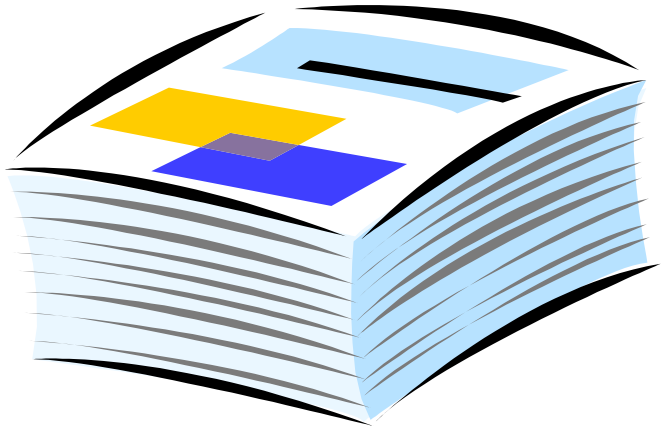
Simulation Examples
~ By Hand
~ Using Excel



Materials from Prof. Ranette Halverson

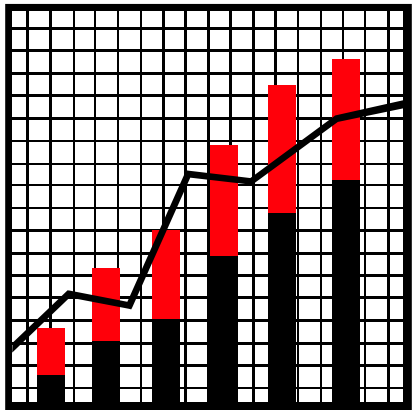
Why do examples by hand or spreadsheet??

- Insight to system
- Hands-on
- Helps with programming

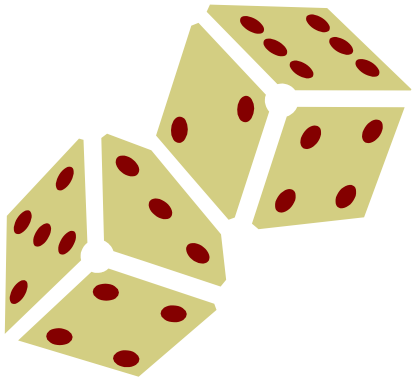
- Complex systems not amenable to spreadsheet simulation

Process

- Determine Characteristics of system
- Construct simulation table
- Generate & compute values



Key Components



- Random Numbers
  - Number: between 0 & 1
  - Variable: some quantity; perhaps from a known distribution
- Descriptive Statistics
  - Values used for describing a systems and making predictions about its behavior

Random Variable

- A quantity determined by some random experiment
- Examples
  - Number of heads obtained when flipping a coin 10 times
  - Number of customers arriving in an hour
  - Maximum length of a queue during the day
  - Shortest service time for a customer for the day

Excel – Random number generators

- =RAND( )
  - Generates real values: 0 <= val < 1
- =RANDBETWEEN (low, high)
  - Generates integers: low <= val <= high
- To use in Excel
  - IF (RAND ( ) < 0.5, 0, 1)
  - IF (A2 <= 0.33, 0, (IF A2 <= 0.66, 1, 2)

Example: Coin Tossing

- Monte Carlo Simulation
- Fair coin → Head/Tail equally likely
- IF (RAND ( ) < 0.5, "H", "T")

Example: Random Service Times

- Integer value 1 to 10, inclusive
  - =RANDBETWEEN (1, 10)
- Integer value with given probability
  - 3 @ 30%; 6 @ 45%, 10 @ 25%
  - Develop cumulative probability
    - 0 - .3 → 3
    - .3 - .75 → 6
    - .75 – 1 → 10
  - IF (A2 <= 0.3, 3, (IF A2 <= 0.75, 6, 10)

Q: Why not? IF (RAND() <= 0.3, 3, (IF RAND()<= 0.75, 6, 10))

Arrival Times

- Arrival Time vs. Inter-Arrival Time
- Arrival time – Clock time of arrival
- Inter-Arrival Time: time between successive arrivals
- Example: Initialize: Clock = 0

Inter-Arrival Time	Arrival Time (Clock)
3	3
7	10
2	12

Simulating Queuing Systems

- Request population
  - Infinite vs. Finite population
- Nature of arrivals
  - Arrival Rate vs. Effective Arrival Rate
- Service mechanism
  - Single vs. Multiple vs. Sequential
- Service time
- Queuing models

