



**STEVENS**  
INSTITUTE OF TECHNOLOGY  
THE INNOVATION UNIVERSITY®

# E355 - Engineering Economics

**Lecture 03:** Understanding the 3 Worths,  
Capitalized Cost and Capitalized  
Recovery

**Chapters:** 5,6,9

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# Lecture Objectives

- After completing this module you should understand the following:
  - Three Worths:
    - Present Worth (PW)
    - Annual Worth (AW)
    - Future Worth (FW)
  - Evaluation of Alternatives based on the time value of money
  - Capitalized costs
  - Capitalized recovery



## Why?

- Understanding the Time-Value of Money impacts your life including but not limited to:
  - ✓ Student Loans, Interest Payments, Future investments(Car, House, Stocks, etc.)
- The amount of a dollar will change with respect to time. Planning for your future requires your knowledge of the 3 Worths.





# Capital Cost & Recovery

## Lecture 3B





- Useful Definitions:
  - Capital Cost
    - *The amount of net investment.*
  - Operating and Maintenance Cost:  $[O+M](i)$ 
    - *Costs incurred through the operation and maintenance of physical plant or equipment needed to provide service.*
  - Capital Recovery Cost:  $CR(i)$ 
    - *The annual payment that will repay the cost of a fixed asset over the useful life of the asset and will provide an economic rate of return on the investment. (Chan S. Park)*
  - Equivalent Uniform Annual Cost:  $EUAC(i)^*$ 
    - $EUAC(i) = CR(i) + [O+M](i)$
    - Refer to lecture 7 (retirements and replacements) also.

\* Also referred to as: “annual equivalent cost” (AEC), OR  
“annual cost” (AC)

## Capitalized Cost

- Capital Equivalent Method

- Uses

- Perpetual project service life
      - $N \rightarrow \infty$
    - Extremely long project service life
      - $N \geq 50$  years

Capitalized Cost is a Present Worth



$$\text{Capitalized Cost} \quad P = \frac{A}{i}$$



## Capitalized Cost – Example 5-6

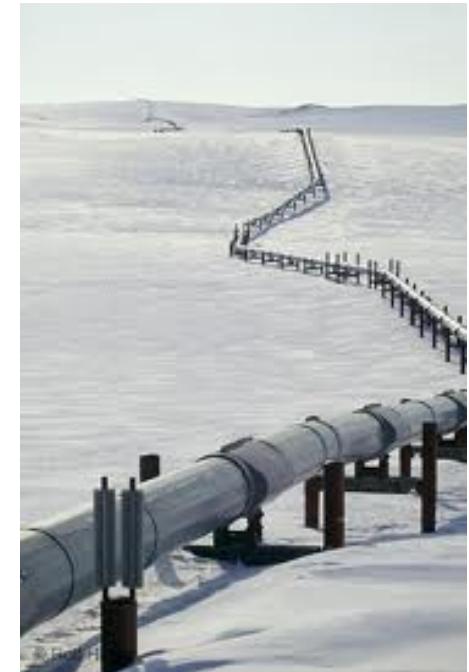
How much should one set aside to pay \$50 per year for maintenance on a gravesite if interest is assumed to be 4%?

