Total Type Error Localization and Recovery with Holes

A Preface

This is the complete formalism demonstrating the *marked lambda calculus*, a judgmental framework for precise bidirectional error localization and recovery that employs gradual typing.

A.1 Organization

Though more is said in each individual section, the overall structure of the document is as follows:

- Section B employs the framework on a gradually typed lambda calculus.
- Section C extends the demonstration with patterned let expressions.
- Section D extends the demonstration with System F-style parametric polymorphism.
- Section E gives a version of the Hazelnut structure editor calculus that uses the marked lambda calculus to solve Hazelnut's deficiency with regards to non-local hole fixes.
- Section F is similar, except that it employs the marking procedure in a roughly incremental fashion.
- Section G additionally gives the rules for constraint generation in relation to the type hole inference work of Section 4.

Note that each of the sections following Section B build upon that same core language.

A.2 Mechanization

Not all parts of the formalism are mechanized in Agda. It is noted in each section whether or not the section has been mechanized and, if so, where to find the relevant definitions and theorems.

As possible, the names of judgments and rules that appear in the mechanization have been made to follow those in this formalism. Refer also to the mechanization's README for more details.

B Marked lambda calculus

The *marked lambda calculus* is a judgmental framework for bidirectional type error localization and recovery. Here, we demonstrate it on a gradually typed lambda calculus with numbers, booleans, and product types, as described in Section 2.1 of the paper.

Mechanization ○

- `core.agda`
- `marking.agda`

B.1 Syntax

$$\begin{array}{rcll}
\mathsf{Type} & \tau & ::= & ? \mid \mathsf{num} \mid \mathsf{bool} \mid \tau \to \tau \mid \tau \times \tau \\
\mathsf{UExp} & e & ::= & x \mid \lambda x : \tau.\, e \mid e\ e \mid \mathsf{let}\ x = e\ \mathsf{in}\ e \mid \underline{n} \mid e + e \\
 & & \mid & \mathsf{tt} \mid \mathsf{ff} \mid \mathsf{if}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e \mid (e, e) \mid \pi_1 e \mid \pi_2 e \mid \llparenthesis\rrparenthesis \\
\mathsf{MExp} & \check{e} & ::= & x \mid \lambda x : \tau.\, \check{e} \mid \check{e}\ \check{e} \mid \mathsf{let}\ x = \check{e}\ \mathsf{in}\ \check{e} \mid \underline{n} \mid \check{e} + \check{e} \\
 & & \mid & \mathsf{tt} \mid \mathsf{ff} \mid \mathsf{if}\ \check{e}\ \mathsf{then}\ \check{e}\ \mathsf{else}\ \check{e} \mid (\check{e}, \check{e}) \mid \pi_1 \check{e} \mid \pi_2 \check{e} \mid \llparenthesis\rrparenthesis \\
 & & \mid & \llparenthesis x \rrparenthesis_{\square} \mid \llparenthesis \check{e} \rrparenthesis_{\nsim} \\
 & & \mid & \llparenthesis \lambda x : \tau.\, \check{e} \rrparenthesis_{:} \mid \llparenthesis \lambda x : \tau.\, \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\to}} \mid \llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \check{e} \\
 & & \mid & \llparenthesis \mathsf{if}\ \check{e}\ \mathsf{then}\ \check{e}\ \mathsf{else}\ \check{e} \rrparenthesis_{\not\sqcup} \\
 & & \mid & \llparenthesis (\check{e}, \check{e}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\times}} \mid \pi_1 \llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\times}} \mid \pi_2 \llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\times}}
\end{array}$$

B.2 Types

$\boxed{\tau_1 \sim \tau_2}$ τ_1 is consistent with τ_2

$$\textsc{TCUnknown1}\ \frac{}{? \sim \tau} \qquad \textsc{TCUnknown2}\ \frac{}{\tau \sim ?} \qquad \textsc{TCRefl}\ \frac{}{\tau \sim \tau}$$

$$\textsc{TCArr}\ \frac{\tau_1 \sim \tau_1' \qquad \tau_2 \sim \tau_2'}{\tau_1 \to \tau_2 \sim \tau_1' \to \tau_2'} \qquad \textsc{TCProd}\ \frac{\tau_1 \sim \tau_1' \qquad \tau_2 \sim \tau_2'}{\tau_1 \times \tau_2 \sim \tau_1' \times \tau_2'}$$

$\boxed{\tau \blacktriangleright_{\to} \tau_1 \to \tau_2}$ τ has matched arrow type $\tau_1 \to \tau_2$

$$\textsc{TMAUnknown}\ \frac{}{? \blacktriangleright_{\to} ? \to ?} \qquad \textsc{TMAArr}\ \frac{}{\tau_1 \to \tau_2 \blacktriangleright_{\to} \tau_1 \to \tau_2}$$

$\boxed{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2}$ τ has matched binary product type $\tau_1 \times \tau_2$

$$\textsc{TMPUnknown}\ \frac{}{? \blacktriangleright_{\times} ? \times ?} \qquad \textsc{TMPProd}\ \frac{}{\tau_1 \times \tau_2 \blacktriangleright_{\times} \tau_1 \times \tau_2}$$

$\boxed{\tau_1 \sqcup \tau_2}$ is a *partial* metafunction $\mathsf{Type} \times \mathsf{Type} \rightharpoonup \mathsf{Type}$ defined as follows:

$$\begin{array}{rcl}
? \sqcup \tau & = & \tau \\
\tau \sqcup ? & = & \tau \\
\mathsf{num} \sqcup \mathsf{num} & = & \mathsf{num} \\
\mathsf{bool} \sqcup \mathsf{bool} & = & \mathsf{bool} \\
(\tau_1 \to \tau_2) \sqcup (\tau_1' \to \tau_2') & = & (\tau_1 \sqcup \tau_1') \to (\tau_2 \sqcup \tau_2') \\
(\tau_1 \times \tau_2) \sqcup (\tau_1' \times \tau_2') & = & (\tau_1 \sqcup \tau_1') \times (\tau_2 \sqcup \tau_2')
\end{array}$$

$\boxed{\tau \text{ base}}$ τ is a base type

$$\textsc{TBNum}\quad \frac{}{\mathsf{num}\ \text{base}} \qquad\qquad \textsc{TBBool}\quad \frac{}{\mathsf{bool}\ \text{base}}$$

