STEVENS
INSTITUTE of TECHNOLOGY
THE INNOVATION UNIVERSITY®

E355 - Engineering Economics

Lecture 02 A: *Understanding Cash Flow Diagrams, Interest Rates and Time Value of Money*
Chapters: 2, 3, 4

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Why study Engineering Economics?

- Sound decisions cannot be made without a diligent analysis of the **financial consequences** of choosing one alternative over another

- Comparing alternatives solely on their **physical characteristics** or performance **does not capture the full financial result** of building purchasing, operating, maintaining and disposing

Lecture Objectives

After completing this module you should understand the following:

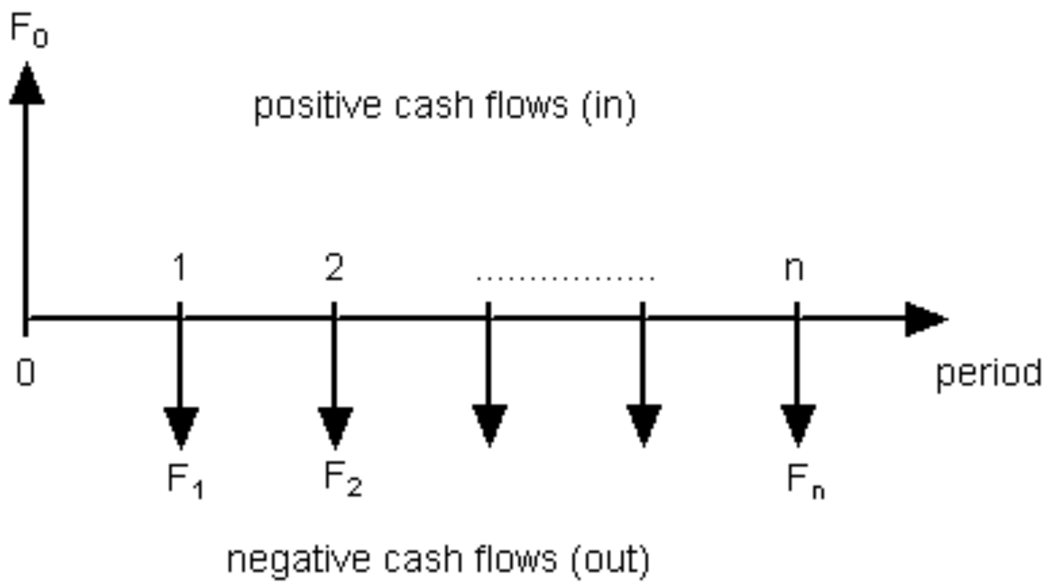
- Cash flow diagram: basics, ‘how to’ and types (arithmetic, geometric gradient)
- Time Value of Money
- Overview of simple and compound interests – calculation methods including continuous compounding
- Nominal, periodic and effective interest rates
- Equivalence calculations with nominal

  and effective interest rates

Why?

- Understand **Long term impacts** of time on economic value. How do you plan for retirement?
- Evaluate different **interest rates** and determine what they mean. Which savings option is best and why?
- **Visualize Economic Values** at different points **in time** to evaluate positive or negative financial impact. This should be a foundation for all economic decisions.

Cash Flow Diagrams (CFD)



Cash Flow Diagrams

Cash Flow Overview

Cash flow is a stream of cash in and/or out of a business, investment or operation

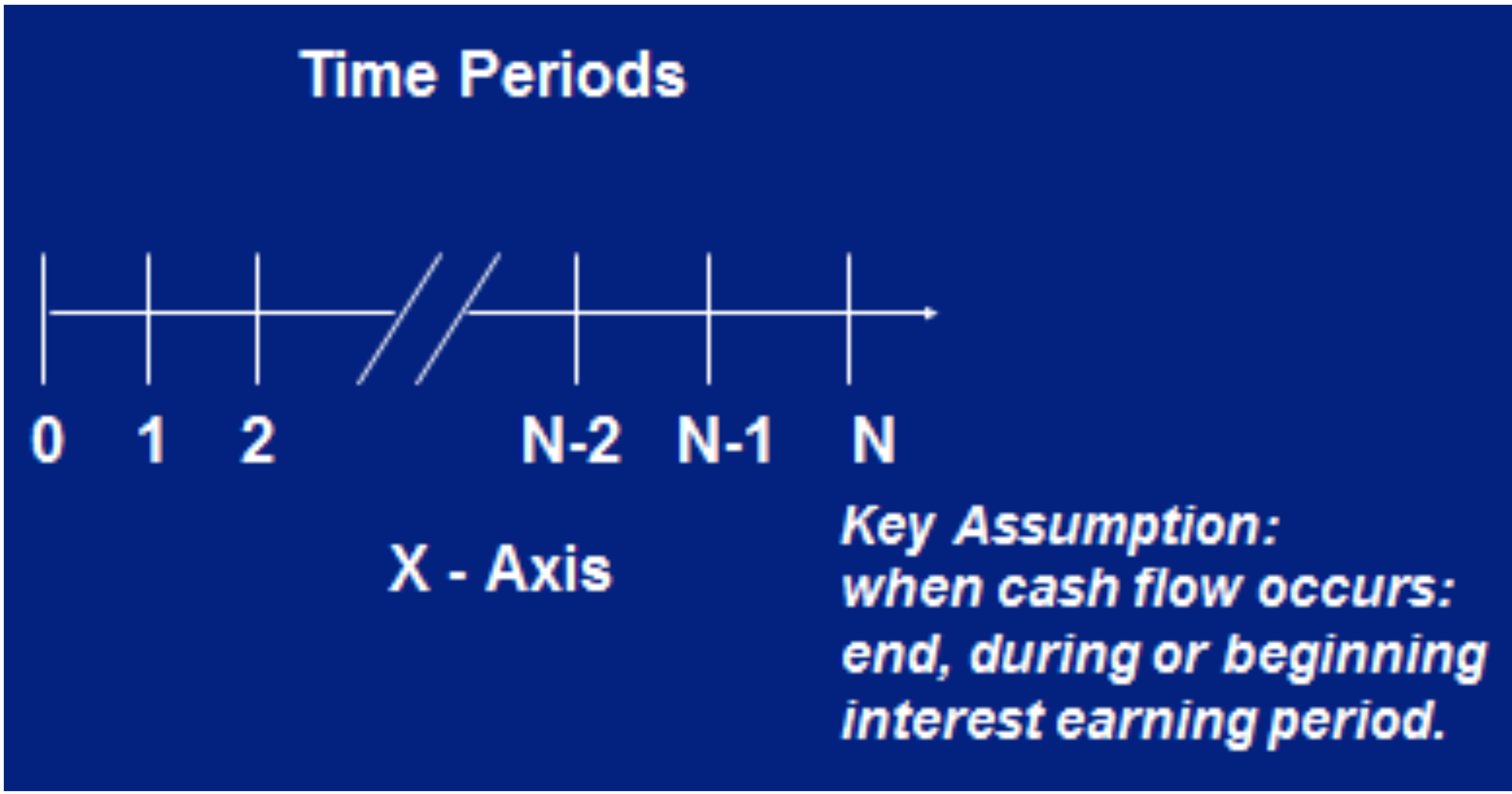
- Cash flows **IN** from **sales** of goods & services
- Cash flows **OUT** from **expenses** from producing those goods & services

