

ALTINBAŞ UNIVERSITY	
DISCRETE MATHEMATICS MIDTERM I	
Course : <i>MATH 211</i> Acad. Year: <i>2021-2022</i> Semester : <i>FALL</i> Date : <i>06.11.2021</i> Time : <i>13:30</i> Duration : <i>180 min</i>	Last Name: _____ Name : _____ Student ID: _____ Signature : _____ <div style="text-align: center; padding-top: 10px;"> 7 QUESTIONS ON 7 PAGES TOTAL 100 POINTS </div>

- Please draw a box around your answers. Show **all** your work! Answers without supporting work will not be given credit.
- You may either print this file, or, work with A4 sheets; but **do not change the order and position** of questions and pages. If you are using A4 papers, put your signature and full name to the left upper corner of the paper. Use the right upper corner of the paper for the Student ID (or any photographed ID).
- Emailed submissions will **not** be accepted. Similarly, late submissions will **not** be accepted.
- When you finish, upload a **pdf** file to UZEM. Keep your original handwritten work in a safe place.

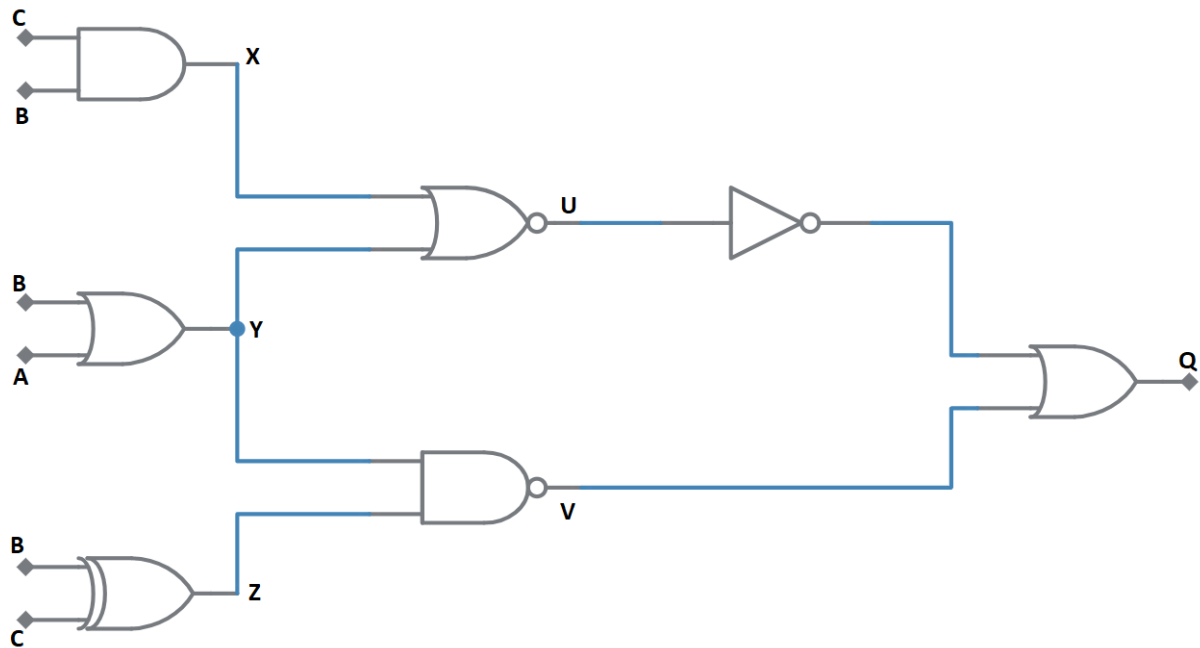
1. (5 pts) By using the truth table given below, prove that the compound statement $(p \oplus q) \oplus r$ is logically equivalent to the compound statement $p \oplus (q \oplus r)$.

p	q	r	$q \oplus r$	$p \oplus (q \oplus r)$	$p \oplus q$	$(p \oplus q) \oplus r$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

Name:

ID:

2. (20 pts) Fill in the table by using the combinatorial circuit. (Note: No explanation is needed. Different orderings in the table will not be given credit.)



A	B	C	X	Y	Z	U	V	Q
0	0	0						
0	0	1						
0	1	0						
0	1	1						
1	0	0						
1	0	1						
1	1	0						
1	1	1						

Name:

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3. (*Total 20 pts*) There is an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. It is assumed that every inhabitant of the island is either a knight or a knave. Below there are 3 inhabitants, who are denoted by A, B and C.

3a. (*10 pts*) What are A, B and C if A says “If B is a knave then C is a knave”, and B says “If C is a knight then A is a knave”? Briefly explain your reasoning.

3b. (*10 pts*) What are A, B and C if A says “B is a knight and C is a knight”, and B says “A is a knight if and only if C is a knave”? Briefly explain your reasoning.

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4. (15 pts) Use rules of inference and logical arguments to show that the following statements imply the conclusion “It snowed”. Explain each step of your reasoning by indicating the rule you applied.

“If it is not snowy or if it is not rainy, then the race of NASCAR will be held and the laser show will go on.”

“If the race of NASCAR is held then the champagne will be sprayed.”

“The champagne was not sprayed.”

Name:

ID:

5. (15 pts) Consider the statement “If n is an integer and $3n + 2$ is odd, then n is odd.”

5a. (5 pts) Prove the above statement by using direct proof technique.

5b. (5 pts) Prove the above statement by using proof by contradiction technique.

5c. (5 pts) Prove the above statement by using proof by contrapositive technique.

6. (*Total 15 pts*) In this question A, B and C denote nonempty sets.

6a. (*5 pts*) Prove or disprove the statement: $(A \cap C) \cup (A - B) = (A \cap B) \cup (A - C)$

6b. (*5 pts*) Prove or disprove the statement: Every injective function $f: A \rightarrow A$ is surjective.

6c. (*5 pts*) Prove or disprove the statement: Every function $f: A \rightarrow B$ can be written as $f = h \circ g$ for some surjection g and injection h .

7. (*Total 10 pts*) Solve the following problems.

7a. (*5 pts*) Find the solution of the recurrence relation $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2$ with $a_0 = 3$ and $a_1 = 6$.

7b. (*5 pts*) Use the bubble sort to sort 6, 2, 3, 1, 5, 4, showing the lists obtained at each step as done in the lecture.