Principal at Risk Conditional Coupon Note

FINA 4354 Group Project

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1 ECONOMIC MOTIVATION

SP 500 drops more than 20% amid the COVID-19 panic sell-off but a total of more than $4 trillion stimulus package has injected confidence into the market. Hence, with the US still being the biggest single engine of global economy [1], we anticipate that the economic will recover gradually when the US reopens for business.

Posited by the rebound view, we recommend on value investments, specifically quality companies with low leverage and high margins [2]. Among the categories of structured investment products, principal at risk securities are usually considered as instruments that are more conservative in the industry [3]. Hence, we are here to provide this Principal at Risk Conditional Coupon Note ('PRCCN') for institutional clients to obtain extra profits by investing in quality companies.

The PRCCN pays the investor a monthly coupon once the value underlying rise, and a quarterly coupon once the underlying outperform the SP 500 index. For unfavorable market condition, the floor rate offers a cushion to constrain the maximum losses. It provides bull-looking investors an optimized strategy to invest in the undervalued stocks. In addition, it may help investors capture the profits once the values of stocks rise or outperform the SP500 market index and provide investors a 'cushion' in this volatile market.

First, noticing that digital options have much lower premiums compared to vanilla ones, we believe the PRCCN, embedded by digital options, would be better at capturing the profits brought by small rebound. Even though the market has experienced a stunning rebound in the past several weeks, we observed that the prices for most small-cap stocks were actually declining in this period, with large-cap back well above fair values [1]. Considering the low possibility of reopening business globally in the short-term, we believe the momentum would be reduced in the coming months. Thus, digital options are preferred in the small rebound since they offer investors a fixed amount if the average of underlying assets generates even a little bit positive return in a month.

Second, we set the strike of the PRCCN as the average return of the last trading week in a month. This Asian nature smooths the payoff calculation and ensures the end-of-month fluctuation would not hurt the overall profiting ability of this product. Furthermore, the bivariate feature of the PRCCN enables clients to invest in more than one asset, which could optimize portfolio profits or hedge idiosyncratic risks.

Third, the PRCCN could serve as a cushion for investors. Although we have witnessed an unprecedented level of stimulus from monetary and fiscal policies globally, there are delays for some corporations in receiving funding, which could lead to an escalation of the possibility of large-scale bankruptcies and trigger a severer recession. More importantly, in the current market with low interest rate (even

negative), the risk appetite of investors are further lowered down. Therefore, the PRCCN is designed to have a fixed floor rate, for the purpose of constraining significant losses in the case of market crashes.

2 PRODUCT DESIGN

2.1 PRODUCT INITIATION

The product would be issued at par. The investor may choose two tradable stocks as the underlying and set the monthly and quarterly floor rate of contract. Once these parameters are determined, the issuer will calculate the monthly and quarterly coupon rate for the product given the market prices of the underlying. Though there is no threshold for the floor rate, the lower the floor rate, the higher the coupon rates will be.

2.2 PERIOD PAYOFF

The product pays coupon after each month and quarter. Monthly coupons are paid if both underlying stock prices are higher than the price of the previous month. Quarterly coupons are paid if either underlying stock outperforms the index return. At maturity, the investor receives the principal.

At month **m**, monthly conditional payoff $C_m = N \times (F_m + D_m \times B_m)$, where

N = Principal, F_m = Monthly Floor rate, B_m = Monthly Coupon Rate,

$$D_m = \begin{cases} 1 & \text{if } \bar{S}^1_m \geq \bar{S}^1_{m-1} \text{ and } \bar{S}^2_m \geq \bar{S}^2_{m-1} \\ 0 & \text{otherwise} \end{cases}$$

$\bar{S}^x_m$ = Arithmetic average of the last 5 closing prices of x before month m ends

At quarter **q**, quarterly conditional payoff $C_q = N \times (F_q + D_q \times B_q)$, where

F_q = Quarterly Floor rate, B_q = Quarterly Coupon Rate

$$D_q = \begin{cases} 1 & \text{if } \bar{S}^1_q / \bar{S}^1_{q-1} > \bar{R}_q / \bar{R}_{q-1} \textbf{ or } \bar{S}^2_q / \bar{S}^2_{q-1} > \bar{R}_q / \bar{R}_{q-1} \\ 0 & \text{otherwise} \end{cases}$$