  X axis = N

  Y axis = $

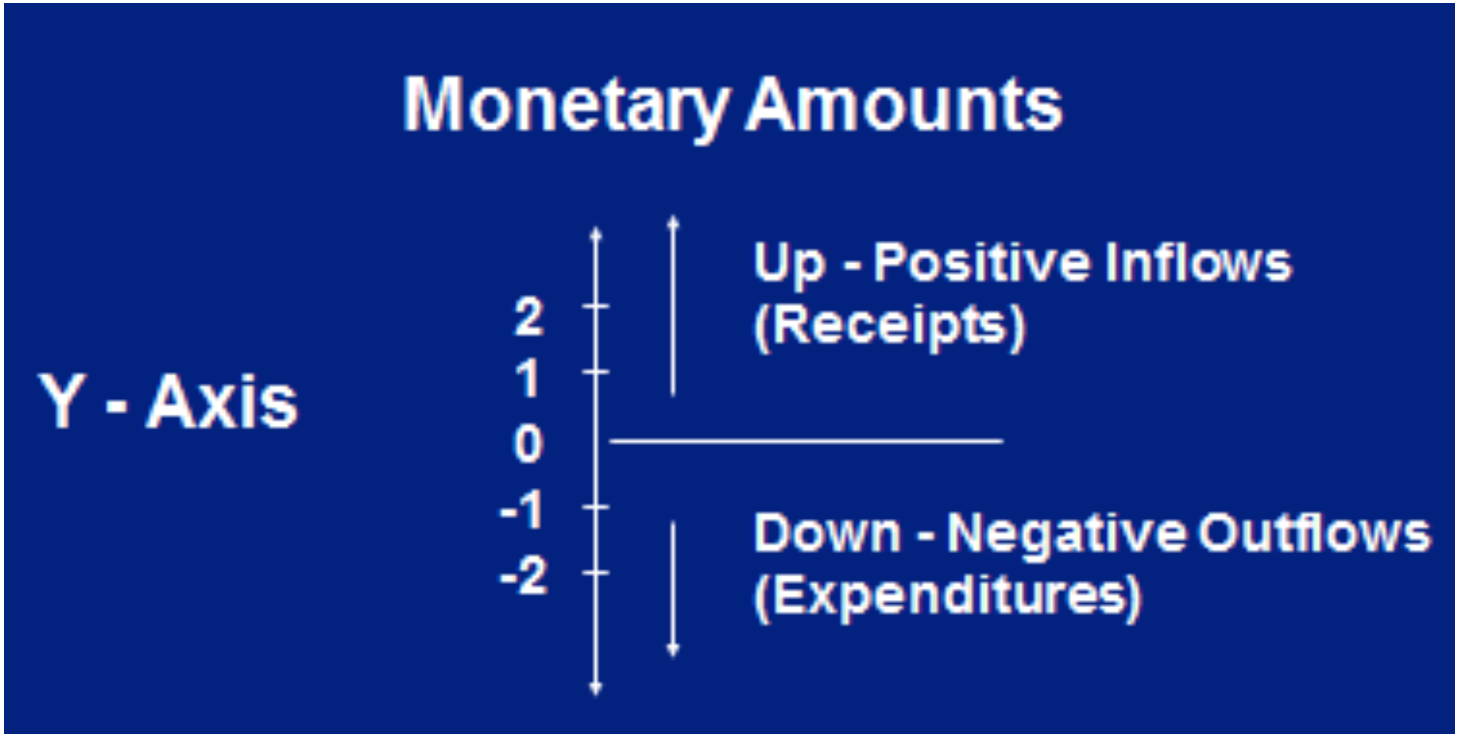
Cash Flow Diagrams

X – Axis Cash Flow Diagram



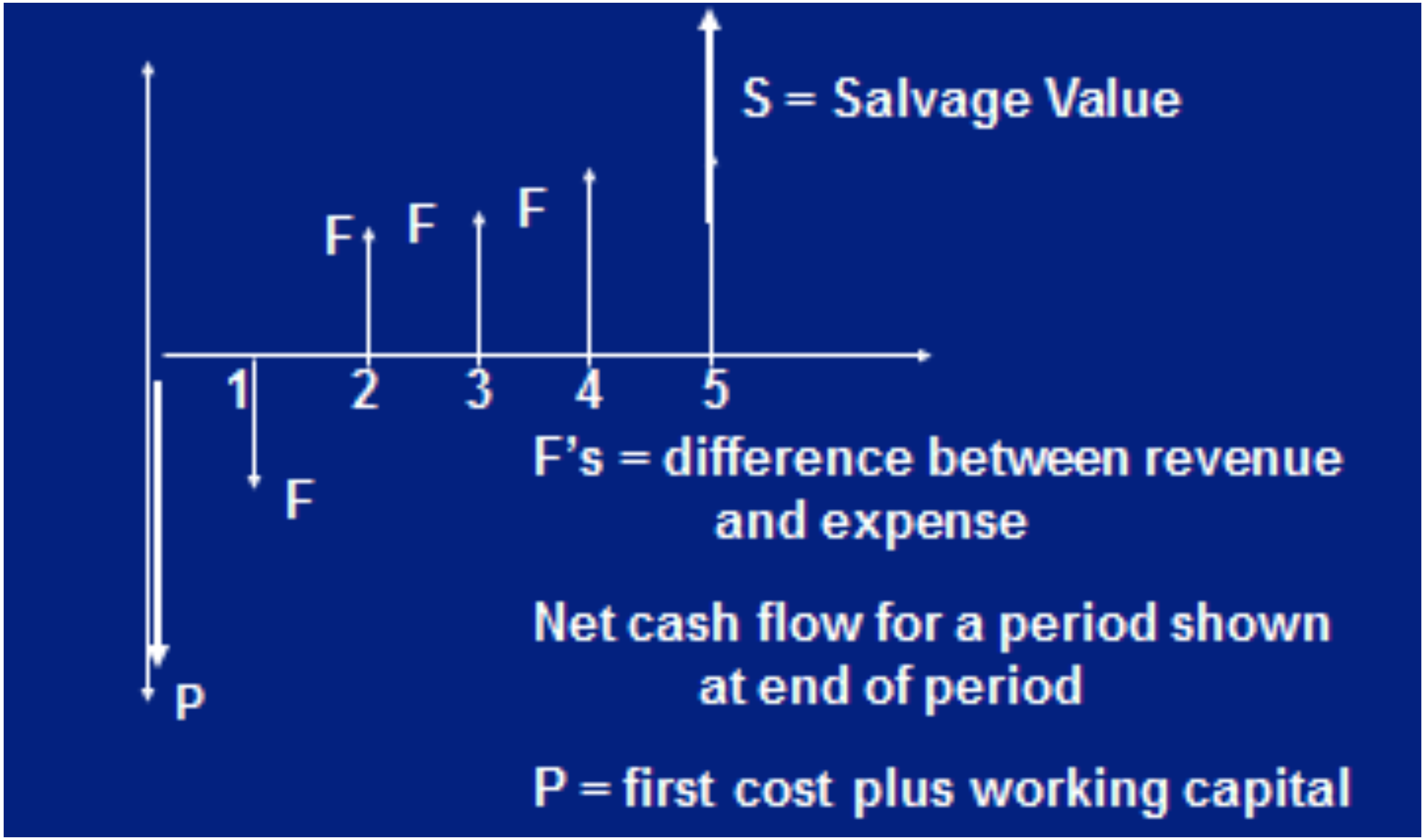
Cash Flow Diagrams

Y – Axis Cash Flow Diagram



Cash Flow Diagrams

Typical Cash Flow for a Project

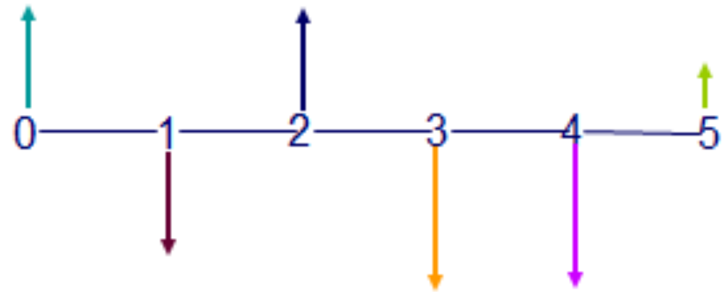


Cash Flow Diagrams

- CFD summarize costs & benefits that occur over time
- CFD illustrates the size, sign and timing of individual cash flows
- Components of CFD
  - A segmented **time-based horizontal line**, divided into time units
  - A **vertical arrow** representing a cash flow is added at the time it occurs
  - Arrow pointing **down for costs** and **up for benefits**

Simple CFD Example

Timing of Cash Flow	Size of Cash Flow
At time zero (now)	Positive $100
1 time period from today	Negative $100
2 time periods from today	Positive $100
3 time periods from today	Negative $150
4 time periods from today	Negative $150
5 time periods from today	Positive $50



Introduction: Nomenclature

Useful Terms:

i	=	Interest Rate
N	=	Number of Periods ($n = 0, 1, \ldots, N$)
P	=	Present Value / Present Worth ($n=0$)
F	=	Future Worth (at some time n)
A	=	Annual Worth / Annual Equivalence
S	=	Salvage Value ($n = N$)

Note:

P = PV = NPV = PW = NPW

F = FW

A = AW = AE (similar to annual cost, AC = EUAC, Equivalent Uniform Annualized Cost)

Cash Flow Diagrams

Drawing a Cash Flow Diagram

- CFD shows when all cash flows occur
- The end of period *t* is the same time as the beginning of period *t+1*
- Rent, lease and insurance payments are usually treated at the beginning-of-period cash flows
- O&M, salvage, revenues and overhauls are assumed to be end-of-period cash flows
- The choice of time 0 is arbitrary

Cash Flow Diagrams

Steps to Solving a Typical Cash Flow Problem:

1. Draw a picture

2. Identify knowns and unknowns

3. Convert all knowns to the same units of time

4. Solve the problem using

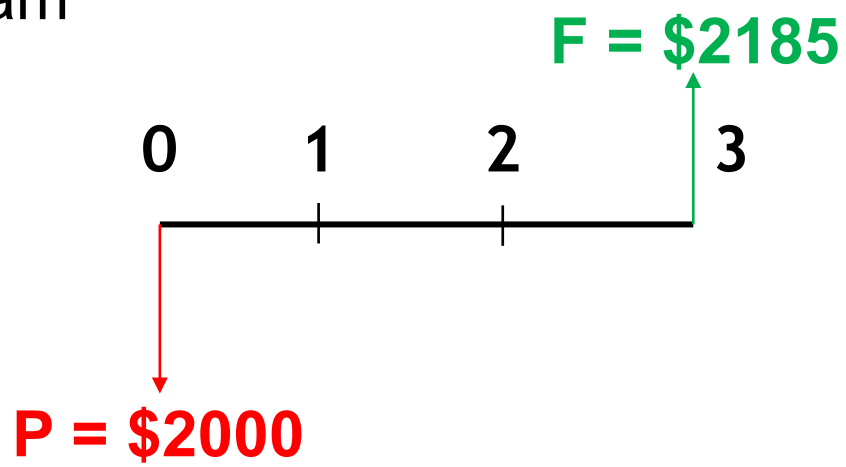
engineering economic techniques

Cash Flow Diagrams

Example: CFD

- Consider an initial investment of $2000
- Investment Period is 3 years
- Interest Rate is 3%
- Future Worth is $2185
- Draw the cash flow diagram

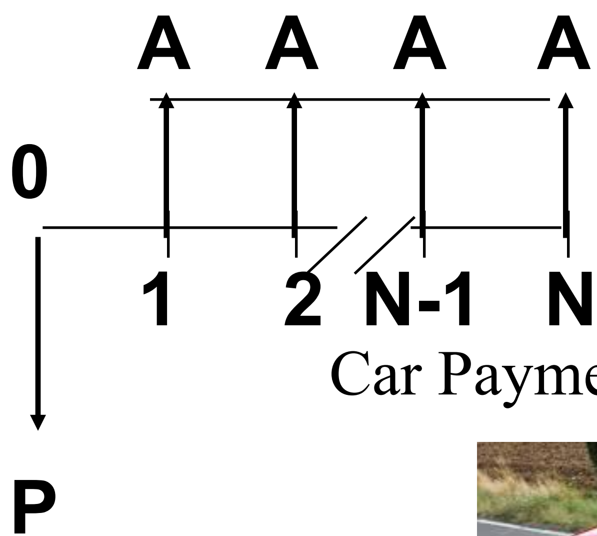
i = 0.03 or 3%
P = $2000
A = $0
F = $2185
N = 3 years



Uniform Series (Annuities)

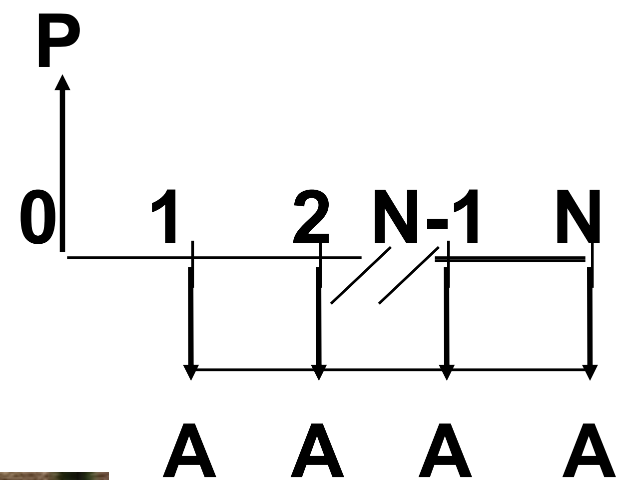
Ordinary Annuities

Lender/Investor Viewpoint



Car Payments

Borrower Viewpoint



Time Value of Money

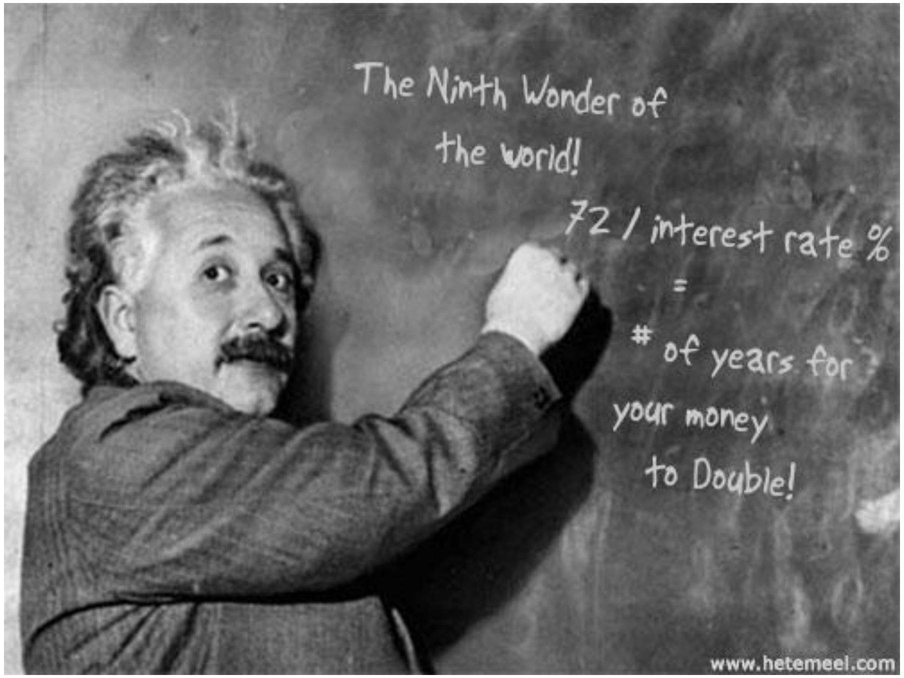
Time Value of Money

- Money has **purchasing power**
- Money has **earning power**
- Money is a **valuable asset**
- People are willing to pay some charges (**interests**) to have money available for their use

Introduction

Albert Einstein once said:

"The most powerful force in the universe is **compound interest**." [1]



1. "Albert Einstein quotes" ThinkExist.com. Aug 13, 2008

Doubling Your Money

Example: i%	(Rule of 74) N Years of doubling	N x i%
1	69.6	70
5	14.2	71
10	7.3	73
20	3.8	76
30	2.6	79

Key Definitions

- **Market Interest Rate**

  Rates of interest paid on deposits and other investments, determined by the interaction of the supply of and demand for funds in the money market. [1]

- **Time Value of Money**

  The idea that a dollar now is worth more than a dollar in the future, even after adjusting for inflation, because a dollar now can earn interest or other appreciation until the time the dollar in the future would be received. [2]

1. “market interest rate” Bank-Street.co.uk. May 20, 2008 <http://bank-street.co.uk/glossary.html>.

