STEVENS
INSTITUTE of TECHNOLOGY
THE INNOVATION UNIVERSITY®

E355 - Engineering Economics

Lecture 03: *Understanding the 3 Worths, Capitalized Cost and Capitalized Recovery*
Chapters: 5,6,9

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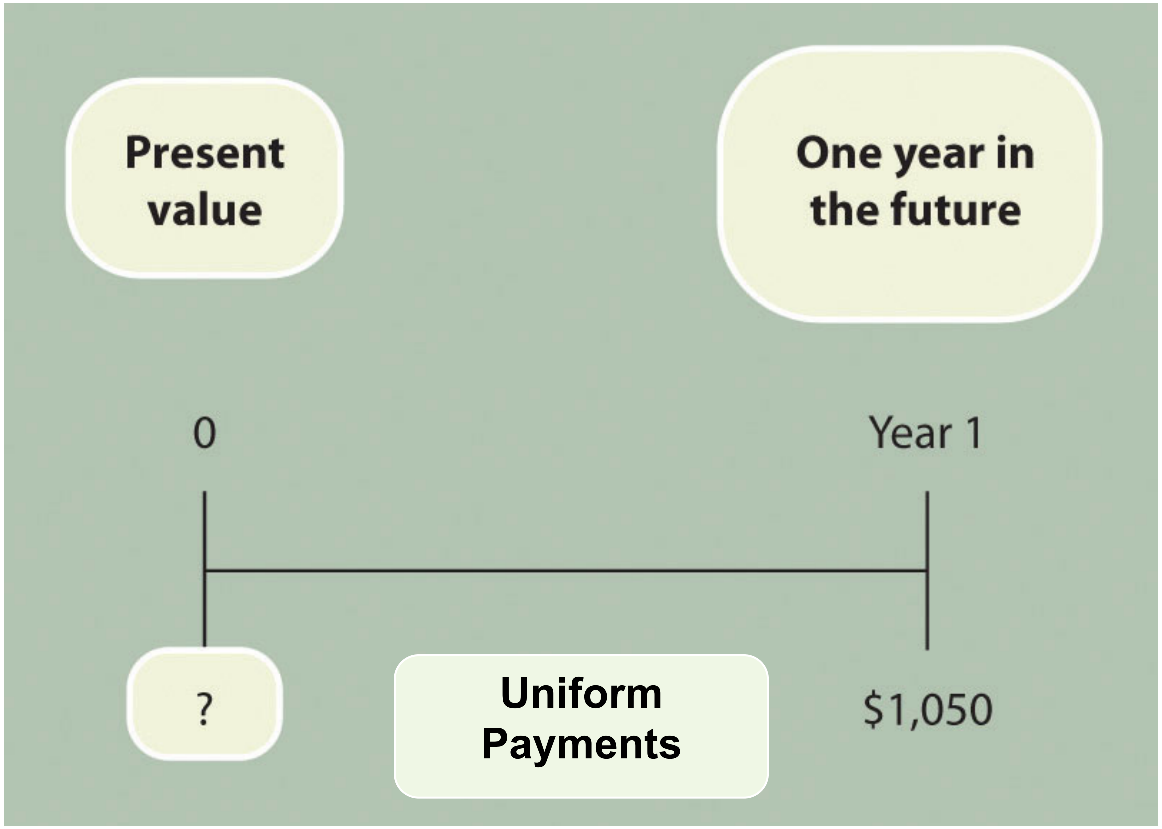
Lecture Objectives

- After completing this module you should understand the following:
  - Three Worths:
    - Present Worth (PW)
    - Annual Worth (AW)
    - Future Worth (FW)
  - Evaluation of Alternatives based on the time value of money
  - Capitalized costs
  - Capitalized recovery

Why?

- Understanding the Time-Value of Money impacts your life including but not limited to:
  - ✓ Student Loans, Interest Payments, Future investments(Car, House, Stocks, etc.)
- The amount of a dollar will change with respect to time. Planning for your future requires your knowledge of the 3 Worths.

Three of the Figures of Merit (FOM)



Overview

Why Equivalence?

Using equivalence formulas allows for the comparison of economic alternatives accounting for time; you can compare the value of an investment today, annually for the next 50 years and when you retire.

Any of the three worths can be converted to any other worth through equivalence.

- PW is most commonly used
- AW is used where annual comparisons are appropriate
- FW used in insurance, pensions, etc.

Present Worth (PW) Definition

- PW = the monetary sum which is equivalent to a future sum when interest is compounded at a given rate
- PW = the discounted value of a future sum when discounted at a given rate

PW = PV = NPW = NPV

- # Present Worth Analysis
  - ## Summary of Key Equations:

$$PW = -P + A(P/A,i,N) + F(P/F,i,N)$$

$$PW = -P + \sum_{n=0}^{N} F(P/F,i,n) + F(P/F,i,N)$$

$$PW = AE(P/A,i,N)$$

$$PW = FW(P/F,i,N)$$

where,

P = initial investment (n = 0)

A = annual cost / revenue (n = 1, 2, …, N)

F = future costs, salvage value or expected income from sale of the item (n = N)

PW = present worth of the investment taking A, F, i and N into account

AE = annual equivalence / worth of the investment taking P, F, i and N into account