B.3 Unmarked expressions

$\boxed{\Gamma \vdash_U e \Rightarrow \tau}$ e synthesizes type τ

$$\textsc{USHole}\quad \frac{}{\Gamma \vdash_U (\!|\,|\!) \Rightarrow ?} \qquad \textsc{USVar}\quad \frac{x : \tau \in \Gamma}{\Gamma \vdash_U x \Rightarrow \tau} \qquad \textsc{USLam}\quad \frac{\Gamma, x : \tau_1 \vdash_U e \Rightarrow \tau_2}{\Gamma \vdash_U \lambda x : \tau_1 .\, e \Rightarrow \tau_1 \to \tau_2}$$

$$\textsc{USAp}\quad \frac{\Gamma \vdash_U e_1 \Rightarrow \tau \qquad \tau \blacktriangleright_{\to} \tau_1 \to \tau_2 \qquad \Gamma \vdash_U e_2 \Leftarrow \tau_1}{\Gamma \vdash_U e_1\ e_2 \Rightarrow \tau_2}$$

$$\textsc{USLet}\quad \frac{\Gamma \vdash_U e_1 \Rightarrow \tau_1 \qquad \Gamma, x : \tau_1 \vdash_U e_2 \Rightarrow \tau_2}{\Gamma \vdash_U \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Rightarrow \tau_2} \qquad \textsc{USNum}\quad \frac{}{\Gamma \vdash_U \underline{n} \Rightarrow \mathsf{num}}$$

$$\textsc{USPlus}\quad \frac{\Gamma \vdash_U e_1 \Leftarrow \mathsf{num} \qquad \Gamma \vdash_U e_2 \Leftarrow \mathsf{num}}{\Gamma \vdash_U e_1 + e_2 \Rightarrow \mathsf{num}}$$

$$\textsc{USTrue}\quad \frac{}{\Gamma \vdash_U \mathsf{tt} \Rightarrow \mathsf{bool}} \qquad \textsc{USFalse}\quad \frac{}{\Gamma \vdash_U \mathsf{ff} \Rightarrow \mathsf{bool}}$$

$$\textsc{USIf}\quad \frac{\Gamma \vdash_U e_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_U e_2 \Rightarrow \tau_1 \qquad \Gamma \vdash_U e_3 \Rightarrow \tau_2 \qquad \tau_3 = \tau_1 \sqcup \tau_2}{\Gamma \vdash_U \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Rightarrow \tau_3}$$

$$\textsc{USPair}\quad \frac{\Gamma \vdash_U e_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_U e_2 \Rightarrow \tau_2}{\Gamma \vdash_U (e_1, e_2) \Rightarrow \tau_1 \times \tau_2} \qquad \textsc{USProjL}\quad \frac{\Gamma \vdash_U e \Rightarrow \tau \qquad \tau \blacktriangleright_{\times} \tau_1 \times \tau_2}{\Gamma \vdash_U \pi_1 e \Rightarrow \tau_1} \qquad \textsc{USProjR}\quad \frac{\Gamma \vdash_U e \Rightarrow \tau \qquad \tau \blacktriangleright_{\times} \tau_1 \times \tau_2}{\Gamma \vdash_U \pi_2 e \Rightarrow \tau_2}$$

$\boxed{\Gamma \vdash_U e \Leftarrow \tau}$ e analyzes against type τ

$$\textsc{UALam}\quad \frac{\tau_3 \blacktriangleright_{\to} \tau_1 \to \tau_2 \qquad \tau \sim \tau_1 \qquad \Gamma, x : \tau \vdash_U e \Leftarrow \tau_2}{\Gamma \vdash_U \lambda x : \tau .\, e \Leftarrow \tau_3} \qquad \textsc{UALet}\quad \frac{\Gamma \vdash_U e_1 \Rightarrow \tau_1 \qquad \Gamma, x : \tau_1 \vdash_U e_2 \Leftarrow \tau_2}{\Gamma \vdash_U \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Leftarrow \tau_2}$$

$$\textsc{UAIf}\quad \frac{\Gamma \vdash_U e_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_U e_1 \Leftarrow \tau \qquad \Gamma \vdash_U e_2 \Leftarrow \tau}{\Gamma \vdash_U \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Leftarrow \tau} \qquad \textsc{UAPair}\quad \frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash_U e_1 \Leftarrow \tau_1 \qquad \Gamma \vdash_U e_2 \Leftarrow \tau_2}{\Gamma \vdash_U (e_1, e_2) \Leftarrow \tau}$$

$$\textsc{UASubsume}\quad \frac{\Gamma \vdash_U e \Rightarrow \tau' \qquad \tau \sim \tau' \qquad e\ \text{subsumable}}{\Gamma \vdash_U e \Leftarrow \tau}$$

$\boxed{e \text{ subsumable}}$ e is subsumable

$$\textsc{USuHole}\quad \frac{}{(\!|\,|\!)\ \text{subsumable}} \qquad \textsc{USuVar}\quad \frac{}{x\ \text{subsumable}} \qquad \textsc{USuAp}\quad \frac{}{e_1\ e_2\ \text{subsumable}} \qquad \textsc{USuNum}\quad \frac{}{\underline{n}\ \text{subsumable}} \qquad \textsc{USuPlus}\quad \frac{}{e_1 + e_2\ \text{subsumable}}$$

$$\textsc{USuTrue}\quad \frac{}{\mathsf{tt}\ \text{subsumable}} \qquad \textsc{USuFalse}\quad \frac{}{\mathsf{ff}\ \text{subsumable}} \qquad \textsc{USuProjL}\quad \frac{}{\pi_1 e\ \text{subsumable}} \qquad \textsc{USuProjR}\quad \frac{}{\pi_2 e\ \text{subsumable}}$$

