Superpowered Optimization in Python With Gurobi and Anaconda

Renan Garcia, Ph.D., Gurobi Optimization
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Renan Garcia, Ph.D.

- Optimization Support Engineer at Gurobi Optimization
- Ph.D. in Industrial and Systems Engineering, Georgia Tech
- Expert in optimization modeling and software development
- Over a decade of experience implementing decision support systems

Michael C. Grant, Ph.D.

- Senior Solution Architect at Continuum Analytics
- Ph.D. in Electrical Engineering, Stanford University
- Subject matter expert in nonlinear convex optimization
- Developer of the CVX modeling framework

Overview

- The Proposition
- Introducing Gurobi Optimization
- About Continuum Analytics & Anaconda
- The Gurobi Python module
- A simple MIP model using Gurobi & Spyder
- Portfolio optimization using Gurobi, Jupyter, Pandas, & Bokeh
- Wrap up / Call to action

The Proposition

- The Gurobi Python module provides the intuitive feel of a *modeling* language inside a powerful *programming* language
- The Anaconda platform delivers a full application development stack as well as a great interactive environment for data analysis and modeling
- Bringing developers and analysts/modelers together using Python means collaboration has never been easier

The Gurobi Optimizer

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Inside the Gurobi Optimizer

- Algorithms for continuous optimization (LP, QP, QCP)
  - Simplex, Barrier
- Algorithms for discrete optimization (MIP, MIQP, MIQCP)
  - Parallel branch-and-bound
- Other algorithms
  - Presolve, parameter tuning, irreducible inconsistent subsystems…
- Command-line interface, full-featured interactive shell
- Programming interfaces
  - C, C++, Java, .NET, Python, MATLAB, R

Gurobi Model Components

- Decision variables
- Objective function
  - minimize $\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{c}^{\mathrm{T}}\mathbf{x} + \alpha$
- Constraints
  - $\mathbf{A}\mathbf{x} = \mathbf{b}$ (linear constraints)
  - $\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$ (bound constraints)
  - some x_h integral (integrality constraints)
  - some x_i lie within second order cones (cone constraints)
  - $\mathbf{x}^{\mathrm{T}}\mathbf{Q}_j\mathbf{x} + \mathbf{q}_j^{\mathrm{T}}\mathbf{x} \leq \beta_j$ (quadratic constraints)
  - some x_k in SOS (special ordered set constraints)

Example: Mixed Integer Program

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Benchmarks – Open Source MIP

- Mittelmann MIPLIB2010 tests, P=1 (>1X means Gurobi wins):
  - http://plato.la.asu.edu/bench.html

	January 2012	January 2013	January 2015	November 2015	% Solved
CBC	10X	13X	17X	26X	55%
SCIP	6X	7X	7X	9X	80%
GLPK	22X	27X	-	-	1%
LPSOLVE	19X	24X	-	-	6%

- GLPK and LPSOLVE are not currently tested in the public benchmarks

Why Gurobi?

- Performance leader
  - Average 2X improvement with each major release
- Consistent track record of innovation
  - First cloud offering, client-server support, distributed algorithms, …
- Outstanding support
  - Direct access to Ph.D.-level optimization experts
- No surprises
  - Flexible licensing and transparent pricing
- Strong academic offering
  - Free full-featured academic licenses, Take Gurobi with You program

