Engineering Computation and Linear Algebra Programming Project

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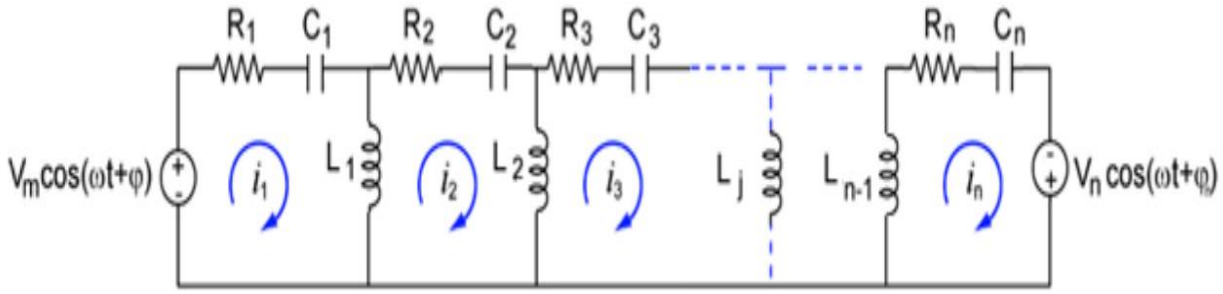
The provided circuit is shown the figure below:



In phasor form voltage sources are:

The following Equations are referred as Eq. (1 & 2), respectively:

$$V_m \cos(\omega t + \phi_1) = V_m \angle \phi_1 \qquad V_n \cos(\omega t + \phi_2) = V_n \angle \phi_2$$

Inductive and capacitive reactances are given by:

The following equations are referred as Eq. (3 & 4), respectively.

$$X_L = \omega L \qquad X_c = \frac{1}{\omega C}$$

Where L and C are inductance & capacitance of the respective inductor and capacitor.

For simplicity, we consider a circuit with three loop. Using KVL:

The following System of Equations are referred as Eq. (5, 6 & 7), respectively.

$$\left.\begin{aligned} \left(R_1 - jX_{C_1} + jX_{L_1}\right) i_1 - jX_{L_1} i_2 &= V_m \angle \phi_1 \\ -jX_{L_1} i_1 + \left(jX_{L_1} + R_2 - jX_{C_2} + jX_{L_2}\right) i_2 - jX_{L_2} i_3 &= 0 \\ -jX_{L_2} i_2 + \left(R_3 - jX_{C_3} + jX_{L_2}\right) i_3 &= V_n \angle \phi_2 \end{aligned}\right\}$$

In Matrix Form: $ZI = V$:

The following systems of Equations are referred as Eq. (8 & 9), respectively.

$$\begin{bmatrix} \left(R_1 - jX_{C_1} + jX_{L_1}\right) & -jX_{L_1} & 0 \\ -jX_{L_1} & \left(jX_{L_1} + R_2 - jX_{C_2} + jX_{L_2}\right) & -jX_{L_2} \\ 0 & -jX_{L_2} & \left(R_3 - jX_{C_3} + jX_{L_2}\right) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_m \angle \phi_1 \\ 0 \\ V_n \angle \phi_2 \end{bmatrix}$$

$$\begin{bmatrix} z_{11} & -z_{12} & 0 \\ -z_{12} & z_{22} & -z_{23} \\ 0 & -z_{23} & z_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_m \angle \phi_1 \\ 0 \\ V_n \angle \phi_2 \end{bmatrix}$$

For n loops:

The following matrix equation is referred as Eq. (10), which is the general form.

$$\begin{bmatrix} z_{11} & -z_{12} & 0 & \cdots & 0 \\ -z_{12} & z_{22} & -z_{23} & \cdots & 0 \\ 0 & -z_{23} & z_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & z_{nn} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_n \end{bmatrix} = \begin{bmatrix} V_m \angle \phi_1 \\ 0 \\ 0 \\ \vdots \\ V_n \angle \phi_2 \end{bmatrix}$$

Where:

The following equations are referred as Eq. (11, 12, 13 & 14)

$$\begin{aligned} z_{11} &= R_1 - jX_{C_1} + jX_{L_1} \\ z_{ii} &= jX_{L_{i-1}} + R_i - jX_{C_i} + jX_{L_i} && 1 < i < n \\ z_{nn} &= R_n - jX_{C_n} + jX_{L_{n-1}} \\ z_{ii+1} &= jX_{L_i} && 1 \leq i < n \end{aligned}$$