Capitalized Cost     $P = \frac{A}{i} = \frac{50}{0.04} = \$1250$



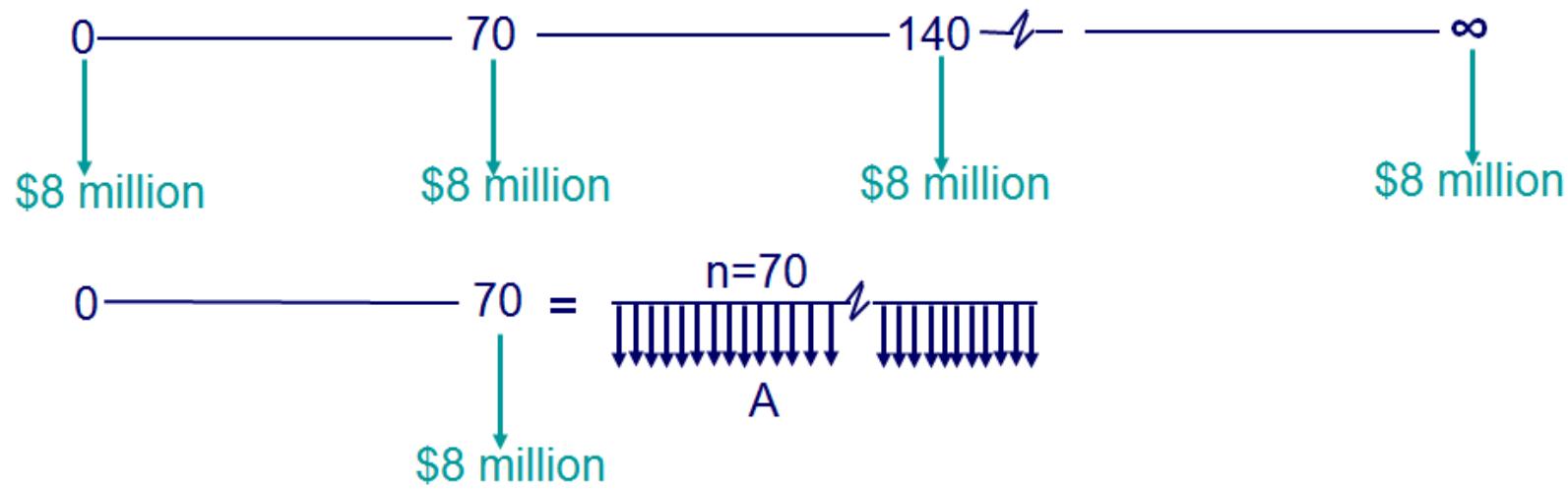
## Capitalized Cost - Example 5-7:

*A city plans to build a pipeline to transport water from a distant watershed area to the city. The pipeline will cost \$8 million and will have an expected life of 70 years. The city expects to keep the water line in service indefinitely. Compute the capitalized cost, assuming 7% interest.*





