An Introduction of EVT for Credit Risk Management in Rare Events

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I. Introduction

According to Houlihan Lokey and Morgan Stanley, COVID-19 has imposed a significant effect on loan products like collateralized loan obligations (CLOs) and fixed-income securities [1, 2]. For investors, the interactions between the returns of these credit instruments, commonly referred as *credit correlation*, is a primary component of portfolio risk management [3]. In addition, credit correlation is a vital aspect in the regulatory requirements in the Basel II Accord for relevant entities [4]. Nevertheless, as financial engineering (FE) was commented as the ultimate culprit for the Global Financial Crisis (GFC), there was a noticeable decline of interest in credit correlation modelling in the quants community [5, 6]. Warned by the skyrocketing credit risk exposures in the current coronavirus crisis, it is expected that more practitioners of FE would realize the importance of more accurate credit risk models in extreme markets.

After the pioneer work on challenging the Gaussian assumption in financial time series [7, 8], accumulated evidences have demonstrated that normal distribution is limited in predicting extreme observations in the financial markets, following which there are a number of debates on the advantages of fat-tailed distributions like stable laws and Student-t [9]. Extreme value theory (EVT) then is proved to nest both distributions and thus offer an elegant solution [9]. Furthermore, while it is inevitable to introduce multivariate settings in portfolio risk management associated with credit crisis, multivariate extreme value theory (MEVT) provides an explicit model to understand the dependence structures of multivariate time series under stress situations, which could differ conspicuously from normal periods [9, 10]. Concurrently, however, any dynamic theory dealing with high-dimensional credit structured products has not yet been available [6]. Catalyzed by the coronavirus credit risks, it is reasonable to believe that a boom of seeking sufficiently rich MEVT-based credit pricing models would emerge in the post-coronavirus industry or academia. At the current stage, MEVT could be applied constructively in areas like *risk aggregation, concentration, and diversification.* This paper presents classical theories for EVT, to hopefully provide insights on credit risk models.

II. Gaussian Copula

Credit pricing model base on Gaussian copula was introduced by Li [11] to manage collateralized deposit obligations (CDO) tranches [12]. Despite its serious drawbacks, the ease of implementation made it predominately used the industry and in understanding default behavior of credit portfolios [12]. With the credit crisis in 2007, however, Gaussian copula model has receive a lot of harsh comments on the fact that it underestimates joint extreme events [6]. We describes it here as a reference for MEVT.

Copula is a multivariate cumulative distribution function, which has standard uniform margnal distributions. We denote it by

$$f : [0,1]^N \to [0,1], (v_1, ...v_N) \to f(v_1, ..., v_N)$$

where $v_i \sim U(0,1), \forall i \in \{1, ..., N\}$ and N is the number of credit risky instruments. By the Sklar's Theorem [13], for standard normal cumulative distribution function (c.d.f.) Φ, there exists a copula function f s.t.,

$$f(v_1, ..v_N) = \Phi_{N,\mathbf{\Sigma}}(\Phi^{-1}(v_1), ..., \Phi^{-1}(v_N))$$

where $\Phi_{N,\Sigma}$ is the multivariate Gaussian c.d.f. on $\mathbb{R}^N$ with mean $\mathbf{0}$ and correlation matrix $\mathbf{\Sigma}$. Utilizing this copula in the geometric Brownian motions of asset valuation model, the joint distribution of correlated default times can be obtained as below [14].

$$\mathbb{P}[\xi_1 \leq t_1, ..., \xi_N \leq t_N] = \Phi_{N,\mathbf{\Sigma}}[\Phi^{-1}(\Psi_1(t_1)), ..., \Phi^{-1}(\Psi_N(t_N))]$$

where ξ_i are random variables (measuring default times) with c.d.f. Ψ_i, and t_i's are specific time spots.