2. "time value of money" InvestorWords.com. WebFinance, Inc. May 20, 2008 <http://www.investorwords.com/4988/time_value_of_money.html>.

Key Definitions

- **Purchasing Power**

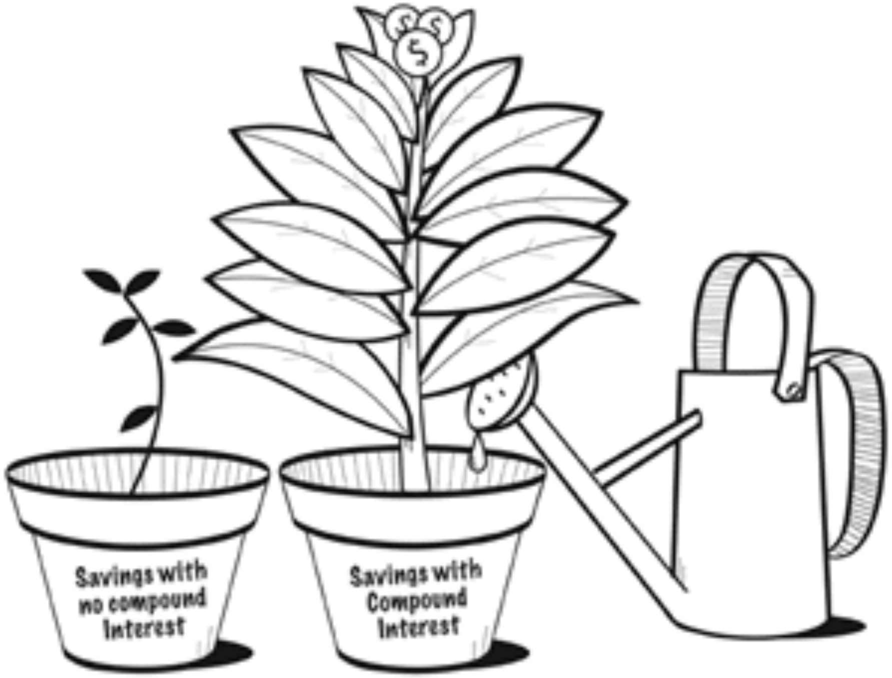
  The value of money, as measured by the quantity and quality of products and services it can buy. [1]

- **Actual Dollars**

  The cash flow measured in terms of dollars at the time of the transaction. [2]

1. "purchasing power" InvestorWords.com. WebFinance, Inc. May 20, 2008 <http://www.investorwords.com/3959/purchasing_power.html>.
2. Park, Chan S. Contemporary Engineering Economics. New Jersey: Pearson Prentice Hall, 2006 (Chapter 3)

Simple & Compound Interest Rates



Two Methods to Calculate Interest

- **Simple Interest**
  - Interest is earned only on principal amount during each period.
  - Interest earned during each interest period does not earn additional interest in the remaining periods.

- **Compounded Interest**
  - Interest is earned during each period based on the TOTAL amount at the end of the previous period.

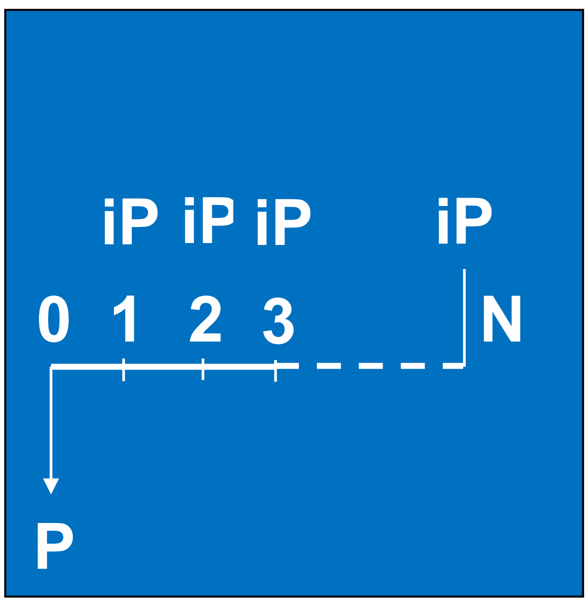
TOTAL = original principal + accumulated interest

Simple Interest

- Interest is computed only on the original sum, and not on accrued interest

Simple interest: F = P (1+iN)

- Derivation



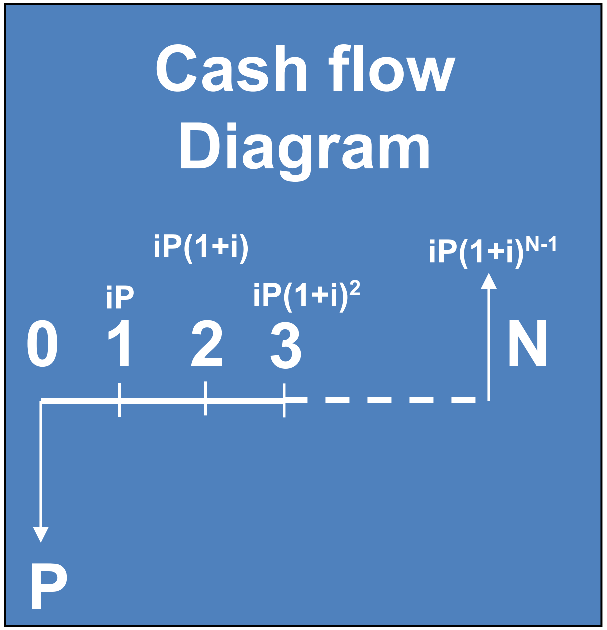
EOY	Bank Balance at End of Year (EOY)
0	P
1	P + iP = P(1 + i)
2	P + iP + iP = P(1 + 2i)
3	P + iP + iP + iP = P(1 + 3i)
.	.
.	.
.	.
N	P + (iP)N = P(1 + iN)