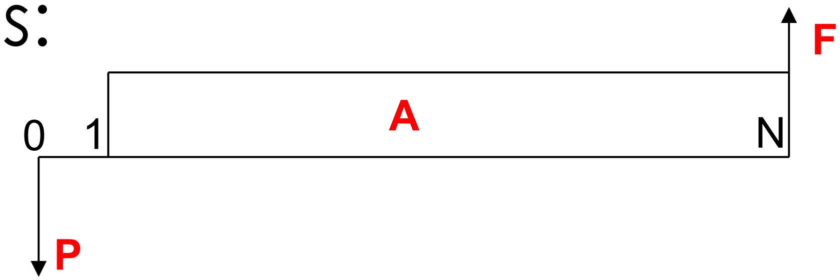
FW = future worth of the investment taking P, A, i and N into account

i = interest rate, MARR

N = project life (n = 1, 2, …, N)

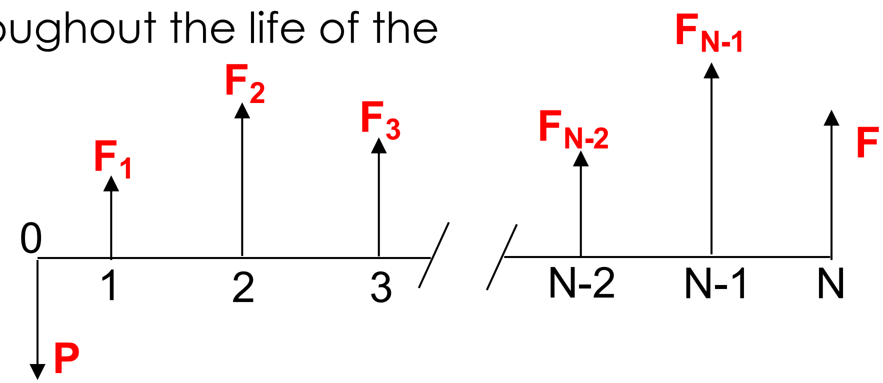
- **Present Worth Analysis:**
  - Using the Key Equations:

$$PW = -P + A(P/A,i,N) + F(P/F,i,N)$$



- Used to calculate the PW from first principles.
- Assumes the net annual costs / revenues / savings are constant throughout the life of the project.

$$PW = -P + \sum_{n=0}^{N} F(P/F,i,n) + F(P/F,i,N)$$



- Used to calculate the PW from first principles.
- Assumes the net annual costs / revenues are NOT constant throughout the life of the project. Each net annual cost / revenue value is treated individually as a "future" value and is discounted to a present worth value using the present worth factor for single payments (P/F, i, N).

Present Worth Analysis

- Single Project Evaluation

  If PW > 0 ➔ ACCEPT

  If PW = 0 ➔ INDIFFERENT

  If PW < 0 ➔ REJECT

- Comparing Multiple Alternatives
  - Revenue Projects:
    - Calculate PW for each alternative
    - Select the option with the largest PW
  - Service Projects:
    - Service projects (equal revenues) are compared on a cost only basis
    - Select the option with the *least negative* PW

Present Worth

- PW Example

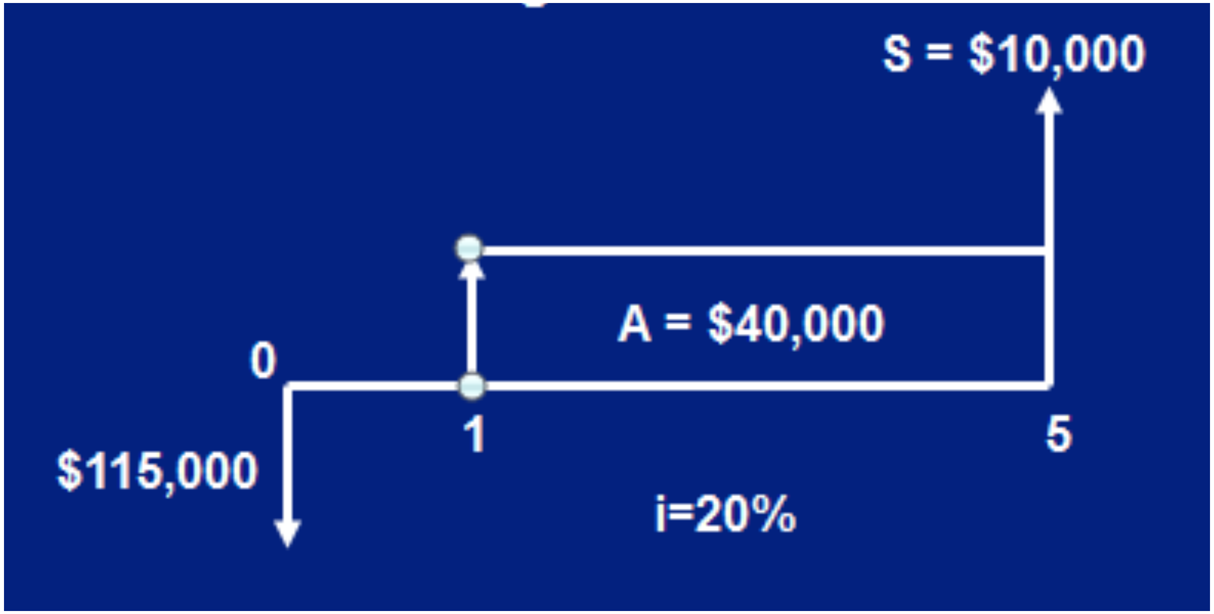
Robot Welder Purchase *(Lang & Merino)*

- Cost of robot welder = $100,000
- Installation cost = $15,000
- Expected annual income = $40,000
- Duration of life = 5 years
- Salvage value = $10,000
- Interest rate = 20%

