University of Colorado - Boulder

ASEN 3801

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ASEN 3801 Lab 3 - Attitude Sensors, Actuators, and Spacecraft Pitch Controls

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UNIVERSITY OF COLORADO **BOULDER**

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I. Preliminary Questions

A. Lab Task 1

1. In order to calculate the angular position of the spacecraft in radians, given measurement data at dt increments in revolutions per minute (RPM), we utilize Equation (1). The integral would be evaluated over the desired interval of time.

$$\theta = \int (\omega_2 - \omega_1) dt \tag{1}$$

$$H_i = H_f \tag{2}$$

$$K I_{rg} \omega_{rg1} = K I_{rg} \omega_{rg2} + I_{sc} \omega_{sc} \tag{3}$$

$$\theta_{sc} = \frac{K I_{rg}}{I_{sc}} \int_0^t (\omega_{rg1} - \omega_{rg2})\, dt [rad] \tag{4}$$

where:
K = gyro sensitivity $\frac{V}{rad/s}$
I_{rg} = moment of inertia about rate gyro kgm^2
I_{sc} = moment of inertia about spacecraft kgm^2
$\omega_{rg1,2}$ = angular velocity of rate gyro initial, final $\frac{rad}{s^2}$
ω_{sc} = angular velocity of spacecraft $\frac{rad}{s^2}$

B. Lab Task 2

2. The required angular acceleration rate of the reaction wheel needed to account for the drag torque is shown in Equation (2).

$$\frac{d\omega}{dt} = \alpha = \frac{Md}{I} \tag{5}$$

where:
α = Angular acceleration of reaction wheel
I = Inertia about reaction wheel spin axis
M_d = disturbance torque

3. If one of us is holding a reaction wheel in our hand, we believe that we will experience a reaction torque in the direction opposing rotation. The gyroscope precession would act in the opposite direction orthogonal to the original rotation about the axis.

4. If we imagine that we are sitting on a stool, holding a spinning bicycle wheel in front of us with the spin axis horizontal, the wheel acts as a CMG. We can control our rotation by raising one hand over the other to initiate gyroscopic precession and change the direction of the angular momentum vector, inducing a rotation about the vertical axis. To rotate left, the wheel should spin away from the controller, with angular momentum vector pointing left. So, we can raise our right hand to tilt spin axis and change angular momentum vector.

C. Lab Task 3

5. In order to measure the spacecraft moment of inertia about the spin axis, we can utilize Equation (3).

$$\tau = I\alpha \tag{6}$$

Here, τ is the applied torque ($N * m$), I is the Moment of Inertia of the spacecraft ($kg * m^2$), and α is the angular rate of rotation of the spacecraft with respect to time ($\frac{rad}{sec}$). We can apply a torque to the spacecraft orthogonal to the axis of free motion, and record data for its angular rate and the time it was applied.

6.

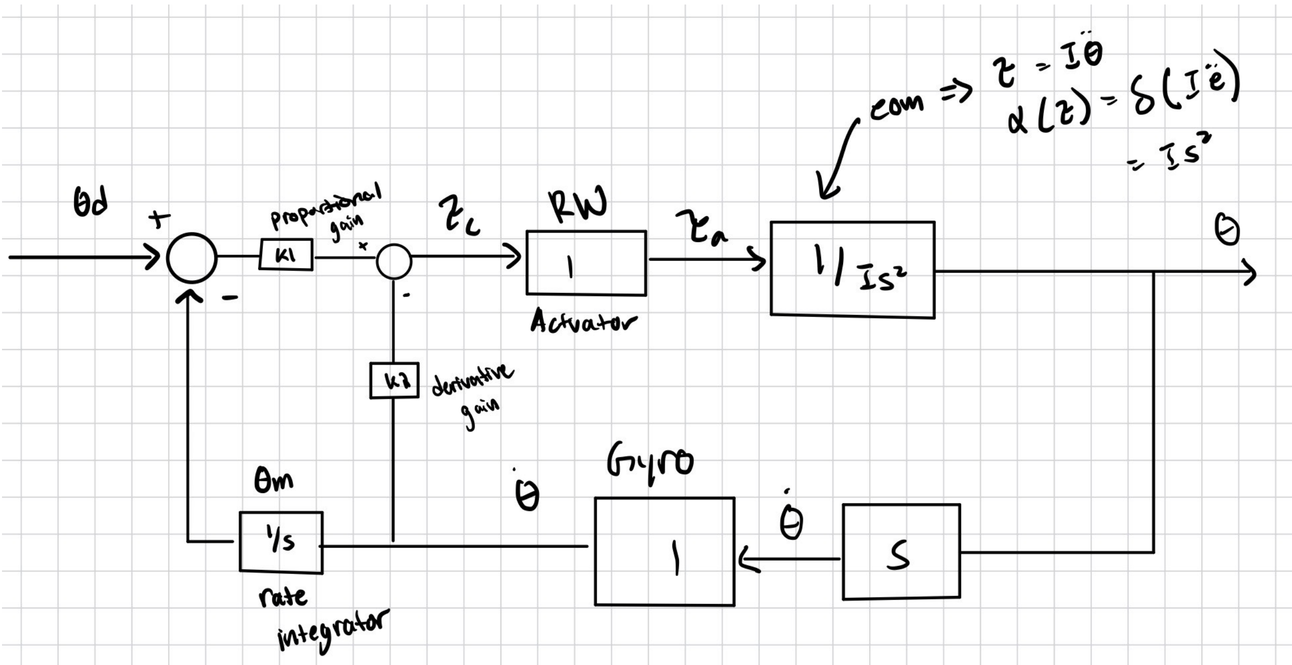

Fig. 1 Single DOF Block Diagram of Closed-Loop System

7. Our control law starts with some angle desired angle. The first step the system takes is that based on the previous angle of the spacecraft, we derive the proportional gain from the difference in the angles. This proportional gain describes the ratio of the previous output θ to the error response determined by the desired angle. From the gyro sensor, we are also able to deduce our derivative gain. The bottom row in the diagram expands this a bit, but in the end can be simplified down to 1 (following the standard rules of multiplying the blocks that feed into each other together). This gives us angular rate, from the s * 1/s blocks, or in other words our derivative gain. Using proportional and derivative gain allows us to calculate the control torque, and after that is fed into our reaction wheel actuator, a final torque then plays out in the dynamics of the system, described by the equations of motion labeled on the block diagram. These dynamical motions determine the final output angular position of the spacecraft. And as per the control law, this angular position is fed back into the system and used to determine the gains required to revert the spacecraft to the position desired.

