University of Colorado - Boulder

ASEN 3200 - Orbital Mechanics

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LAB A2: Spinning Spacecraft

Author: *Ben Klementovich*

Author: *Blair Schulze*

Author: *Hayden Gebhardt*

Professor: Casey Heidrich

Ann and H.J. Smead
Aerospace Engineering Sciences
UNIVERSITY OF COLORADO **BOULDER**

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I. Abstract

The purpose of this lab is to see the effects of external torques, and energy dissipation on the spin stability of various spacecraft. This will be done through visualization in the form of spacecraft video files that show representations of angular velocity and angular momentum, as well as an experiment done in the lab to tie together the visualization of torque as well as the mathematics behind it. Using these tools as well as the knowledge gained in class the groups will take multiple different results and try to derive the external forces that create these results. This will allow us to have a better understanding of spin stability and torques not only intuitively but also mathematically.

II. Introduction

We will have two main parts to this lab: Firstly a spacecraft represented as a spinning rectangle with an angular momentum vector and an angular velocity vector. These will be in the form of downloadable mp4 files that can be viewed, there are three types of files for each body, an inertial frame, a body fixed frame, and a polhode plot. Secondly, there will be an in-class experiment in which the groups will spin up a bicycle wheel and attach it to a hanging chain via a small axis, then multiple measurements from the speedometer and a ruler will be taken.

These experiments will allow us to discover how prolate and oblate bodies interface with spin stability. This is relevant to spacecraft design and analysis as it allows us to have more control over the spinning motions involved in space. This can be used to stabilize an out-of-control craft or adjust and alter the current stability of a craft in order to position instruments to be more effective.

III. Spacecraft Animations

A. Preliminary Questions

1. What conditions are required for the angular momentum vector of a spacecraft to move as seen from an inertial frame?

The angular momentum vector of a spacecraft is often moving in an inertial frame, this is because the only condition needed for this to be true in an inertial frame is that the angular velocity vector is not aligned with the angular momentum vector. So although in a body frame, the angular momentum vector can be completely unchanging, in an inertial frame the body can be spinning in such a way that the angular momentum vector moves as well.

2. What do the terms oblate and prolate mean? Explain what we mean by spin near the major or minor axis. How can these be distinguished by observing spacecraft motion?

Prolate can be thought of as a human, or something tall and slender. As long as the axis of symmetry has a smaller inertia than the other two axes, the object is prolate. The opposite is true for an oblate body, its axis of symmetry inertia is large whereas the other axis inertias are smaller. The major axis is the longer axis, which has a smaller moment of inertia because the rest of the body's mass is closer to this axis. The other axis is therefore the minor one. So when we say spinning closest to an axis it means which axis (using the right-hand rule) is the angular velocity vector closest to. When looking at a spacecraft motion we can think of it as a domino (close representation to the mp4 files we were given). If instead of space the domino was spinning flat on a table it would be spinning about its minor axis, whereas if you were to spin it while it was standing up it would be spinning about its major axis. This can be imagined when looking at a spaceraft and then the observer can estimate what case the spacecraft is closest to.

3. How does the angular velocity vector move in the body frame when the body is rigid (no internal dissipation)? How does energy dissipation change this motion?

When the is no internal dissipation the angular velocity vector will trace out a circle, that is because it is spinning with the object and as long as there are no external torques or internal energy dissipation there will be no deviation in magnitude or path of the vector. If there is energy dissipation then the spinning body will seek out a low state of energy which is a spin about its major axis, the angular velocity vector will change according to what orientation the spacecraft was originally spinning in. This motion tends to have the angular velocity vector trace out a spiral.

4. If a spacecraft is spinning, describe how thrust must be applied to move the angular momentum vector in a desired direction.

In order to move the angular momentum vector's direction one must apply thrust perpendicularly, and if you wish to change the magnitude of the vector you need to apply it in parallel. If you wish to change both you can apply it at any non-orthogonal angle.

B. Description of Simulations

The general motion of these simulations will be described in an inertial frame as an observer would see it.

Case A: The spacecraft begins slowly spinning about its major axis with its major axis close to being aligned with the inertial y-axis. Then as the torque is applied the spacecraft tumbles and the axis of rotation is constantly changing until there is no longer torque applied. Finally, it ends up slowly spinning perfectly about its major axis as its angular momentum and angular velocity vectors are aligned about the -x, -z intermediate axis.