## Capitalized Cost - Example 5-7:

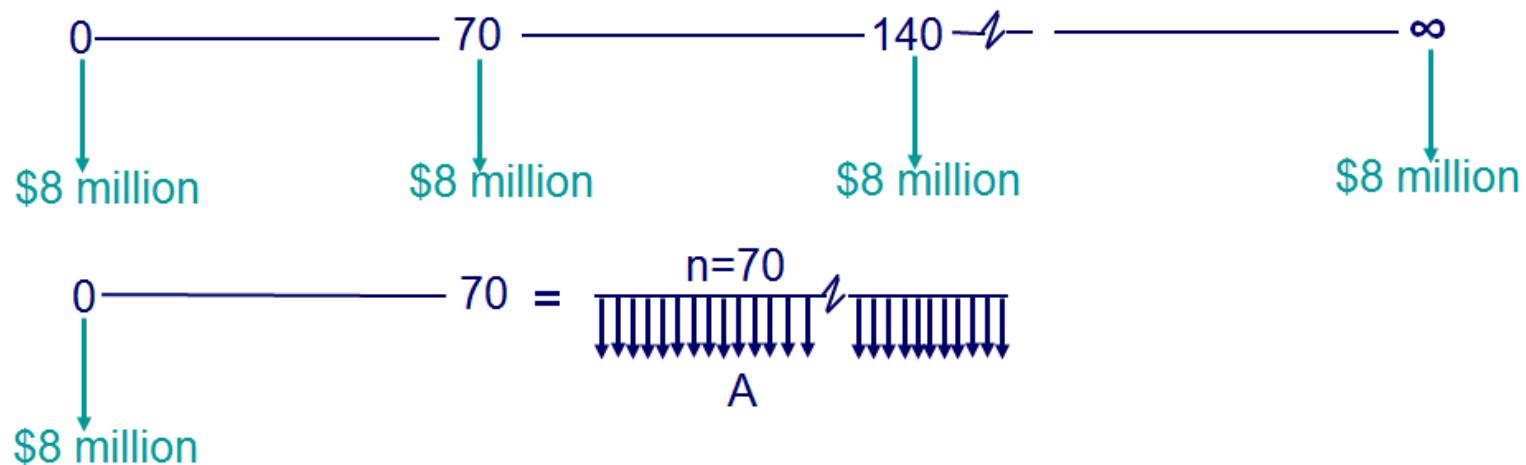


$$A = 8 \text{ million } \textcircled{A/F, 7\%, 70} = \$4960$$

$$\text{Capitalized Cost } P = 8 \text{ million} + \frac{A}{i} = \frac{4960}{0.07} = \$8,071,000$$



## Capitalized Cost – Example 2

*Alternate Solution 1*

$$A = 8 \text{ million } (A/P, 7\%, 70) = \$565,000$$

$$\text{Capitalized Cost} \quad P = \frac{A}{i} = \frac{565000}{0.07} = \$8,071,000$$



## Capitalized Cost – Example 2

*Alternate Solution 2*

$$i_{70 \text{ yr}} = (1 + 7\%)^{70} - 1 = 113.989$$

Capitalized Cost  $P = 8 \text{ million} + \frac{A}{i}$

$$= 8 \text{ million} + \frac{8 \text{ million}}{113.989} = \$8,071,000$$



## Highway Project (L/M 8.4)

- Initial cost : \$100 million
- Annual maintenance expense: \$ 3 million
- Overhaul expenditures every five year : \$5 million
- What is the capitalized cost for MARR of 10%?