About Continuum Analytics & Anaconda

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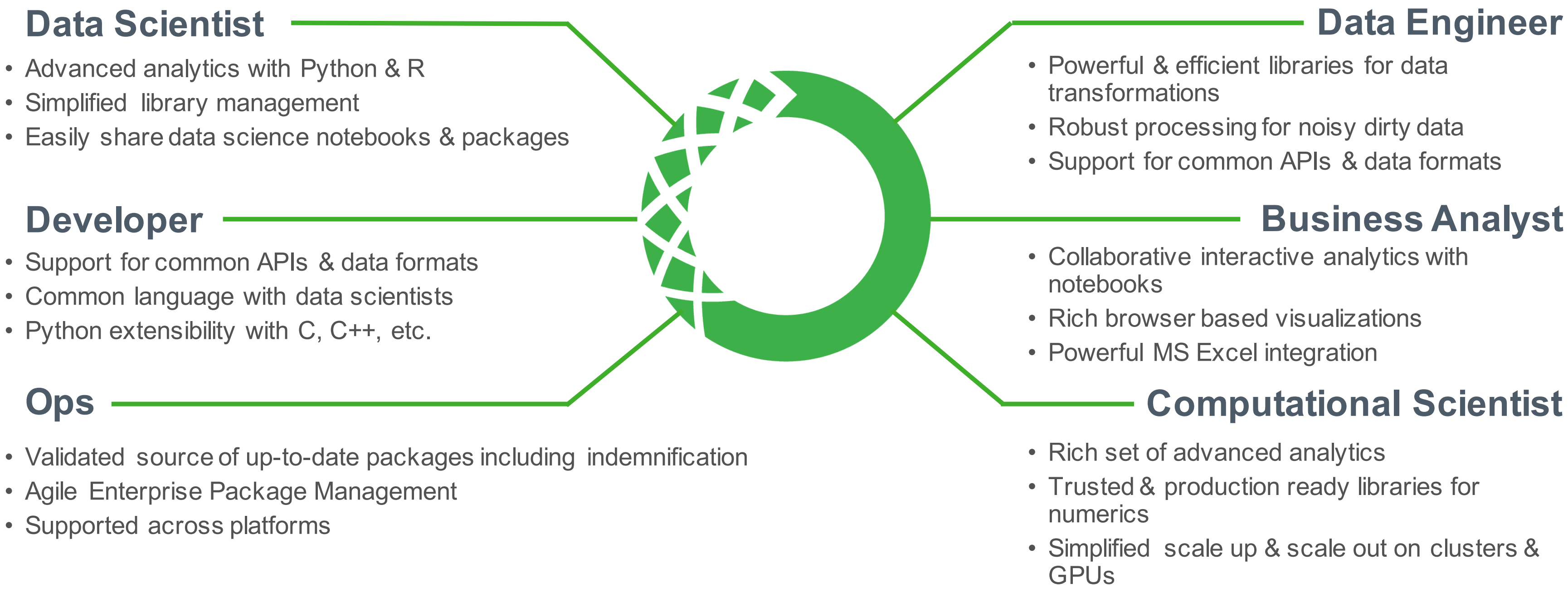
Continuum Analytics

- Founded in 2012 by Travis Oliphant and Peter Wang
- Key contributors to the Python open source ecosystem
  - Travis: primary developer, NumPy; founding contributor, SciPy
  - PyTables, Pandas, Jupyter/IPython, Matplotib
  - New projects: Blaze, Numba, Conda
- Commercial efforts:
  - Enterprise software & support
  - Training: Python for Science, Python for Finance
  - Consulting: custom software development

Anaconda for Data Science

Empowering Everyone on the Team



Data Scientist

- Advanced analytics with Python & R
- Simplified library management
- Easily share data science notebooks & packages

Developer

- Support for common APIs & data formats
- Common language with data scientists
- Python extensibility with C, C++, etc.

Ops

- Validated source of up-to-date packages including indemnification
- Agile Enterprise Package Management
- Supported across platforms

Data Engineer

- Powerful & efficient libraries for data transformations
- Robust processing for noisy dirty data
- Support for common APIs & data formats

Business Analyst

- Collaborative interactive analytics with notebooks
- Rich browser based visualizations
- Powerful MS Excel integration

Computational Scientist

- Rich set of advanced analytics
- Trusted & production ready libraries for numerics
- Simplified scale up & scale out on clusters & GPUs

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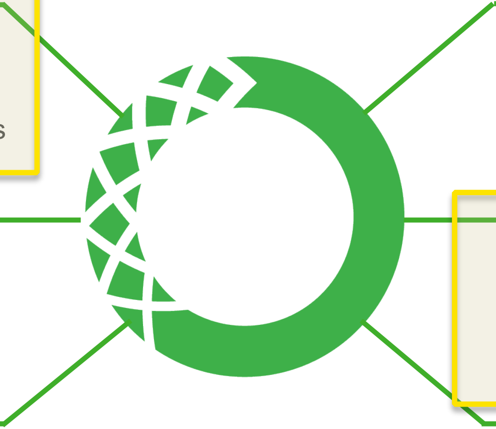

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Why Python?