Present Worth

Robot Welder Purchase

•Cash Flow Diagram



Present Worth

Robot Welder Purchase

PW = A (P / A, i, N) + S (P / F, i, N) - P

= \$ 40,000 (P / A, 20, 5) + 10,000 (P / F, 20,5) - 115,000

= \$ 40,000 x 2.991 + 10,000 x 0.4019 - 115,000

= \$ 119,640 + 4,019 - 115,000

= \$ 8,659 **in year 0**

Since the PW > 0 this is a good investment

How to Find the Factors

20% **Compound Interest Factors** **20%**

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor Find *F* Given *P*	Present Worth Factor Find *P* Given *F*	Sinking Fund Factor Find *A* Given *F*	Capital Recovery Factor Find *A* Given *P*	Compound Amount Factor Find *F* Given *A*	Present Worth Factor Find *P* Given *A*	Gradient Uniform Series Find *A* Given *G*	Gradient Present Worth Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.200	.8333	1.0000	1.2000	1.000	0.833	0	0	1
2	1.440	.6944	.4545	.6545	2.200	1.528	0.455	0.694	2
3	1.728	.5787	.2747	.4747	3.640	2.106	0.879	1.852	3
4	2.074	.4823	.1863	.3863	5.368	2.589	1.274	3.299	4
5	2.488	$\boxed{.4019}$	.1344	.3344	7.442	$\boxed{2.991}$	1.641	4.906	5
6	2.986	.3349	.1007	.3007	9.930	3.326	1.979	6.581	6
7	3.583	.2791	.0774	.2774	12.916	3.605	2.290	8.255	7
8	4.300	.2326	.0606	.2606	16.499	3.837	2.576	9.883	8
9	5.160	.1938	.0481	.2481	20.799	4.031	2.836	11.434	9
10	6.192	.1615	.0385	.2385	25.959	4.192	3.074	12.887	10
11	7.430	.1346	.0311	.2311	32.150	4.327	3.289	14.233	11
12	8.916	.1122	.0253	.2253	39.581	4.439	3.484	15.467	12
13	10.699	.0935	.0206	.2206	48.497	4.533	3.660	16.588	13
14	12.839	.0779	.0169	.2169	59.196	4.611	3.817	17.601	14
15	15.407	.0649	.0139	.2139	72.035	4.675	3.959	18.509	15
16	18.488	.0541	.0114	.2114	87.442	4.730	4.085	19.321	16
17	22.186	.0451	.00944	.2094	105.931	4.775	4.198	20.042	17
18	26.623	.0376	.00781	.2078	128.117	4.812	4.298	20.680	18
19	31.948	.0313	.00646	.2065	154.740	4.843	4.386	21.244	19
20	38.338	.0261	.00536	.2054	186.688	4.870	4.464	21.739	20
21	46.005	.0217	.00444	.2044	225.026	4.891	4.533	22.174	21
22	55.206	.0181	.00369	.2037	271.031	4.909	4.594	22.555	22
23	66.247	.0151	.00307	.2031	326.237	4.925	4.647	22.887	23
24	79.497	.0126	.00255	.2025	392.484	4.937	4.694	23.176	24
25	95.396	.0105	.00212	.2021	471.981	4.948	4.735	23.428	25

Present Worth

Present Worth Conversion Shortcuts

- To convert PW to AW:
  - Multiply by A/P factor
    - P(A/P, i, n) = A
- To convert PW to FW
  - Multiply by F/P factor
    - P(F/P, i, n) = F

Annual Worth

- The annual equivalent worth measures the worth of an investment by determining equal payments on an annual basis.

AW = AE = AV = -AC

- **Annual Equivalent Worth Analysis:**
  - Summary of Key Equations:

$$AW = -P(A/P,i,N) + A + F(A/F,i,N)$$

$$AW = -P(A/P,i,N) + \sum_{n=1}^{N} F(P/F,i,N)(A/P,i,N) + F(A/F,i,N)$$

$$AW = -P(A/P,i,N) + \sum_{n=1}^{N} F(F/P,i,N-n)(A/F,i,N) + F(A/F,i,N)$$

$$AW = PW(A/P,i,N)$$

$$AW = FW(A/F,i,N)$$

where,

P = initial investment (n = 0)

A = annual cost / revenue (n = 1, 2, …, N)

F = future costs, salvage value or expected income from sale of the item (n = N)

PW = present worth of the investment taking A, F, i and N into account

AW = annual equivalence / worth of the investment taking P, F, i and N into account

FW = future worth of the investment taking P, A, i and N into account

i = interest rate, MARR

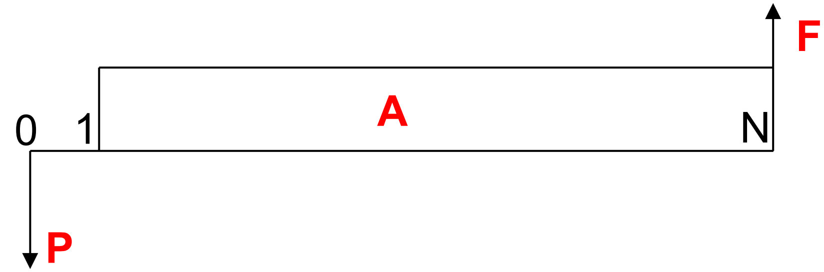
N = project life (n = 1, 2, …, N)

• Annual Equivalent Worth Analysis:

– Using the Key Equations:

$$AE = -P(A/P,i,N) + A + F(A/F,i,N)$$

- Used to calculate the AE from first principles.
- Assumes the net annual costs / revenues / savings are constant throughout the life of the project.



$$AE = PW(A/P,i,N)$$

- Short cut method.
- The present worth has been calculated.
- The AE can then be calculated (equivalence calculation) using the capital recovery factor for an equal payment series (A/P, i, N).