8. Using our given values, we can calculate the damping coefficient and the natural frequency. We can calculate our damping coefficient using our max overshoot equation, which is given by:

$$M_p = exp(\frac{\zeta}{\sqrt{1-(\zeta)^2}}\pi) \tag{7}$$

We can re-arrange to solve for ζ, which provides us with:

$$\zeta = \frac{-ln(0.1)}{\sqrt{(\pi)^2 + ln(0.1)^2}} \tag{8}$$

Evaluating this equation gives us a damping ratio of $\zeta = 0.591$.
Next, we can evaluate the natural frequency by using the percent overshoot equation, which has the form of:

$$P = exp(-\zeta w_n t_s) \tag{9}$$

Re-arranging to solve for w_n, we get:

$$w_n = \frac{ln(0.1)}{\zeta t_s} \tag{10}$$

Evaluating, we get a natural frequency of $w_n = 3.379$ rad/s.

Subsequently, if we evaluate our transfer function from the above block diagram, we get that our damping coefficients are:

$$Iw_n^2 = K_1 = 11.4461I \tag{11}$$

$$2\zeta w_n I = K_2 = 4I \tag{12}$$

II. Experiment and Analysis

A. Lab Task 1

1A

A As the spinning disk inside the gyro assembly moves as we rotate the gyro about the positive z-axis, we noticed that the disk tilts downwards (away from the battery pack). When rotated around the negative z-axis, it tilts upwards (towards the battery pack). Around the positive y-axis, it tilts downwards, and around the negative y-axis, it tilts upwards. When rotated around the x-axis, however, there is no noticeable reaction.

B When rotating around the positive x-axis, we can feel a torque pushing in the positive x-axis. We feel no torque when the gyroscope is rotating about the positive z-axis. When the gyroscope is rotating about the positive y-axis, there is a torque pointing in the positive y-direction. When rotating about the negative y-axis, the torque points in the negative y-direction.

1B When rotating about the positive z-axis, there is a precession, and the disk tilts downwards. The disk is spinning clockwise relative to the y-axis.

1C When the gyro is rotated about the z-axis the disc becomes unstable and applies a torque in the opposite direction of rotation (see Appendix 19). The torque applied to the reaction wheel acts at a 90^o offset to the body y-axis. The disc is rotating clockwise with respect to the y-axis

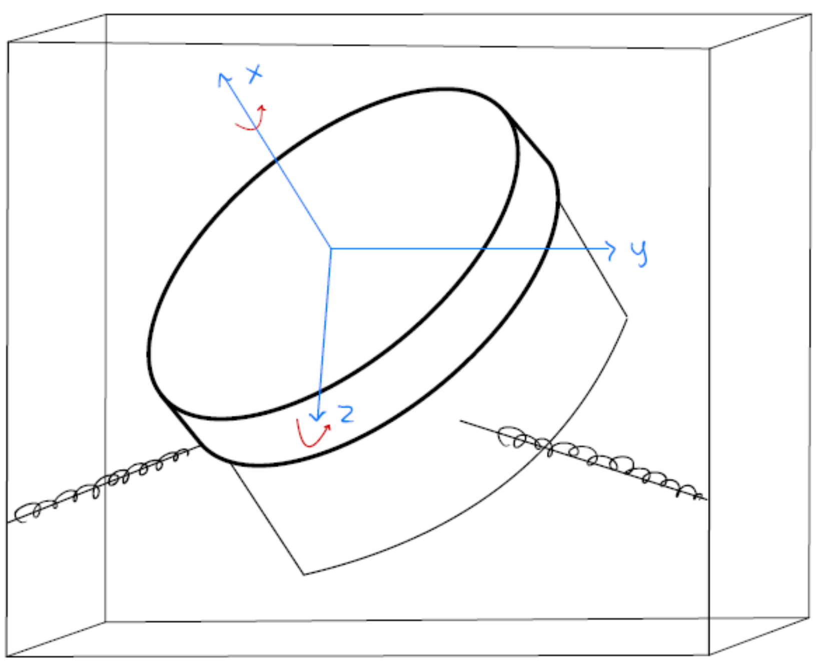

Fig. 2 Diagram of Physical Rate Gyro with Defined Body Coordinate System

B. Lab Task 2

A We defined the positive x-axis as pointing out of the group member's chest. In order to make this a right-handed coordinate system, the positive y-axis would be pointing out of the group member's left shoulder, and the positive z-axis would be pointing out the top of their head. We started by spinning the wheel around the negative y-axis (counterclockwise). When we turned the wheel along the x-axis, in other words, tilting it to the left or right, we found ourselves moving in that same direction. When we inverted the wheel's spin, a torque was induced on the person holding the wheel in the opposite direction.

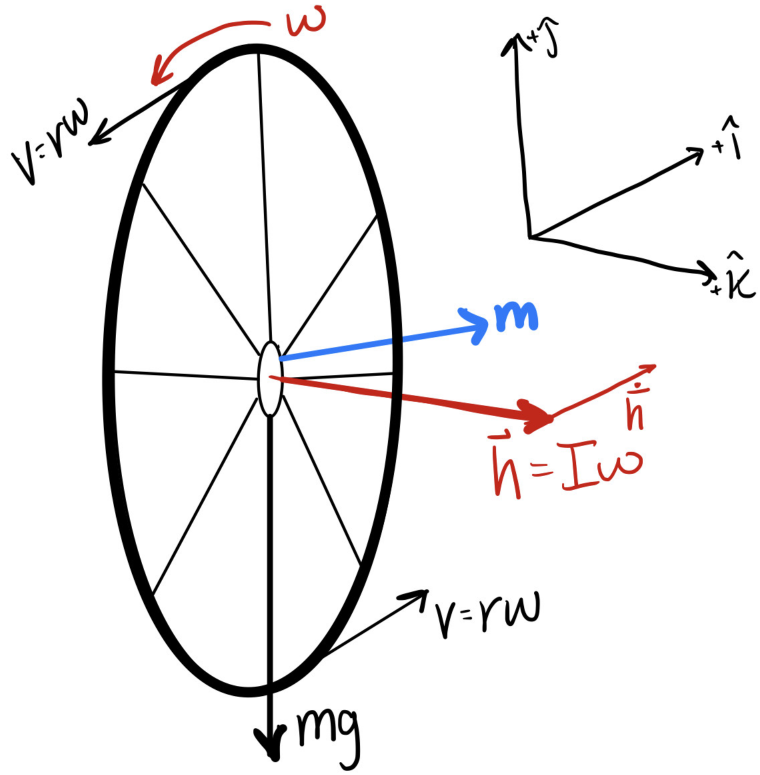

Fig. 3 Angular momentum, change in angular momentum, and moment vector of spinning gyroscope

When attempting to track a group member by spinning the gyroscope, we noticed that, after a few seconds, it was hard to keep up with them. This is likely because the movement on the spinner is caused by the initial turning of the bicycle wheel. Thus, it resists change in its orientation, and will move you if need be in order to counteract this change.

