Numerical Analysis: MATLAB

Given Ellipses:

$$(x + y + 2)^2 + (x + 3)^2 = 5$$

$$2(x + 3)^2 + \left(\frac{y}{3}\right)^2 = 4$$

To evaluate the intersection points of the ellipses using Newton-Raphson's Method.

Mathematical Simplification of the Equations:

First Ellipse's Equation:

$$(x + y + 2)^2 + (x + 3)^2 = 5$$
$$\Rightarrow x^2 + y^2 + 4 + 2xy + 4x + 4y + x^2 + 9 + 6x = 5$$
$$\Rightarrow y^2 + (2x + 4)y + (2x^2 + 10x + 8) = 0$$

Solving for y: Using quadratic formula:

$$y = \frac{-(2x + 4) \pm \sqrt{(2x + 4)^2 - 4(2x^2 + 10x + 8)}}{2}$$
$$\Rightarrow y = \frac{-(2x + 4) \pm \sqrt{4x^2 + 16x + 16 - 8x^2 - 40x - 32}}{2}$$
$$\Rightarrow y = \frac{-(2x + 4) \pm \sqrt{-4x^2 - 24x - 16}}{2}$$
$$\Rightarrow y = \frac{-(2x + 4) \pm 2\sqrt{-x^2 - 6x - 4}}{2}$$
$$\Rightarrow f_1(x) = \frac{-(2x + 4) + 2\sqrt{-x^2 - 6x - 4}}{2}$$
$$f_2(x) = \frac{-(2x + 4) - 2\sqrt{-x^2 - 6x - 4}}{2}$$

For real y, discriminant must by greater than zero. i.e. $-4x^2 - 24x - 16 > 0$

$$\Rightarrow x^2 + 6x + 4 < 0$$

we has

We have two roots of x for the above relation, i.e. $x_1 = -5.2361, \quad x_2 = -0.7639$

$$\Rightarrow x_1 \le x \le x_2$$
$$\Rightarrow -5.2361 \le x \le -0.7639$$

Now taking second Ellipse's Equation:

$$2(x + 3)^2 + \left(\frac{y}{3}\right)^2 = 4$$
$$\Rightarrow \left(\frac{y}{3}\right)^2 = 4 - 2(x + 3)^2$$
$$\Rightarrow y = \pm 3\sqrt{4 - 2(x + 3)^2}$$

$$\Rightarrow f_3(x) = 3\sqrt{4-2(x+3)^2}$$
$$f_4(x) = -3\sqrt{4-2(x+3)^2}$$

Finding range of x: for range of x, $4-2(x+3)^2 \geq 0$

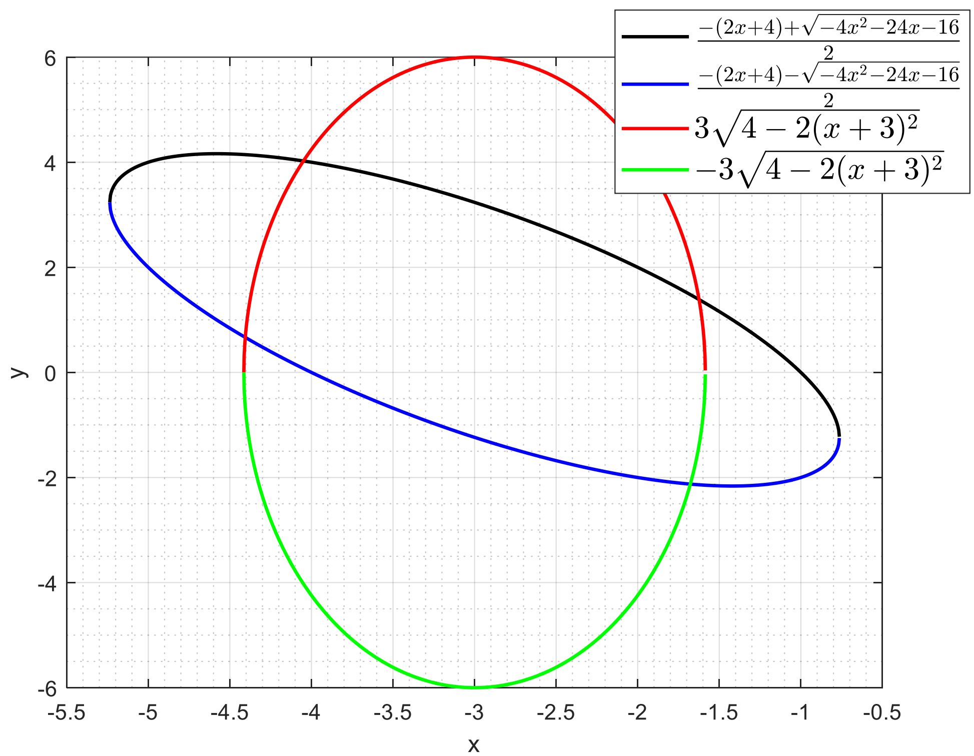
$$\Rightarrow (x+3)^2 \leq 2$$
$$\Rightarrow x^2+6x+9-2 \leq 0$$
$$\Rightarrow x^2+6x+7 \leq 0$$

We have two roots of the above relation: $x_1 = -4.4142$, $x_2 = -1.5858$

$$\Rightarrow x_1 \leq x \leq x_2$$
$$\Rightarrow -4.4142 \leq x \leq -1.5858$$

Implementation of Newton-Raphson's Method:

Consider the following figure:



We see that there are four intersection points, i.e. two Blue-Red, one Black-Red and one Black-Green.

