HW02 Solutions

① 20 point

Arraylist indexing → constant time → $\Theta(1)$

assume that * lenght = n

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c = c\left[\sum_{i=1}^{n-1} n - \sum_{i=1}^{n-1} i\right]$$

$$= c \cdot \left[n \cdot (n-1) - \frac{(n-1) \cdot n}{2}\right]$$

$$= \frac{c \cdot (n-1) \cdot n}{2} = \Theta(n^2)$$

worst case : for ( ) for ( ) c #1 $\boxed{n^2}$

best case : (sorted array) for (--) for (---) ~~if (comparison)~~ $\boxed{n^2}$ // always perform this part different from default buble sort//

$$= c_2 n^2 \le \frac{n^2 - n}{2} \le c_1 n^2 \quad \text{[formal def]}$$

$$= c_1 = 1, \quad n_0 = 1, \quad c_2 = 1/4$$

$$= -n < n^2 \checkmark \qquad \frac{n^2}{2} \le n^2 - n \quad n > 1 \checkmark$$

Linked List indexing = linear time

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} cn = c\left[\sum_{i=1}^{n-1} n^2 - n\sum_{i=1}^{n-1} i\right]$$

$$= c\left[(n-1) \cdot n^2 - \frac{n^2 \cdot (n-1)}{2}\right]$$

$$= c\,\frac{n^2 \cdot (n-1)}{2} = \Theta(n^3) \quad \text{best} \ne \text{worst} \quad \text{different}$$

formal definition $c_2 n^3 \le \dfrac{n^3 - n^2}{2} \le c_1 n^3$

$c_1 = 1 \quad n_0 = 1 \quad c_2 = 1/4$

$-n^2 < n^3 \quad n > 1 \checkmark$

$\frown n)$

2 20 point

$n^{2.56}$, $\log n!$, $n\log n$, $\log\log n^2$

$n^{2.56} \rightarrow 2^{\log n^{2.56}} \rightarrow \underline{2^{2.56\log n}}$

$\log n! \rightarrow \log n! \geq \log 2^n = n = \underline{2^{\log n}}$

$\log n! \leq \log n^n = n\log n = \underline{2^{\log n + \log\log n}}$

$n\log n \rightarrow 2^{\log(n \cdot \log n)} \rightarrow \underline{2^{\log n + \log\log n}}$

$\log\log n^2 \rightarrow 2^{\log(\log\log n)^2} \rightarrow 2^{\log(\cancel{\quad} 2\log\log n)}$

$\rightarrow \underline{2^{\log 2 + \log\log\log n}}$

growth rate

$\log\log n^2 < \log n! < n\log n < n^{2.56}$

3 15

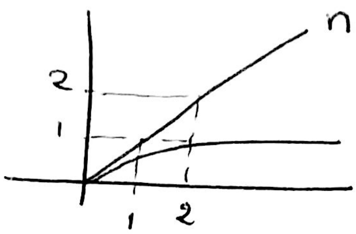
a) n^2 is is $O(2^n)$

$n^2 \leq 2^n . c$

$\log n^2 \leq \log 2^n + \log c$

$2 \log n \leq n . \log 2 + \log c$ $\quad c=1 \quad n_0=1$ } sadece sayıları (c, n_0) vermek yeterli değil tanımdaki eşitsizliği her zaman sağladığını göstermek zorundayız.

$2 \log n \leq n \longrightarrow$



b) $n!$ is in $\Omega(2^n)$

$2^n . c \leq n! \longrightarrow \underbrace{\lim_{n \to \infty} \frac{2^n}{n!}}_{5/\infty = 0} . c < 1$ $\quad c = 1 \quad n_0 = 4$

c) $\log n$ is in $\Theta(\log_{64} n)$

$c_2 . \log_{64} n \leq \log n \leq c_1 . \log_{64} n$

$c_2 . \frac{\log n}{\log 64} \leq \frac{\log n}{\log 2} \leq \frac{c_1 \log n}{\log 64}$

$\frac{c_2}{6} \leq 1 \leq \frac{c_1}{6}$ $\quad c_2 = 1 \quad c_1 = 7 \quad n_0 = 1$

Scanned by CamScanner

(4) 10P

$2n^2 - 4n + 9 = \Theta(n^2)$

$c_2 n^2 \le 2n^2 - 4n + 9 \le c_1 n^2$

$n = 1$, $c_2 = 1$, $c_1 = 8$

$1 < 2 - 4 + 9 < 8$ ✓

$n = k$, T

$c_2 k^2 \le 2k^2 - 4k + 9 \le c_1 k^2$

$n = k+1$

$c_2(k+1)^2 \le 2(k+1)^2 - 4(k+1) + 9 \le c_1(k+1)^2$

$c_2k^2 + 2c_2k + c_2 \le 2k^2 + 4k + 2 - 4k - 4 + 9 \le c_1k^2 + 2c_1k + c_1$

$c_2k^2 + 2c_2k + c_2 \le \underbrace{(2k^2 - 4k + 9)} + 4k - 2 \le c_1k^2 + 2c_1k + c_1$

$\downarrow$

$2c_2k + c_2 \le 4k - 2 \le 2c_1k + c_1$

$c_1 = 2$
$c_2 = 1/2$
$n_0 = 1$

15p

(5) Equivalence Relation

✓ Reflexive

✓ Symmetrix

✓ Transitive

Big-O is not an equivalence relation

- Transitive [if aRb and bRc then aRc]

if $n = O(n^2)$ and $n^2 = O(n^3)$ $n = O(n^3)$ ✓

- Symmetric

$n = O(n^2)$ ✗

$n^2 \neq O(n)$

- Reflexive

$g(n) = O(g(n))$ ✓

(6) a) $\frac{n^2}{5}$ (loop time)

10p

b) inner $\sum_{j=0}^{i} c$ i 2n olacak $\boxed{n}$

outer i kendinin 4 katı halde büyüyecek

k	i
1. için	1
2. için	4
3. için	16
⋮	⋮

$4^k = i \rightarrow \log_4 i = k$

i 2n'e doğru büyüyecek

$\log_4 2n = \boxed{\log n}$

$n \log n$

7

$$\sum_{i=1}^{\log n} \sum_{j=i}^{i+5} \sum_{k=1}^{i^2} c$$

10P

$$= c \cdot \sum_{i=1}^{\log n} \sum_{j=i}^{i+5} i^2$$

$$= c \cdot \sum_{i=1}^{\log n} 5 i^2$$

$$= 5 \cdot c \sum_{i=1}^{\log n} i^2$$

$$= 5c \left[\frac{\log n \, (\log n + 1)(2\log n + 1)}{6} \right]$$

$n = 10$ 8sn

$n = 160$?

find c

calculate n = 160 for

$\log_2 10 = 3.32 \rightarrow \dfrac{5 \cdot c \quad 3.32 \cdot 4.32 \, (3.322 + 1)}{6} = 10$

$\log_2 160 = 7.32 \Rightarrow$