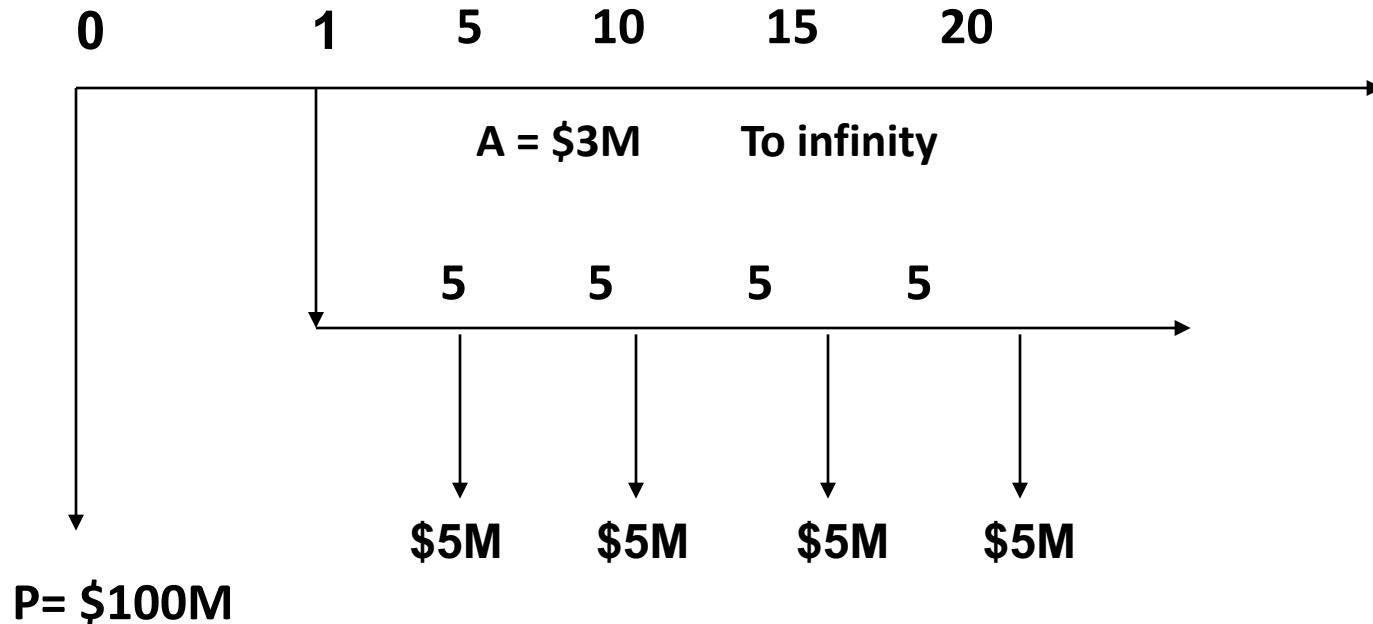


# Example 8.4

## Highway Project - Cash Flow Diagram



Cash flow diagram





## Example 8.4

### Highway Project - Cash Flow Table

EOY	Symbol	First Cost (\$M)	O & M Costs (\$M)	Overhaul Costs (\$M)
0	P	100	-	-
1- infinity	A	-	3	-
5, 10, 15, .....	F	-	-	5
... infinity				



$$\begin{aligned}CC &= P + A ( P / A, i, \text{infinity} ) + F ( A / F, i, p ) / i \\&= 100 + 3( P / A, 10, \text{infinity} ) + 5 ( A / F, 10, 5 ) / 0.1 \\&= 100 + 3 \times 10 + (5 \times 0.1638 \times 10) \\&= 100 + 30 + 8 \\&= \$138M\end{aligned}$$



# Show Video

# Capitalized Cost

(3min)



## Capitalized Recovery

- A uniform series representing the difference between the equivalent annual cost of the first cost and the equivalent annual worth of the salvage value

- $EUAC = P (A/P, i, n) - S (A/F, i, n) + O\&M$
- $CR_{EUAC} = (P-S)(A/F, i,n)+Pi$
- $CR_{EUAC} = (P-S)(A/P,i, n)+Si$

Where  $P$  = initial cost;  $S$ =salvage value

All three equations yield the same answer. **This is also known as the capital recovery cost of the project.**

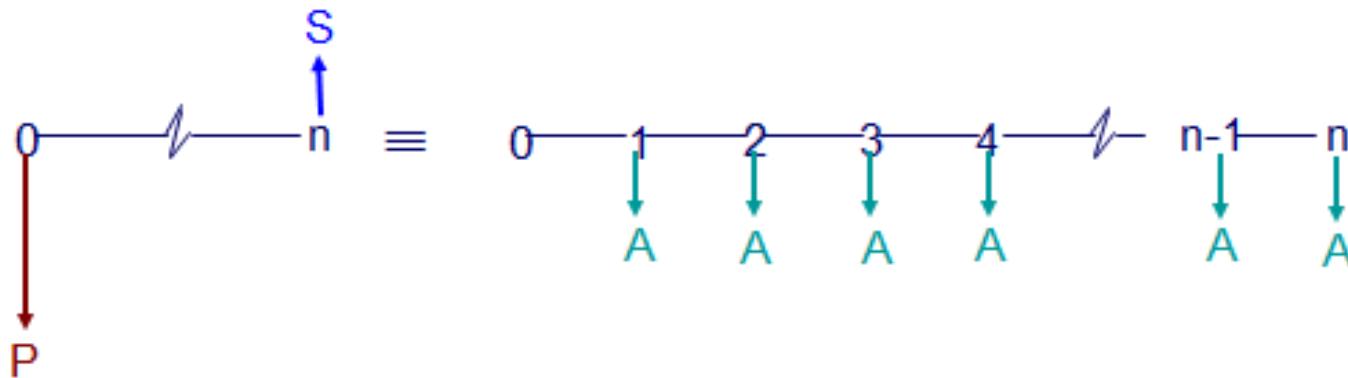
- Capital Recovery cost for infinite horizons