In order to improve the experiment, we recommend having multiple trials with multiple different group members in order to really see and feel the effect that the bicycle wheel has when acting as a gyroscope.

C. Lab Task 3

1. Section 3.1

3.1a.

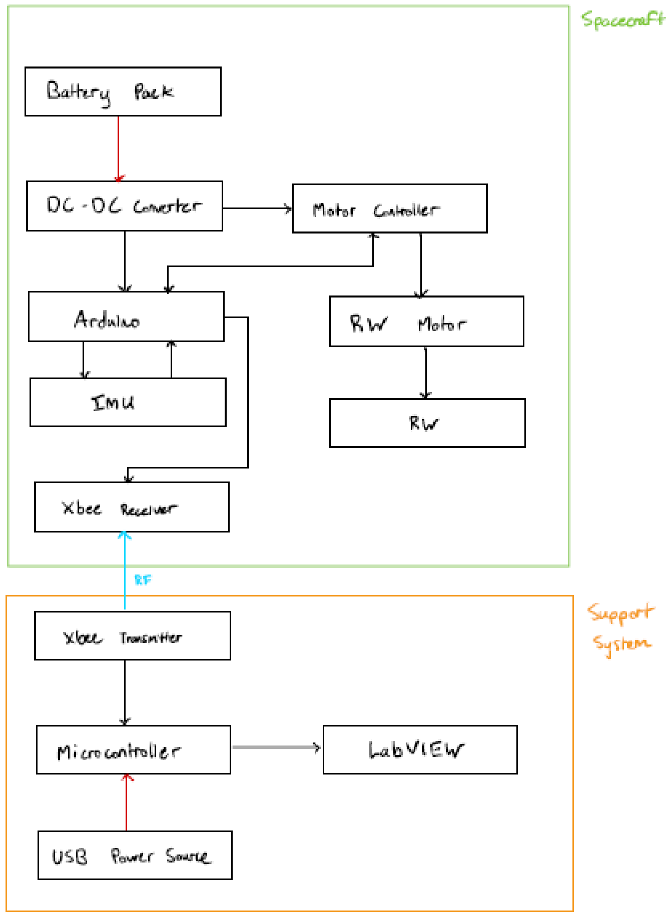

Fig. 4 Block Diagram of MEMS Gyro Setup

The battery pack provides the desired voltage values in order to operate the motors. The DC-DC converter temporarily stores electrical energy for the purpose of converting direct current (DC) from one voltage level to another. The motor controller tells the motor what to do. The reaction wheel motor spins up when told to do so in order to make the reaction wheel spin, producing a torque and spinning the module. The Arduino reads an input from the motor controller and the DC-DC converter in order to produce an output to the IMU. The inertial measuring unit measures the specific angular rate of the system. The XBee receiver and transmitter communicate the signal from the Arduino to the microcontroller. A microcontroller has an input/output connected to the LabVIEW that allowed for user input to be communicated to the spacecraft system via the XBee. Finally, the USB power source provides power to the microcontroller so it can do its job.

3.1b. When being pushed in a manual, sinusoidal wave by the user, the system is very smooth, and all error comes from the user. The gyro outputs a voltage proportional to the angular rate. As the angular rate increases, the voltage increases as well. This value is determined based off of the gyro's sensitivity, measured in volts/radian/second.

3.1c. **Figures**

Test	Adj. Scale Factor	Bias
Manual	-.8646	-.0301
.1Hz	-.8614	-.0303
.2Hz	-.8623	-.0302
.4Hz	-.8626	-.0298
Mean	-0.8627	-0.0301
STD	0.0013	2.0560e-04

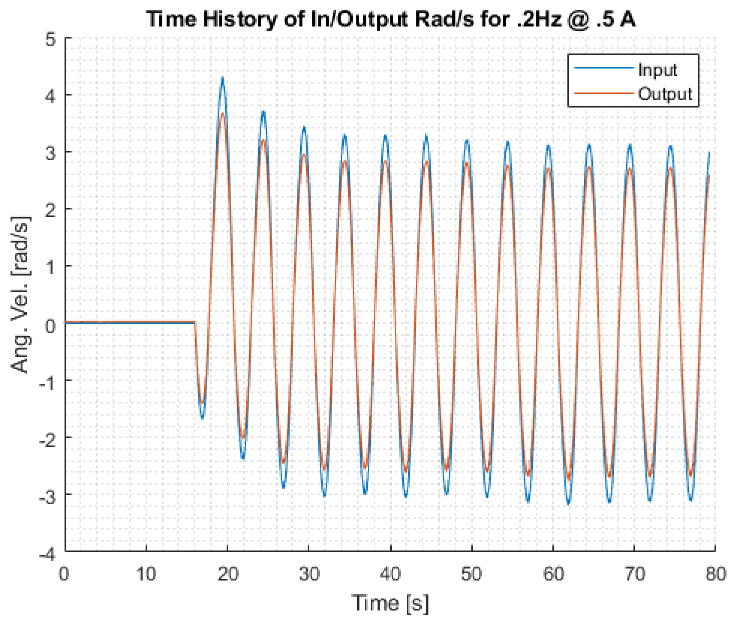

Fig. 5 Time History of Angular Velocity 3.1.c.1

3.1d. With the data we gathered through four separate trials, we were able to fairly accurately calculate the sensitivity of the MEMS Gyro. Looking at 11 and **??**, we would consider it to be accurate for all rates that we experimented with. Neither of the plots had large error spikes around any specific range of input rates. Our manual test was not particularly jarring towards the MEMS Gyro and we did not see any issues with the data. The gyro bias should be easily and consistently repeatable, which is important to allow for double checking this specific device, and also in that the practice can be done for any generic MEMS Gyro device, allowing for more accurate data from said generic MEMS Gyro devices. According to our data, 9 and 10, the error between measured and true position was fairly consistent, and the error followed a sinusoidal curve. The error changes over time as most of the error in the data comes from the amplitude of the peaks. The "true" position, calculated from commanded input rates is larger in magnitude that the "measured" position which is where most of the error comes from. This is because the "measured" positional data deals with reality, and the commanded rates do not. In reality, the "measured" position falls behind at the peaks and troughs due to limitations in the hardware.

