STEVENS INSTITUTE of TECHNOLOGY
THE INNOVATION UNIVERSITY®

E355 - Engineering Economics

Lecture 02 B: *Understanding Cash Flow Diagrams, Interest Rates and Time Value of Money*

Chapters: 2, 3, 4

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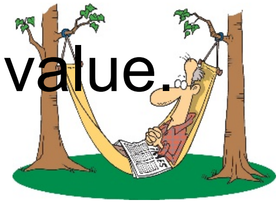
Lecture Objectives

After completing this module you should understand the following:

- Cash flow diagram: basics, ‘how to’ and types (arithmetic, geometric gradient)
- Time Value of Money
- Overview of simple and compound interests – calculation methods including continuous compounding
- Nominal, periodic and effective interest rates
- Equivalence calculations with nominal and effective interest rates

Why?

- Understand Long term impacts of time on economic value. How do you plan for retirement?



- Evaluate different interest rates and determine what they mean. Which savings option is best and why?
- Visualize Economic Values at different points in time to evaluate positive or negative financial impact. This should be a foundation for all economic decisions.

Economic Equivalence

Economic Equivalence:

Definition:

The process of comparing two different cash amounts at different points in time.

Can assess:

- Single Payments
- Series of Payments

Economic Equivalence:

Guiding Principles

- Equivalence calculations made to compare alternatives require a common time basis.
- Equivalence depends on interest rate.
- Equivalence calculations may require the conversion of multiple payment cash flows to a single cash flow.
- Equivalence is maintained regardless of point of view.

Equivalence

- Consider two investment opportunities: A and B. If you are indifferent to one or the other in making a choice between them, they are said to be equivalent.

- Equivalence allows us to convert from any P(Present Value) to any F (Future Value) and vice versa.

Equivalence Show Video:

Time Value of Money

What is *i*?

MARR

Minimum **A**ttractive **R**ate of **R**eturn

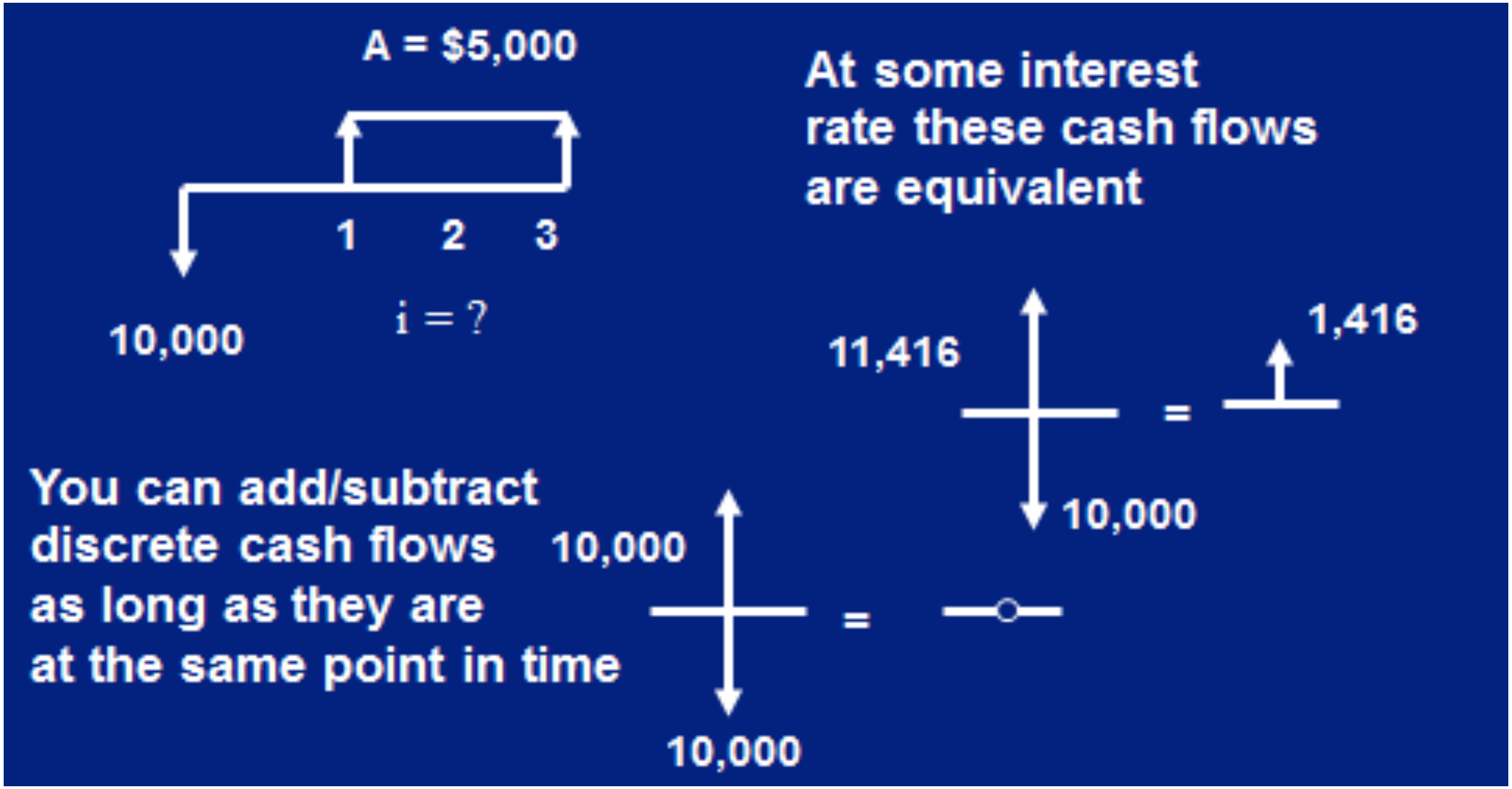
- The interest rate that the company wants to earn on its investments.
- Selection is usually a policy decision made by top management.
- MARR can change over the life of a project.

