Approximation of Measurement Error Using Differentials

by Sophia

WHAT'S COVERED

In this lesson, you will apply differentials to situations in which there could be measurement errors. For example, we can examine the effect that a measurement error on each side of a square could have on its area. Specifically, this lesson will cover:

1. Comparing the Differential to the Maximum Error
2. Applying Differentials to Situations Involving Measurement

1. Comparing the Differential to the Maximum Error

Let's say a square piece of material is to have sides 10" long, but each side could have a measurement error of at most 0.25". What is the greatest possible error in measuring its area?

To answer this question, we need to look at a few scenarios:

- If the sides are all 0.25" too large, then the area would be $(10.25)^2 = 105.0625 \text{ in}^2$.
- If the sides are all 0.25" too small, then the area would be $(9.75)^2 = 95.0625 \text{ in}^2$.
- If the sides are all perfectly measured, then the area would be $10^2 = 100 \text{ in}^2$.
- When the sides are all 0.25" too large, the error is $105.0625 - 100 = 5.0625 \text{ in}^2$.
- When the sides are all 0.25" too small, the error is $100 - 95.0625 = 4.9375 \text{ in}^2$.

This means that the maximum error is 5.0625 in^2.

Think about how differentials are related to this situation:

- If the sides were measured accurately, the area would be $10^2 = 100 \text{ in}^2$.
- The goal is to determine the change in *A* when the side changes by at most 0.25".

Now, let's examine this situation using differentials.

Let $A(x) = x^2$, which is the area of a square whose length is *x*.

Then, the differential of *A* is $dA = 2xdx$.

The situation above suggests that we want to find *dA* when $x = 10$ (length of a side) and $dx = 0.25$ (change in *x*).

This gives $dA = 2(10)(0.25) = 5\text{ in}^2$, which is very close to the errors 4.9375 in^2 and 5.0625 in^2.

BIG IDEA

If $f(x)$ is a function that depends on an x-variable that has a possible error of *dx* units, then the differential *df* will provide an estimate of the maximum error in computing *f*.

2. Applying Differentials to Situations Involving Measurement

Now that we see how useful the differential is, let's apply them to situations involving measurement.

EXAMPLE A sphere is to be designed with a radius of 2.5 feet. What is the estimate for the maximum error when measuring the volume of the sphere if the possible error in measuring the radius is 0.1 feet?

The volume of a sphere is $V(r) = \frac{4}{3}\pi r^3$, which has derivative $V'(r) = 4\pi r^2$. Thus, the differential is $dV = 4\pi r^2 dr$.

Now, let $r = 2.5$ and $dr = 0.1$. Then, the maximum error in estimating the volume is $dV = 4\pi(2.5)^2(0.1) \approx 7.854\text{ ft}^3$.

TRY IT

Suppose you have a cube with sides that are 8 inches.

Use differentials to estimate the maximum error in the surface area of this cube with an error of no more than 0.2" on each side. +

$S = 6x^2$, $dS = 12xdx$, $dS = 19.2\text{ in}^2$

WATCH

In this video, differentials are used to estimate the maximum error in the volume of a cube.

SUMMARY

In this lesson, you learned that **differentials can be used to estimate the maximum error** in computing *f* when its input variable, *x*, has known maximum errors. Specifically, if $f(x)$ is a function that depends on an x-variable that has an error of *dx* units, then the differential *df* will provide an estimate of the maximum error in computing *f*. Next, you **applied differentials to situations involving measurement**, such as estimating the error in calculating the volume of a sphere and surface area of a cube.

SOURCE: THIS WORK IS ADAPTED FROM CHAPTER 2 OF *CONTEMPORARY CALCULUS* BY DALE HOFFMAN.