PHAS2443 : Practical Mathematics II 2018

Homework 2

1. Consider a particle, initially at rest in space, which then starts to fall radially towards the centre of a nearby planet, under the influence of the planet's gravitational field. The particle moves in one dimension and its position is described by the coordinate *z*. In an appropriate system of units, the particle's equations of motion are the following, in which position *z* and velocity *v* vary with time *t* according to:

$$\frac{dz}{dt} = v$$

$$\frac{dv}{dt} = \frac{-1}{(1+z)^2}$$

(where $z=0$ corresponds to the surface of the planet, and the planet has unit radius - thus $1+z$ is the distance from the planet centre).

(a) Use an appropriate form of the forward Euler integration algorithm described in the lectures, in order to integrate these two first-order differential equations and obtain *z* and *v* as a function of time, from $t=0$ to $t=t_{max}=8$, using a time step $\Delta t=0.1$. Use the following initial / boundary conditions for $t=0$: $z(0)=3$ and $v(0) = 0$. Make a plot of the particle's position as a function of time, with *t* plotted on the horizontal axis and *z* on the vertical axis. Make also a plot of the particle's *specific energy* as a function of time, where this quantity is given by $E(t) = \frac{1}{2} v(t)^2 - \frac{1}{(1+z(t))}$. Comment on the conservation of energy *E* during the particle's motion, according to this integration scheme.

10 marks

(b) Repeat part **(a)**, but now use the Simple Verlet scheme described in the class exercises 4, i.e.:

$$z(t+\Delta t) = 2\, z(t) - z(t-\Delta t) + \Delta t^2\, F(z(t)),$$

with velocity as a function of time derived from the *z* as follows:

$$v(t) = \frac{z(t+\Delta t) - z(t-\Delta t)}{2\,\Delta t} \quad \text{for } t>0.$$

For initial conditions, you may assume $z=3$ for times $t\leqslant 0$ and $v=0$ for $t\leqslant 0$. Compute velocity values for as many time steps as the algorithm allows.

Comment on 'how well' *E* is conserved for this scheme, compared to the result from part **(a)**.

9 marks

(c) Extending your results now to a larger value of t_{max}, for the scheme which you think is the more accurate of the two (i.e. Forward Euler or Simple Verlet), estimate the time at which the particle strikes the planet's surface, to within an accuracy of Δt. (Except for t_{max}, use the

same parameters and initial conditions for the problem as described above).
Estimate also, assuming perfect conservation of energy, the magnitude (absolute value) of the particle's velocity just before it strikes the planet's surface.
8 marks

Solutions are due by **5pm on Monday February 19th**. Make a copy of your final solutions with the output deleted (`Cell|Delete All Output`), name that file `yourname_hw2` with an appropriate substitution for `yourname`, upload that file to Moodle **and submit it**. The file will be evaluated using `Evaluation|Evaluate Notebook`, so make sure the final file you submit will produce the results you expect when that is done. If you submit a file without deleting the output, the output will be deleted and regenerated for purposes of assessment.
Please read the submission instructions on the 'Homeworks' tab, and note in particular that an uploaded homework is *not* submitted until you click on 'Submit Assignment' in Moodle.