$$AE = FW(A/F,i,N)$$

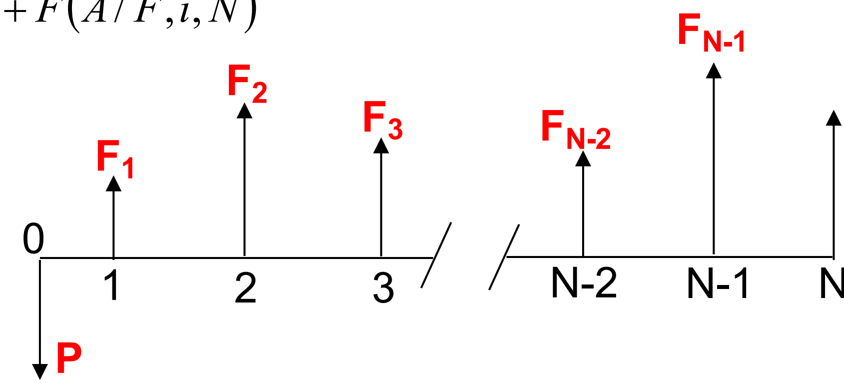
- Short cut method.
- The future worth has been calculated.
- The AE can then be calculated (equivalence calculation) using the sinking fund factor for a single payment series (A/F, i, N).

Annual Equivalent Worth Analysis:

– Using the Key Equations:

$$AW = -P(A/P,i,N) + \sum_{n=1}^{N} F(P/F,i,N)(A/P,i,N) + F(A/F,i,N)$$

- Used to calculate the AW from first principles.
- Assumes the net annual costs / revenues are NOT constant throughout the life of the project.



- Each net annual cost / revenue value is treated individually as a "future" value and is discounted to a present value using the present worth factor for single payments (P/F, i, N). This is then equated to an annual equivalence (AE) using the capital recovery factor (A/P,I,N).

Annual Equivalent Worth Analysis

- Single Project Evaluation

  If AW > 0 → ACCEPT

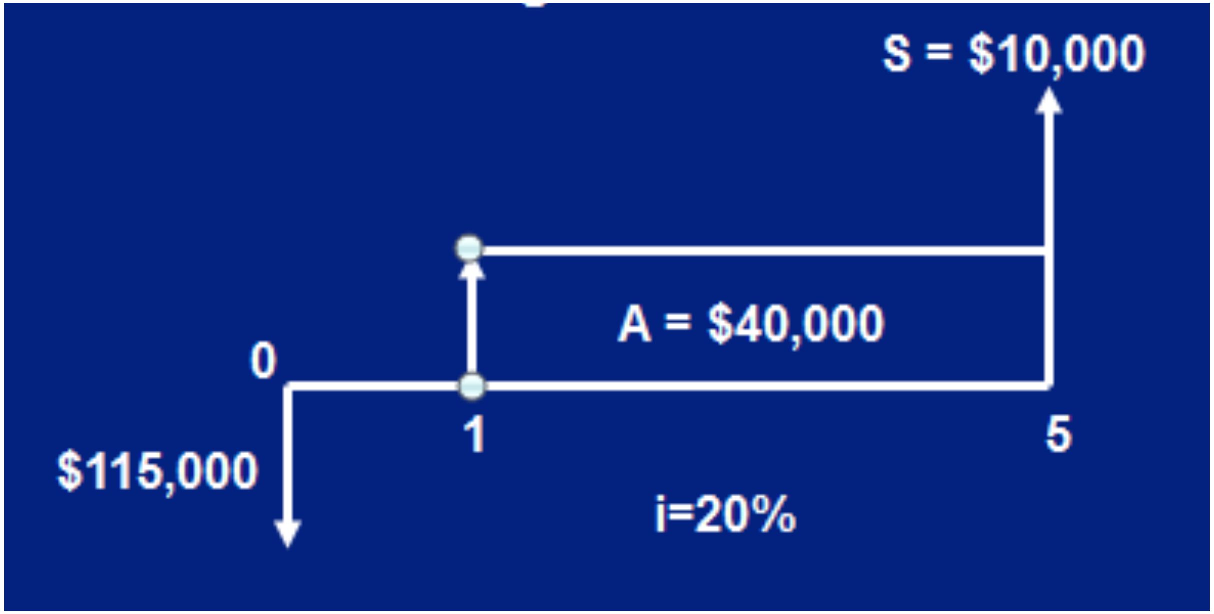
  If AW= 0 → INDIFFERENT

  If AW< 0 → REJECT

- Comparing Multiple Alternatives
  - Revenue Projects:
    - Calculate AW for each alternative
    - Select the option with the largest AW
  - Service Projects:
    - Service projects (equal revenues) are compared on a cost only basis
    - Select the option with the *least negative* AW

Robot Welder Purchase

- Cash Flow Diagram



Robot Welder Purchase

$$AW = A + S\,(A/F, i, N) - P\,(A/P, i, N)$$

$$= A + S\,(A/F, 20,5) - P\,(A/P, 20,5)$$

$$= \$\,40{,}000 + 10{,}000(0.1344) - 115{,}000(0.3344)$$

$$= \$\,40{,}000 + 1{,}344 - 38{,}456$$

= $ 2,889 **<u>for</u>** 5 years

OR

$$AW = P(A/P, 20\%, 5) = 8{,}659(0.3344) = \$2{,}889$$

(conversion shortcut – already had P from first example)

Annual Worth Conversion Shortcuts

- To convert AW to PW:
  - Multiply by P/A factor
    - A(P/A, i, n) = P
- To convert AW to FW
  - Multiply by F/A factor
    - A(F/A, i, n) = F

Future Worth

- The value of an asset or cash at a specified date in the future that is equivalent in value to a specified sum today.