$\bar{S}^x_q$ = Arithmetic average of the last 5 closing prices of x before quarter q ends

R = Benchmark, e.g. SP500

3 PRICING METHODOLOGY

The product can be decomposed into a 1-year zero-coupon note and a series of forward-started digital Asian options and a zero-coupon note.

$$\begin{aligned} PV &= \sum_{m=1}^{12} PV(C_m) + \sum_{q=1}^{4} PV(C_q) + PV(N) \\ &= N \times \left(\sum_{m=1}^{12} e^{-mr_m} F_m + \sum_{m=1}^{12} B_m \times DC^M(m) + \sum_{q=1}^{4} e^{-qr_q} F_q + \sum_{q=1}^{4} B_q \times DC^Q(q) + e^{-r} \right) \end{aligned}$$

where

$DC^M(m)$, $DC^Q(q)$ are prices of a 1 month and 1 quarter Asian bivariate digital calls

Given the bivariate and Asian nature of the product, it is difficult to determine the closed form formula. We, therefore, assume the assets follow correlated Geometric Brownian Motion and choose Monte Carlo method under Black-Scholes framework to conduct pricing. After setting the monthly and quarterly floor rates i.e. F_m and F_q, we use multivariate optimization to find the coupon rates i.e. B_m and B_q such that the product will be sold at par.

4 HEDGING STRATEGY

Since the embedded options are exotic in nature and thinly traded in the market, static hedging might be rather costly if not infeasible in practice, and dynamic Delta hedging might be preferred for this product. However, due to the digital nature of the embedded options, the deltas near the settlement date would become unstable mathematically (it may even go infinity if the spot price is near the money). Therefore, approximations are conducted: 1) The deltas of embedded monthly Asian multivariate digital calls are approximated by monthly Asian multivariate call spreads, and; 2) The deltas of embedded quarterly Asian multivariate digital calls are approximated by monthly Asian multivariate call spreads:

$$\begin{aligned} \Delta DC^M &\approx \Delta V1\left(\bar{S}^1_{m-1} - \epsilon^M_1, \bar{S}^2_{m-1} - \epsilon^M_2\right) - \Delta V1\left(\bar{S}^1_{m-1} + \epsilon_1, \bar{S}^2_{m-1} + \epsilon_2\right) \\ \Delta DC^Q &\approx \Delta V2\left(\bar{S}^1_{q-1} - \epsilon^Q_1, \bar{S}^2_{q-1} - \epsilon^Q_2, \bar{R}_{q-1} - \epsilon^Q_2\right) - \Delta V2\left(\bar{S}^1_{q-1} + \epsilon^Q_1, \bar{S}^2_{q-1} + \epsilon^Q_2, \bar{R}_{q-1} + \epsilon^Q_2\right) \end{aligned}$$

where:

$\Delta DC^{M/Q}$: The delta of the monthly/quarterly embedded options

$\Delta V1(S_1, S_2)$: The delta of the Asian multivariate call with strikes S_1, S_2

$\Delta V2(S_1, S_2, R)$: The delta of the Asian multivariate call with strikes S_1, S_2, R

ϵ^X_y : Adjustments to mimic the payoff of the digitals

In other words, we "de-digitalise" the digital option by using a call spread formed by two non-digital calls. For each $V1(S_1, S_2)$ and $V2(S_1, S_2)$, their deltas can be denoted by:

$$\Delta V1 = \left(\frac{\partial V1}{\partial S_1} \quad \frac{\partial V1}{\partial S_2}\right)^T \qquad \Delta V2 = \left(\frac{\partial V2}{\partial S_1} \quad \frac{\partial V2}{\partial S_2} \quad \frac{\partial V2}{\partial R}\right)^T$$

For each partial derivative, it can be approximated by the central difference method. For example, $\frac{\partial V1}{\partial S_1}$ can be approximated by

$$\frac{\partial V1}{\partial S_1} \approx \frac{V1(S_1 + h/2, S_2) - V1(S_1 - h/2, S_2)}{h}$$

where h is the tick size of the underlying. The $V1$ and $V2$ can be valued with Monte Carlo simulation, by assuming that the assets follow correlated geometric Brownian motion processes.

