

As an abuse of notation, write

$$(a_1, \dots, a_n) \in \text{Data}$$

to mean that there is a row in Data such that when restricted to an understood set of attributes, the resulting tuple is (a_1, \dots, a_n) .

Denote by

$$O((a_1, \dots, a_n), a) := (\text{number of occurrences of } (a_1, \dots, a_n, a) \text{ in Data})$$

where a corresponds to an attribute not in the set of attributes understood to correspond to (a_1, \dots, a_n) .

With this notation, the support of a functional dependency $F = \text{Domain} \rightarrow \text{Attribute}$ is given by

$$\text{supp}(F) = \frac{\sum_{(a_1, \dots, a_n) \in \text{Data}} \max_a O((a_1, \dots, a_n), a)}{\sum_{(a_1, \dots, a_n) \in \text{Data}} \sum_a O((a_1, \dots, a_n), a)} \quad (1)$$

We shall use the notation

$$(a_1, \dots, \hat{a}_i, \dots, a_n) = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$$

i.e. the hat indicates the non-presence of a_i in the tuple.

With this in mind, we quickly see that

$$O((a_1, \dots, \hat{a}_i, \dots, a_n), a) = \sum_{a_i} O((a_1, \dots, a_i, \dots, a_n), a)$$

As a result, we find that

$$\begin{aligned} \sum_{(a_1, \dots, \hat{a}_i, \dots, a_n) \in \text{Data}} \sum_a O((a_1, \dots, \hat{a}_i, \dots, a_n), a) &= \sum_{(a_1, \dots, \hat{a}_i, \dots, a_n) \in \text{Data}} \sum_{a_i} \sum_a O((a_1, \dots, a_i, \dots, a_n), a) \\ &= \sum_{(a_1, \dots, a_i, \dots, a_n) \in \text{Data}} \sum_a O((a_1, \dots, a_i, \dots, a_n), a) \end{aligned}$$

In other words, if Domain' is any subset of Domain and $F' = \text{Domain}' \rightarrow \text{Attribute}$, we see that the denominator of $\text{supp}(F')$ in Equation 1 is the same as that for $\text{supp}(F)$.

Likewise,

$$\begin{aligned} \sum_{(a_1, \dots, \hat{a}_i, \dots, a_n) \in \text{Data}} \max_a O((a_1, \dots, \hat{a}_i, \dots, a_n), a) &= \sum_{(a_1, \dots, \hat{a}_i, \dots, a_n) \in \text{Data}} \max_a \sum_{a_i} O((a_1, \dots, a_i, \dots, a_n), a) \\ &\leq \sum_{(a_1, \dots, \hat{a}_i, \dots, a_n) \in \text{Data}} \sum_{a_i} \max_a O((a_1, \dots, a_i, \dots, a_n), a) \\ &= \sum_{(a_1, \dots, a_i, \dots, a_n) \in \text{Data}} \max_a O((a_1, \dots, a_i, \dots, a_n), a) \end{aligned}$$

In other words, if Domain' is any subset of Domain and $F' = \text{Domain}' \rightarrow \text{Attribute}$, we see that the numerator of $\text{supp}(F')$ in Equation 1 is the same as that for $\text{supp}(F)$.

Together,

$$\text{supp}(F') \leq \text{supp}(F)$$