Based on:

- Cost of Capital
- Risk

Equivalence

Equivalence Cash Flow Diagrams



Manipulating Rates of Return

- Aligning time horizons
- Adding and subtracting cash flows
- Manipulating gradients

Single Cash Flow

- Compound Amount Factor
  - Used to find the future worth (FW) of a present value
  - Find F given P, i and N

$$F = P(1 + i)^N = P(F/P, i, N)$$

Same as Compound Interest

Single Payment Compound Amount Factor (TABLE FACTOR)

Single Cash Flow

- **Present Worth Factor**
  - Used to find the present worth (PW) of a future value.
  - Find P given F, i and N.
  - Opposite of compounding
  - Known as “discounting”.

$$P = F(1 + i)^{-N} = F(P/F, i, N)$$

From Compound Interest Formula

Single Payment Discount Amount Factor (TABLE FACTOR)

Uniform Series

Uniform Series Notation:

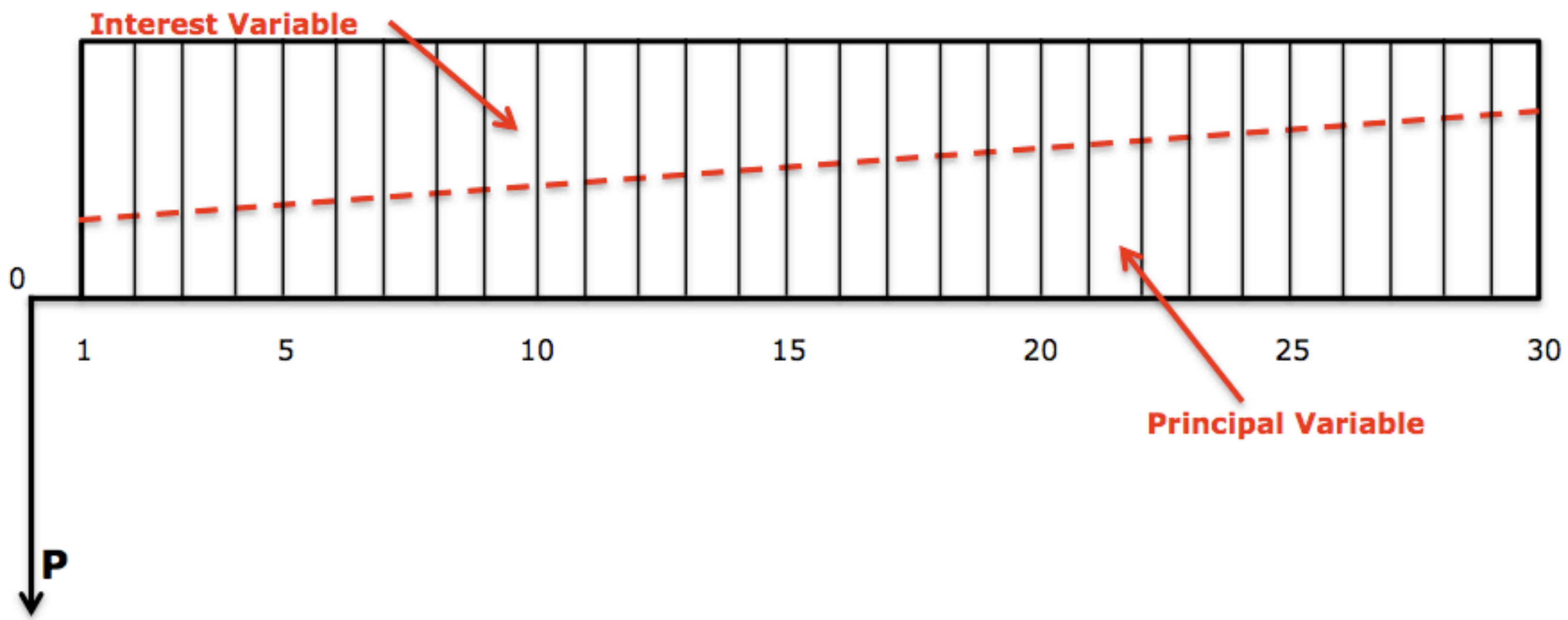
A = an end-of-period cash flow in a uniform series, continuing for n periods

