7.5 A Black–Scholes path generation engine

The Black–Scholes engine will produce paths from the risk-neutral Black–Scholes process. The paths will be an array of spot values at the times specified by the product. We allow the possibility of variable interest rates and dividend rates, as well as variable but deterministic volatility. The stock price therefore follows the process

$$dS_t = (r(t) - d(t))S_t dt + \sigma(t)S_t dW_t, \tag{7.1}$$

with S_0 given. To simulate this process at times $t_0, t_1, \ldots, t_{n-1}$, we need n independent $N(0, 1)$ variates W_j and we set

$$\log S_{t_0} = \log S_0 + \int_0^{t_0} \left(r(s) - d(s) - \frac{1}{2}\sigma(s)^2 \right) ds + \sqrt{\int_0^{t_0} \sigma(s)^2 ds}\, W_0, \tag{7.2}$$

and put

$$\log S_{t_j} = \log S_{t_{j-1}} + \int_{t_{j-1}}^{t_j} \left(r(s) - d(s) - \frac{1}{2}\sigma(s)^2 \right) ds + \sqrt{\int_{t_{j-1}}^{t_j} \sigma(s)^2 ds}\, W_j. \tag{7.3}$$