디지털통신 과제 1 - 3

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1.2) $x(t) = \text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & -\frac{T}{2} \le 0 \le \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$

$X(w) = FT\{x(t)\} = \int_{\infty}^{\infty} x(t) e^{-jwt} dt$

$X(w) = \int_{-\frac{T}{2}}^{\frac{T}{2}} 1 \cdot e^{-jwt} dt$

$= \frac{1}{-jw} \left[e^{-jwt} \right]_{-\frac{T}{2}}^{\frac{T}{2}}$

$= \frac{1}{jw} \left[e^{jw\frac{T}{2}} - e^{-jw\frac{T}{2}} \right]$

$= \frac{2}{w} \left[\frac{e^{jw\frac{T}{2}} - e^{-jw\frac{T}{2}}}{2j} \right]$

$X(w) = \frac{2}{w} \sin\left(\frac{wT}{2}\right)$

$S(w) = |X(w)|^2 = \left| \frac{2}{w} \sin\left(\frac{wT}{2}\right) \right|^2$

$= \frac{4}{w^2} \sin^2\left(\frac{wT}{2}\right)$

$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(w)|^2 dw$

$\int_{-\frac{T}{2}}^{\frac{T}{2}} 1^2 dt = [t]_{-\frac{T}{2}}^{\frac{T}{2}} = T$

1.12)

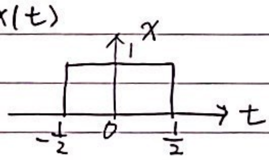
$x(t) \xleftrightarrow{FT} \text{sinc}(f)$

$R_x(\tau) \xleftrightarrow{FT} |\text{sinc}(f)|^2$

$x(t) * x(t) \xleftrightarrow{FT} |\text{sinc}(f)|^2$

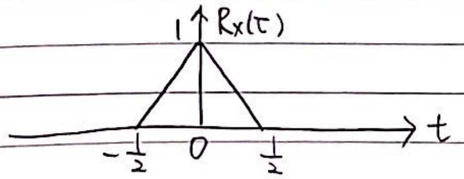
$R_x(\tau) = x(t) * x(t)$

$\text{sinc}(f) \xleftrightarrow{FT}$



$x(t) = \text{rect}(t)$

$R_x(\tau) = \text{rect}(t) * \text{rect}(t)$



1.15)

a) normalized average power in $x(t)$

: $\int_{-\infty}^{\infty} |G_x(f)|^2 df$

$0 \le f \le 10$ kHz

$\int_{-10k}^{10k} |G_x(f)|^2 df = 2\int_0^{10k} |G_x(f)|^2 df$

$= 2\int_0^{10k} |10^{-6} f^2|^2 df = 2\int_0^{10k} |10^{-12} f^4| df$

$= \left[\frac{2}{5} \times 10^{-12} f^5 \right]_0^{10k} = \frac{2 \times 10^{-12}}{5} (10^{20} - 0)$

$= 0.4 \times 10^8$

b) normalized power from 5 kHz to 6 kHz

$\int_{-6k}^{-5k} |G_x(f)|^2 df + \int_{5k}^{6k} |G_x(f)|^2 df$

$= 2\int_{5k}^{6k} |G_x(f)|^2 df = \left[\frac{2 \times 10^{-12}}{5} f^5 \right]_{5k}^{6k}$

$= \frac{2 \times 10^{-12}}{5} (7776 \times 10^{15} - 3125 \times 10^{15})$

$= 0.4 \times 4651 \times 10^3 = 1860.4 \times 10^3$

3.5)

$P_B = Q(x) \le 10^{-3}$.

we need $x \ge 3.09$

$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ where

$E_b = A^2 T$ and $A = 1$ for bipolar signalling

$\sqrt{\frac{2E_b}{N_0}} \ge 3.09 \Rightarrow \frac{E_b}{N_0} \ge 4.77$

$\frac{N_0}{2} = 10^{-3}$

$\rightarrow E_b \ge 4.77 \times 2 \times 10^{-3}$

$R_b = \frac{1}{E_b} \le 104.8$ bits/sec

3.6)

$$r_0 = \frac{S_1+S_2}{2} + \frac{\sigma^2}{S_1-S_2} \ln\left(\frac{P(S_2)}{P(S_1)}\right)$$

$\sigma^2 \equiv$ variance of AWGN $= 0.1$

$S_1 = 1$, $S_2 = -1$

$P(S_2) = 1 - P(S_1)$

a) $P(S_1) = 0.5$

$P(S_2) = 1 - 0.5 = 0.5$

$r_0 = \frac{0.1}{2} \ln 1 = 0\,V.$

b) $P(S_1) = 0.7$

$P(S_2) = 1 - 0.7 = 0.3$

$r_0 = \frac{0.1}{2} \ln\left(\frac{3}{7}\right) = -0.042\,V.$

c) $P(S_1) = 0.2$

$P(S_2) = 1 - 0.2 = 0.8$

$r_0 = \frac{0.1}{2} \ln(4) = 0.0693\,V.$

3.8) a) $R_b = 10$ Mbits/s

16 level $\rightarrow \log_2 16 = 4$

minimum bandwidth $= \frac{R_b}{n} = \frac{10M}{4}$

$= 2.5MHz$

b) $BW = \frac{R_b}{n}(1+\alpha)$

$2.5M = 1.375 \times 10^6 (1+\alpha)$

$1+\alpha = \frac{2.5}{1.375} = 1.8182$

$\alpha = 0.8182$

3.12) $L = 32$, $f_m = 100kHz$

$f_s = 2f_m = 200\,kHz$

$r = 0.6$

$32 = 2^n$ $\quad n = 5.$

a) $r = 0$

$R_b = nf_s = 5 \times 200 \times 10^3 = 1 \times 10^6.$

$R_b = 1Mbps.$

b) $B.W = \frac{R_b}{2}[1+r]$

$= \frac{1 \times 10^6}{2}[1+0.6]$

$= 800 \times 10^3 = 800\,kHz$

c) 8-ary

$n = \log_2 8 = 3$

a-) $R_b = nf_s = 3 \times 200k$

$= 600$ kbps

b-) $B.W = \frac{R_b}{6}[1+r]$

$= 100 \times 10^3 \times 1.6$

$= 160 \times 10^3 = 160\,kHz$