Examples

- Automobile loans, mortgage payments, insurance premium, rents
- Estimated future costs and benefits

Debt Management: Commercial Loans

- **Amortized Loan**
  - Sample Cash Flow Diagram:



Equal (Uniform) Series

- **Compound-Amount Factor**
  - Used to find the future worth (FW) of an annuity
  - Find F given A, i and N

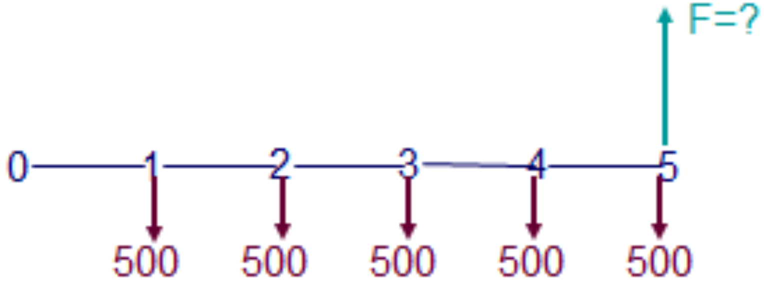
$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i, N)$$

Equal (Uniform) Payment Series Compound Amount Factor (TABLE FACTOR)

Uniform Series

Example 4-1: Uniform Series Compound Interest Formulas

$500 were deposited in a credit union (pays 5% compounded annually) at the end of each year for 5 years, how much do you have after the 5th deposit?



$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(F/A, i, n)$$

$$= 500(F/A, 5\%, 5) = 500(5.526) = \$2673.00$$

Equal (Uniform) Series

- **Sinking-Fund Factor**
  - Used to find the annual worth (AW) of a future value.
  - Find A given F, i and N
  - Sinking Fund:
    - *Interest bearing account into which a fixed sum is deposited each interest period*
    - Uses
      - Replacing fixed assets
      - Retiring corporate bonds

Equal (Uniform) Payment Series Sinking-Fund Factor (TABLE FACTOR)

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$$

Equal (Uniform) Series

- **Capital Recovery (Annuity) Factor**
  - Used to find the annual worth (AW) of a present value.
  - Find A given P, i and N.
  - Capital Recovery Factor:
    - *Used to determine the revenue requirements needed to address the upfront capital costs for projects.*
  - Annuity
    - *A level stream of cash flows for a fixed period of time.* (Chan S. Park)