Case B: The spacecraft starts by very slowly spinning about its intermediate axis, with the craft rotating about its major axis 360 degrees every few circles that are traced out by the angular velocity vector. Then an external torque is applied and the craft enters a quick tumble where it ends up spinning again about an intermediate axis with a much faster spin rate. The spin rate is roughly two 360-degree spins about the major axis for every circle that is traced out by the angular velocity vector.

Case C: The spacecraft has a very slow spin rate throughout the entire motion, it begins spinning about its major axis with its angular velocity vector aligned with the positive z-axis. Then as the energy dissipates the craft slowly begins to spin about its minor axis until it is very close to the end of the animation.

Case D: The spacecraft begins by slowly spinning about its minor axis unstably, then it accelerates and spins about a transverse axis for a short time until it returns to spinning about its minor axis. This process forms a cycle that is constantly repeated.

Case E: The spacecraft begins spinning about its intermediate axis not aligned with any particular inertial frame axis. Then as a torque is applied, the spin rate of the spacecraft increases rapidly as the axis of rotation remains constant. The increase in spin rate eventually stops and the animation is restarted.

C. Analysis of Results

Case A: The body is initially in torque-free motion and then a torque that seems to be about the -x axis in the inertial frame is applied which causes the body to lose rotational energy and then it returns to spinning with its angular velocity vector and angular momentum vector aligned in a maximum rotational energy state. The torque is negative and acting about the body frame z-axis which causes the angular momentum vector to become more and more negative in the z-axis until it is aligned with the negative z-axis. There is internal energy dissipation in this case but only for a short period of time when the angular momentum vector approaches the body fixed x-axis. That being said, the angular velocity vector starts aligned with the transverse axis, then travels to the minor axis for a short time, and ends up aligned with the major axis.

Case B: There is a torque present that is introduced after the start of the motion. The angular momentum vector is initially precessing about the negative x-axis in the body fixed frame, then after the torque is applied it begins to precess about the positive z-axis in the body fixed frame. Energy dissipation is not present, the energy only increases over the course of the motion due to an external torque. The angular momentum vector starts by precessing closely around the minor axis(negative-x) in a state close to minimum internal energy. Then an external torque is applied which causes the angular momentum vector to precess about the major axis(positive-z) and is almost in a maximum internal energy state.

Case C: The angular momentum vector does not change in either magnitude or direction, so there is no torque applied in this case. There is internal energy dissipation present, shown by the slowly decreasing size of the kinetic energy ellipsoid. The spin begins around the major axis and slowly shifts to be around the minor axis of the object in the inertial frame animation.

Case D: There are no torques present in this model since the angular momentum vector does not change in magnitude or direction over time. Since there are no torques acting on this body the angular momentum ellipsoid in the polhode plot does not undergo any changes throughout the animation, so there is no internal energy dissipation. In this model, the spin precesses counterclockwise about the z-axis, while the angular velocity vector precesses around the z-axis closer to the x-y plane.

Case E: There is a torque present in this rotation such that the angular momentum increases in magnitude but not direction. The torque acts in the same direction as the angular momentum vector, causing the magnitude of the angular momentum vector to grow and the angular velocity vector to speed up along its same precession path. There is no internal dissipation in this case since the blue angular momentum ellipsoid in the polhode animation increases over time at the same rate as the pink angular kinetic ellipsoid. The spin of this object is about halfway between the major and minor axes, and while this direction stays constant throughout the process the angular velocity of the spin is increased after the torque is applied.

IV. Bicycle Wheel Precesssion

A. Model for Precession

Wheel Diameter: 26 inches
Axle Length: 7.5 inches
Wheel mass: 8.2 lbs

The model used to determine the gyroscopic precession rate versus the wheel spin rate, as well as the equation for precession period can be seen below:

$$T = \frac{2\pi}{\omega_p} \tag{1}$$

$$\omega_p = \frac{L_{axle} g}{\omega_s r_{wheel}^2} \tag{2}$$

Please see section IV.C for a detailed derivation of the model.

