Problem Set 8

By Hayden Orth; GitHub: haydenorth

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Abstract

This article contains my solutions to Problem Set 8 for the graduate Computational Physics course. Problem Set 8 investigates Fourier Transform and ordinary differential equations methods.

1 Problem 1: Fourier Transforms of Musical Instruments

Problem 1 involves taking the Fourier transform of waveforms produced by two different instruments: a trumpet and a piano. The program loads the data from a .txt file and plots it. Figures 1 and 2 show the waveforms of the piano and trumpet, respectively. Then, the program uses SciPy's Fast Fourier Transform (FFT) routine to take the Fourier transform of the waveforms. Then, I take the absolute square of the Fourier coefficients.

Next, in order for the frequency data to be meaningful, I must calculate the frequency resolution. I also calculate the Nyquist frequency to ensure that I include all frequencies relevant in the spectrum. Both the frequency resolution and the Nyquist frequency are calculated using the inverse of the sampling rate, which gives the time separation of the samples of the waveform data. Finally, using the frequency resolution and the Nyquist frequency, I create an array of frequency values and plot the magnitude of the coefficients of these frequency values. The resulting plots are shown in Figures 3 and 4.

We can see from the Fourier coefficient plots that the sound from the trumpet includes many harmonics of a particular frequency, while the sound from the piano is mainly at only one frequency. I found that the piano's fundamental and almost entirely dominating frequency was around 523 Hz, indicating that it is a C5. The trumpet's spectrum includes more contributions from the integer harmonics of its fundamental frequency, which I found to be around 1046 Hz, or C6. This makes sense, as a trumpet's sound comes from air resonating in its tubes, resulting in integer multiples in its frequency spectrum. The piano's sound is produced by a vibrating string which vibrates much more precisely at a single frequency.

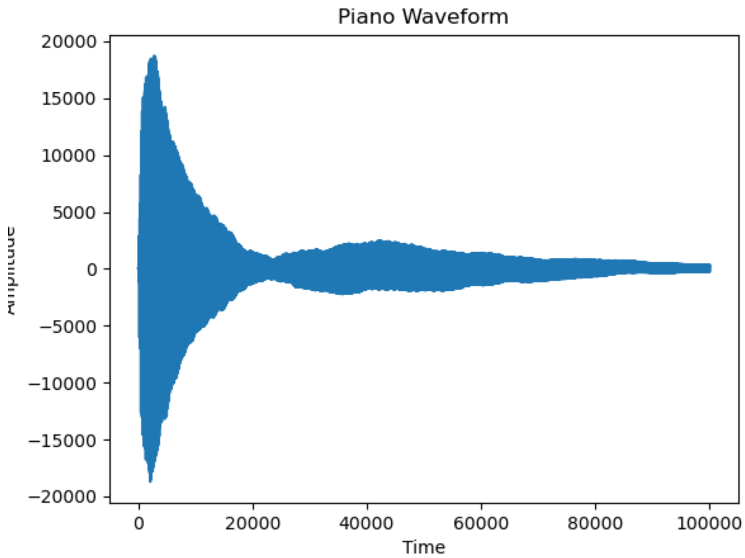

Figure 1: Plot of waveform data for the piano.

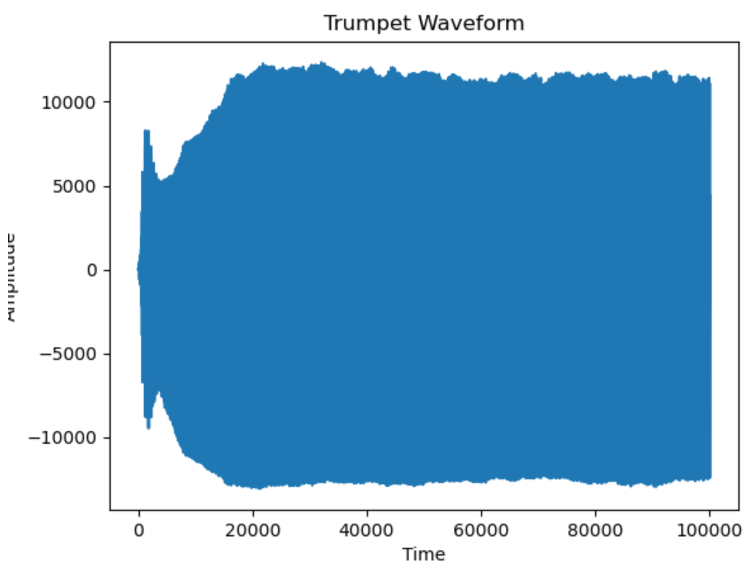

Figure 2: Plot of waveform data for the trumpet.

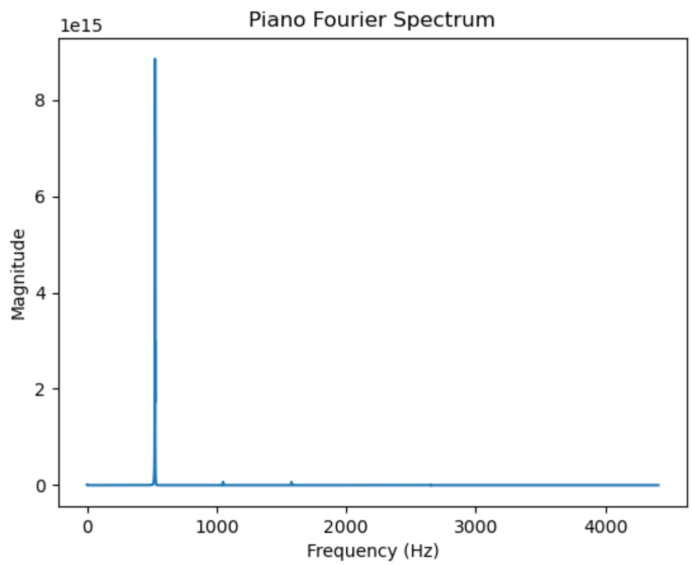

Figure 3: Plot of the Fourier spectrum of the piano's waveform.