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P, i, N)$$

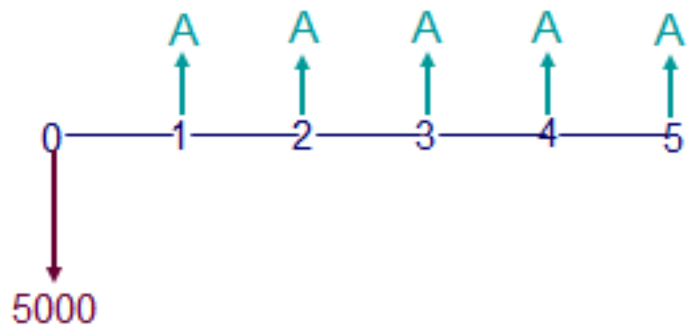
Equal (Uniform) Payment Series Capital Recovery Factor

(TABLE FACTOR)

Uniform Series Example

Example 4-3: Uniform Series Compound Interest Formulas

A machine costs $5000 and has a life of 5 years. If interest rate is 8%, how much must be saved every year to recover the investment?



$$A = P(A/P, i, n)$$
$$= 5000(A/P, 8\%, 5) = 5000(0.2505) = \$1252.00$$

Uniform Series Example

- **Present-Worth Factor**
  - Used to find the present worth (PW) of an annuity.
  - Find P given A, i and N.
  - Uses:
    - *Used to determine the what should be invested now in order to withdraw A dollars at the end of each of the next N periods.* (Chan S. Park)

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A, i, N)$$

Equal (Uniform) Payment Series Present Worth Factor

(TABLE FACTOR)

  - Excel Formula: A = PMT(i,N,P,F,Type)
    - Type = 0 (payments at end of period)
    - Type = 1 (payments at start of period)
    - For this class, assume Type = 0 unless otherwise specified

Equal Uniform Series

- **Present-Worth Factor**
  - Used to find the present worth (PW) of an annuity.
  - Find P given A, i and N.
  - Uses:
    - *Used to determine the what should be invested now in order to withdraw A dollars at the end of each of the next N periods.* (Chan S. Park)

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A,i,N)$$

Equal (Uniform) Payment Series Present Worth Factor

(TABLE FACTOR)

  - Excel Formula: A = PMT(i,N,P,F,Type)
    - Type = 0 (payments at end of period)
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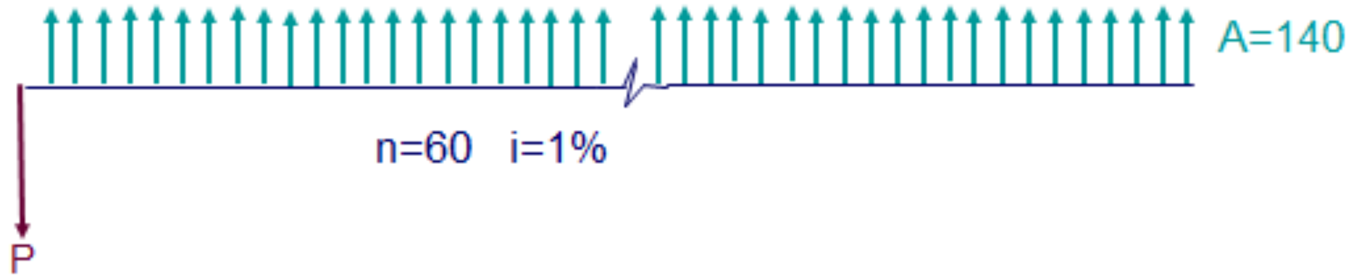
    - For this class, assume Type = 0 unless otherwise specified

Uniform Series Example

Example 4-4 – Uniform Series Compound Interest Formulas

Should you spend \$6800 to buy a contract that pays \$140 at the end of each month for 5 years if the desired return is 1% per month?



$$P = A(P/A, i, n)$$

$$= 140(P/A, 1\%, 60) = 140(44.955) = \$6293.70$$

The contract only worth \$6293.70, so reject the offer.

Equivalence Calculations: Effective Interest Rates

- **Payment Period = Compounding Period.**
  - Identify # compounding periods per year, M
    - M = K (payment period)
    - M = CK, therefore, C = 1
  - Calculate effective interest rate per period (periodic interest rate),
  - Determine # of compounding periods,
    - N = M × (number of years)
  - Calculate PW, AW or FW using i and N

Equivalence Calculations: Effective Interest Rates

- **Payment Period < ‘OR’ > Compounding Period.**
  - Identify # compounding periods per year (M), the number of payment periods per year (K), the number of interest periods per payment period (C).
  - Calculate effective interest rate per period (periodic interest rate),
    - Discrete: $i = \left(1 + \frac{r}{M}\right)^{C} - 1$
    - Continuous: $i = e^{r/K} - 1$
  - Determine # of compounding periods,
    - N = K × (number of years)
  - Calculate PW, AW or FW using i and N

Effective Rate per Payment Period

Supplemental Example

Suppose that you make quarterly deposits in a savings account that earns 9% interest compounded monthly.