B. Experiment

Starting speed (mph)	Period (s)
24	6.65
30	6.81
22	5.72
30	6.49
23	6.13

Diagram:

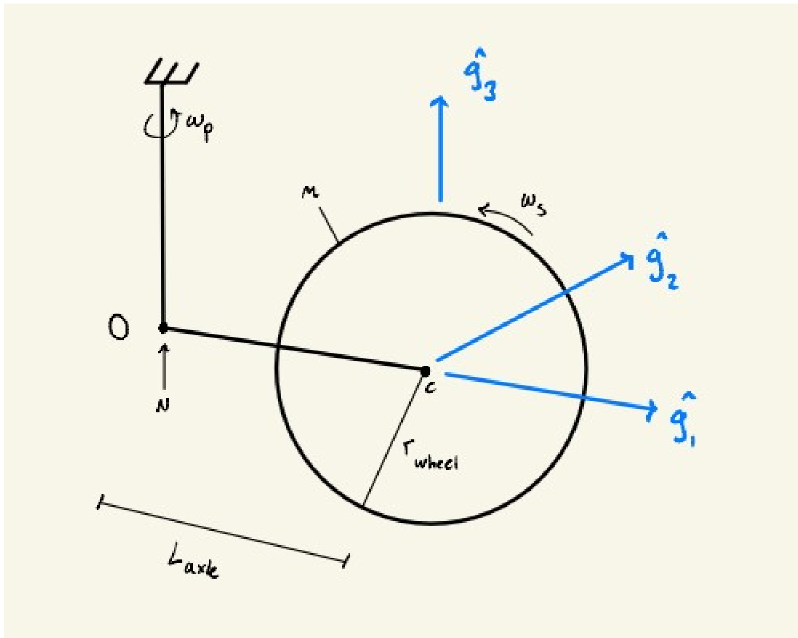

Fig. 1 Caption

Procedure: For this part of the lab, we spun a bicycle wheel vertically clockwise about a central axis along a rope. The speed of the rotating wheel was controlled using a hand-held motor such that the initial speed of each of our trials was consistently between around 25-30 mph. The period measurements were taken manually using a stopwatch.

Observations: For a wheel spun counterclockwise about its horizontal axis, the precession rate about the central vertical axis of the lab structure is also counterclockwise. Increasing the wheel spin rate, ω_p, caused an increase in the period of precession, and the inverse relationship between period and precession rate caused the precession rate of the system to decrease.

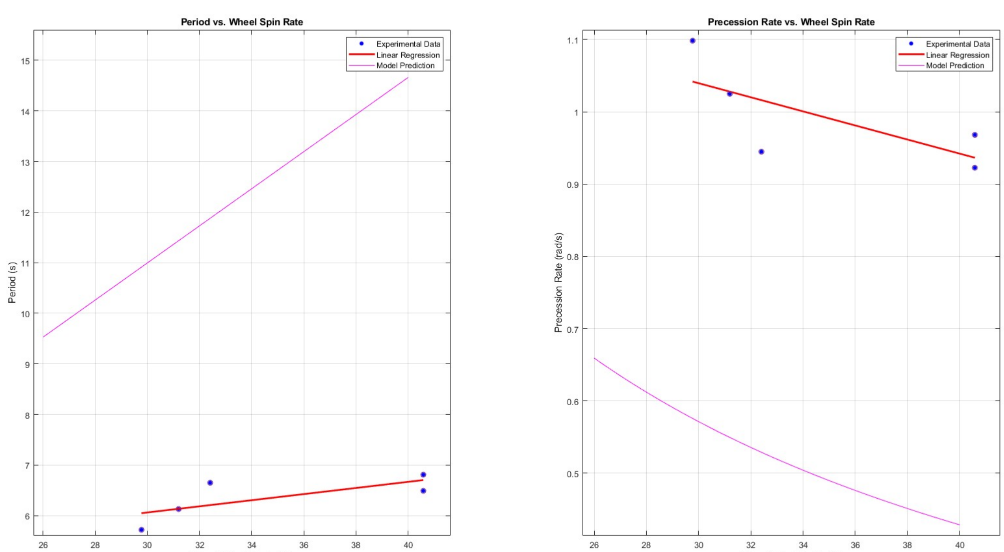

Fig. 2 Experimental Data vs. Derived Model

C. Analysis of Results

To derive the gyroscopic precession rate as a function of wheel spin, we assume that the only forces acting on the wheel are gravity, and an applied torque. We assume that friction and pendular motion are negligible and that all of the wheel's mass is allocated along its rim (wheel modeled as a hoop). From these assumptions, we can reduce the relationship of our variables to:

$$\omega_p = \frac{\tau}{L_{axle}} = \frac{L_{axle} g}{\omega_s r_{wheel}^2} \tag{3}$$

A comparison of the predicted precession rate and the measured precession rate can be seen in the table below. The main source of error in these measurements can be attributed to human error. The offset seen between the experimental and model curves is largely due to timing inaccuracies in the speed and period measurements.