- $CR = Pi$

**Capitalized Recovery is an Annual Worth**



## Annual Cash Flow Calculations



$$\text{EUAC} = P(A/P, i, n) - S(A/F, i, n) \quad (6-1)$$

$$\text{EUAC} = (P - S)(A/F, i, n) + P(i) \quad (6-3)$$

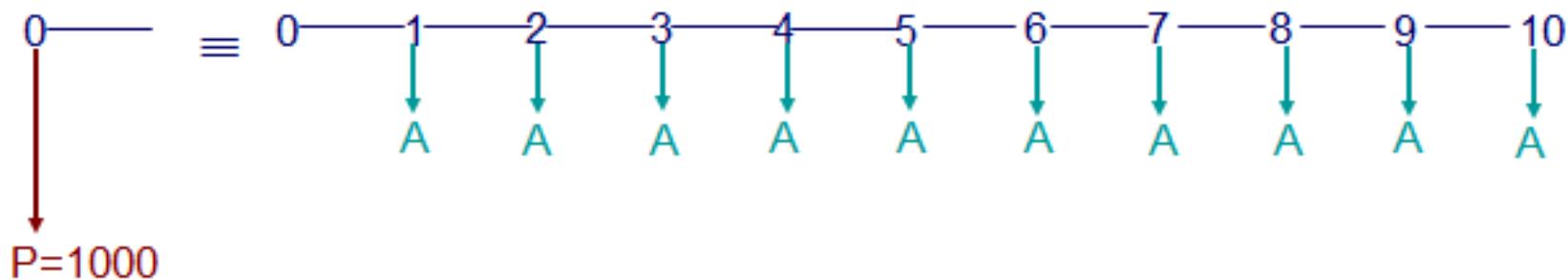
$$\text{EUAC} = (P - S)(A/P, i, n) + S(i) \quad (6-4)$$



## Annual Cash Flow – Example 6-1

*Simplest Case: convert a present sum  $P$  to a series of equivalent uniform end-of-period cash flows.*

A student bought \$1000 worth of furniture. What is the equivalent annual cost if it is expected to last 10 years and the interest rate is 7%?



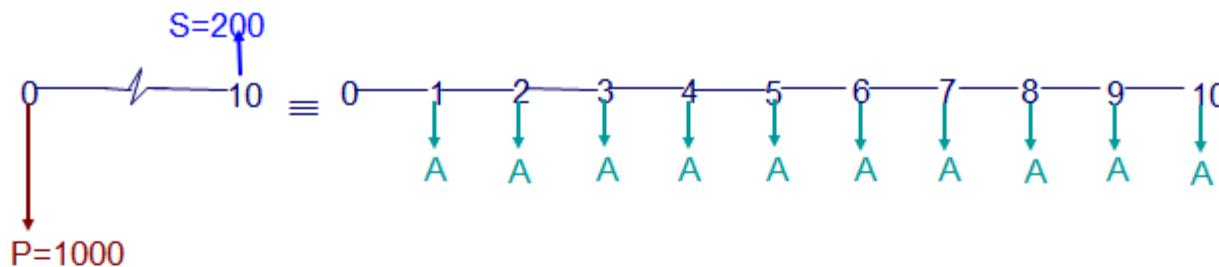
$$\begin{aligned} \text{EUAC} &= 1000(A/P, 7\%, 10) \\ &= \$142.40 \end{aligned}$$



## Annual Cash Flow – Example 6-2

*With Salvage Value at the end of an assets useful life, the EUAC decreases.*

A student bought \$1000 worth of furniture. What is the equivalent annual cost if it is expected to last 10 years and can be sold for \$200?



$$\text{EUAC} = 1000(A/P, 7\%, 10) - 200(A/F, 7\%, 10) = \$127.92$$

$$\text{EUAC} = (1000 - 200)(A/P, 7\%, 10) + 200(7\%) = \$127.92$$

$$\text{EUAC} = (1000 - 200)(A/F, 7\%, 10) + 1000(7\%) = \$127.92$$

**\*\*Note:** Other equations can be substituted and used

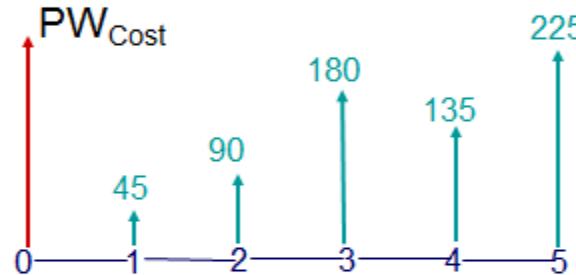


## Annual Cash Flow – Example 6-3

*Irregular series of payments: 2 Steps*

- 1) Use single payment present worth factors to compute the present worth of cost for the 5 years. Assume 7% interest and end-of-year disbursements.
- 2) With the PW cost known, use the capital recovery factor to compute EUAC