FW = NFW = NFV = FV

- **Future Worth Analysis:**
  - Summary of Key Equations:

$$FW = -P(F/P,i,N) + A(F/A,i,N) + F$$

$$FW = -P(F/P,i,N) + \sum_{n=0}^{N} F(F/P,i,N-n) + F$$

$$FW = AE(F/A,i,N)$$

$$FW = PW(F/P,i,N)$$

where,

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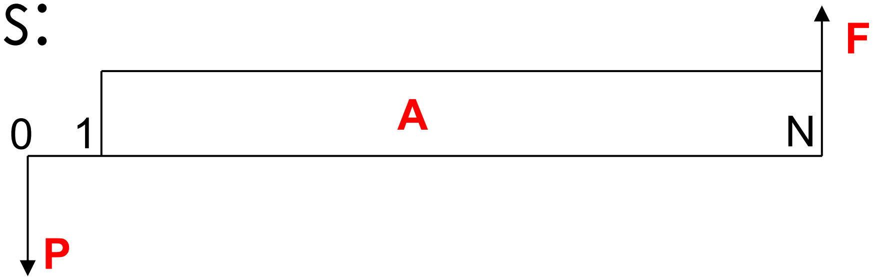
i = interest rate, MARR

N = project life (n = 1, 2, …, N)

Future Worth Analysis:

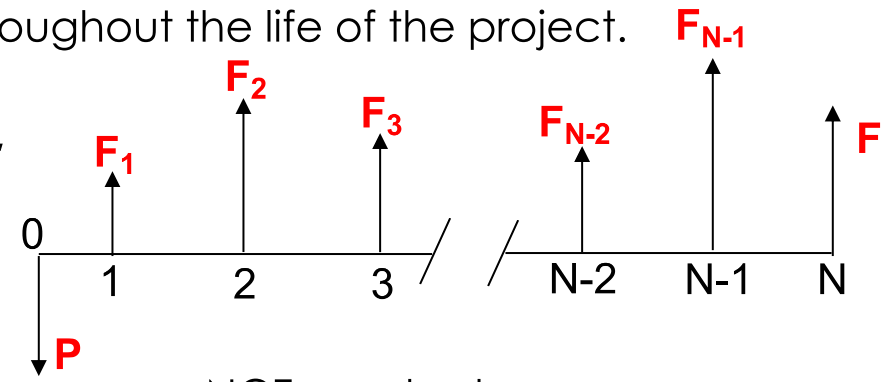
–Using the Key Equations:

$$FW = -P(F/P,i,N) + A(F/A,i,N) + F$$



- Used to calculate the FW from first principles.
- Assumes the net annual costs / revenues / savings are constant throughout the life of the project.

$$FW = -P(F/P,i,N) + \sum_{n=0}^{N} F(F/P,i,N-n) + F$$



- Used to calculate the FW from first principles.
- Assumes the net annual costs / revenues are NOT constant throughout the life of the project. Each net annual cost / revenue value is treated individually as a "present" value and is compounded to a future value (over "N-n" years) using the compound amount factor for single payments (F/P, i, N).

Future Worth Analysis

- Single Project Evaluation

  If FW > 0 ➔ ACCEPT

  If FW = 0 ➔ INDIFFERENT

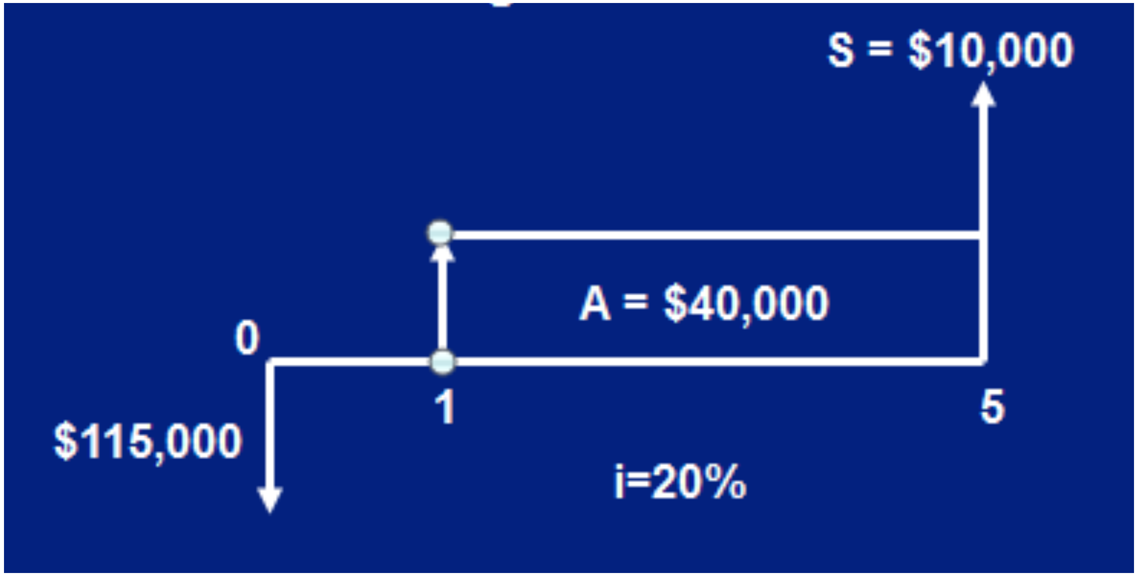
  If FW < 0 ➔ REJECT

- Comparing Multiple Alternatives
  - Revenue Projects:
    - Calculate FW for each alternative
    - Select the option with the largest FW
  - Service Projects:
    - Service projects (equal revenues) are compared on a cost only basis
    - Select the option with the *least negative* FW