B.4 Marking

$\boxed{\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau}$ e is marked into $\check{e}$ and synthesizes type τ

$$\textsc{MKSHole}\ \frac{}{\Gamma \vdash (\!|\,|\!) \looparrowright (\!|\,|\!) \Rightarrow ?} \qquad \textsc{MKSVar}\ \frac{x : \tau \in \Gamma}{\Gamma \vdash x \looparrowright x \Rightarrow \tau} \qquad \textsc{MKSFree}\ \frac{x \notin \mathrm{dom}(\Gamma)}{\Gamma \vdash x \looparrowright (\!| x |\!)_{\square} \Rightarrow ?}$$

$$\textsc{MKSLam}\ \frac{\Gamma,\ x : \tau_1 \vdash e \looparrowright \check{e} \Rightarrow \tau_2}{\Gamma \vdash \lambda x : \tau_1.\, e \looparrowright \lambda x : \tau_1.\, \check{e} \Rightarrow \tau_1 \to \tau_2}$$

$$\textsc{MKSAp1}\ \frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Rightarrow \tau \qquad \tau \blacktriangleright_{\to} \tau_1 \to \tau_2 \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Leftarrow \tau_1}{\Gamma \vdash e_1\ e_2 \looparrowright \check{e}_1\ \check{e}_2 \Rightarrow \tau_2}$$

$$\textsc{MKSAp2}\ \frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Rightarrow \tau \qquad \tau \blacktriangleright_{\not\to} \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Leftarrow ?}{\Gamma \vdash e_1\ e_2 \looparrowright (\!| \check{e}_1 |\!)^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \check{e}_2 \Rightarrow ?}$$

$$\textsc{MKSLet}\ \frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma,\ x : \tau_1 \vdash e_2 \looparrowright \check{e}_2 \Rightarrow \tau_2}{\Gamma \vdash \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \looparrowright \mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2 \Rightarrow \tau_2} \qquad \textsc{MKSNum}\ \frac{}{\Gamma \vdash \underline{n} \looparrowright \underline{n} \Rightarrow \mathsf{num}}$$

$$\textsc{MKSPlus}\ \frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Leftarrow \mathsf{num} \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Leftarrow \mathsf{num}}{\Gamma \vdash e_1 + e_2 \looparrowright \check{e}_1 + \check{e}_2 \Rightarrow \mathsf{num}} \qquad \textsc{MKSTrue}\ \frac{}{\Gamma \vdash \mathsf{tt} \looparrowright \mathsf{tt} \Rightarrow \mathsf{bool}} \qquad \textsc{MKSFalse}\ \frac{}{\Gamma \vdash \mathsf{ff} \looparrowright \mathsf{ff} \Rightarrow \mathsf{bool}}$$

$$\textsc{MKSIf}\ \frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Rightarrow \tau_1 \qquad \Gamma \vdash e_3 \looparrowright \check{e}_3 \Rightarrow \tau_2 \qquad \tau_3 = \tau_1 \sqcup \tau_2}{\Gamma \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \looparrowright \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Rightarrow \tau_3}$$

$$\textsc{MKSInconsistentBranches}\ \frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Rightarrow \tau_1 \qquad \Gamma \vdash e_3 \looparrowright \check{e}_3 \Rightarrow \tau_2 \qquad \tau_1 \nsim \tau_2}{\Gamma \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \looparrowright (\!| \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 |\!)_{\not\sqcup} \Rightarrow ?}$$

$$\textsc{MKSPair}\ \frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Rightarrow \tau_2}{\Gamma \vdash (e_1, e_2) \looparrowright (\check{e}_1, \check{e}_2) \Rightarrow \tau_1 \times \tau_2} \qquad \textsc{MKSProjL1}\ \frac{\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau \qquad \tau \blacktriangleright_{\times} \tau_1 \times \tau_2}{\Gamma \vdash \pi_1 e \looparrowright \pi_1 \check{e} \Rightarrow \tau_1} \qquad \textsc{MKSProjL2}\ \frac{\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau \qquad \tau \blacktriangleright_{\not\times}}{\Gamma \vdash \pi_1 e \looparrowright \pi_1 (\!| \check{e} |\!)^{\Rightarrow}_{\blacktriangleright_{\not\times}} \Rightarrow ?}$$

$$\textsc{MKSProjR1}\ \frac{\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau \qquad \tau \blacktriangleright_{\times} \tau_1 \times \tau_2}{\Gamma \vdash \pi_2 e \looparrowright \pi_2 \check{e} \Rightarrow \tau_2} \qquad \textsc{MKSProjR2}\ \frac{\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau \qquad \tau \blacktriangleright_{\not\times}}{\Gamma \vdash \pi_2 e \looparrowright \pi_2 (\!| \check{e} |\!)^{\Rightarrow}_{\blacktriangleright_{\not\times}} \Rightarrow ?}$$

$\boxed{\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau}$ e is marked into $\check{e}$ and analyzes against type τ

MKALam1
$$\frac{\tau_3 \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \tau \sim \tau_1 \qquad \Gamma,\ x : \tau \vdash e \looparrowright \check{e} \Leftarrow \tau_2}{\Gamma \vdash \lambda x : \tau.\, e \looparrowright \lambda x : \tau.\, \check{e} \Leftarrow \tau_3}$$

MKALam2
$$\frac{\tau_3 \blacktriangleright_{\not\rightarrow} \qquad \Gamma,\ x : \tau \vdash e \looparrowright \check{e} \Leftarrow ?}{\Gamma \vdash \lambda x : \tau.\, e \looparrowright (\!|\lambda x : \tau.\, \check{e}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\rightarrow}} \Leftarrow \tau_3}$$

MKALam3
$$\frac{\tau_3 \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \tau \nsim \tau_1 \qquad \Gamma,\ x : \tau_1 \vdash e \looparrowright \check{e} \Leftarrow \tau_2}{\Gamma \vdash \lambda x : \tau.\, e \looparrowright (\!|\lambda x : \tau.\, \check{e}|\!)_{:} \Leftarrow \tau_3}$$

MKALet
$$\frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma,\ x : \tau_1 \vdash e_2 \looparrowright \check{e}_2 \Leftarrow \tau_2}{\Gamma \vdash \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \looparrowright \mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2 \Leftarrow \tau_2}$$

MKAIf
$$\frac{\Gamma \vdash e_1 \looparrowright \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Leftarrow \tau \qquad \Gamma \vdash e_3 \looparrowright \check{e}_3 \Leftarrow \tau}{\Gamma \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \looparrowright \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Leftarrow \tau}$$

MKAPair1
$$\frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash e_1 \looparrowright \check{e}_1 \Leftarrow \tau_1 \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Leftarrow \tau_2}{\Gamma \vdash (e_1, e_2) \looparrowright (\check{e}_1, \check{e}_2) \Leftarrow \tau}$$