Thus, the issuer can approximately replicate the Delta profiles of this product by buying or selling the underlying according to the partial deltas so as to hedge its market exposure.

5 CASE STUDY

Many investors may believe that quality companies are undervalued due to the sell-off during the recent crisis. To test the performance of the PRCCN under similar situations, two defensive stocks, PepsiCo (PEP) and Johnson & Johnson (JNJ) are chosen to be the underlying assets, and three different historical time periods are selected to mimic the current environment. The S&P 500 returns are chosen as the benchmark for quarterly coupons. The monthly and quarterly floor rates are set to be 2%, and the monthly coupon rate (B_m) and quarterly bonus rate (B_q) are optimized such that the product was sold at par.

TABLE 1: PERFORMANCE SCENARIOS

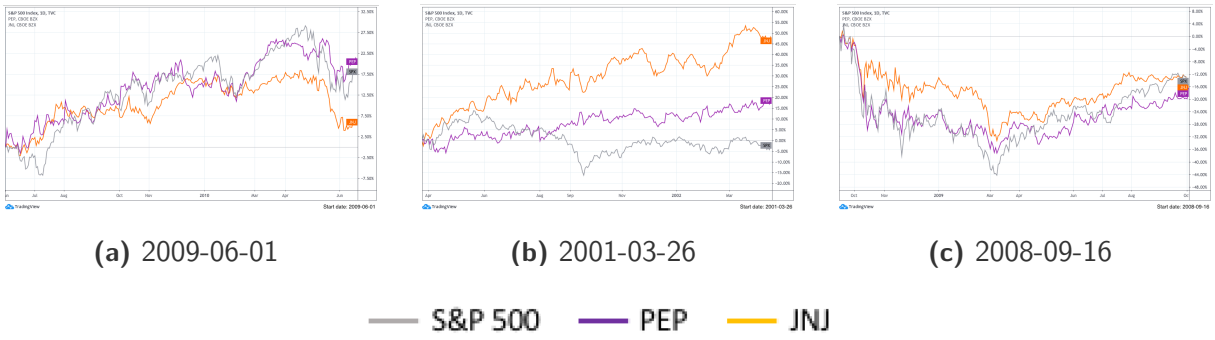
Start Date	Monthly Coupon (B_m)	Quarterly Bonus (B_q)	Product Return (%)	Average Return of Stocks (%)	Benchmark Return (%)
2009-06-01	4.5	7	21.89	13.86	14.39
2001-03-26	4.5	6.5	17.89	35.01	-1.52
2008-09-16	4.5	6	1.46	-15.03	-14.13

The first period is from 1 June 2009 to 30 May 2010, nearly one year after the bankruptcy of the Lehman Brothers. Investors might have anticipated that the stock market had already reached the bottom and that it would bounce back. The results in table 1 show that the PRCCN gained a total return of 21.89%, outperforming the underlying by around 8%. The bivariate natures of the product enabled investors to capture the out-performance of PEP with respect to the U.S. market and avoid the loss caused from the slight drop in underlying stocks after May 2010.

The second test is conducted assuming that the contract started from 26th March, 2001, when the U.S. underwent a bear market. The underlying defensive stocks turned out to perform strongly

with respect to the market and realized extremely high total returns. Although the PRCCN limited the upside potential, it still gained a satisfactory return compared to the market.

The third test is conducted assuming that the contract started from 16^{th} September, 2008, which is the onset of the 2008 financial crisis. Under such an unfavoured market condition, the underlying stocks suffered from huge capital loss while the PRCCN provided investors a 'cushion' and constrained the losses.



(a) 2009-06-01 **(b)** 2001-03-26 **(c)** 2008-09-16

FIGURE 1: COMPARISON BETWEEN DIFFERENT STATES.

6 INVESTOR RISK ANALYSIS

The PRCCN's investments are subject to the risks inherent in all underlying securities, including the fact that the value of holdings may fluctuate. Investors should also note that the PRCCN does not amount to a capital guarantee in respect of an investor's original investment in the note. Moreover, Investors who purchase this product are exposed to the credit risk of the product issuer or counterparties. There is no assurance of protection against a default by the product issuer in respect of its repayment obligations.

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