Future Worth

Robot Welder Purchase

- Cash Flow Diagram



Future Worth

Robot Welder Purchase

FW = A (F/A, i, N) + S – P (F/P, i, N)

= A (F/A, 20,5) + S – P (F/P, 20, 5)

= \$ 40,000 (7.442) + 10,000 – 115,000 (2.488)

= \$ 297,680 + 10,000 – 286,120

= \$ 21,560 **in** 5 years

OR

FW = P(F/P,20%,5) = 8,659(2.4883) = \$21,560

(conversion shortcut – already had P from first example)

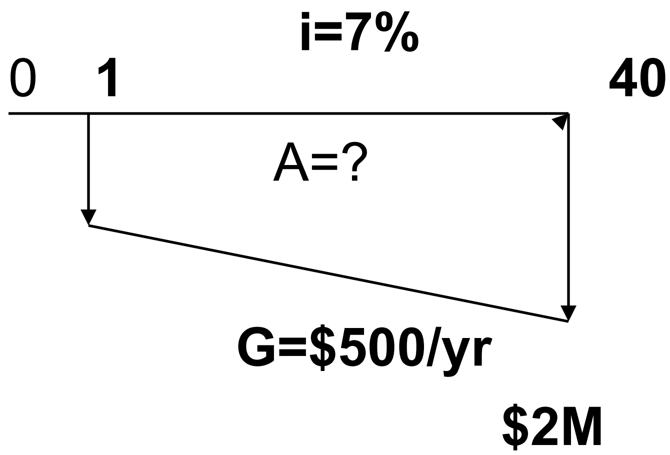
Robot Welder Purchase

Summary of Worths

- PW = $8, 659 in year 0
- AW = $2,889 for 5 years
- FW = $21,560 in year 5

Example problem 2

How much would you need to deposit annually in your 401k at 7%, to have $2 million in 40 years? Assume you will be financially able to increase the amount $500/year.



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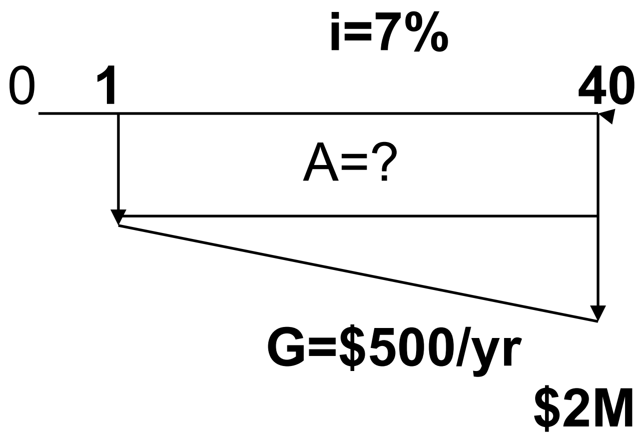


A= F(A/F, i, N) - G(A/G,i,N)

A = 2M(A/F, .07, 40) - 500(A/G, .07, 40)

A = 2M(.0050) - 500(11.4230)

A = $4290

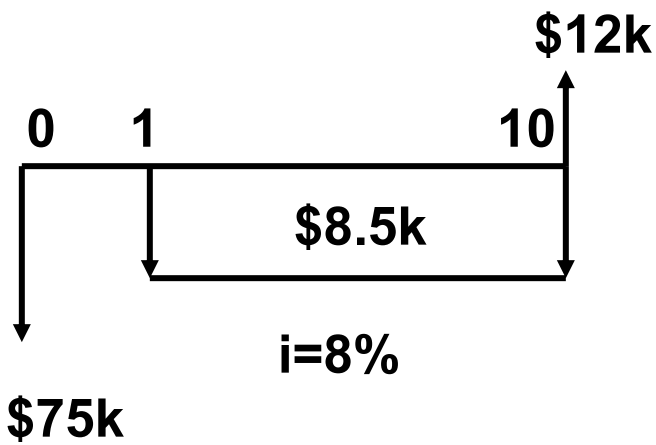
Example Problem 3

You are comparing the cost of two light bulldozers for your construction company. Assuming an interest rate of 8%, make a procurement recommendation.

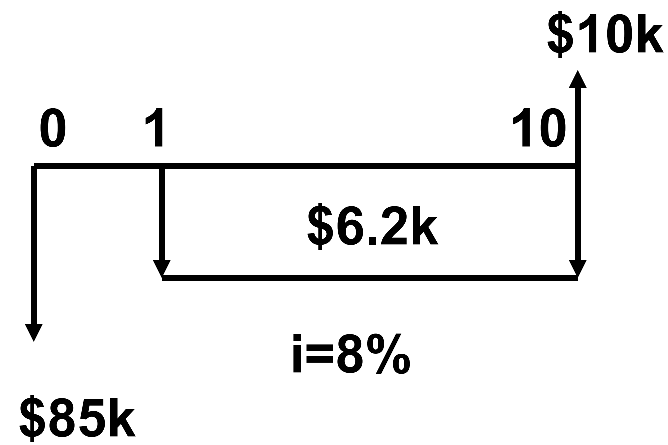
	Cat	John Deere
Initial Cost	$75k	$85k
Salvage @ 10 years	$12k	$10k
Operating Cost/year	$8.5k	$6.2k

Example Problem 3

Cat



John Deere



PW = -75k - 8.5k (P/A,8%,10)
+ 12k (P/F,8%,10)
= - $125,865.18

PW = -85k - 6.2k (P/A,8%,10)
+ 10k (P/F,8%,10)
= - $119,287.57

Example Problem 3

Which bulldozer would you buy?