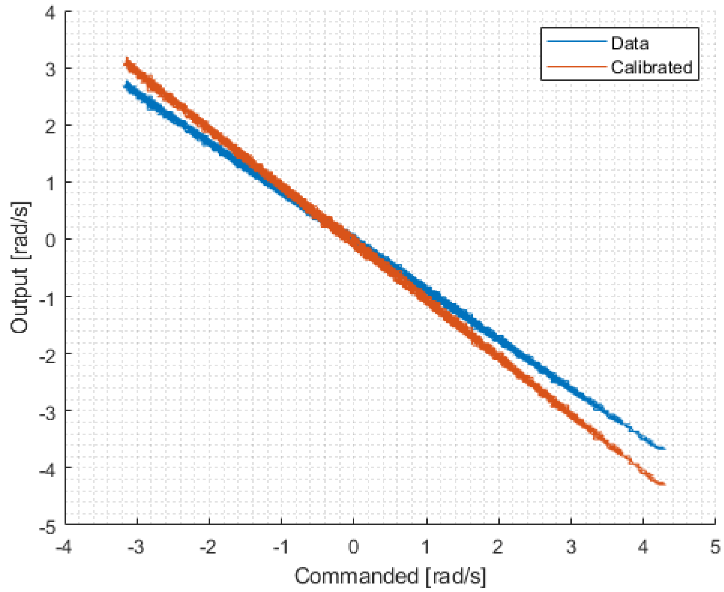

Fig. 6 Commanded vs Output Angular Velocities 3.1.c.2

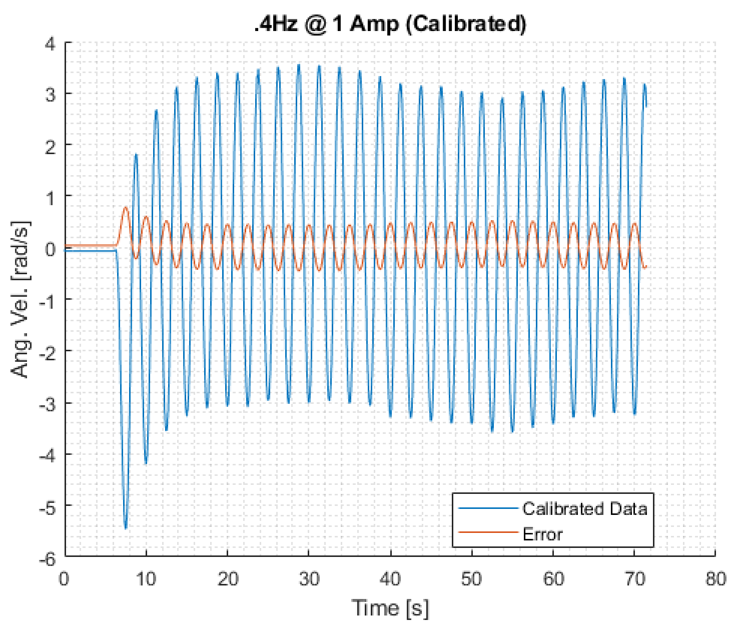

Fig. 7 Angular Velocity vs Time 1 [3.1.c4ab1]

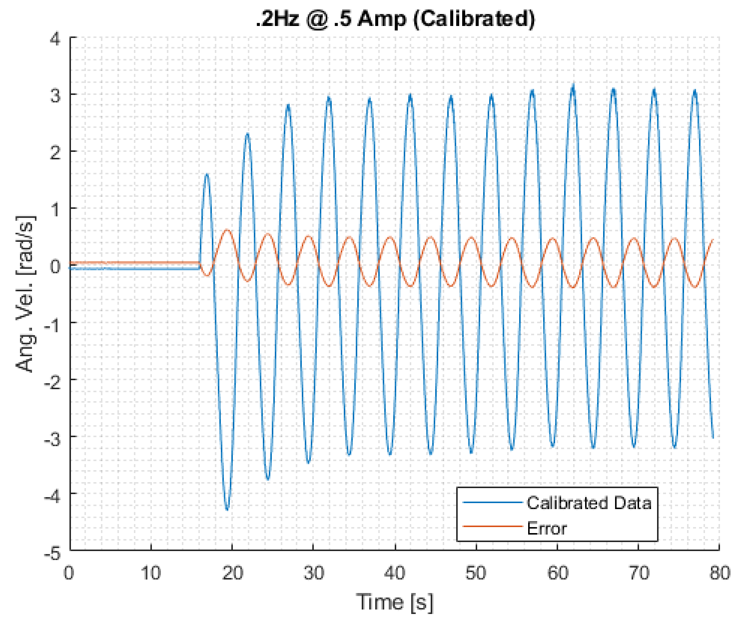

Fig. 8 Angular Velocity vs Time 2 [3.1.c4ab2]

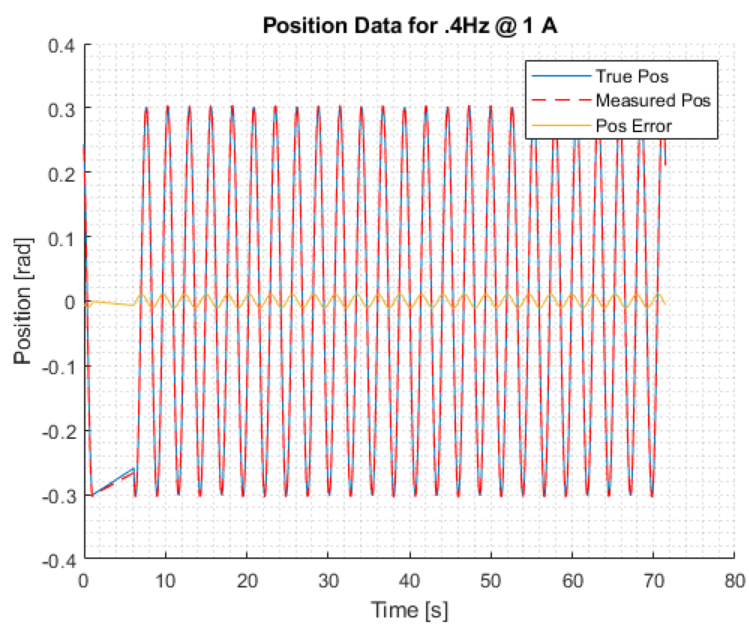

Fig. 9 Angular Position and Error 1 [3.1.c4cd1]

