1) Find the formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues

$$\left\{-3,\ 2,\ -\frac{4}{3},\ \frac{8}{9},\ -\frac{16}{27}, \ldots\right\}$$

2) Determine whether the sequence converges or diverges. If it converges, find the limit

$$a_n = \frac{5}{n+2}$$

3) Determine whether the geometric series is convergent or divergent. If it convergent, find its sum

$$\sum_{n=1}^{\infty} 12 \cdot (0{,}73)^{n-1}$$

4) Use the Integral Test whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{(3n-1)^4}$$

answers 10-12

#1 $a_1 = -3$

$r = -\frac{2}{3}$

$a_n = -3\left(-\frac{2}{3}\right)^{n-1}$

#2 $a_n = \frac{5}{n+2} = \frac{5/n}{(n+2)/n} = \frac{5/n}{1+2/n}$

$a_n \to \frac{0}{1+0} = 0$ as $n \to \infty$

converges

#3 $a = 12$

$r = 0.73$

$|r| = 0.73 < 1$ converges

to $\frac{a}{1-r} = \frac{12}{1-0.73} = \frac{12}{0.27} = \frac{400}{9}$

#4 $\int_1^\infty \frac{1}{(3\cancel{x}-\cancel{4})^4}\, d\cancel{x} = \lim_{\cancel{t}\to\infty} \int_1^t \frac{1}{(3x-1)^4}\, dx$

$= \lim_{t\to\infty} \int_1^{\cancel{t}} (3x-1)^{-4}\, dx = \lim_{t\to\infty} \left[\frac{(3x-1)^{-3}}{-\cancel{4}3 \cdot 3} \right]_1^t$

$= -\frac{1}{\cancel{12}\,9} \lim_{t\to\infty} \frac{1}{(3x-1)^3}\Big|_1^t = -\frac{1}{\cancel{12}\,9}\left[\cancel{\frac{1}{(3t-1)^3}} - \frac{1}{8}\right] =$

0

$= + \cancel{\frac{1}{96}}\ \frac{1}{72}$

5) Limit Comparison Test

$$\sum_{n=4}^{\infty} \frac{n^2}{n^3-3}$$

6) Determine whether the following series converges or diverges (p-series Test)

$$\sum_{n=1}^{\infty} \frac{1}{n^{1.04}}$$

#5

$b_n = \frac{n^2}{n^3} = \frac{1}{n}$

$\sum_{n=4}^{\infty} \frac{1}{n}$ will diverge because it is a harmonic series ~~ex~~.

or P-ser. test

$\underbrace{\frac{n^2}{n^3} < \frac{n^2}{n^3 - 3}}_{\text{less than } n^3}$

$\sum_{n=1}^{\infty} \frac{n^2}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n}$

~~co~~ diverge

#6

p - Series Test

$p = 1.04$

$\underbrace{1.04 > 1}_{\text{converges}}$

7. (a) (4 pts.) Does the sequence below converge or diverge? If it converges, what does it converge to?

$$a_n = e^{1/n} \cos \frac{1}{n}$$

$\lim_{n\to\infty} e^{1/n} \cos\frac{1}{n} = \lim_{n\to\infty} e^{1/n} \cdot \lim_{n\to\infty} \cos\frac{1}{n}$

$1 \cdot 1 = 1$

(b) (4 pts.) Does the series below converge or diverge?

$$\sum_{n=1}^{\infty} e^{1/n} \cos \frac{1}{n}$$

diverges

$\lim \Rightarrow \sum_{n=1}^{\infty} e^{1/n} \cos 1/n$

$\lim_{n\to\infty} (e^{1/n} \cdot \cos(\frac{1}{n})) = 1 \neq 0$

$\underbrace{e^{0}}_{1}$ $\quad \cos 0 = 1$

diverge

8. (4 pts.) Give an example of a geometric series that converges to $-\frac{1}{2}$.

$S = \frac{a}{1-r}$ $\quad |r| < 1$

$\frac{a}{1-r} = -\frac{1}{2}$

guess and check

$a = -\frac{1}{4}$

$r = \frac{1}{2}$

$\frac{-1/4}{1-1/2} = -\frac{1}{4} \cdot \frac{2}{1} = -\frac{1}{2}$

9. (5 pts.) Determine if the following series converges or diverges.

$$\sum_{n=0}^{\infty}(-1)^{n+1}\frac{n}{e^n}$$

#9(cont)

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n \frac{d}{dn}}{e^n \frac{d}{dn}} = \lim_{n\to\infty} \frac{1}{e^{\infty}} = 0$$

$$\frac{n+1}{e^{n+1}} \leq \frac{n}{e^n}$$

$$\frac{n+1}{\cancel{e^n}\cdot e} \leq \frac{n}{\cancel{e^n}}$$

$$n+1 \leq e\cdot n$$

$$n \leq e\cdot n - 1$$

yes

#10 diverges

Limit comparison test

#11 diverges

Series Divergence Test

#12 converges

[illegible] Series Test

10. (5 pts.) Determine if the following series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{n}{1+n^2}$$

$b_n = \frac{n}{n^2} = \frac{1}{n}$ $\Rightarrow$ harmonic diverges

$a_n = \frac{n}{1+n^2}$

$\lim_{n\to\infty} \left[\frac{n}{1+n^2} \cdot \frac{n}{1} \right] = \lim_{n\to\infty} \left[\frac{n^2}{1+n^2} \right] = \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$

$= \lim_{n\to\infty} \frac{1}{\frac{1}{n^2}+1} = 1$

Both diverge

11. (5 pts.) Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{n}{\ln n}$$

$\lim_{n \to \infty} S_n$

$\lim_{n \to \infty} \sum_{n=2}^{\infty} \frac{n}{\ln n} = \frac{1}{1/n} = n \to \infty$

diverges

12. (6 pts.) Classify the following series as absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=0}^{\infty}(-1)^n \frac{n}{n^2-n}$$

$$a_n = \frac{n}{n^2-n} = \frac{1}{n-1}$$

$$\lim_{n\to\infty} \frac{1}{n-1} = \frac{1}{\infty} = 0 \quad ✓$$

$$a_{n+1} \overset{?}{<} a_n$$

$$\frac{1}{(n+1)-1} \overset{?}{<} \frac{1}{n-1}$$

$$\frac{1}{n} < \frac{1}{n-1}$$

yes

converges