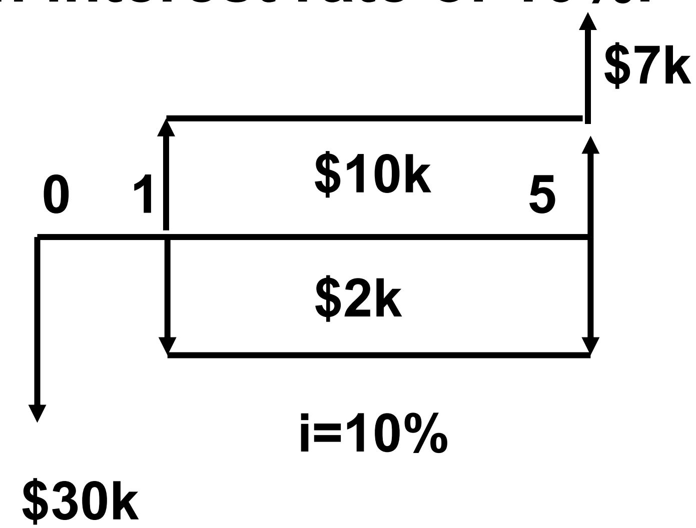
CAT = PW = -$125,565

John Deere = PW = -$119,287

Choose Lowest Cost:
John Deere

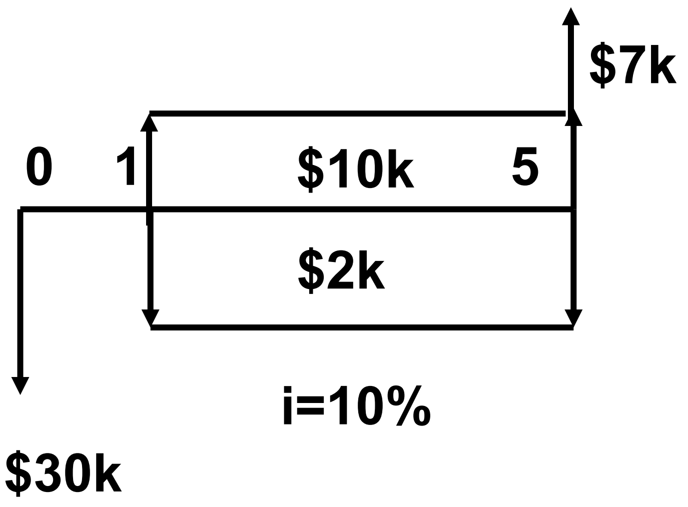
Example Problem 6-1

A machine will cost $30,000 to purchase. Annual operating and maintenance costs will be $2000. The machine will save $10,000 per year in labor costs. The salvage value of the machine after 5 years will be $7000. Draw the cash flow diagram and calculate the machine's annual worth for an interest rate of 10%.



Example Problem 6-1

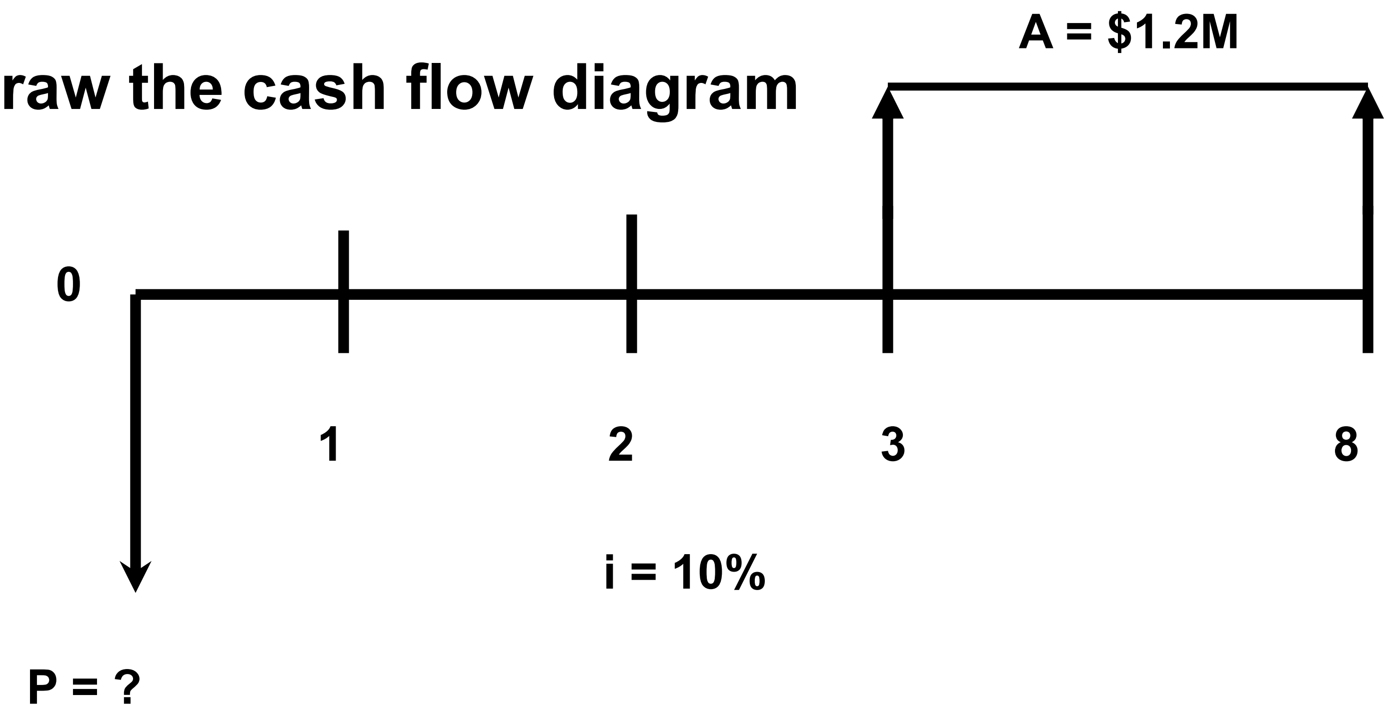
$$AW = -P\,(A/P, i, N) + (A\text{-}A) + F\,(A/F, i, N)$$
$$AW = -30k\,(A/P, 10\%, 5) + (10k\text{-}2k) + 7k(A/F, 10\%, 5)$$
$$AW = -30k\,(.2638) + 8000 + 7k(.1638)$$
$$AW = \$1232$$