- Designed for readability
- Functional, object-oriented, declarative
- Great glue language for legacy C/Fortran
- Rich standard library
- Incredible community library
- Great online resources
- Great name

"The second-best language for everything"
— Peter Wang, CTO

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The Anaconda Python Distribution

- Windows, Mac, Linux; 32/64-bit
- Python 2.x, 3.x — your choice (though do pick 3.x!)
- ~150 packages in the default installation
- **`conda`** package and environment management
  - ~350 packages available with a simple **`conda install`**
  - Create isolated “silos” for multiple projects
- Like R? Anaconda has that too

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conda package management

Install	`conda install pandas numpy=1.9 gurobi`
Update	`conda update gurobi`
List	`conda list`
Search	`conda search gurobi`
Remove	`conda remove matplotlib`

conda environment management

Create	`conda create -n lp_project anaconda gurobi`	

Select	`source activate lp_project`
	`activate lp_project`
		(Linux/Mac)
		(Windows)

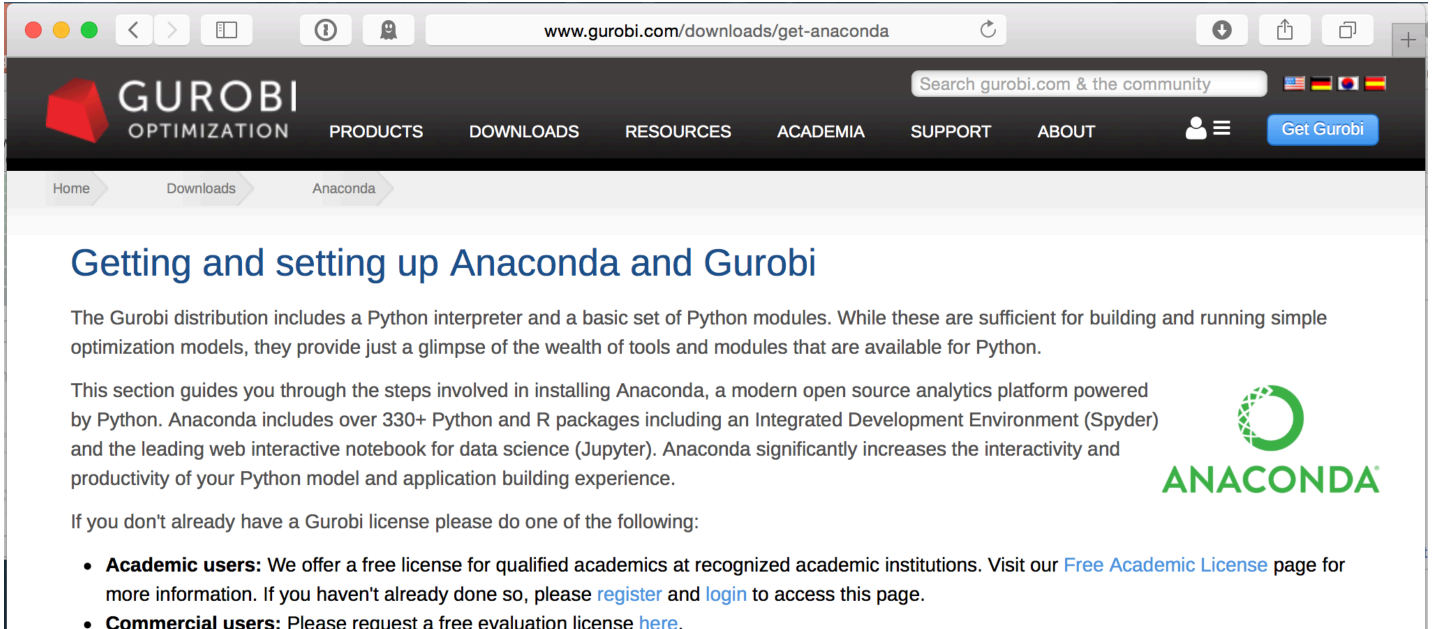
Return to default	`source deactivate`
	`deactivate`
		(Linux/Mac)
		(Windows)

Remove	`conda remove -n lp_project --all`	

Obtaining Anaconda

From Continuum: https://www.continuum.io/downloads

From Gurobi: http://www.gurobi.com/downloads/get-anaconda



Getting and setting up Anaconda and Gurobi

The Gurobi distribution includes a Python interpreter and a basic set of Python modules. While these are sufficient for building and running simple optimization models, they provide just a glimpse of the wealth of tools and modules that are available for Python.