Year	Maintenance and Repair Cost
1	45
2	90
3	180
4	135
5	225



$$\begin{aligned}
 PW_{\text{Cost}} &= 45(P/F, 7\%, 1) + 90(P/F, 7\%, 2) + 180(P/F, 7\%, 3) \\
 &\quad + 135(P/F, 7\%, 4) + 225(P/F, 7\%, 5) = \$531
 \end{aligned}$$

$$EUAC = 531(A/P, 7\%, 5) = \$130$$

\*assuming 7% interest and end-of-year disbursements



## Annual Cash Flow – Example 6-5

*Choosing between 2 alternatives*

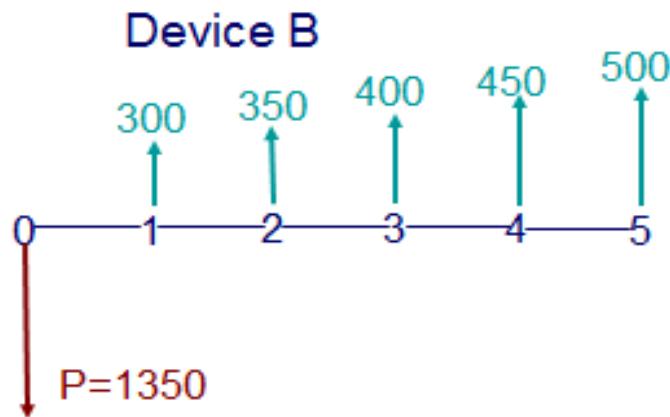
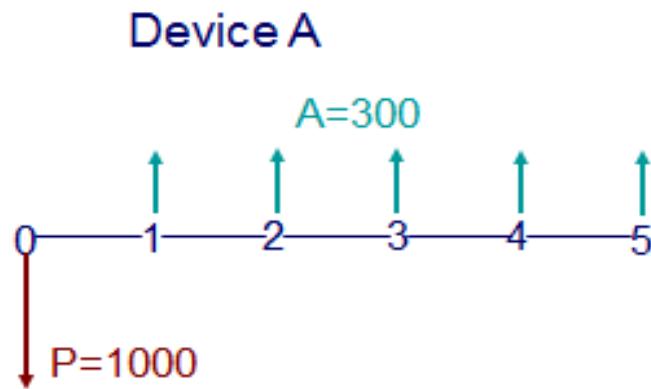
A firm is considering which of two devices to install to reduce costs. Both devices have useful lives of 5 years with no salvage value. Device A costs \$1000 and can be expected to result in \$300 savings annually. Device B costs \$1350 and will provide costs savings of \$300 the first year; however, savings will increase \$50 annually, making the second year savings \$350, the third year savings \$400 and so forth. With interest at 7%, which device should the firm purchase?





## Annual Cash Flow – Example 6-5

*Choosing between 2 alternatives*



$$\begin{aligned} \text{EUAW}_A &= -1000(A/P, 7\%, 5) + 300 \\ &= -1000(0.2439) + 300 = \$56.1 \end{aligned}$$

$$\begin{aligned} \text{EUAW}_B &= -1350(A/P, 7\%, 5) + 300 + 50(A/G, 7\%, 5) \\ &= -1350(0.2439) + 300 + 50(1.865) = \$64.0 \end{aligned}$$

**To maximize EUAW, select DEVICE B**



## Annual Cash Flow: *Infinite Analysis Period*

- $\text{EUAC}_{\text{infinite analysis period}} = P(A/P,i,\infty) + \text{any other annual costs}$
- When  $n = \infty$ , we have:
  - $A = Pi$ , and therefore  $(A/P,i,\infty) = i$
  - $\text{EUAC}_{\text{infinite analysis period}} = Pi + \text{any other annual costs}$



## Annual Cash Flow – Example 6-9

*Infinite Analysis Period*

In the construction of an aqueduct to expand the water supply in a city, there are two alternatives for a particular portion of the aqueduct. Either a tunnel can be constructed through a mountain, or a pipeline can be laid to go around the mountain. If there is permanent need for the aqueduct, should the tunnel or the pipeline be selected for this particular portion of the aqueduct?

