CMPSC 130B Winter 2021 Homework #4

Due March 8, 2021, 11:59 pm

Instructions for the teams for Homework 4:

- We will again allow working in groups for this homework.
- Each team may contain upto 2 members.
- Both team members should submit the homework through gradescope so we can keep track of your scores. Both submissions should be identical.
- Please indicate members of your team in this sheet (even if you decide to work alone).
- Feel free to look for teammates through piazza posts.

Problem Set:

1. (10 points) Define the classes P, NP, Co-NP, and EXP.

2. (10 points) For each of the problems given below, write the decision version, the search version, and the optimization versions. Also, considering the decision version of the problems, show that each of them belongs to NP.
   a. Set cover
   b. Hamiltonian cycle
   c. Clique
   d. 3-SAT
   e. Subset-sum

3. (10 points) Mark these as true, false, not known. (Note that $\subset$ denotes proper subset.)
   a. P $\subset$ $(NP \cap Co-NP)$
   b. P $\subseteq NP \subseteq$ EXP
   c. (P = NP) $\Rightarrow$ (NP = Co-NP)
   d. 2-SAT $\in$ P
   e. 2-Color $\in$ P

4. (10 points) Consider the reduction of 3-SAT to Independent Set discussed in class. Apply this reduction to the following 3-SAT instance:
   $(x_1 \vee x_2 \vee x_4) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee \overline{x_3} \vee x_4) \wedge (x_1 \vee x_3 \vee \overline{x_4})$
   Show the resulting graph? How many nodes and edges are there in the graph? Indicate, if any, the independent set of size 4 in the graph.

5. (10 points) The Subset-Sum Decision Problem (SSDP) is defined as follows:
   "Given n natural numbers $w_1, \ldots, w_n$ and an integer W, is there a subset that adds up to exactly W?"

We proved in lectures that SSDP is NP-complete. We would like to show that the following decision problems are also NP-complete by reduction from SSDP.

- (4 points) "Given n integers $w_1, \ldots, w_n$ and an integer W, is there a subset of size k that adds up to exactly W?"
- (6 points) "Given n integers $w_1, \ldots, w_n$, is there a way to divide them into two subsets of equal value?"

6. (10 points) Consider the 3SAT decision problem of checking if a Boolean formula in conjunctive normal form (CNF) with three literals per clause is satisfiable.
   - (4 points) Give an example of a 3SAT formula that is satisfiable under at least two different assignments to its variables. An assignment is a row in the truth table for the formula.
   - (6 points) Consider the corresponding decision problem MULTIPLE-3SAT: "The input Boolean formula in CNF with 3 literals per clause is satisfiable under at least three different assignments."
     Prove that MULTIPLE-3SAT is NP-Complete by reduction from 3SAT.

7. (10 points) We are trying to assemble an excursion to Mars and, in wanting to inhabit the planet quickly, wish to send as many people as possible. But humans do not always like each other and we would like to limit the total amount of discord in the group we put on the spaceship. The input to your algorithm is an nxn discord matrix (n is the number of interested people) that captures the love/hate relationship. Every value in this matrix is between 0 and 1, with 0 indicating love and 1 indicating hate. Thankfully, the diagonal entries are all 0. Also, assume that the entries are symmetric. The other input is the maximum allowable discord in the chosen team (chosen by summing all pairwise values on the team). Your goal is to output the maximum number of people that can be put on the spaceship given the discord matrix and the maximum allowable discord.
   a. Show that this problem belongs to NP.
   b. Either show that the problem belongs to P or that it is NP-complete by reducing a known NP-complete to this problem. (Please check with us regarding your choice of the known NP-complete problem.)