Compound Interest

- Equation

Compound interest: $F = P(1 + i)^N$

- Derivation

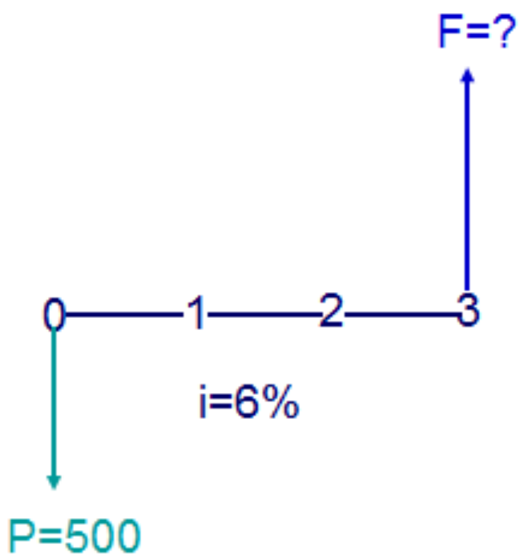


EOY	Bank Balance at End of Year (EOY)
0	P
1	$P + iP = P(1 + i)$
2	$P(1+i) + i[P(1+i)] = P(1 + i)^2$
3	$P(1+i)^2 + i[P(1+i)^2] = P(1 + i)^3$
.	.
.	.
.	.
N	$P(1+i)^{N-1} + i[P(1+i)^{N-1}] = P(1 + i)^N$

Compound Interest | Example

Example 3-5 – Single Payment Compound Interest

$500 were deposited in a saving account (pays 6% compounded annually) for 3 years



$$F = P(1+i)^n = 500(1+0.06)^3$$

$$= \$595.50$$

Or

$$F = P(F/P, i, n) = 500(F/P, 6\%, 3)$$

$$= 500(1.191) = \$595.50$$

Five Types of Cash Flows

1. Single Cash Flow

- Equivalence relations: **P** and **F**
- Single present or future cash flow

2. Equal (Uniform) Series

- Equivalence relations: **P**, **F** and **A**
- Series of cash flows of equal amounts at regular intervals

3. Linear (Arithmetic) Gradient Series

- Equivalence relations: **P**, **F** and **A**
- Fixed amount (G) increase or decrease at regular intervals

Five Types of Cash Flows

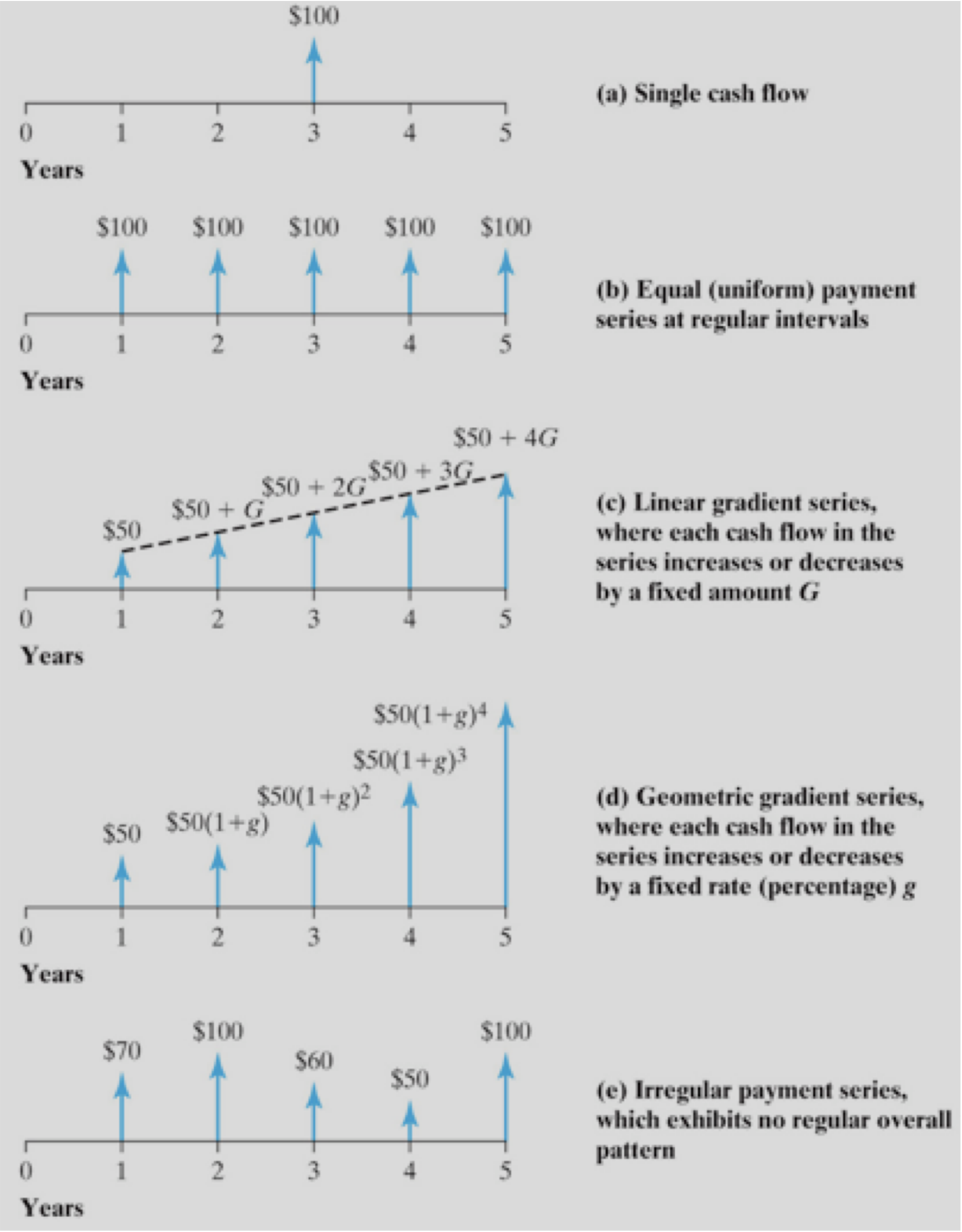
4. Geometric Gradient Series

- Equivalence relations: **P**, **F** and **A**
- Fixed % rate (g) increase or decrease at regular intervals