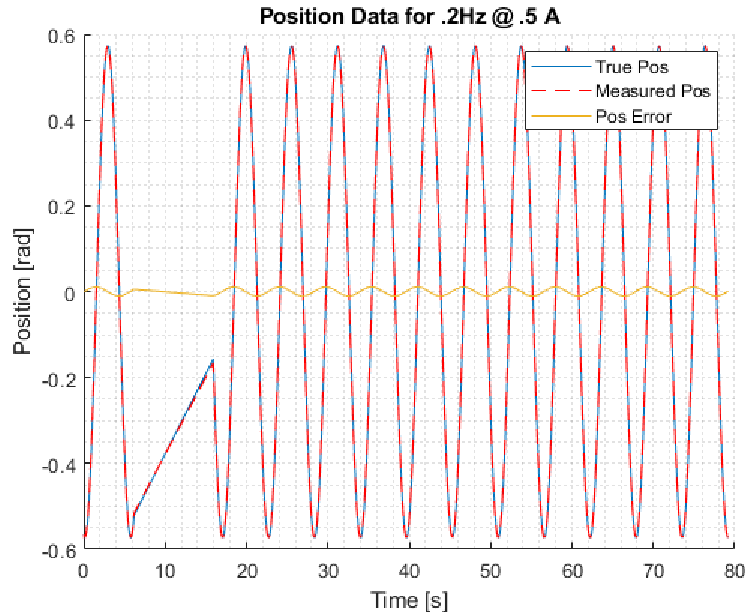

Fig. 10 Angular Position and Error 2 [3.1.c4cd2]

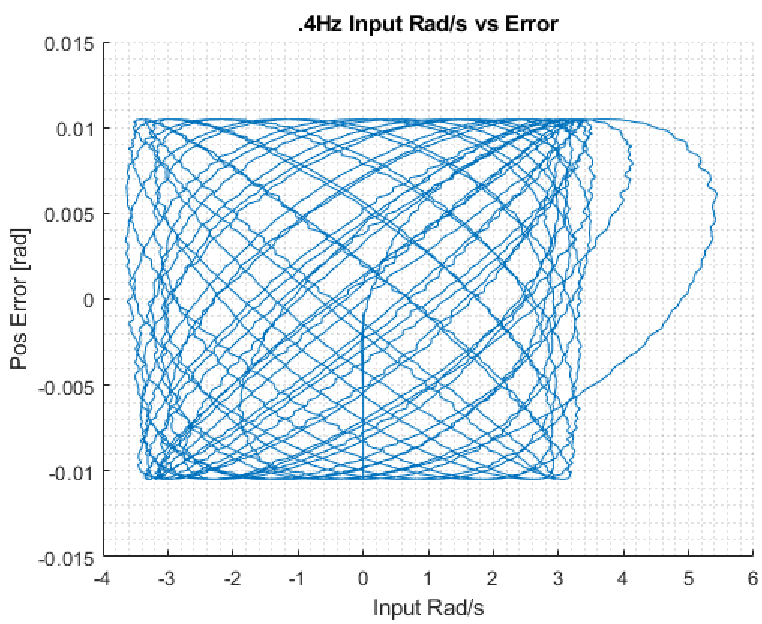

Fig. 11 Commanded Input Rate vs Positional Error 1 [3.1.c4e1]

2. Section 3.2

3.2a.