Effective Rate per Payment Period

Supplemental Example

Given:

r= 9%

M= 12 interests periods per year

K= 4 quarterly payments per year

C= 3 interest periods per quarter

Find: i

$$i = \left(1 + \frac{r}{CK}\right)^{C} - 1$$

Effective Rate per Payment Period

Supplemental Example (cont'd)

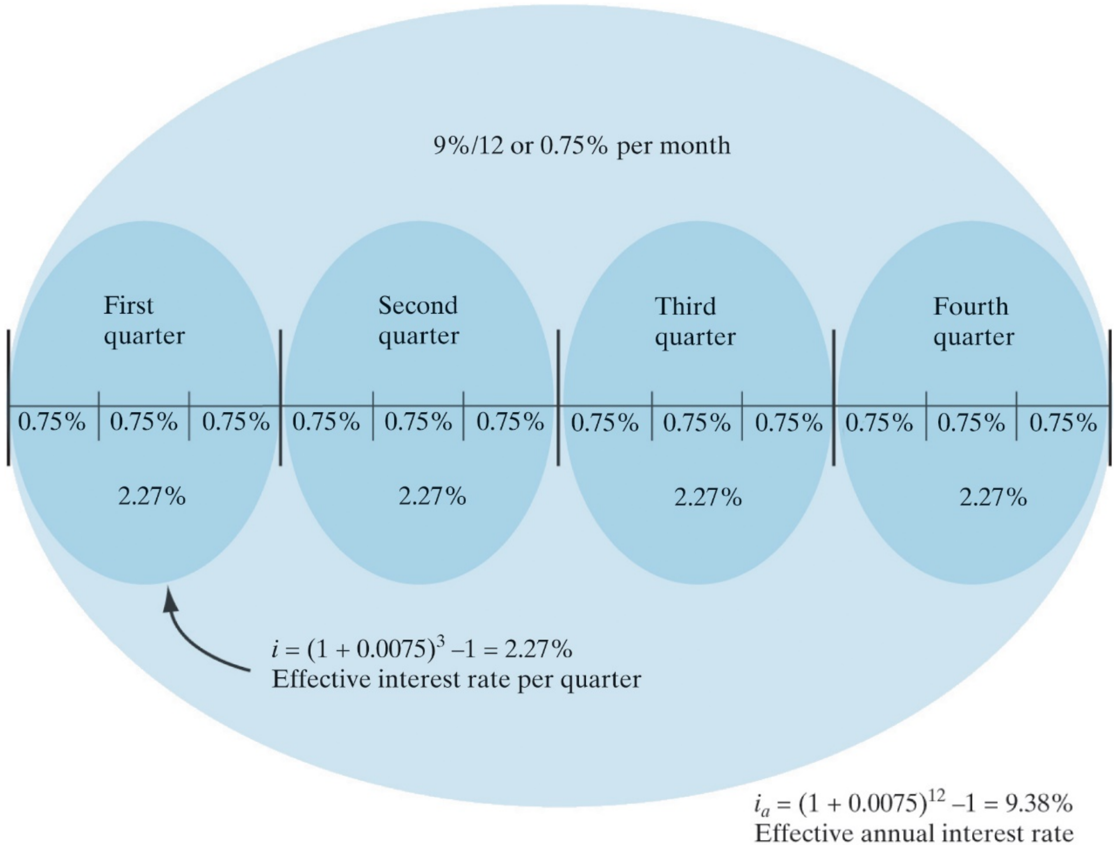
Suppose that you make quarterly deposits in a savings account that earns 9% interest compounded monthly.

$$i = \left[1 + \frac{0.09}{12}\right]^3 - 1 \quad = 2.27\%$$

Effective Rate per Payment Period

Supplemental Example (cont'd)

This figure illustrates the relationship between the nominal and effective interest rates per payment period.



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