5. Irregular (Mixed) Series

- Equivalence relations: **P**, **F** and **A**
- No regular overall pattern (patterns may exist in portions)

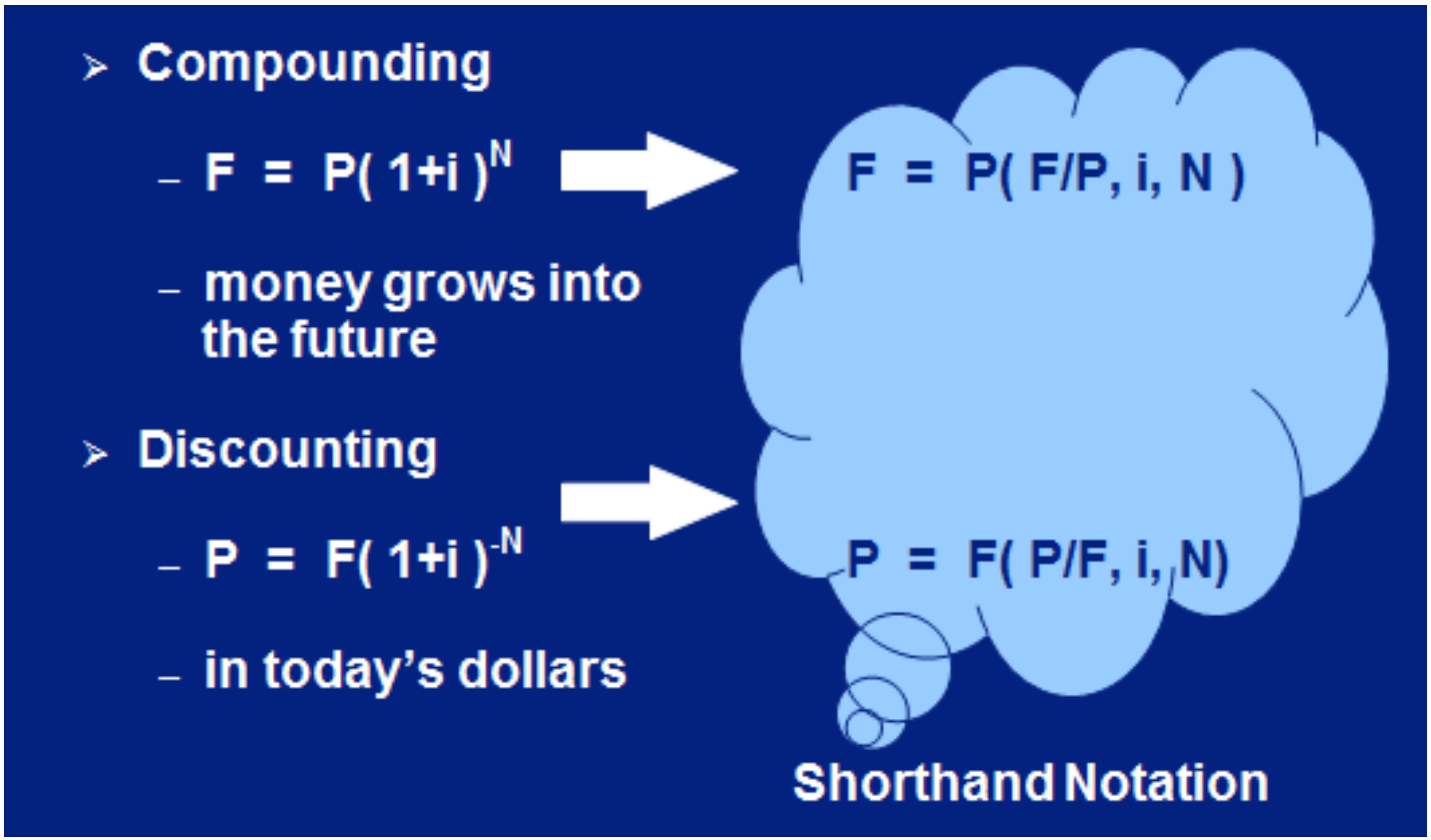
Five Types of Cash Flows



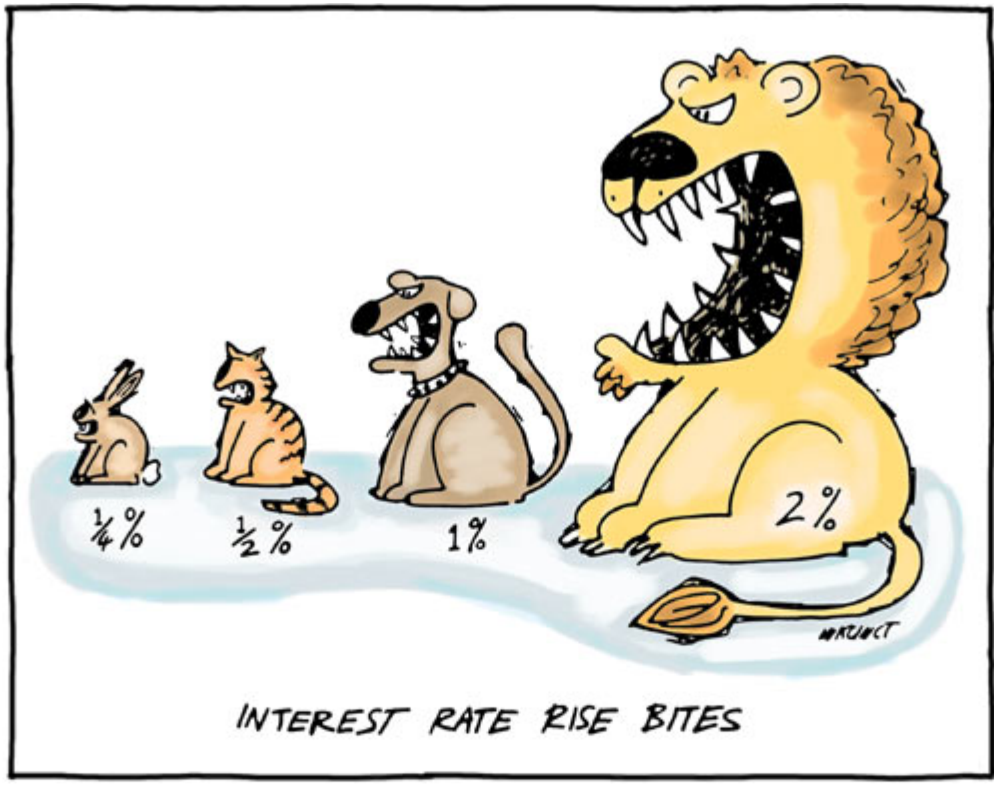
Five Types of Cash Flows
(Chan S. Park, Figure 3.10)