This section guides you through the steps involved in installing Anaconda, a modern open source analytics platform powered by Python. Anaconda includes over 330+ Python and R packages including an Integrated Development Environment (Spyder) and the leading web interactive notebook for data science (Jupyter). Anaconda significantly increases the interactivity and productivity of your Python model and application building experience.

If you don't already have a Gurobi license please do one of the following:

- **Academic users:** We offer a free license for qualified academics at recognized academic institutions. Visit our Free Academic License page for more information. If you haven't already done so, please register and login to access this page.
- **Commercial users:** Please request a free evaluation license here.

The Gurobi Python Module

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Building Models with Gurobi

- Option 1: use a modeling language
  - Very easy to build models
    - Optimization modeling constructs built into language
  - Attractive choice for non-programmers
- Option 2: use a full programming language
  - Much more powerful and flexible development environment
    - Complete access to solver functionality
    - Richer set of language features
  - Natural choice when deploying and/or integrating with applications

Gurobi Python Environment

- High-level optimization modeling constructs embedded in Python
- Design goals:
  - Require minimal programming skills to get started
  - Bring "feel" of a modeling language to the Python interface
  - Allow for code that is easy to write and maintain
  - Maintain unified design across all of our interfaces
  - Remain lightweight and efficient compared to solver alone
  - Support all solver and programming needs

Essential Gurobi Python Constructs

- Objects represent model components (`Model`, `Var`, `Constr`)
- Overloaded operators
  - Arithmetic (+, -, ×, ÷), constraints (≤, =, ≥)
- Aggregate sum operator (`quicksum`)
- Python provides the rest for representing data, indices and subscripts
  - Lists, tuples, dictionaries, loops, generator expressions, …

Ex: $x_i + y_i \le 5, \forall i \in I$ $\Leftrightarrow$

```
for i in I:
    m.addConstr(x[i] + y[i] <= 5)
```

Building a Gurobi Model: Six Steps

Create a model	`m = Model()`
Add variables	`x = m.addVar(...)`
Commit changes	`m.update()`
Set objective	`m.setObjective(...)`
Add constraints	`m.addConstr(...)`
Optimize	`m.optimize()`

Installing the Gurobi Python Module

Add the channel	`conda config --add channels gurobi`
Install Gurobi	`conda install gurobi`
Retrieve license	`grbgetkey xxxxxxxx-xxxx-xxxx-xxxx-xxxxxxxxxxxx`
Test license	`>>> import gurobipy`
Test token server	`gurobi_cl --tokens`