For Black-Red intersection, we have:

$$f_{Br}(x) = f_3(x) - f_1(x) = 3\sqrt{4-2(x+3)^2} - \frac{-(2x+4)+2\sqrt{-x^2-6x-4}}{2} = 0$$

For Blue-Red intersection, we have:

$$f_{br}(x) = f_3(x) - f_2(x) = 3\sqrt{4-2(x+3)^2} - \frac{-(2x+4)-2\sqrt{-x^2-6x-4}}{2} = 0$$

For Blue-Green intersection, we have:

$$f_{bg}(x) = f_4(x) - f_2(x) = -3\sqrt{4-2(x+3)^2} - \frac{-(2x+4) - 2\sqrt{-x^2-6x-4}}{2}$$

Blue-Black Ellipse correspond to First Equation, while Red-Green Ellipse corresponds to Second Equation.

Using Newton-Raphson method, we got the following solution:

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Black-Blue Insection: -4.0491 4.0238
Black-Blue Insection: -1.6241 1.3867
Blue-Red Intersection: -4.4055 0.6663
Blue-Red Intersection: -1.6775 -2.1256
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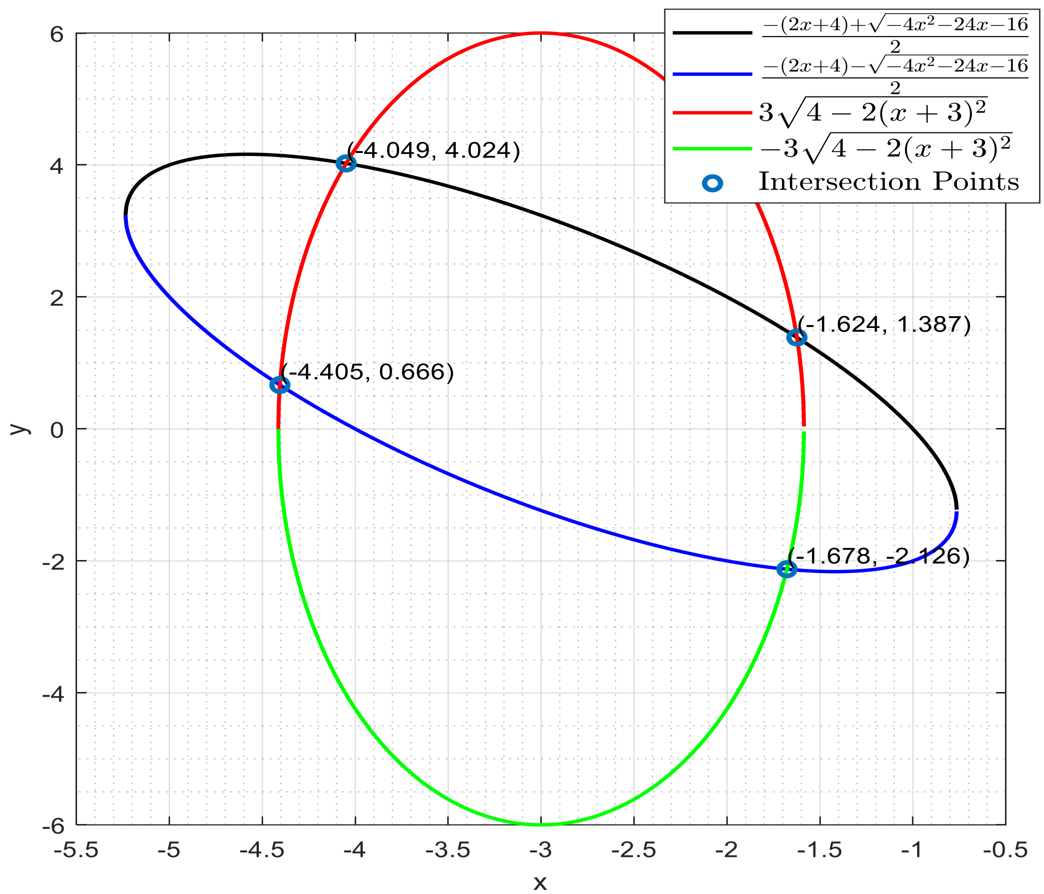

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