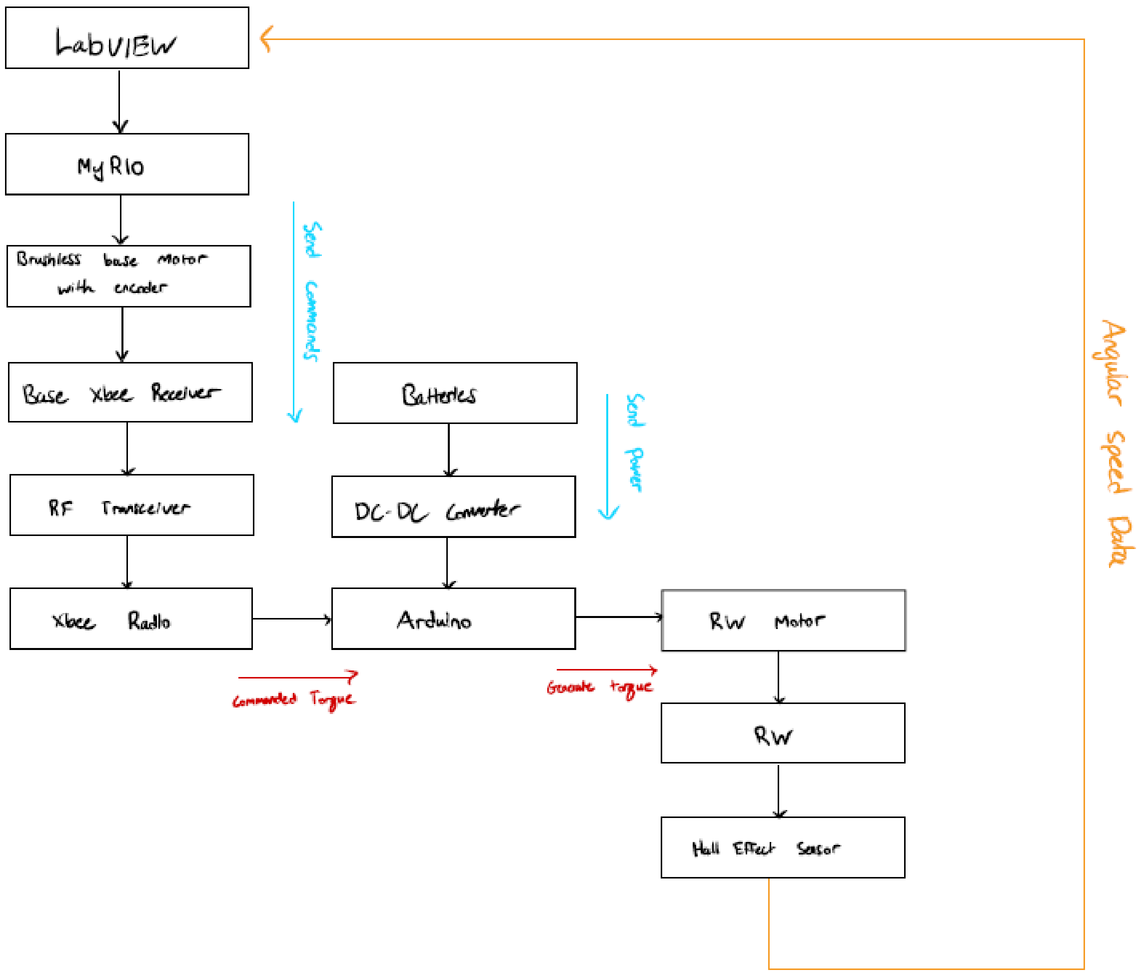

Fig. 12 Block Diagram of Reaction Wheel Torque Setup

The LabVIEW provides an interface for the user to input desired values for the system to abide by. The MyRIO is a reconfigurable I/O that allows for the implementation of multiple design concepts. The base motor with encoder receives a signal from the MyRIO, and sends it through the XBee Receiver, RF transmitter/receiver, and XBee Radio transmitter. This signal gets sent to the Arduino, which gets power from the DC-DC converter that increases the voltage from the batteries that power the Arduino. The Arduino then interprets this signal and sends it to the reaction wheel motor, which will spin up the reaction wheel. This reaction wheel physically moves the reaction wheel. The reaction wheel affects the Hall Effect Sensor, which detects the presence and magnitude of a magnetic field, and the output is directly proportional to the strength of the magnetic field. This is then compared to the desired input in the LabVIEW, and the cycle repeats again.

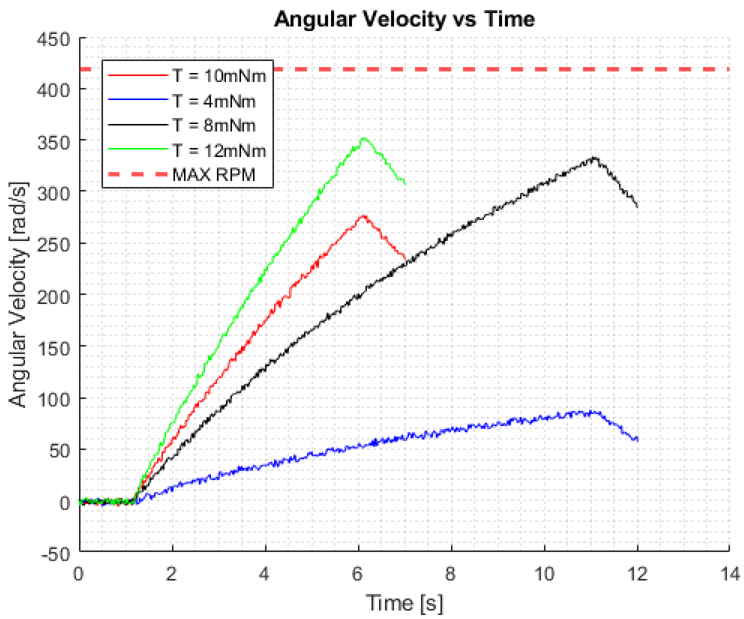

Fig. 13 Angular Velocity vs Time

3.2c. Below is a plot of the angular velocity as a function of time of the reaction wheel as a result of some torque acting on it. If we project those lines all the way up to the upper limit, or the max rpm, we can infer that the reaction wheel can resist an aerodynamic torque for about about 8 to 16 seconds, depending on the torque, before it would be to fast. In our cases, we remove control before we hit this limit, as shown on the dips in the plot. The angular momentum capacity of the wheel can be calculated using L = I*w. We determined that the wheel's moment of ineria is 0.2546 kgm^2 by calculating the applied torque from the actual current. Accounting for correct scaling, we took the average among several tests and fit the data. Knowing this moment of inertia, as well as the maximum angular velocity of 418.867 rad/sec (as depicted in the graph), we can determine that our maximum angular momentum is 106.64 kgm^2/s.

3. Section 3.3

3.3a. Our calculations for the Moments of Intertia were done for both the reaction wheel as well as the spacecraft, using the same method. We had the option of doing our calculations using the actual torque, or the commanded torque. For the sake of getting more accurate measurements, we calculated our actual applied torque from the actual current (in mNm/amp), to actual Newtons using the torque constant of 25.5 mNm/A. From there we took the average torque scalars for each test from the data with only positive current. Next, we took fits of the angular velocity for each test, and used those to find the moments of inertia for the reaction wheel and spacecraft. This was done by dividing the average torque by the angular velocities for each test, and from there taking the average of each tests' calculated MoI.

Our reaction wheel moment of inertia came out to a magnitude of 0.2546 kgm^2, and our spacecraft moment of inertia came out to a magnitude of 7.1263 kgm^2. This was then used in our calculation of our gains, as shown later.

3.3b.

a. For a proportional gain of K1 = 50 mNm, the spacecraft returns close to the initial angle, but it takes a while and overshoots the whole time. With a proportional gain of K1 = 100 mNm, the spacecraft overshoots a bit more, and the reaction wheel spins up much faster in order to counteract the displacement of one radian. Any gain higher than K1 = 100 mNm causes the reaction wheel to speed up too quickly, above 4000 RPM.

b. In order to bring the system to a stop without paying attention to angular position, the K2 gain needs to be large while the K1 gain is zero. When we implemented this, we saw that the system was damped and stopped quickly.

c. For gains values of K1 = 75 mNm and K2 = 15 mNm, the reaction wheel fairly quickly returned to the angular position that was inputted (0.5 radians), and didn't overshoot much at all. Every 10 seconds, according to the input frequency of 0.05 Hz, the spacecraft rotated to the expected reference height of 0.5 radians, giving it the expected period. When we increase K1 to 100 mNm and leave the K2 value at 15 mNm, slight wobbling and overshoot occurs. When K1 = 100 mNm and K2 is increased to 30 mNm, there is basically no oscillation when traveling back to the reference position. For a K1 value of 50 mNm and a K2 value of 30 mNm, the system returns to the position slowly, but with no oscillation or overshoot. Thus, a high K1 value will have high overshoot by itself but will return the system to its reference quickly, and a K2 will slow the system response but prevent overshoot or oscillation around the reference angle.