A Simple MIP Model using the Spyder IDE

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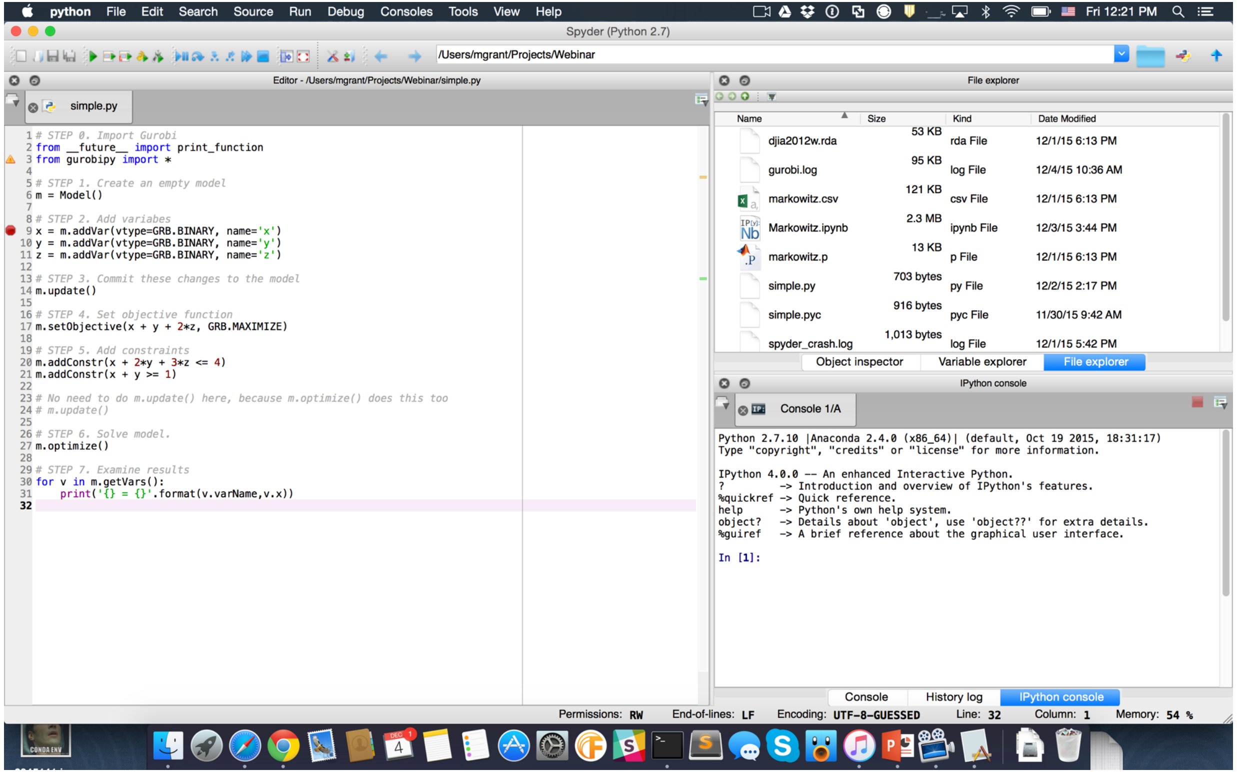
Spyder

- **S**cientific **Py**thon **D**evelopment **E**nvi**R**onment
- Familiar *integrated development environment* (IDE) for users of MATLAB, Visual Studio, etc.
- Built-in editor, file explorer, documentation browser
- Graphical interface for debugging (ipdb)
- IPython console for interactive code execution

http://pythonhosted.org/spyder

Spyder Demo



```
# STEP 0. Import Gurobi
from __future__ import print_function
from gurobipy import *

# STEP 1. Create an empty model
m = Model()

# STEP 2. Add variabes
x = m.addVar(vtype=GRB.BINARY, name='x')
y = m.addVar(vtype=GRB.BINARY, name='y')
z = m.addVar(vtype=GRB.BINARY, name='z')

# STEP 3. Commit these changes to the model
m.update()

# STEP 4. Set objective function
m.setObjective(x + y + 2*z, GRB.MAXIMIZE)

# STEP 5. Add constraints
m.addConstr(x + 2*y + 3*z <= 4)
m.addConstr(x + y >= 1)

# No need to do m.update() here, because m.optimize() does this too
# m.update()

# STEP 6. Solve model.
m.optimize()

# STEP 7. Examine results
for v in m.getVars():
    print('{} = {}'.format(v.varName,v.x))

```

Name	Size	Kind	Date Modified
djia2012w.rda	53 KB	rda File	12/1/15 6:13 PM
gurobi.log	95 KB	log File	12/4/15 10:36 AM
markowitz.csv	121 KB	csv File	12/1/15 6:13 PM
Markowitz.ipynb	2.3 MB	ipynb File	12/3/15 3:44 PM
markowitz.p	13 KB	p File	12/1/15 6:13 PM
simple.py	703 bytes	py File	12/2/15 2:17 PM
simple.pyc	916 bytes	pyc File	11/30/15 9:42 AM
spyder_crash.log	1,013 bytes	log File	12/1/15 5:42 PM

```
Python 2.7.10 |Anaconda 2.4.0 (x86_64)| (default, Oct 19 2015, 18:31:17)
Type "copyright", "credits" or "license" for more information.

IPython 4.0.0 -- An enhanced Interactive Python.
?         -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help      -> Python's own help system.
object?   -> Details about 'object', use 'object??' for extra details.
%guiref   -> A brief reference about the graphical user interface.

In [1]:
```

