Time Series Forecasting

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Weak Stationarity

- No systematic change in mean (i.e. no trend)
- No systematic change in variation
- No periodic fluctuations

The properties of one section of a data are much like the properties of the other sections of the data.

For a non-stationary time series, we will do some transformations to get stationary time series.

Autocovariance function

$$\gamma(s,t) = Cov(X_s, X_t) = E[(X\_s - \mu_s)(X_t - \mu_t)]$$

$$\gamma(t,t) = E[(X_t - \mu_t)^2] = Var(X_t) = \sigma_t^2$$

$$\gamma_k = \gamma(t, t+k) \approx c_k$$

Covariance

X, Y are two random variables.

Measures the linear dependence between two random variables

$$Cov(X,Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

Estimation of the covariance

We have a paired dataset

$$(x_1, y_1), (x_2, y_2), \ldots., (x_n, y_n)$$

Estimation of covariance

$$s_{xy} = \frac{\sum_{t=1}^{N}(x_t - \bar{x})(y_t - \bar{y})}{N-1}$$

Autocovariance coefficients

Autocovariance coefficients at different lags $\gamma_k = Cov(X_t, X_{t+k})$

$$c_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{N}$$

where

$$\bar{x} = \frac{\sum_{t=1}^{N} x_t}{N}$$

The autocorrelation function (ACF)

The autocorrelation coefficient between X_t and X_{t+k} is defined to be

$$-1 \le \rho_k = \frac{\gamma_k}{\gamma_0} \le 1$$

Estimation of autocorrelation coefficient at lag k

$$r_k = \frac{c_k}{c_0} \qquad r_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2}$$

Random walk

Location at previous step(or a price of the stock yesterday)

Location at time t (or a price of a stock today)

$$X_t = X_{t-1} + Z_t$$

$Z_t \sim$ Normal(μ, σ^2)

White noise (residual)

Removing the trend

$X_t = X_{t-1} + Z_t$

$X_t - X_{t-1} = Z_t$

$\nabla X_t = X_t - X_{t-1}$

$\nabla X_t = Z_t$

Purely Random Process

Moving average processes

Intuition

$\boldsymbol{X_t}$ is a stock price of a company

Each daily announcement of the company is modeled as a noise

Effect of the daily announcements (noises Z_t) on the stock price (X_t) might last few days (say 2 days)

Moving average processes

Stock price is linear combination of the noises that affects it

$$X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$$

Moving average model of order 2 (MA(2))

$X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \ldots + \theta q Z_{t-q}$

Z_i are i.id. & $Z_i \sim Normal\ (\mu, \sigma^2)$

ACF of MA(q) cuts off at lag q

References

- https://d3c33hcgiwev3.cloudfront.net/_6f27156fa030a6e18372e06cf96d7b82_Week-1---slides--White.pdf?Expires=1582329600&Signature=f98~0mfntfXwmFHN~0MCWhd1RMzHdvSNeOfHq~xlkppsyTlgY1nxlka~PF2ww2E-rQyNGtqEBmY6oLSMvqqGRdQt4zawqXJOel-1wDuyqONbyvY3n3LvtF9H5Tj1p0B7WygiYnJaTAjObDW~ndQiDTNyh-xCwTluEz-kwp~hheU_&Key-Pair-Id=APKAJLTNE6QMUY6HBC5A