MKAPair2
$$\frac{\tau \blacktriangleright_{\not\times} \qquad \Gamma \vdash e_1 \looparrowright \check{e}_1 \Leftarrow ? \qquad \Gamma \vdash e_2 \looparrowright \check{e}_2 \Leftarrow ?}{\Gamma \vdash (e_1, e_2) \looparrowright (\!|(\check{e}_1, \check{e}_2)|\!)^{\Leftarrow}_{\blacktriangleright_{\not\times}} \Leftarrow \tau}$$

MKAInconsistentTypes
$$\frac{\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau' \qquad \tau \nsim \tau' \qquad e\ \text{subsumable}}{\Gamma \vdash e \looparrowright (\!|\check{e}|\!)_{\nsim} \Leftarrow \tau}$$

MKASubsume
$$\frac{\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau' \qquad \tau \sim \tau' \qquad e\ \text{subsumable}}{\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau}$$

B.5 Marked expressions

$\boxed{\Gamma \vdash_M \check{e} \Rightarrow \tau}$ $\check{e}$ synthesizes type τ

MSHole
$$\frac{}{\Gamma \vdash_M (\!|\,|\!) \Rightarrow ?}$$

MSVar
$$\frac{x : \tau \in \Gamma}{\Gamma \vdash_M x \Rightarrow \tau}$$

MSFree
$$\frac{x \notin \mathrm{dom}(\Gamma)}{\Gamma \vdash_M (\!|x|\!)_{\square} \Rightarrow ?}$$

MSLam
$$\frac{\Gamma,\ x : \tau_1 \vdash_M \check{e} \Rightarrow \tau_2}{\Gamma \vdash_M \lambda x : \tau_1.\, \check{e} \Rightarrow \tau_1 \to \tau_2}$$

MSAp1
$$\frac{\Gamma \vdash_M \check{e}_1 \Rightarrow \tau \qquad \tau \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \Gamma \vdash_M \check{e}_2 \Leftarrow \tau_1}{\Gamma \vdash_M \check{e}_1\ \check{e}_2 \Rightarrow \tau_2}$$

MSAp2
$$\frac{\Gamma \vdash_M \check{e}_1 \Rightarrow \tau \qquad \tau \blacktriangleright_{\not\rightarrow} \qquad \Gamma \vdash_M \check{e}_2 \Leftarrow ?}{\Gamma \vdash_M (\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\rightarrow}}\ \check{e} \Rightarrow ?}$$

MSLet
$$\frac{\Gamma \vdash_M \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma,\ x : \tau_1 \vdash_M \check{e}_2 \Rightarrow \tau_2}{\Gamma \vdash_M \mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2 \Rightarrow \tau_2}$$

MSNum
$$\frac{}{\Gamma \vdash_M \underline{n} \Rightarrow \mathsf{num}}$$

MSPlus
$$\frac{\Gamma \vdash_M \check{e}_1 \Leftarrow \mathsf{num} \qquad \Gamma \vdash_M \check{e}_2 \Leftarrow \mathsf{num}}{\Gamma \vdash_M \check{e}_1 + \check{e}_2 \Rightarrow \mathsf{num}}$$

MSTrue
$$\frac{}{\Gamma \vdash_M \mathsf{tt} \Rightarrow \mathsf{bool}}$$

MSFalse
$$\frac{}{\Gamma \vdash_M \mathsf{ff} \Rightarrow \mathsf{bool}}$$

MSIf
$$\frac{\Gamma \vdash_M \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_M \check{e}_2 \Rightarrow \tau_1 \qquad \Gamma \vdash_M \check{e}_3 \Rightarrow \tau_2\ \tau_3 = \tau_1 \sqcup \tau_2}{\Gamma \vdash_M \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Rightarrow \tau_3}$$

MSInconsistentBranches
$$\frac{\Gamma \vdash_M \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_M \check{e}_2 \Rightarrow \tau_1 \qquad \Gamma \vdash_M \check{e}_3 \Rightarrow \tau_2 \qquad \tau_1 \nsim \tau_2}{\Gamma \vdash_M (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\not\sqcup} \Rightarrow ?}$$

MSPair
$$\frac{\Gamma \vdash_M \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_M \check{e}_2 \Rightarrow \tau_2}{\Gamma \vdash_M (\check{e}_1, \check{e}_2) \Rightarrow \tau_1 \times \tau_2}$$

MSProjL1
$$\frac{\Gamma \vdash_M \check{e} \Rightarrow \tau \qquad \tau \blacktriangleright_{\times} \tau_1 \times \tau_2}{\Gamma \vdash_M \pi_1 \check{e} \Rightarrow \tau_1}$$

MSProjL2
$$\frac{\Gamma \vdash_M \check{e} \Rightarrow \tau \qquad \tau \blacktriangleright_{\not\times}}{\Gamma \vdash_M \pi_1 (\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\times}} \Rightarrow ?}$$

MSProjR1
$$\frac{\Gamma \vdash_M \check{e} \Rightarrow \tau \qquad \tau \blacktriangleright_{\times} \tau_1 \times \tau_2}{\Gamma \vdash_M \pi_2 \check{e} \Rightarrow \tau_2}$$

MSProjR2
$$\frac{\Gamma \vdash_M \check{e} \Rightarrow \tau \qquad \tau \blacktriangleright_{\not\times}}{\Gamma \vdash_M \pi_2 (\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\times}} \Rightarrow ?}$$

