University of Colorado Boulder

ASEN 3112: Structures

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Lab 4: Beam Deflection

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Abstract

The purpose of this lab was to explore the mechanisms of buckling and the accuracy of using Euler columns to predict the behavior for buckling. Two different samples, a rectangular and square cross section, were tested while recording loading and lateral deflection. These experimental values were then compared to our analytical approximations to validate the accuracy of this model.

Contents

1 Results

1.1 Question 1: Buckling Load

Predicted buckling load of the aluminum square hollow cross section:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{1}$$

Using the dimensions of the square hollow beam:

$$\frac{\pi^2 * 10,000,000 * (\frac{0.25^4}{12} - \frac{0.125^4}{12})}{11.0625^2} = 246.1179\ psi \tag{2}$$

The predicted buckling load of the rectangular solid cross section is equivalent to equation 2:

$$\frac{\pi^2 * 10,000,000 * (\frac{0.125^3}{12})}{11.0625^2} = 131.2629\ psi \tag{3}$$

A table comparing the predicted and experimental buckling loads for each beam can be found below:

	Predicted Buckling Load	Experimental Buckling Load
Hollow Beam	246.1179 psi	**239.241** psi
Solid Beam	131.2629 psi	**89.873** psi

Table 1: Comparison of experimental and analytical data.

The percentage error between the predicted and experimental buckling loads are: **-2.7941**%, **-31.5321**% for the hollow and solid beams, respectively. The error between these results can be attributed to many things, but the largest source of error is likely due to inaccuracies in the voltage and displacement calculations. These measurements were made with the naked eye, rather than a more accurate instrument. An additional source of error could come from the clamps that held each beam in the test section. The beams were gripped with contact screws, which could change the overall deflection of the beam.

1.2 Question 2: Post-Buckling Behavior

To determine the predicted lateral deflection that corresponds to the initiation of plastic post-buckling, we first calculated the yield strain from the given data:

$$\epsilon_y = \frac{\sigma_{yield}}{E} \tag{4}$$

The result of this calculation was a yield strain of **0.0035**. From this, we calculated the lateral deflection:

$$\epsilon = k(x)y \tag{5}$$

where

$$k(x) = v''(x) \tag{6}$$

and

$$v(x) = \delta \sin\frac{\pi x}{L} \tag{7}$$

Using these relations, we can compute the curvature of the test member, equation 6, by differentiating equation 7 twice, with respect to x:

$$v''(x) = -\frac{\pi^2\delta}{L^2}\sin\frac{\pi x}{L} \tag{8}$$

Substituting equation 6 into equation 5 yields the following:

$$\epsilon = \frac{\pi^2\delta}{L^2}\sin\frac{\pi x}{L}y \tag{9}$$

We can drop the negative sign, as the deflection of the beam can occur in both the positive and negative y-direction. From this, we can equate epsilon to the desired lateral deflection:

$$\sigma = \frac{\epsilon L^2}{\pi^2 \sin\frac{\pi x}{L}y} \tag{10}$$

These equations allowed for the calculation of predicted lateral deflections at plastic yield by substituting the yield strain and (x[1], y) data for each test member:

[1]The 'x' data is represented as $\frac{1}{2}$ the length of the test member.

Test Article	Yield Strain	'x' data	'y' data
Hollow Beam	0.0035	5.53125 in	0.125 in
Solid Beam	0.0035	5.53125 in	0.0625 in

Table 2: Comparison of experimental and analytical data.

Finally, from these values, the predicted lateral deflections at plastic yield are **0.1875** inches and **0.0875** inches for the hollow and solid beams, respectively. A plot of the predicted and experimental loads can be found below. The vertical bars correspond to the yielding horizontal displacement for buckling:

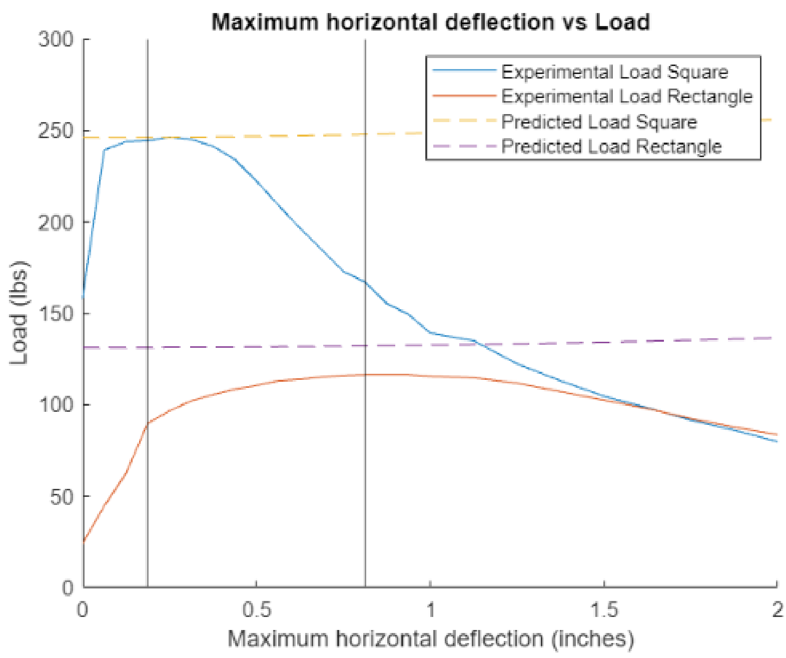

Figure 1: Predicted and experimental applied loads vs. lateral beam deflections

1.3 Question 3: Design Study

In this design study, we analyzed two beams with identical cross sections and material composition as the two members we tested. This time, however, we varied their lengths. The following figure illustrates the buckling load for both beams under the following considerations: (1) simply supported - simply supported, (2) fixed - fixed boundary conditions. The third component of these plots identifies the load that will result in beam yielding, for a compressive loading scenario:

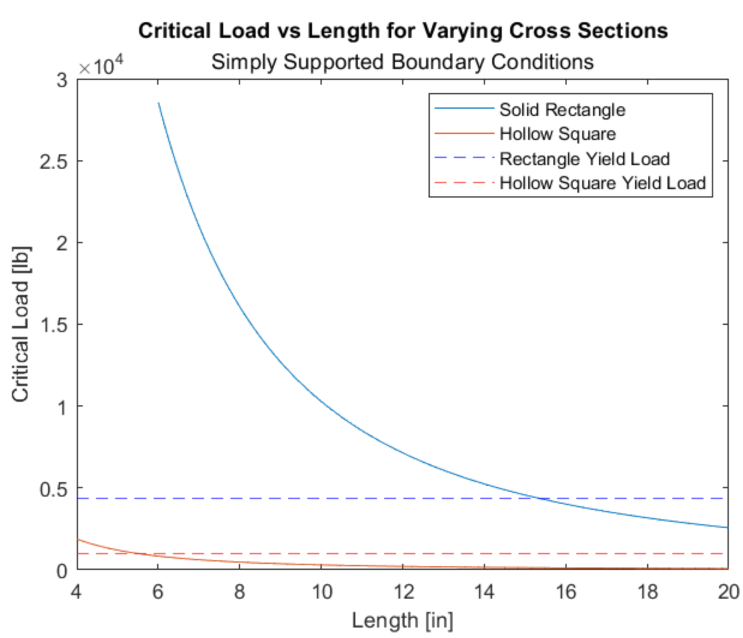

Figure 2: Length vs Critical Load for Simply Supported Boundary Conditons

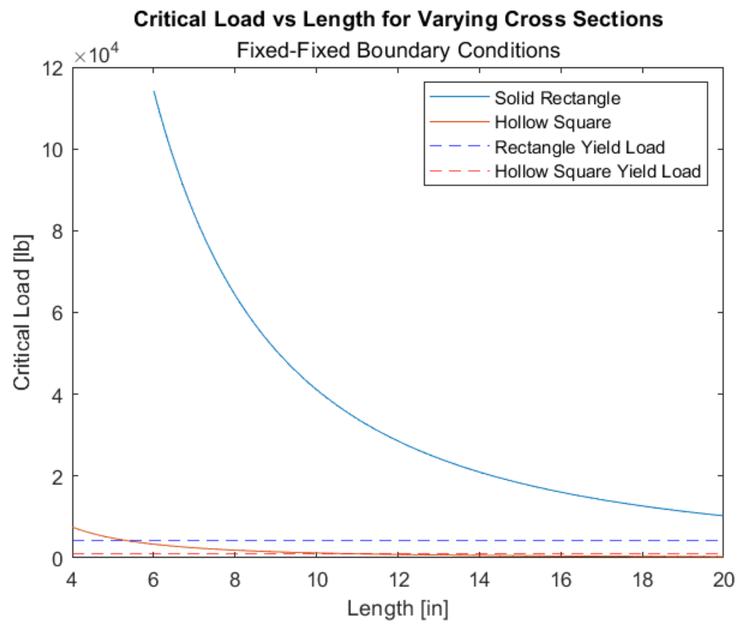

Figure 3: Length vs Critical Load for Fixed-Fixed Boundary Conditons

As can be seen in the plots, the solid rectangular cross section has a larger critical buckling load for both boundary conditions. Based on our calculations, it also has a larger yield load, meaning that it will be able to take much higher loads before transitioning into plastic deformation. These results indicate that for these cross sections, the square rectangle is more resilient in these loading scenarios, making it more versatile in situations where large compressive loads are being applied.

2 Appendix

2.1 Participation Report

Name	Contribution	Signature
Andrew Vo	Report, Experiment, Analysis, Code	AV
Connor Triplett	Report, Experiment, Analysis, Code	CT
Bo Iacobbo	Report, Experiment, Analysis, Code	BI
Chaney Sullivan	Report, Experiment, Analysis, Code	CS
Hayden Gebhardt	Report, Experiment, Analysis, Code	HG
Darius Mirhosseini	Report, Experiment, Analysis, Code	DM

2.2 MATLAB Code

```
clear;clc;close all;

LR = linspace(6,20,100); %in
LS = linspace(4,20,100); %in
E = 10000000; %psi
YS = 35000;
Yrec = YS*(.125*1);
Ysq = YS*(0.25^2-(0.25-0.0625)^2);
Irec = (1/12)*(0.125)*1^3;
Isq = (0.25^4)/12 - ((0.25-2*0.0625)^4/12);
PcrRec = (pi^2*E*Irec)./LR.^2;
PcrSq = (pi^2*E*Isq)./LS.^2;

figure
plot(LR,PcrRec);
hold on
plot(LS,PcrSq);
yline(Yrec,'b--');yline(Ysq,'r--');
xlabel('Length [in]');ylabel('Critical Load [lb]');title('Critical Load vs Length for ...
    Varying Cross Sections');
subtitle('Simply Supported Boundary Conditions');
legend('Solid Rectangle','Hollow Square','Rectangle Yield Load','Hollow Square Yield Load');

PcrffR = (pi^2*E*Irec)./(LR/2).^2;
PcrffSq = (pi^2*E*Isq)./(LS/2).^2;

figure
plot(LR,PcrffR);
hold on
plot(LS,PcrffSq);
yline(Yrec,'b--');yline(Ysq,'r--');
xlabel('Length [in]');ylabel('Critical Load [lb]');title('Critical Load vs Length for ...
    Varying Cross Sections');
subtitle('Fixed-Fixed Boundary Conditions');
legend('Solid Rectangle','Hollow Square','Rectangle Yield Load','Hollow Square Yield Load');
```

```
clc
clear
close all

data_rectangle = load('rectangle_beam_data.csv');
data_square = load('square_beam_data.csv');

load_square = data_square(:,2)./0.00237;
load_rectangle = data_rectangle(:,2)./0.00237;

P_cr_square = 246.1179;
P_cr_rectangle = 131.2629;
L = 11.0625;

P_square = P_cr_square.*(1+(pi^2/(8*L^2).*(data_square(:,1)).^2));
P_rectangle = P_cr_rectangle.*(1+(pi^2/(8*L^2).*(data_rectangle(:,1)).^2));

critical_exp_index_square = find(load_square>P_cr_square);
critical_exp_index_rectangle = find(load_rectangle==max(load_rectangle));

figure
hold on
plot(data_square(:,1),load_square)
plot(data_rectangle(:,1),load_rectangle)
plot(data_square(:,1),P_square,'--')
plot(data_rectangle(:,1),P_rectangle,'--')
```

```
xline(data_square(critical_exp_index_square-1,1),'k')
xline(data_rectangle(critical_exp_index_rectangle(1),1),'k')
legend('Experimental Load Square','Experimental Load Rectangle','Predicted Load ...
    Square','Predicted Load Rectangle')
title('Maximum horizontal deflection vs Load')
xlabel('Maximum horizontal deflection (inches)')
ylabel('Load (lbs)')
```