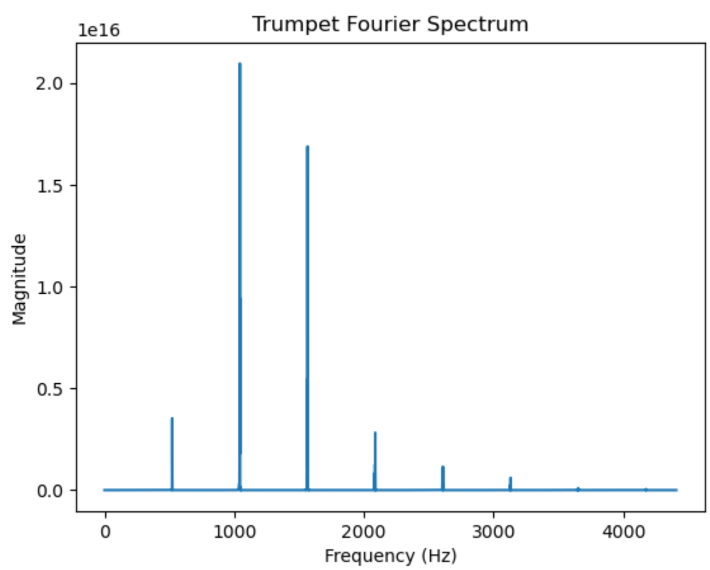

Figure 4: Plot of the Fourier spectrum of the trumpet's waveform.

2 Problem 2: Lorenz Equations

Problem 2 investigates the Lorenz equations, which is a set of three ordinary differential equations. The program solves the Lorenz equations and then plots y vs. t and z vs. x. First, I created a function f that contains the set of differential equations. For particular values of t, x, y, and z, the function returns dx/dt, dy/dt, and dz/dt. Then, I create an array of time values to be used to solve the equations. I then use the Radau method via SciPy's solve_ivp routine to solve the differential equations. Passing the function f, the time endpoints, the initial conditions, the time values, and the required tolerance to the routine, along with indicating to use the Radau method, solves the differential equations. I can then extract the values of x, y, and z from the solution object that the solve_ivp routine returns. Finally, the program plots y vs. t and z vs. x, which can be seen in Figures 5 and 6, respectively.

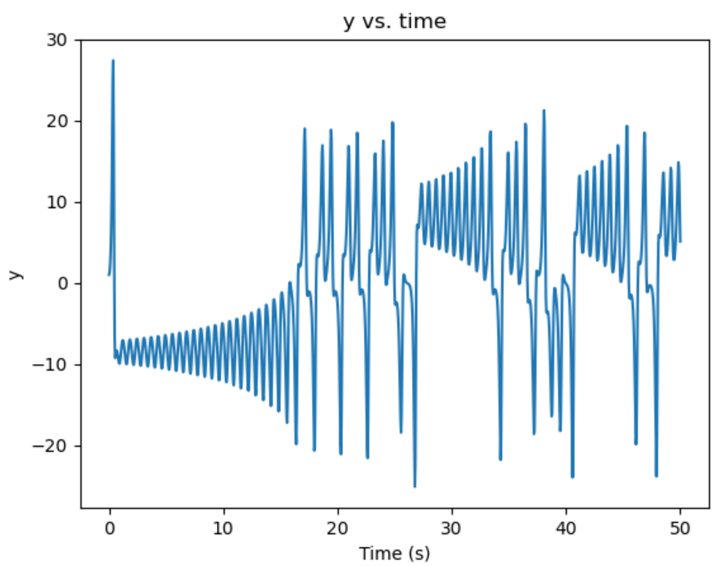

Figure 5: Plot of y values as a function of time.

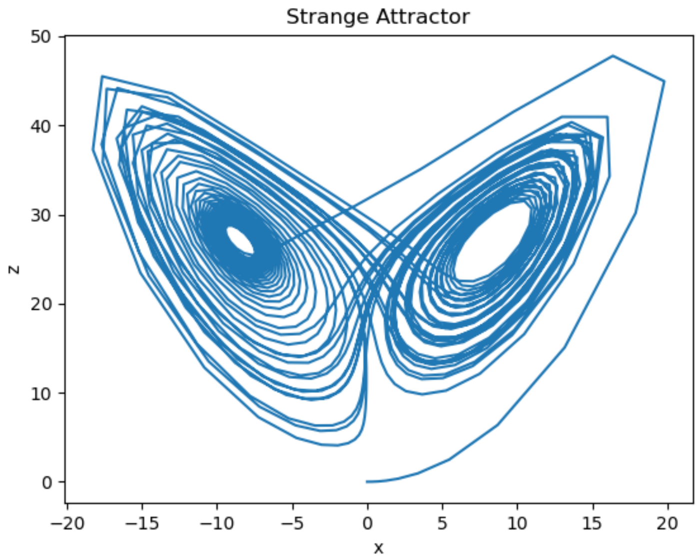

Figure 6: Plot of z values as a function of x values, creating the "strange attractor".