Wheel Spin Rate (rad/s)	32.4913	40.6141	29.7837	40.6141	31.1375
Predicted Precession Rate (rad/s)	0.5275	0.4220	0.5755	0.4220	0.5505
Experimental Precession Rate (rad/s)	0.9448	0.9226	1.0985	0.9681	1.0250
Error	0.7911	1.1863	0.9088	1.2941	0.8621

V. Conclusions and Recommendations

The purpose of this lab was to see the effects of external torques, and energy dissipation on the spin stability of various spacecraft. This was done through visualization in the form of spacecraft video files that show representations of angular velocity and angular momentum, as well as an experiment completed in lab to tie together the visualization of torque as well as the mathematics behind it. Using these tools as well as the knowledge gained in class, multiple different results were recorded and compared to the mathematical models. This facilitated the understanding of spin stability and torques not only intuitively but also mathematically. In reflection, it would have been extremely beneficial to ensure the accuracy of the recorded experimental data for wheel speed and period.

VI. Acknowledgements

Name	Contribution	Percent Contribution
Blair Schulze	Bicycle wheel Precession, some simulation analysis	100%
Hayden Gebhardt	Bicycle Wheel Precession, diagram	100%
Ben Klementovich	Spacecraft Animations, introduction/abstract/conclusion	100%

VII. Appendix

A. MATLAB Code

```matlab
% Hayden Gebhardt, Blair Schulze, Ben Klementovich
% ASEN 3200 - Lab 2
% 02/24/2023

%% Housekeeping

clc
clear all
close all

%% Plots

g = 9.81; % [m/s]

d_wheel = 0.6604; % [m]
r_wheel = d_wheel / 2; %[m]
l_axle = 0.1905; % [m]

m_wheel = 3.71946; % [kg]

%% Experimental Data

T = [6.65, 6.81, 5.72, 6.49, 6.13]; % [s]

omega_s = [10.7, 13.4, 9.83, 13.4, 10.3]; % [m/s]
omega_s = omega_s ./ r_wheel; % [rad/s]

omega_p = ((2*pi) ./ T); % [rad/s]

coefficients = polyfit(omega_s, T, 1);
coefficients2 = polyfit(omega_s, omega_p, 1);

x_fit = linspace(min(omega_s), max(omega_s), 1000);
y_fit = polyval(coefficients , x_fit);
y_fit2 = polyval(coefficients2 , x_fit);

%% Model Data

omega_s_th = linspace(26, 40, 1000);
omega_p_th = (l_axle * g) ./ (omega_s_th * (r_wheel^2));

period_th = (2*pi) ./ omega_p_th;

figure(1)
subplot(1, 2, 1)
plot(omega_s, T, 'b.', 'MarkerSize', 15);
hold on
plot(x_fit, y_fit, 'r-', 'LineWidth', 2); % Plot fitted line.
hold on
plot(omega_s_th, period_th, 'm')
hold on
scatter(omega_s, T)
grid on;
ylabel("Period (s)")
```

```
xlabel("Wheel Spin Rate (rad/s)")
legend("Experimental Data", "Linear Regression", "Model Prediction")
title("Period vs. Wheel Spin Rate")

subplot(1, 2, 2)
plot(omega_s, omega_p, 'b.', 'MarkerSize', 15);
hold on
plot(x_fit, y_fit2, 'r-', 'LineWidth', 2); % Plot fitted line.
hold on
plot(omega_s_th, omega_p_th, 'm')
hold on
scatter(omega_s, omega_p)
grid on;
ylabel("Precession Rate (rad/s)")
xlabel("Wheel Spin Rate (rad/s)")
legend("Experimental Data", "Linear Regression", "Model Prediction")
title("Precession Rate vs. Wheel Spin Rate")
```