Time Series Forecasting

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Outline

- AR model
- MA model
- ARMA model
- ARIMA model

Autoregressive (AR) model

An autoregressive (AR) model predicts future behavior based on past behavior. It’s used for forecasting when there is some correlation between values in a time series and the values that precede and succeed them. You only use past data to model the behavior, hence the name autoregressive.

In an AR model, the value of the outcome variable (Y) at some point t in time is — like “regular” linear regression — directly related to the predictor variable (X). Where simple linear regression and AR models differ is that Y is dependent on X and previous values for Y.

AR models are also called conditional models, Markov models, or transition models.

Stochastic model (optional)

A **stochastic model** represents a situation where uncertainty is present. In other words, it’s a model for a process that has some kind of randomness.

The opposite is a **deterministic model**, which predicts outcomes with 100% certainty. Deterministic models always have a set of equations that describe the system inputs and outputs exactly. On the other hand, stochastic models will likely produce different results every time the model is run.

AR(p) Models

An **AR(p)** model is an autoregressive model where specific lagged values of $\mathbf{y_t}$ are used as predictor variables. Lags are where results from one time period affect following periods.

The value for "p" is called the order. For example, an AR(1) would be a "first order autoregressive process." The outcome variable in a first order AR process at some point in time t is related only to time periods that are one period apart (i.e. the value of the variable at t – 1). A second or third order AR process would be related to data two or three periods apart.

AR(p) model equation

The AR(p) model is defined by the equation:

$$y_t = \delta + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \ldots + \varphi_p y_{t-1} + A_t$$

Where:

- y_{t-1}, $y_{t-2} \ldots y_{t-p}$ are the past series values (lags)
- A_t is white noise (i.e. randomness)
- and δ is defined by the following equation

$$\delta = \left(1 - \sum_{i=1}^{p} \phi_i\right) \mu \,,$$

where μ is the process mean

Moving Average (MA) model

Moving-average model (MA model), also known as moving-average process, is a common approach for modeling univariate time series. The moving-average model specifies that the output variable depends linearly on the current and various past values of a stochastic (imperfectly predictable) term.

The notation MA(q) refers to the moving average model of order q:

$$X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

where μ is the mean of the series, the $\theta_1, \ldots, \theta_q$ are the parameters of the model and the $\varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_{t-q}$ are white noise error terms.

Moving average in statistics (optional)

In statistics, a moving average (rolling average or running average) is a calculation to analyze data points by creating a series of averages of different subsets of the full data set. It is also called a moving mean (MM) or rolling mean and is a type of finite impulse response filter. Variations include: simple, and cumulative, or weighted forms.

Autoregressive Moving Average (ARMA) model

An ARMA model, or Autoregressive Moving Average model, is used to describe weakly stationary stochastic time series in terms of two polynomials. The first of these polynomials is for autoregression, the second for the moving average.

Often this model is referred to as the ARMA(p,q) model; where:

- p is the order of the autoregressive polynomial
- q is the order of the moving average polynomial

ARMA equation

The equation is given by:

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}.$$

Where:

- φ = the autoregressive model's parameters
- θ = the moving average model's parameters
- c = a constant
- ε = error terms (white noise)

Difference between ARMA and ARIMA models

What sets ARMA and ARIMA apart is differencing. An ARMA model is a stationary model; If your model isn’t stationary, then you can achieve stationarity by taking a series of differences. The “I” in the ARIMA model stands for integrated; It is a measure of how many non-seasonal differences are needed to achieve stationarity. If no differencing is involved in the model, then it becomes simply an ARMA.

References

- Stochastic-model
- Autoregressive-model
- Moving-average_model
- Moving_average
- arma-model