Permissions: RW End-of-lines: LF Encoding: UTF-8-GUESSED Line: 32 Column: 1 Memory: 54 %

Portfolio Optimization in Jupyter

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Modern Portfolio Theory In One Slide

- First offered by Harry Markowitz in 1952
- Recognized by a Nobel Memorial Prize in Economics
- Associates with each stock an *expected return* and a risk or *volatility* measure, and a matrix of *correlations* between pairs of stocks
- The goal is to study the tradeoff between *risk* and *return*
  - Minimize risk for a given target (expected) return
  - Maximize return given an upper bound on risk
- The most basic models are *quadratic programs* (QPs)

(okay, two slides)

$$\begin{array}{ll} \text{minimize} & x^T \Sigma x \\ \text{subject to} & \vec{1}^T x = 1 \\ & \rho^T x = \gamma \\ & x \geq 0 \end{array}$$

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j \\ \text{subject to} & \sum_{i=1}^n x_i = 1 \\ & \sum_{i=1}^n \rho_i x_i = \gamma \\ & x_i \geq 0 \quad i = 1, 2, \ldots, n \end{array}$$

x_i	amount of investment allocated to stock i
ρ_i	expected return for stock i
σ_{ii}	variance of the performance for stock i
$\sigma_{ij}/\sqrt{\sigma_{ii}\sigma_{jj}}$	correlation between stocks i and j

Jupyter (formerly IPython)

- A web application for interactive data analysis
- Create notebooks containing live code, text, equations, and visualizations
- A great way to prototype new algorithms, document experiments, share reproducible results
- Originally for Python, now there are kernels for over 50 different languages

http://www.jupyter.org

Pandas

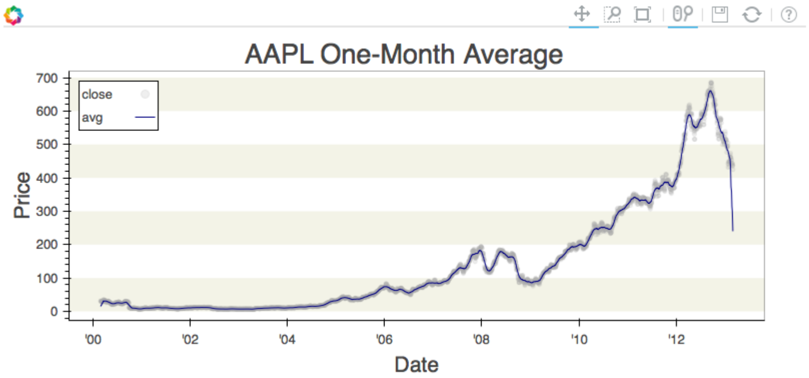
- Fast and efficient DataFrame and Series objects
- Read/write CSV, HDF5, Excel, SQL, plain text
- Missing data handling
- Slicing, fancy indexing, subsetting
- Merges, joins
- Split-apply-combine operations (groupby)

http://pandas.pydata.org

Bokeh

Interactive visualization for web browsers

- High performance with streaming data and big data
- In-browser interactivity on otherwise *static* pages
- Bindings for Python, Scala, Julia, R