$\boxed{\Gamma \vdash_M \check{e} \Leftarrow \tau}$ $\check{e}$ analyzes against type τ

$$\textsc{MALam1}\quad \frac{\tau_3 \blacktriangleright_{\to} \tau_1 \to \tau_2 \qquad \tau \sim \tau_1 \qquad \Gamma,\ x : \tau \vdash_M \check{e} \Leftarrow \tau_2}{\Gamma \vdash_M \lambda x : \tau.\, \check{e} \Leftarrow \tau_3}$$

$$\textsc{MALam2}\quad \frac{\tau_3 \blacktriangleright_{\not\to} \qquad \Gamma,\ x : \tau \vdash_M \check{e} \Leftarrow ?}{\Gamma \vdash_M \llparenthesis \lambda x : \tau.\, \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\to}} \Leftarrow \tau_3}$$

$$\textsc{MALam3}\quad \frac{\tau_3 \blacktriangleright_{\to} \tau_1 \to \tau_2 \qquad \tau \nsim \tau_1 \qquad \Gamma,\ x : \tau_1 \vdash_M \check{e} \Leftarrow \tau_2}{\Gamma \vdash_M \llparenthesis \lambda x : \tau.\, \check{e} \rrparenthesis_{:} \Leftarrow \tau_3}$$

$$\textsc{MALet}\quad \frac{\Gamma \vdash_M \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma,\ x : \tau_1 \vdash_M \check{e}_2 \Leftarrow \tau_2}{\Gamma \vdash_M \mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2 \Leftarrow \tau_2}$$

$$\textsc{MAIf}\quad \frac{\Gamma \vdash_M \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_M \check{e}_1 \Leftarrow \tau \qquad \Gamma \vdash_M \check{e}_2 \Leftarrow \tau}{\Gamma \vdash_M \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Leftarrow \tau}$$

$$\textsc{MAPair1}\quad \frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash_M \check{e}_1 \Leftarrow \tau_1 \qquad \Gamma \vdash_M \check{e}_2 \Leftarrow \tau_2}{\Gamma \vdash_M (\check{e}_1, \check{e}_2) \Leftarrow \tau}$$

$$\textsc{MAPair2}\quad \frac{\tau \blacktriangleright_{\not\times} \qquad \Gamma \vdash_M \check{e}_1 \Leftarrow ? \qquad \Gamma \vdash_M \check{e}_2 \Leftarrow ?}{\Gamma \vdash_M \llparenthesis (\check{e}_1, \check{e}_2) \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\times}} \Leftarrow \tau}$$

$$\textsc{MAInconsistentTypes}\quad \frac{\Gamma \vdash_M \check{e} \Rightarrow \tau' \qquad \tau \nsim \tau' \qquad \check{e}\ \mathsf{subsumable}}{\Gamma \vdash_M \llparenthesis \check{e} \rrparenthesis_{\nsim} \Leftarrow \tau}$$

$$\textsc{MASubsume}\quad \frac{\Gamma \vdash_M \check{e} \Rightarrow \tau' \qquad \tau \sim \tau' \qquad \check{e}\ \mathsf{subsumable}}{\Gamma \vdash_M \check{e} \Leftarrow \tau}$$

$\boxed{\check{e}\ \mathsf{subsumable}}$ $\check{e}$ is subsumable

$$\textsc{MSuHole}\quad \frac{}{\llparenthesis\rrparenthesis\ \mathsf{subsumable}} \qquad \textsc{MSuVar}\quad \frac{}{x\ \mathsf{subsumable}} \qquad \textsc{MSuFree}\quad \frac{}{\llparenthesis x \rrparenthesis_{\square}\ \mathsf{subsumable}}$$

$$\textsc{MSuAp1}\quad \frac{}{\check{e}_1\ \check{e}_2\ \mathsf{subsumable}} \qquad \textsc{MSuAp2}\quad \frac{}{\llparenthesis \check{e}_1 \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \check{e}_2\ \mathsf{subsumable}}$$

$$\textsc{MSuNum}\quad \frac{}{\underline{n}\ \mathsf{subsumable}} \qquad \textsc{MSuPlus}\quad \frac{}{\check{e}_1 + \check{e}_2\ \mathsf{subsumable}} \qquad \textsc{MSuTrue}\quad \frac{}{\mathsf{tt}\ \mathsf{subsumable}} \qquad \textsc{MSuFalse}\quad \frac{}{\mathsf{ff}\ \mathsf{subsumable}}$$

$$\textsc{MSuInconsistentBranches}\quad \frac{}{\llparenthesis \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \rrparenthesis_{\not\sqcup}\ \mathsf{subsumable}}$$

$$\textsc{MSuProjL1}\quad \frac{}{\pi_1 \check{e}\ \mathsf{subsumable}} \qquad \textsc{MSuProjL2}\quad \frac{}{\pi_1 \llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\times}}\ \mathsf{subsumable}}$$

$$\textsc{MSuProjR1}\quad \frac{}{\pi_2 \check{e}\ \mathsf{subsumable}} \qquad \textsc{MSuProjR2}\quad \frac{}{\pi_2 \llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\times}}\ \mathsf{subsumable}}$$

$\boxed{\check{e}\ \mathsf{markless}}$ $\check{e}$ has no marks

$$\textsc{MLHole}\quad \frac{}{\llparenthesis\rrparenthesis\ \mathsf{markless}} \qquad \textsc{MLVar}\quad \frac{}{x\ \mathsf{markless}} \qquad \textsc{MLLam}\quad \frac{\check{e}\ \mathsf{markless}}{\lambda x : \tau.\, \check{e}\ \mathsf{markless}}$$

$$\textsc{MLAp}\quad \frac{\check{e}_1\ \mathsf{markless} \qquad \check{e}_2\ \mathsf{markless}}{\check{e}_1\ \check{e}_2\ \mathsf{markless}}$$

$$\textsc{MLLet}\quad \frac{\check{e}_1\ \mathsf{markless} \qquad \check{e}_2\ \mathsf{markless}}{\mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2\ \mathsf{markless}} \qquad \textsc{MLNum}\quad \frac{}{\underline{n}\ \mathsf{markless}}$$

$$\textsc{MLPlus}\quad \frac{\check{e}_1\ \mathsf{markless} \qquad \check{e}_2\ \mathsf{markless}}{\check{e}_1 + \check{e}_2\ \mathsf{markless}} \qquad \textsc{MLTrue}\quad \frac{}{\mathsf{tt}\ \mathsf{markless}} \qquad \textsc{MLFalse}\quad \frac{}{\mathsf{ff}\ \mathsf{markless}}$$

$$\textsc{MLIf}\quad \frac{\check{e}_1\ \mathsf{markless} \qquad \check{e}_2\ \mathsf{markless} \qquad \check{e}_3\ \mathsf{markless}}{\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3\ \mathsf{markless}}$$

$$\textsc{MLPair}\quad \frac{\check{e}_1\ \mathsf{markless} \qquad \check{e}_2\ \mathsf{markless}}{(\check{e}_1, \check{e}_2)\ \mathsf{markless}} \qquad \textsc{MLProjL}\quad \frac{\check{e}\ \mathsf{markless}}{\pi_1 \check{e}\ \mathsf{markless}} \qquad \textsc{MLProjR}\quad \frac{\check{e}\ \mathsf{markless}}{\pi_2 \check{e}\ \mathsf{markless}}$$

