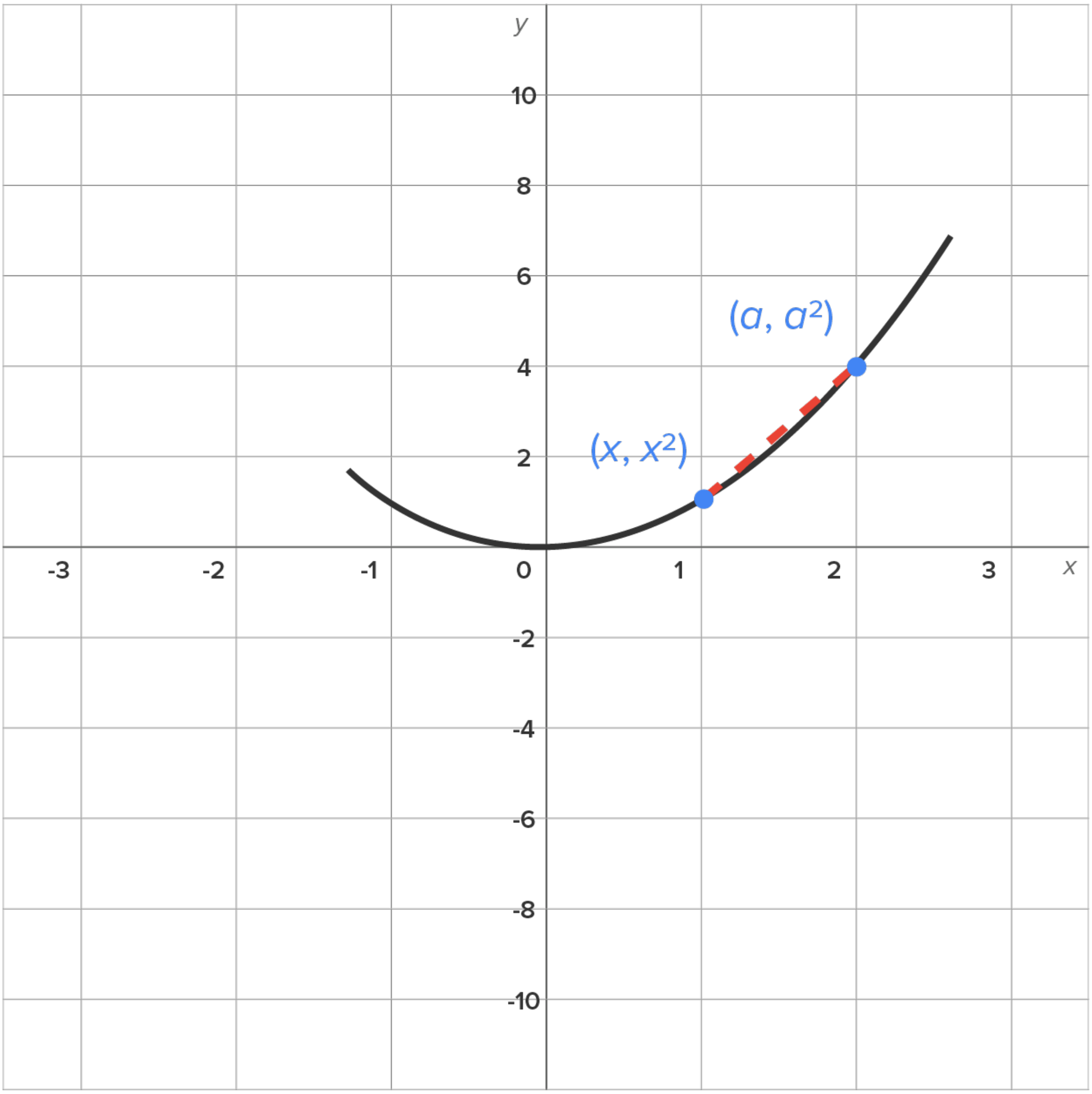
2. Slope of a Line Between Two Points on a Curve

In this course, we will frequently be examining the slope between two points that are on the same curve.

For example, let's look at the graph of $y = x^2$, which contains the points (x, x^2) and (a, a^2), where $a \neq x$ (meaning they are two different points).



$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$m = \frac{a^2 - x^2}{a - x}$	Substitute the two points.
$m = \frac{(a-x)(a+x)}{a-x}$	Factor the numerator.
$m = a + x$	Remove the common factors.

This means that for any values of a and x, the slope of the line between these points is $a + x$.

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3. Applications of Slope to Real-Life Situations

3a. Population

Suppose that in 1980, the population of a small town was 3,192 people. Then, in 1990, a count was taken again and the population was found to be 5,362. On average, at what rate did the population grow each year?

If we represent the information with two ordered pairs, (1980, 3192) and (1990, 5362), the slope between these points gives the average growth per year. Here is how:

$$\text{Slope} = m = \frac{\Delta\text{people}}{\Delta\text{time}}$$

$$\text{Slope} = \frac{5362 \text{ people} - 3192 \text{ people}}{\text{year } 1990 - \text{year } 1980} = \frac{2170 \text{ people}}{10 \text{ years}}$$

$$\text{Slope} = 217 \text{ people/year}$$

BIG IDEA

In general, the units of the slope are $\frac{\Delta y \text{ units}}{\Delta x \text{ units}}$.

3b. Temperature

Did you know that the Fahrenheit and Celsius temperature scales are linearly related? Let's investigate.

The temperature at which water freezes is 0° C and 32° F.
The temperature at which water boils is 100° C and 212° F.

We can represent this information as two ordered pairs: (0, 32) and (100, 212).

Then, the slope of the line formed by these two points is $m = \frac{212 - 32\ (°\text{F})}{100 - 0\ (°\text{C})} = \frac{1.8\ (°\text{F})}{1\ (°\text{C})}$.

The slope is 1.8, and as we see in the computation, the slope means that the Fahrenheit scale goes up by 1.8° for every 1° increase in the Celsius scale.

SUMMARY

In this lesson, you learned about an important concept in calculus, the **slope of a line**, which is a measure of the steepness of a line. The slope between two points also represents the average rate of change between the points. You learned that the slope can be found using the formula of rise over run, which is the difference in y divided by the difference in x. There are four types of slope: positive, negative, zero (horizontal line), and undefined (vertical line). You also learned how to use the slope formula to calculate the **slope of a line between two points on a curve**. Lastly, you explored several **applications of slope to real-life situations**, using slope to calculate the rate of **population** growth and the linear relationship between Fahrenheit and Celsius **temperature** scales.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 0 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.

TERMS TO KNOW

Slope
The ratio of the change in y to the change in x; measure of the steepness of a line.

FORMULAS TO KNOW

Slope

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