Assume a 6% interest rate.





## Annual Cash Flow – Example 6-9

*Infinite Analysis Period*

	Tunnel	Pipeline
Initial cost	\$5.5 million	\$5 million
Maintenance	0	0
Useful life	Permanent	50 years
Salvage value	0	0

$$\text{EUAC}_{\text{Tunnel}} = P(i) = 5.5 \text{ million}(6\%) = \$330,000$$

$$\text{EUAC}_{\text{Pipeline}} = 5 \text{ million}(A/P, 6\%, 50) = \$317,000$$

**For fixed output, minimize EUAC. Select the Pipeline.**

Example:

$$P = \$100 \text{ million}$$

$$i = 10\%$$

$$\text{Volume} = 20 \text{ million gallons}$$

Assume Infinite Life



$$\text{Then } CR = 100 \times 0.10 = \$10 \text{ million / year}$$

$$CR (\text{ Price }) = \$10 \text{ million / 20 million} = \$.50 / \text{Gallon}$$



## Supplemental Problem (L/M 8.12)

Sym's Lock has just purchased a new key cutting machine for \$15,000. Maintenance for the machine is estimated at \$1,500 per year. The useful life of the machine is 20 years, and the expected average number of keys cut in a day is 40. If Sym's Lock wants a rate of return of 10%, what should be the minimum price charged to cut a key?





$$CR = P(A/P, i, N) - S(A/F, i, N)$$

$$CR = (\$15,000)(A/P, 10, 20) - \$0$$

$$CR = (\$15,000)(0.1175) - \$0 = \$1,763$$

$$\text{Total AC} = CR + [O\&M]$$

$$= \$1,763 + \$1,500$$

$$= \$3,263$$



$$\text{Total \# keys/year} = (40)(365) = 14,600 \text{ keys/year}$$

$$\text{Minimum price} = \$3,263 / 14,600 = \$0.22/\text{key}$$

## Supplemental Problem (L/M 8.6) Quebec Hydroelectric Plant

Calculate the Capital Recovery Cost

Initial cost of Phase 1 = \$14 Billion

Rate of Return = 11.5%

Life cycle = 50 years = assume infinite life



Calculate the Capital Recovery Cost

$$= CR = Pi = \$14 \text{ Billion (.115)}$$

$$= \$1.61 \times 10^9$$



## Supplemental Problem (L/M 8.13)

Metro transit is considering building an additional bridge to relieve traffic on an extremely congested river crossing. The costs of construction of the bridge have been bid at \$50 million. Operating and maintenance costs have been estimated at 44 million per year. Traffic conditions on the bridge are expected to average 13,000 cars per day. If the desired rate of return is 12%, what should be the minimum amount of toll per crossing?





$$CR = Pi = 50M (.12) = 6M$$

$$\text{Total AC}^* = CR - [O\&M]$$

$$= \$6M + 4M$$

$$= \$10M$$



$$\text{Total \# cars/year} = (13000)(365) = 4.745M$$

$$\text{Minimum toll} = \$10M / 4.745M = \$2.10/\text{car}$$

\*Remember true form of equation should be remembered as Total AC = CR +/- A

## Merck/Prescription Drugs

- \$300 to \$500 million R & D costs per new drug
- Short life: 5 to 6 years after 17 year patent expires
- What is CR for a pill?



## Hotel or Cruise Ship

- \$300 to \$500 million construction costs per ship/hotel
- High operating costs
- Long life: 15-25 years
- What is CR per room?





# Questions?

ANY  
QUESTIONS?  
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