Compounding & Discounting Summary

Short hand vs. Long hand equations



Nominal, Periodic, and Effective Interest Rates



Definitions

- **Periodic Rate**

  Rate for the specified compounding period

- **Nominal Rate**

  Periodic rate multiplied by the number of compounding periods per year

- **Effective Rate**

  Annual rate equivalent to the periodic rate compounded for the number of periods per year

Periodic Interest Rates

- The interest rate per compounding period is called a periodic interest rate (or periodic rate).
- The interest the lender will charge on the amount you borrow. If the lender also charges fees, the periodic interest rate will not be the true interest rate.

$$i_m = \frac{r}{M}$$ where,

i_m = periodic interest rate

r = nominal interest rate

M = # of compounding (interest) periods per year

Nominal Interest Rate

Also known as “annual percentage rate (APR)”

Definition:

- Yearly cost of a loan including interest, insurance, and the origination fee, expressed as a percentage. Does not consider the effect of any compounding.
- Example:

18% APR, compounded monthly = 1.5% per month

2.0% per month = 24% APR

- Does NOT represent the amount of interest earned in a year.
- r = nominal interest rate

Effective Interest Rates

Effective Interest Rate: actual rate

- Also known as "annual percentage yield (APY)"
- Represents the interest earned in a year.

Definition:

- Rate actually earned or paid in one year, taking into account the affect of compounding.

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1$$

where,

i_a = effective annual interest rate

r = nominal interest rate

M = # of compounding (interest) periods per year

Difference Between Nominal, Periodic and Effective Rates

MP4 in folder (2 min)

Nominal and Effective Rates - Example

Compounding	Interest Periods/Yr			Effective Rate
Period	M	r	r / M	i
Annual	1	10	10	10
Semiannual	2	10	5	10.25
Quarterly	4	10	2.5	10.38
Monthly	12	10	0.833	10.47
Daily	360 / 365	10	0.002	10.51
Continuous	Infinity	10	0	10.52

Kinds of Interest Rates

MP4 in folder (2 min)

Example 3-10 Nominal & Effective Interest Rates

"If I give you \$50, you owe me \$60 on the following Monday."

a) Weekly interest rate = (\$60 - \$50) / 50 = 20%

Nominal annual rate = 20% * 52 = 1040%

b) Effective annual rate

c) End-of-the-year balance

$$i_a = (1+i)^m - 1 = (1+20\%)^{52} - 1 = 1310400\%$$

$$F = P(1+i)^n = 50(1+20\%)^{52} = \$655,200$$

Continuous Compounding

Definition:

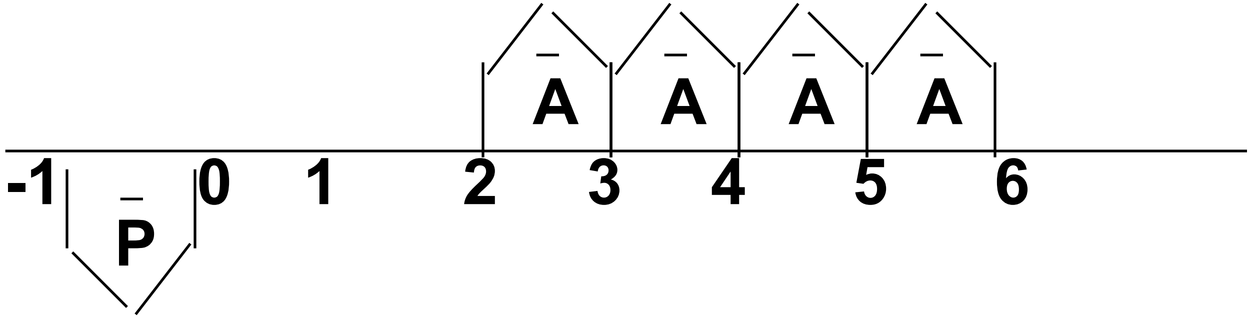
The process of calculating interest and adding it to existing principal and interest at infinitely short time intervals.

$$\boxed{i_a = e^r - 1}$$

where,

r = nominal interest rate

Continuous Cash Flow



Note: continuous cash flows extend over the period

Continuous Compounding

Example:

What is the effective annual rate for 9% APR compounded continuously?

$$i_a = e^r - 1$$

$$i_a = e^{0.09} - 1$$

$$i_a = 0.09417$$

Continuous Compounding

Example 3-11:

If you were to deposit $2,000 in a bank that pays 5% nominal interest, compounded continuously, how much would be in the account at the end of 2 years?

$$F = P(e^{r \cdot n}) = 2000e^{(0.05)(2)} = \$2210.40$$

Or

$$i = e^{r} - 1 = 5.1271\%$$

$$F = P(1+i)^{2} = 2000(1.051271)^{2} = \$2210.34$$

Conclusion
Compound Interest Video

MP4 in folder (3 min)