B.6 Mark erasure

$\boxed{\check{e}^{\square}}$ is a metafunction MExp → UExp defined as follows:

$$
\begin{array}{rcl}
\llparenthesis\rrparenthesis^{\square} & = & \llparenthesis\rrparenthesis \\
x^{\square} & = & x \\
\llparenthesis x \rrparenthesis_{\square}{}^{\square} & = & x \\
(\lambda x : \tau .\, \check{e})^{\square} & = & \lambda x : \tau .\, (\check{e}^{\square}) \\
\llparenthesis \lambda x : \tau .\, \check{e} \rrparenthesis_{:}{}^{\square} & = & \lambda x : \tau .\, (\check{e}^{\square}) \\
\llparenthesis \lambda x : \tau .\, \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright\not\to}{}^{\square} & = & \lambda x : \tau .\, (\check{e}^{\square}) \\
(\check{e}_1\ \check{e}_2)^{\square} & = & (\check{e}_1^{\square})\ (\check{e}_2^{\square}) \\
(\llparenthesis \check{e}_1 \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\to}\ \check{e}_2)^{\square} & = & (\check{e}_1^{\square})\ (\check{e}_2^{\square}) \\
(\mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2)^{\square} & = & \mathsf{let}\ x = (\check{e}_1^{\square})\ \mathsf{in}\ (\check{e}_2^{\square}) \\
\underline{n}^{\square} & = & \underline{n} \\
(\check{e}_1 + \check{e}_2)^{\square} & = & (\check{e}_1^{\square}) + (\check{e}_2^{\square}) \\
\mathsf{tt}^{\square} & = & \mathsf{tt} \\
\mathsf{ff}^{\square} & = & \mathsf{ff} \\
(\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3)^{\square} & = & \mathsf{if}\ (\check{e}_1^{\square})\ \mathsf{then}\ (\check{e}_2^{\square})\ \mathsf{else}\ (\check{e}_3^{\square}) \\
\llparenthesis \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \rrparenthesis_{\not\sqcup}{}^{\square} & = & \mathsf{if}\ (\check{e}_1^{\square})\ \mathsf{then}\ (\check{e}_2^{\square})\ \mathsf{else}\ (\check{e}_3^{\square}) \\
(\check{e}_1, \check{e}_2)^{\square} & = & (\check{e}_1^{\square}, \check{e}_2^{\square}) \\
\llparenthesis (\check{e}_1, \check{e}_2) \rrparenthesis^{\Leftarrow}_{\blacktriangleright\not\times}{}^{\square} & = & (\check{e}_1^{\square}, \check{e}_2^{\square}) \\
(\pi_1 \check{e})^{\square} & = & \pi_1(\check{e}^{\square}) \\
(\pi_1 \llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\times})^{\square} & = & \pi_1(\check{e}^{\square}) \\
(\pi_2 \check{e})^{\square} & = & \pi_2(\check{e}^{\square}) \\
(\pi_2 \llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\times})^{\square} & = & \pi_2(\check{e}^{\square}) \\
\llparenthesis \check{e} \rrparenthesis_{\nsim}{}^{\square} & = & \check{e}^{\square}
\end{array}
$$

B.7 Metatheorems

Theorem B.1 (Marking Totality).

1. *For all* Γ *and* e*, there exist* $\check{e}$ *and* τ *such that* $\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau$ *.*
2. *For all* Γ*,* e*, and* τ*, there exists* $\check{e}$ *such that* $\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau$ *.*

Theorem B.2 (Marking Well-Formedness).

1. *If* $\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau$ *, then* $\Gamma \vdash_{M} \check{e} \Rightarrow \tau$ *and* $\check{e}^{\square} = e$*.*
2. *If* $\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau$ *, then* $\Gamma \vdash_{M} \check{e} \Leftarrow \tau$ *and* $\check{e}^{\square} = e$*.*

Theorem B.3 (Marking of Well-Typed/Ill-Typed Expressions).

1. (a) *If* $\Gamma \vdash_{U} e \Rightarrow \tau$ *and* $\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau$ *, then* $\check{e}$ markless.

   (b) *If* $\Gamma \vdash_{U} e \Leftarrow \tau$ *and* $\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau$ *, then* $\check{e}$ markless.

2. (a) *If there does not exist* τ *such that* $\Gamma \vdash_{U} e \Rightarrow \tau$ *, then for all* $\check{e}$ *and* τ' *such that* $\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau'$ *, it is not the case that* $\check{e}$ markless.

   (b) *If there does not exist* τ *such that* $\Gamma \vdash_{U} e \Leftarrow \tau$ *, then for all* $\check{e}$ *and* τ' *such that* $\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau'$ *, it is not the case that* $\check{e}$ markless.

Theorem B.4 (Marking Unicity).

1. *If* $\Gamma \vdash e \looparrowright \check{e}_1 \Rightarrow \tau_1$ *and* $\Gamma \vdash e \looparrowright \check{e}_2 \Rightarrow \tau_2$ *, then* $\check{e}_1 = \check{e}_2$ *and* $\tau_1 = \tau_2$*.*
2. *If* $\Gamma \vdash e \looparrowright \check{e}_1 \Leftarrow \tau$ *and* $\Gamma \vdash e \looparrowright \check{e}_2 \Leftarrow \tau$ *, then* $\check{e}_1 = \check{e}_2$*.*

B.8 Alternative conditional rules

There are alternative ways to formulate error localization in conditionals. Below, we provide two alternatives to the rules above.

B.8.1 Localize to second

In this formulation, we always select the first branch as "correct" and localize errors to the second.

