CMPSC 130B Winter 2021 Homework #3

Due Feb 26, 2021, 11:59 pm

Instructions for the teams for Homework 3:

- Considering the course feedback from the class, to increase the student interaction and motivation to solve problems, we will allow working in groups for homework 3.
- Each team may contain upto 2 members.
- Both team members should submit the homework through gradescope so we can keep track of your scores. Both submissions should be identical.
- Please indicate members of your team in this sheet (even if you decide to work alone).
- Feel free to look for teammates through piazza posts.

Problem Set:

1. (10 points) Extend the LCS recurrence (as discussed in class) to k given strings. The length of each string is n. What are the dimensions of the memoization structure maintained by your solution? What is the time complexity of your solution?

2. (10 points) Write the recurrence for finding common substring without gaps of k given strings. The length of each string is n. What are the dimensions of the memoization structure maintained by your solution? What is the time complexity of your solution?

3. (10 points) Suppose we want to use dynamic programming to solve the prefix coding problem of k symbols by sorting the symbols (based on frequency) and searching over (all i) trees consisting of optimal left subtree for the first i symbols and optimal right subtree for the remaining k-i symbols.
    a. Write down the recurrence.
    b. Is this scheme a legal prefix coding?
    c. Is it optimal? If so, present a proof. If not, present a counterexample by showing that the expected cost of the encoding is higher than an optimal encoding.

4. (10 points) You are given a sequence of n integers and asked to find the decreasing subsequence with the smallest sum. For example, in the input sequence (-20, 30, 50, 0, -10, 40, -30, 20, 10), the answer would be (-20, -30). Set up a dynamic programming solution for the problem. What are the dimensions of the memoization structure maintained by your solution? What is the time complexity of your solution?

5. (10 points) You are given an n x n board and a game piece. You must move the piece from the bottom edge of the board to the top edge of the board. At each step, the piece can move in one of three ways:
    a. the square immediately above,
    b. the square that is one up and one to the left (but only if the piece is not already in the leftmost column),

c. the square that is one up and one to the right (but only if the piece is not already in the rightmost column).

The move from square x to square y costs c(x,y) dollars, which is specified for all legal moves. Give a dynamic programming algorithm that figures out the set of moves that will move the piece from the leftmost square on the bottom row to any square on the top row while minimizing the cost. What are the dimensions of the memoization structure maintained by your solution? What is the time complexity of your solution?

6. An assembly line has n sequential stages with stage i performed by robot R_i that costs c_i and that can fail with probability p_i. If we use a single robot at each stage then the probability of success, assuming that the robots fail independent of each other, is $\prod_{i=1}^{n}(1-p_i)$. Thanks to some venture fundraising, we have some extra money to purchase robots to reduce the failure probability at any stage. This allows us to use m_i copies of robot R_i in parallel at stage i. The cost now increases to $c_i m_i$ but the probability of failure at stage i reduces to $(p_i)^{m_i}$. Your task is to use the available budget B wisely so that the success probability of the entire assembly line is maximized. Develop a dynamic programming solution. What are the dimensions of the memoization structure maintained by your solution? What is the time complexity of your solution?