3.3c. We ended up calculating out the following values for K1 and K2: 81.317 mNm/rad and 28.463 mNm/rad^2, respectively. This was done using the equations of motion briefly denoted in the block diagram earlier, and expanded out below. We used our second-order equation as well as our equation for torque using gains, derived in part (or alternatively, expressed) in the block diagram and the corresponding transfer functions. The values were solved using the experimentally calculated moments of inertia discussed earlier, as well as using the natural frequency and damping ratios calculated. That is explained in the following figure.

$$\tau_c = I\ddot{\theta} = -K_p \underbrace{\Delta\theta}_{(\theta_m - \theta_d)} - K_D \underbrace{\Delta\dot{\theta}}_{(\dot{\theta}_m - 0)}$$

$$\dot{\theta}_d = 0$$

$$I\ddot{\theta} = -K_p(\theta_m - \theta_d) - K_D(\dot{\theta}_m - \cancelto{0}{\dot{\theta}_d})$$

$$\ddot{\theta} + \frac{K_D}{I}\dot{\theta} + \frac{K_p}{I}\theta = 0$$

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

$$K_D = I \cdot 2\zeta\omega_n = (7.1263) \cdot 2(0.5912)(3.378) = \boxed{28.463\ \frac{mNm}{rad^2}}$$

$$K_p = I \cdot \omega_n^2 = (7.1263)) \cdot (3.378)^2 = \boxed{81.317\ \frac{mNm}{rad}}$$

$$|moi \cdot 1000| = 7.1263$$

Fig. 14 Gains calculations

Below is how we calculated our poles from the settling time and overshoot. These poles then will allow us to predict the step response, as well as calculate those gains above to test.

5% $\tau_s < 1.5\,s$

overshoot < 10%

$$OS = e^{-\frac{\zeta}{\sqrt{\pi^2 - \zeta^2}}} \Rightarrow \zeta = \frac{-\ln(OS)}{\sqrt{\pi^2 + \ln(OS)^2}}$$

$$\zeta = \frac{-\ln(0.1)}{\sqrt{\pi^2 + \ln(0.1)^2}} = \boxed{0.5912}$$

$$1.5 = \frac{-\ln(5\%)}{\zeta \omega_n} \Rightarrow 1.5 = \frac{-\ln(0.05)}{0.5912 \cdot \omega_n} \Rightarrow \omega_n = \boxed{3.378 \tfrac{rad}{s}}$$

$$\text{poles} = -\zeta \pm \sqrt{\zeta^2 - 1}\ \omega_n$$

$$= -0.5912 \pm \sqrt{0.5912^2 - 1}\ (3.378)$$

$$= \boxed{-0.5912 \pm 2.724i}$$

Fig. 15 Poles Determination

3.3d.

We can start by predicting the response of our system given the poles calculated above, as well as the inputted initial condition. The response of a system given certain eigenvalues can be described using the equation below:

$$\theta(t) = A_1 * exp(\lambda t) \tag{13}$$

This was then fed into matlab and plotted over a reasonable time scale to yield the following plot.

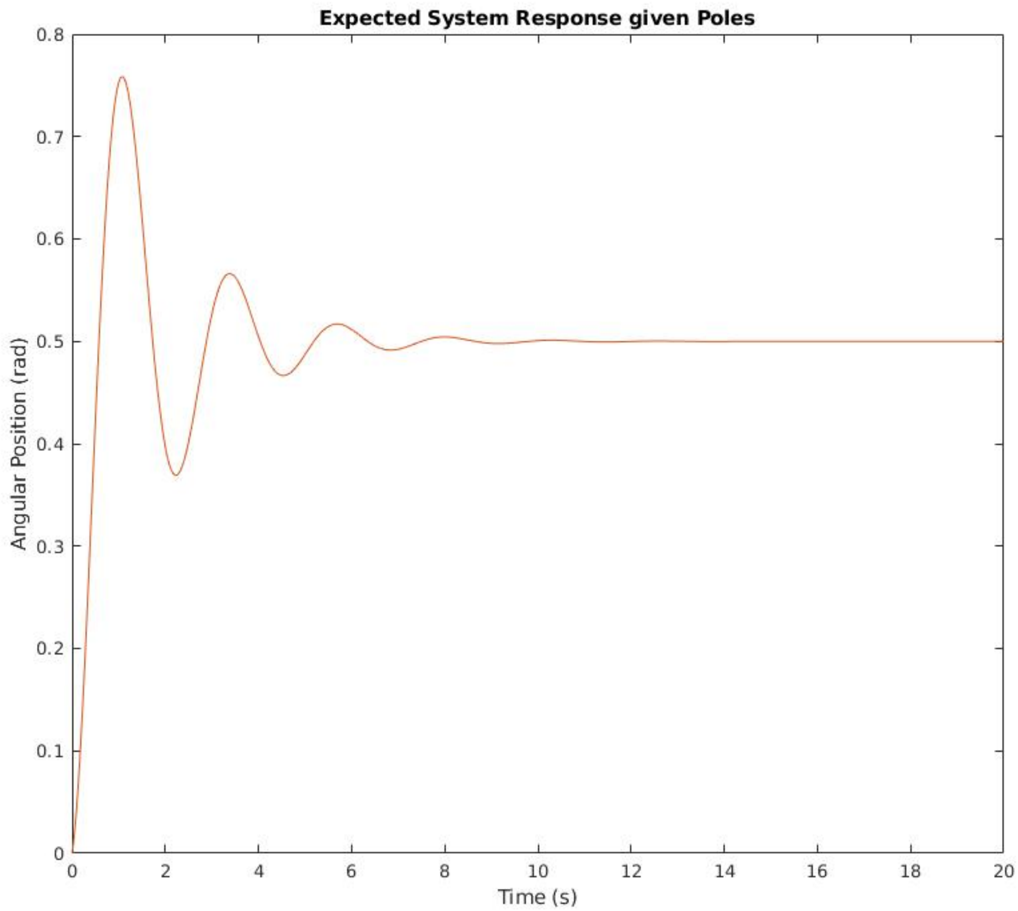

Fig. 16 Expected System Response

Our real data with these inputted gains isn't quite as pretty, but in fact returns to the desired start a fair bit quicker. That is demonstrated below in the following two plots of our actual data:

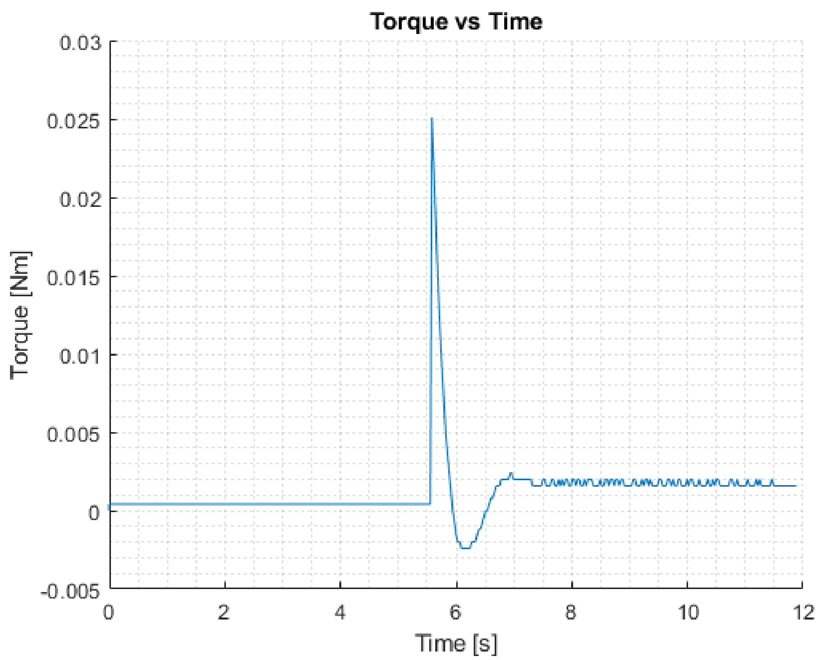

Fig. 17 Torque Response

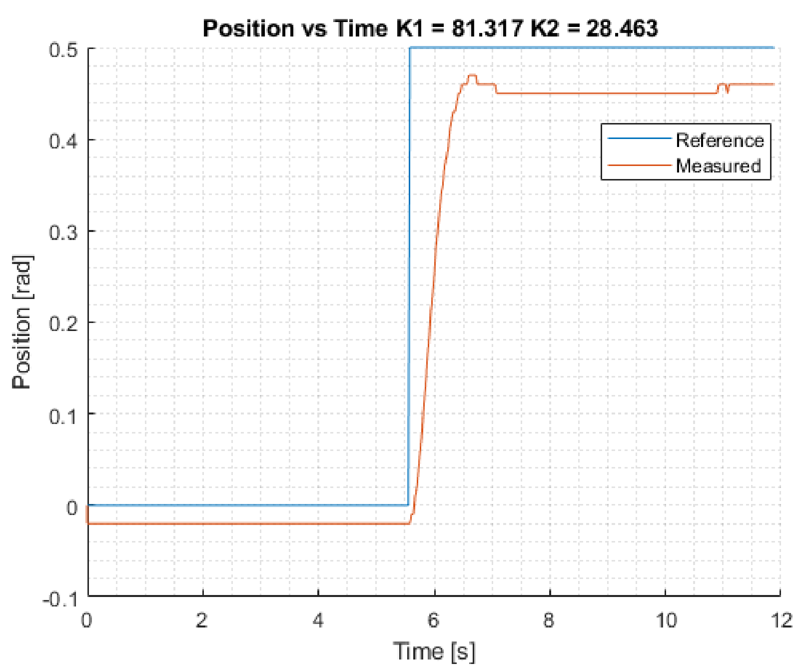

Fig. 18 Spacecraft Position

Above it is evident that the gains very nicely allow the spacecraft to input a proper response torque to quickly return it to the reference angle. Therefore we can infer that the reaction wheel is likely reacting in accordance with the torque we are seeing in the plot, which would make sense due to the angular rate change for the spacecraft. As more torque is applied to the wheel, the greater its angular velocity, and the more the spacecraft is able approach the reference angle. There is the slight oscillation shown in both plots that line up with what we expect to see given the overshoot and settling time requirements discussed earlier

III. Acknowledgements

A. Participation Table

X's are in place for those who have acknowledged the listed numbers. 0's are to indicate someone who did not participate in the listed task, 1's explain that someone was involved in the task. A 2 means someone was a leader in the task.

Names	Plan	Model	Experiment	Results	Report	Code	ACK
Alex Havens	1	1	1	2	1	2	X
Drew Brabec	2	1	1	1	1	1	X
Hayden Gebhardt	1	2	2	1	2	1	X
Winnie Regan	1	1	1	2	1	1	X

Appendix

B. Derivations

$$N = \{\hat{x}, \hat{y}, \hat{z}\}$$

$$\mathcal{E} = \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$$

$$M = {}^{N}\frac{dL}{dt} = {}^{\mathcal{E}}\frac{dL}{dt} + \omega \times L$$

Choose body axis $\hat{e}_i$ to be principal axis s.t.

$$L_i = \Sigma I_i \omega_i$$

$$L = I_x \dot{\omega}_1 \hat{e}_1 + I_y \dot{\omega}_2 \hat{e}_2 + I_z \dot{\omega}_3 \hat{e}_3 + \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ \omega_1 & \omega_2 & \omega_3 \\ I_x\omega_1 & I_y\omega_2 & I_z\omega_3 \end{vmatrix}$$

$$\Rightarrow L_x = I_x \dot{\omega}_1 - (I_y - I_z)\,\omega_2\omega_3$$

$$L_y = I_y \dot{\omega}_2 - (I_z - I_x)\,\omega_3\omega_1 \qquad \dot{\omega} = 0$$

$$L_z = I_z \dot{\omega}_3 - (I_x - I_y)\,\omega_1\omega_2$$

$$L_z = -(I_x - I_y)\,\omega_1\omega_2$$

$$I_y = \frac{1}{2}MR^2 \qquad I_x = \frac{1}{4}MR^2$$

Fig. 19 Justification of Rate Gyro Behavior about Z-axis

C. MATLAB Code