$\boxed{\Gamma \vdash_{U} e \Rightarrow \tau}$ e synthesizes type τ

$$\text{USIF'}\ \frac{\Gamma \vdash_{U} e_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_{U} e_2 \Rightarrow \tau \qquad \Gamma \vdash_{U} e_3 \Leftarrow \tau}{\Gamma \vdash_{U} \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Rightarrow \tau}$$

$\boxed{\Gamma \vdash e \hookrightarrow \check{e} \Rightarrow \tau}$ e is marked into $\check{e}$ and synthesizes type τ

$$\text{MKSIF'}\ \frac{\Gamma \vdash e_1 \hookrightarrow \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash e_2 \hookrightarrow \check{e}_2 \Rightarrow \tau \qquad \Gamma \vdash e_3 \hookrightarrow \check{e}_3 \Leftarrow \tau}{\Gamma \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \hookrightarrow \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Rightarrow \tau}$$

$\boxed{\Gamma \vdash_{M} \check{e} \Rightarrow \tau}$ $\check{e}$ synthesizes type τ

$$\text{MSIF'}\ \frac{\Gamma \vdash_{M} \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_{M} \check{e}_2 \Rightarrow \tau \qquad \Gamma \vdash_{M} \check{e}_3 \Leftarrow \tau}{\Gamma \vdash_{M} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Rightarrow \tau}$$

B.8.2 Localize to first

In this formulation, we always select the second branch as "correct" and localize errors to the first.

$\boxed{\Gamma \vdash_{U} e \Rightarrow \tau}$ e synthesizes type τ

$$\text{USIF''}\ \frac{\Gamma \vdash_{U} e_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_{U} e_3 \Rightarrow \tau \qquad \Gamma \vdash_{U} e_2 \Leftarrow \tau}{\Gamma \vdash_{U} \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Rightarrow \tau}$$

$\boxed{\Gamma \vdash e \hookrightarrow \check{e} \Rightarrow \tau}$ e is marked into $\check{e}$ and synthesizes type τ

$$\text{MKSIF''}\ \frac{\Gamma \vdash e_1 \hookrightarrow \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash e_3 \hookrightarrow \check{e}_3 \Rightarrow \tau \qquad \Gamma \vdash e_2 \hookrightarrow \check{e}_2 \Leftarrow \tau}{\Gamma \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \hookrightarrow \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Rightarrow \tau}$$

$\boxed{\Gamma \vdash_{M} \check{e} \Rightarrow \tau}$ $\check{e}$ synthesizes type τ

$$\text{MSIF''}\ \frac{\Gamma \vdash_{M} \check{e}_1 \Leftarrow \mathsf{bool} \qquad \Gamma \vdash_{M} \check{e}_3 \Rightarrow \tau \qquad \Gamma \vdash_{M} \check{e}_2 \Leftarrow \tau}{\Gamma \vdash_{M} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Rightarrow \tau}$$

C Extension: patterned let expressions

In this section, we describe an extension of the marked lambda calculus for destructuring let expressions, as described in Section 2.3 of the paper.

MECHANIZATION ×

C.1 Syntax

$$
\begin{array}{rccl}
\textsf{Type} & \tau & ::= & \cdots \mid ?^{\Rightarrow} \\
\textsf{UExp} & e & ::= & \cdots \mid \textsf{let } p = e \textsf{ in } e \\
\textsf{MExp} & \check{e} & ::= & \cdots \mid \textsf{let } p = \check{e} \textsf{ in } \check{e} \\
\textsf{UPat} & p & ::= & \_ \mid x \mid (p, p) \mid p : \tau \\
\textsf{MPat} & \check{p} & ::= & \_ \mid x \mid (\check{p}, \check{p}) \mid \check{p} : \tau \\
 & & \mid & (\!|\check{p}|\!)_{\nsim} \mid (\!|(\check{p}, \check{p})|\!)^{\Leftarrow}_{\blacktriangleright_{\not\times}}
\end{array}
$$

C.2 Types

$\boxed{\tau_1 \sim \tau_2}$ τ_1 is consistent with τ_2

TCUNKNOWNSWITCH1
$$\frac{}{?^{\Rightarrow} \sim \tau}$$

TCUNKNOWNSWITCH2
$$\frac{}{\tau \sim ?^{\Rightarrow}}$$

$\boxed{\tau \blacktriangleright_{\rightarrow} \tau_1 \rightarrow \tau_2}$ τ has matched arrow type $\tau_1 \rightarrow \tau_2$

TMAUNKNOWNSWITCH
$$\frac{}{?^{\Rightarrow} \blacktriangleright_{\rightarrow} ?^{\Rightarrow} \rightarrow ?^{\Rightarrow}}$$

$\boxed{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2}$ τ has matched binary product type $\tau_1 \times \tau_2$

TMPUNKNOWNSWITCH
$$\frac{}{?^{\Rightarrow} \blacktriangleright_{\times} ?^{\Rightarrow} \times ?^{\Rightarrow}}$$

$\boxed{\tau_1 \sqcup \tau_2}$ is a *partial* metafunction Type × Type ⇀ Type defined as follows:

$$
\begin{array}{rcl}
 & \vdots & \\
?^{\Rightarrow} \sqcup \tau & = & ?^{\Rightarrow} \\
\tau \sqcup ?^{\Rightarrow} & = & ?^{\Rightarrow}
\end{array}
$$

C.3 Unmarked patterns

$\boxed{\Gamma \vdash_U p \Rightarrow \tau}$ p synthesizes type τ

USPWILD
$$\frac{}{\Gamma \vdash_U \_ \Rightarrow ?^{\Rightarrow}}$$

USPVAR
$$\frac{}{\Gamma \vdash_U x \Rightarrow ?^{\Rightarrow}}$$

USPPAIR
$$\frac{\Gamma \vdash_U p_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_U p_2 \Rightarrow \tau_2}{\Gamma \vdash_U (p_1, p_2) \Rightarrow \tau_1 \times \tau_2}$$

USPANN
$$\frac{\Gamma \vdash_U p \Leftarrow \tau \dashv \Gamma'}{\Gamma \vdash_U p : \tau \Rightarrow \tau}$$

$\boxed{\Gamma_1 \vdash_U p \Leftarrow \tau \dashv \Gamma_2}$ p analyzes against type τ producing context Γ_2

$$\frac{}{\Gamma \vdash_U \_ \Leftarrow \tau \dashv \Gamma}\;\textsc{UAPWild}$$

$$\frac{}{\Gamma \vdash_U x \Leftarrow \tau \dashv \Gamma,\; x : \tau}\;\textsc{UAPVar}$$

$$\frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash_U p_1 \Leftarrow \tau_1 \dashv \Gamma_1 \qquad \Gamma_1 \vdash_U p_2 \Leftarrow \tau_2 \dashv \Gamma_2}{\Gamma \vdash_U (p_1, p_2) \Leftarrow \tau \dashv \Gamma_2}\;\textsc{UAPPair}$$

$$\frac{\Gamma \vdash_U p \Leftarrow \tau' \dashv \Gamma' \qquad \tau \sim \tau'}{\Gamma \vdash_U p : \tau' \Leftarrow \tau \dashv \Gamma'}\;\textsc{UAPAnn}$$