http://bokeh.pydata.org

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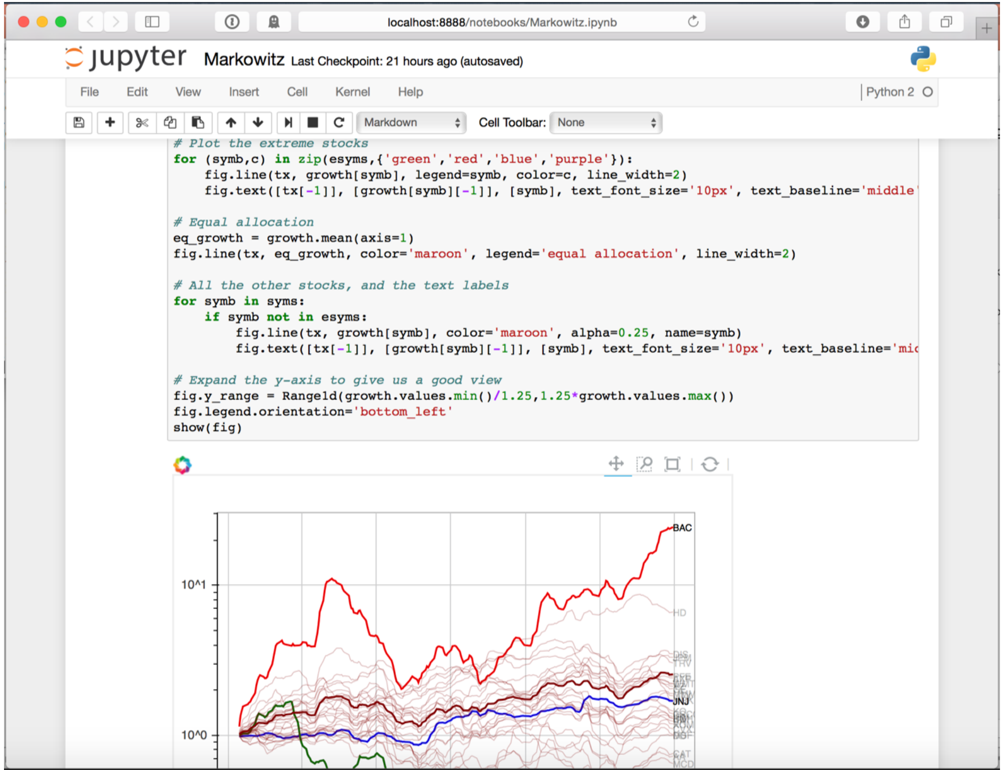
Jupyter Demo

https://notebooks.anaconda.org/mcg/portfolio



```
# Plot the extreme stocks
for (symb,c) in zip(esyms,{'green','red','blue','purple'}):
    fig.line(tx, growth[symb], legend=symb, color=c, line_width=2)
    fig.text([tx[-1]], [growth[symb][-1]], [symb], text_font_size='10px', text_baseline='middle'

# Equal allocation
eq_growth = growth.mean(axis=1)
fig.line(tx, eq_growth, color='maroon', legend='equal allocation', line_width=2)

# All the other stocks, and the text labels
for symb in syms:
    if symb not in esyms:
        fig.line(tx, growth[symb], color='maroon', alpha=0.25, name=symb)
        fig.text([tx[-1]], [growth[symb][-1]], [symb], text_font_size='10px', text_baseline='mic

# Expand the y-axis to give us a good view
fig.y_range = Range1d(growth.values.min()/1.25,1.25*growth.values.max())
fig.legend.orientation='bottom_left'
show(fig)
```

Closing thoughts

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Developers vs. Analysts?

Analysts benefit from Jupyter:

- Interactive, iterative development
- Notebooks combine code, results, visualizations, documentation

Developers benefit from Spyder:

- Traditional IDE organization
- Graphical debugging

Both benefit from *straightforward collaboration:*

- Code from notebooks readily inserted into standard Python modules

Wrap up

- The Gurobi Python interface brings the expressive power of a modeling language into the powerful Python ecosystem
- Spyder offers a traditional IDE for Python development, including full-featured editing and graphical debugging
- IPython and the Jupyter notebook system provide a powerful paradigm for interactive development, collaboration, and sharing
- The Anaconda platform delivers a suite of powerful open-source tools and connects to world-class commercial software like Gurobi
- Need training, consulting, enterprise support? Continuum can help

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Thank you!

Download Gurobi and/or Anaconda and try them for yourself:

www.gurobi.com/get-anaconda

Email: sales@gurobi.com

Twitter: @Gurobi

Sales: sales@continuum.io

Twitter: @ContinuumIO

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