III. Extreme Value Theory

Denote $X_1, ..., X_m$ as m i.i.d. random variables, and $M_m = \min\{X_1, ..., X_m\}$. Then with a *location series* $b_m \in \mathbb{R}$ and a *scale factors* $a_m > 0$, the distribution of the sequence of $\frac{M_m - b_m}{a_m}$ would converges to a non-degenerated distribution as $m \to \infty$. This distribution is usually referred as generalized extreme value (GEV) distribution and it belongs to either one of the following families [9].

i. Gumbel: $G_* = 1 - \exp(-\exp(x)), \forall x \in (-\infty, \infty)$

ii. Fréchet ($k < 0$): $G_* = \begin{cases} 1 - e^{[-(1+kx)^{\frac{1}{k}}]} & \text{if } x < -\frac{1}{k}, \\ 1 & \text{otherwise} \end{cases}$

iii. Weibull ($k > 0$): $G_* = \begin{cases} 1 - e^{[-(1+kx)^{\frac{1}{k}}]} & \text{if } x > -\frac{1}{k}, \\ 0 & \text{otherwise} \end{cases}$

There are generally four methods to estimate parameters of GEV distribution, namely, Hill and Pickands estimators, method of blocks, peaks of threshold (POT), and declustering. Depending on the estimating methods and correlation assumption of underlying returns, the calculation of corresponding VaR may vary. With the GEV models, the range of estimated VaRs, usually derived by RiskMetrics and GARCH model, are further expanded.

IV. Multivariate Extreme Value Copula

Assume the dimension under consideration is k. Denote m i.i.d. random vectors $\boldsymbol{X}_1, ..., \boldsymbol{X}_m \in \mathbb{R}^k$, where $\boldsymbol{X}_i = X_{ij}, \forall i \in \{1, ..., m\}, j \in \{1, ..., k\}$. Then the minima of each component is $M_j^m = \min\{X_{1j}, ..., X_{mj}\}$, which gives the k-dimensional minima vector $\boldsymbol{M}_m = [M_1^m, ..., M_k^m]^T$. By the one-dimensional case, each normalized M_j^m converges to a GEV distribution, denoted by G_j. Again, adopting the Sklar's Theorem [13], we can find a unique copula function f, s.t., after a specific transformations (an analogy to normalization in the one-dimensional case), the joint c.d.f. of the k-variate minima converges to a distribution, G, with $\boldsymbol{x} = [x_1, ..., x_k]^T \in \mathbb{R}^k$, s.t., $G(\boldsymbol{x}) = f(G_1(x_1), ..., G_k(x_k))$ [6].

Under the Basel II framework, the following two measures are concerned, which involve the use of MEVT to calculate $\mathrm{VaR}^\alpha(\sum_{j=1}^k X_j)$ [6].

i. $\mathcal{D}(\alpha) = \mathrm{VaR}^\alpha(\sum_{j=1}^k X_j) - \sum_{j=1}^k \mathrm{VaR}^\alpha(X_j)$

ii. $\mathcal{C}(\alpha) = \frac{\mathrm{VaR}^\alpha(\sum_{j=1}^k X_j)}{\sum_{j=1}^k \mathrm{VaR}^\alpha(X_j)}$

where $\mathcal{D}(\alpha)$ and $\mathcal{C}(\alpha)$ are measuring diversification and concentration, respectively.

V. Conclusion

The credit risk management has long been faced with challenges of rare events, especially after the subprime mortgage crisis. Financial engineers and mathematicians, who constructed credit derivative instruments, have also been criticized post-crisis, resulting in a decrement in the research of credit risk models. Even though literature on counterparty risk evaluation has been inverting the trend, there is no doubt that efforts for credit correlation has diminished [5]. However, shirking or evading cannot prevent the recurrence of rare events like COVID-19 pandemic. While loan instruments still maintain its remarkable volume in the financial markets, more robust approaches shall be accessed to preclude excessive credit risks.

The MEVT method discussed in this paper is expected to seize nonlinear relationships in extreme situations, surpassing widely implemented measurements of credit correlation patterns like GARCH [15]. Additionally, it has been a hallucination that modern FE could offer better comprehension and control of complicated credit products, which would still exist for a period without a better understanding of MEVT to simplify the over-sophisticated FE-products [6]. Nevertheless, the extreme value copula mentioned in section IV is usually referred as classical MEVT. More non-classical methods that could also blossom can be found in the statistical and computational literature [6].

Besides more accurate prediction of VaRs for the purpose of risk management and hedging, MEVT could be applied to asset allocation. Conservative investors could then have portfolios excluding credit-risky assets exposed to rare events at the first place. This idea of measuring and restricting extreme risks could potentially generalize to other risks. For instance, the negative exposures due to operational and political risks for crude oil future contracts in extremal conditions shall be excluded, to prevent the reenactment of the BoC crude oil "bao" case.

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