Example Problem 5 - Manipulation

Digitel a small communications company is considering whether to replace the network routers. Through the increased speed and traffic, they believe the company will expect an increase in revenues of $1.2M starting 3 years from now and continuing for 5 years thereafter. Digitel uses 10% APR compounded daily for the value of their money. How much should Digitel be willing to pay for these routers?

1. Draw the cash flow diagram



2. Calculate the effective interest rate

$$i = (1 + r/M)^{M} - 1$$
$$= (1+0.1/365)^{365} - 1 = 10.52\ \%$$

1) Align time horizons

Start at year 1

2) Manipulate gradients

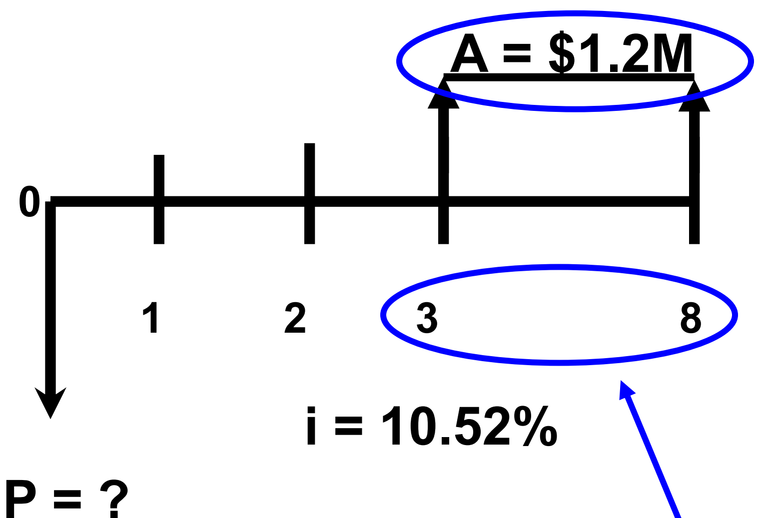
3) Add/subtract cash flow

only one cash flow at any point on line (x axis)

Annual payments must run from year 1 to N

So, change annual income to a lump sum

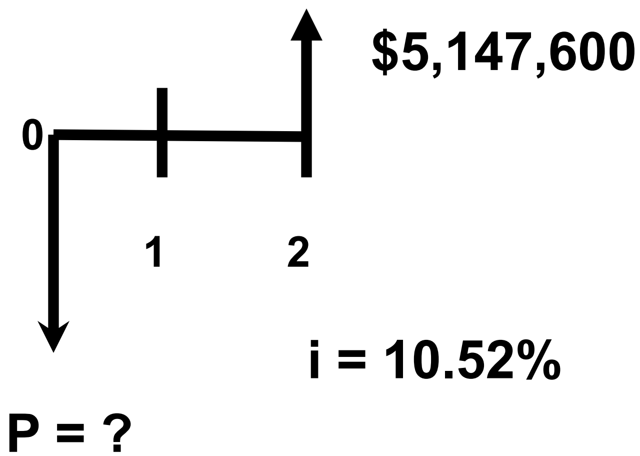
Note: N = 6



$$
\begin{aligned}
P &= A\,(P/A,\, i,\, N)\\
&= 1.2M(P/A,.1052,6)\\
&= 1.2M[(1 + i)^N - 1)/\, i(1 + i)^N]\\
&= 1.2M[(1.1052^6 - 1)/(0.1052 * 1.1052^6)]\\
&= \$5{,}147{,}600
\end{aligned}
$$

must be 1 to n

- Move the lump sum to the present
- Thus the final answer is



$$
\begin{aligned}
P &= \$5.1476M(P/F,\ 10.52\%,2) \\
&= \$5.1476M\ [1/(1 + I)^{N}] \\
&= \$5{,}147600(1.1052)^{-2} \\
&= \$4{,}214{,}275
\end{aligned}
$$

Figures of Merit (FOM) Summary

- **Present Worth**

$$PW = -P + A(P/A,i,N) + F(P/F,i,N)$$

$$PW = -P + \sum_{n=0}^{N} F(P/F,i,n) + F(P/F,i,N)$$

$$PW = AE(P/A,i,N)$$

$$PW = FW(P/F,i,N)$$

- **Future Worth:**

$$FW = -P(F/P,i,N) + A(F/A,i,N) + F$$

$$FW = -P(F/P,i,N) + \sum_{n=0}^{N} F(F/P,i,N-n) + F$$

$$FW = AE(F/A,i,N)$$

$$FW = PW(F/P,i,N)$$

- **Annual Equivalence Worth:**

$$AE = -P(A/P,i,N) + A + F(A/F,i,N)$$

$$AE = -P(A/P,i,N) + \sum_{n=1}^{N} F(P/F,i,N)(A/P,i,N) + F(A/F,i,N)$$

$$AE = -P(A/P,i,N) + \sum_{n=1}^{N} F(F/P,i,N-n)(A/F,i,N) + F(A/F,i,N)$$

$$AE = PW(A/P,i,N)$$

$$AE = FW(A/F,i,N)$$