

# Stochastic Processes

MATH 447

Nicholas Hayek

*Taught by Prof. Louigi Addario-Berry*

## CONTENTS

I	Introduction	3
---	--------------	---

We assume working knowledge of probability and no knowledge of measure theory (though a grasp of analysis is essential). See these [MATH 356 course notes](#) from 2023, also taught by Louigi!

# I Introduction

Before providing definitions, we give some examples of stochastic processes:

**Eg. 1.1** A simple random walk:  $S_{i+1} = S_i + X_i$ , where  $X_i \sim \text{Ber}(p)$  and  $S_0 = 0$ . We might ask: does  $S_i$  ever return to 0, i.e.

$$\mathbb{P}(\exists i > 0 : S_i = 0)$$

**Eg. 1.2** A branching process: as in asexual reproduction, we have an initial node. Each node  $n$  has a number of children  $X_n$ , where  $\frac{X_n}{2} \sim \text{Ber}(p)$ . We denote  $Z_i$  to be the number of individuals in the  $i$ -th generation. We might ask: does  $Z_i$  ever have no children, i.e.

$$\mathbb{P}(\exists i > 0 : Z_i = 0)$$

**Eg. 1.3** Choose  $k$  independent random points in the square  $[0, \sqrt{k}]^2$ . On average, then, there is 1 point within any unit square  $U \subseteq [0, \sqrt{k}]^2$ .