Arrivals & Services

Generally defined by a distribution (random)

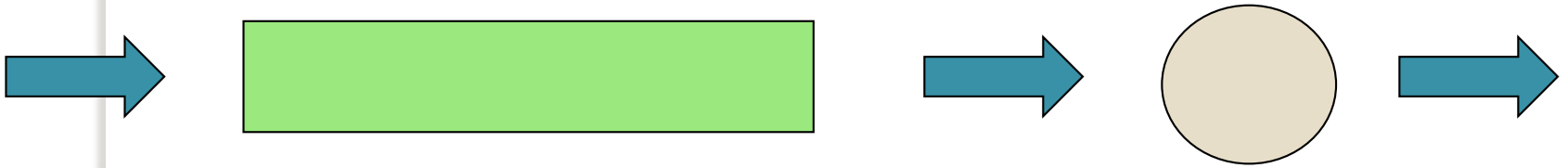
- Arrivals
  - Time between arrivals – inter-arrival time
- Service
  - Service times

Arrival rate must be less than the service rate. What if it is not? Unstable, explosive

Queue Basics

- System State
  - Number & status of entities (units)
- Event
  - Circumstance that causes a change in system state
- Clock
  - Relative time

Single Server & Queue



What are the state variables?

What are the events?

Future Events List (FEL)

- Can Generate Events
  - up-front
    - Before simulation begins
    - OK for small/short simulations
  - on-the-fly
    - As needed
    - Used for professional/complex simulations
- Generate Inter-arrival times & Service times

Brief Example

Cust #	IAT	A-time *Clock	S-begin *Clock	S-time	S-end *Clock
1	0	0	0	2	2
2	2	2	2	1	3
3	4	6	6	3	9
4	3	7	9	2	11
5	2	9	11	1	12
6	6	15	15	4	19

Other simulation items

- What else can we keep track of during the simulation?
  - Wait time in queue
  - Time in system
  - Server idle time

- Calculate these for previous example.

Other simulation items

- What can we calculate at the end of simulation?
  - λ: Average inter-arrival time
  - S: Average service time
  - U: Server utilization (% busy)
  - RT: Response Time
  - N: Average queue length
- Calculate for previous example.

Common Stats to Calculate

- Customer
  - Time in queue, Time in system, Probability of waiting in queue, Inter-arrival time
  - Averages, max, min
- Server
  - Utilization, Service times (max, min, average)
- Queue
  - Length (current, average, max, min)

System State vs. Performance Measure * Current vs. After Simulation

1. Current queue length
2. Server status (busy, idle)
3. Customer wait time

1. Average, max, min queue length
2. Average, min, max service time; utilization
3. Average wait time, max, min

Statistics – Performance Measures

Average Wait time for a customer

$$= \frac{\text{total time customers wait in queue}}{\text{total number of customers}}$$

Average wait time of those who wait

$$= \frac{\text{total time of customers who wait in queue}}{\text{number of customers who wait}}$$

More Statistics

Proportion of server busy time

$$= \frac{\text{number of time units server busy}}{\text{total time units of simulation}}$$

Average service Time

$$= \frac{\text{total service time}}{\text{number of customers serviced}}$$

More Statistics

Average time customer spends in system

$$= \frac{\text{total time customers spend in system}}{\text{total number of customers}}$$

Probability a customer has to wait in queue

$$= \frac{\text{number of customers who wait}}{\text{total number of customers}}$$

Server Utilization

- % of time the server is busy serving customers
- If there is 1 server
  - SU = TI = (service mean)/(inter-arrival mean)
- If there are N servers
  - SU = 1/N * (service mean)/(inter-arrival mean)

Weighted Averages

- Necessary when unequal probability of values.
- Example: Service times: 20% take 5 minutes, 38% take 8 minutes, 42% take 11 minutes.
- What is the average service time (AST)?
  - Is it (5 + 8 + 11) / 3 = 8 ???

- Correct answer:

AST = .2 * 5 + .38 * 8 + .42 * 11

= 1 + 3.04 + 4.62

= 8.66

Class Exercises

- 3-Coin Toss Simulation
- Small Store Simulation
- Spreadsheet template on CANVAS

More resources & tools:
Open source queueing tool: https://pypi.org/project/queueing-tool/
Commercial Anylogic: https://www.anylogic.com/