C.4 Pattern marking

$\boxed{\Gamma \vdash p \looparrowright \check{p} \Rightarrow \tau}$ p is marked into $\check{p}$ and synthesizes τ

$$\frac{}{\Gamma \vdash \_ \looparrowright \_ \Rightarrow ?^{\Rightarrow}}\;\textsc{MKSPWild}$$

$$\frac{}{\Gamma \vdash x \looparrowright x \Rightarrow ?^{\Rightarrow}}\;\textsc{MKSPVar}$$

$$\frac{\begin{array}{c}\Gamma \vdash p_1 \looparrowright \check{p}_1 \Rightarrow \tau_1 \\ \Gamma \vdash p_2 \looparrowright \check{p}_2 \Rightarrow \tau_2\end{array}}{\Gamma \vdash (p_1, p_2) \looparrowright (\check{p}_1, \check{p}_2) \Rightarrow \tau_1 \times \tau_2}\;\textsc{MKSPPair}$$

$$\frac{\Gamma \vdash_U p \Leftarrow \tau \dashv \Gamma' \qquad \Gamma \vdash p \looparrowright \check{p} \Leftarrow \tau \dashv \Gamma''}{\Gamma \vdash p : \tau \looparrowright \check{p} : \tau \Rightarrow \tau}\;\textsc{MKSPAnn1}$$

$$\frac{\Gamma \vdash_U p \not\Leftarrow \tau \dashv \Gamma' \qquad \Gamma \vdash p \looparrowright \check{p} \Leftarrow ? \dashv \Gamma''}{\Gamma \vdash p : \tau \looparrowright \llparenthesis \check{p} \rrparenthesis_{\not\sim} : \tau \Rightarrow \tau}\;\textsc{MKSPAnn2}$$

$\boxed{\Gamma_1 \vdash p \looparrowright \check{p} \Leftarrow \tau \dashv \Gamma_2}$ p is marked into $\check{p}$ and analyzes against τ producing Γ_2

$$\frac{}{\Gamma \vdash \_ \looparrowright \_ \Leftarrow \tau \dashv \Gamma}\;\textsc{MKAPWild}$$

$$\frac{}{\Gamma \vdash x \looparrowright x \Leftarrow \tau \dashv \Gamma,\; x : \tau}\;\textsc{MKAPVar}$$

$$\frac{\begin{array}{c}\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash p_1 \looparrowright \check{p}_1 \Leftarrow \tau_1 \dashv \Gamma_1 \\ \Gamma_1 \vdash p_2 \looparrowright \check{p}_2 \Leftarrow \tau_2 \dashv \Gamma_2\end{array}}{\Gamma \vdash (p_1, p_2) \looparrowright (\check{p}_1, \check{p}_2) \Leftarrow \tau \dashv \Gamma_2}\;\textsc{MKAPPair1}$$

$$\frac{\begin{array}{c}\tau \not\blacktriangleright_{\times} \qquad \Gamma \vdash p_1 \looparrowright \check{p}_1 \Leftarrow ? \dashv \Gamma_1 \\ \Gamma_1 \vdash p_2 \looparrowright \check{p}_2 \Leftarrow ? \dashv \Gamma_2\end{array}}{\Gamma \vdash (p_1, p_2) \looparrowright \llparenthesis (\check{p}_1, \check{p}_2) \rrparenthesis^{\Leftarrow}_{\not\blacktriangleright_{\times}} \Leftarrow \tau \dashv \Gamma_2}\;\textsc{MKAPPair2}$$

$$\frac{\tau \sim \tau' \qquad \Gamma \vdash p \looparrowright \check{p} \Leftarrow \tau' \dashv \Gamma'}{\Gamma \vdash p : \tau' \looparrowright \check{p} : \tau' \Leftarrow \tau \dashv \Gamma'}\;\textsc{MKAPAnn1}$$

$$\frac{\tau \nsim \tau' \qquad \Gamma \vdash p \looparrowright \check{p} \Leftarrow \tau' \dashv \Gamma'}{\Gamma \vdash p : \tau' \looparrowright \llparenthesis \check{p} : \tau' \rrparenthesis_{\not\sim} \Leftarrow \tau \dashv \Gamma'}\;\textsc{MKAPAnn2}$$

C.5 Marked patterns

$\boxed{\Gamma \vdash_M \check{p} \Rightarrow \tau}$ $\check{p}$ synthesizes type τ

$$\frac{}{\Gamma \vdash_M \_ \Rightarrow ?^{\Rightarrow}}\;\textsc{MSPWild}$$

$$\frac{}{\Gamma \vdash_M x \Rightarrow ?^{\Rightarrow}}\;\textsc{MSPVar}$$

$$\frac{\Gamma \vdash_M \check{p}_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_M \check{p}_2 \Rightarrow \tau_2}{\Gamma \vdash_M (\check{p}_1, \check{p}_2) \Rightarrow \tau_1 \times \tau_2}\;\textsc{MSPPair}$$

$$\frac{\Gamma \vdash_M \check{p} \Leftarrow \tau \dashv \Gamma'}{\Gamma \vdash_M \check{p} : \tau \Rightarrow \tau}\;\textsc{MSPAnn}$$

$\boxed{\Gamma_1 \vdash_M \check{p} \Leftarrow \tau \dashv \Gamma_2}$ $\check{p}$ analyzes against type τ producing context Γ_2

MAPWild
$$\frac{}{\Gamma \vdash_M \_ \Leftarrow \tau \dashv \Gamma}$$

MAPVar
$$\frac{}{\Gamma \vdash_M x \Leftarrow \tau \dashv \Gamma,\ x : \tau}$$

MAPPair1
$$\frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash_M \check{p}_1 \Leftarrow \tau_1 \dashv \Gamma_1 \qquad \Gamma_1 \vdash_M \check{p}_2 \Leftarrow \tau_2 \dashv \Gamma_2}{\Gamma \vdash_M (\check{p}_1, \check{p}_2) \Leftarrow \tau \dashv \Gamma_2}$$

MAPPair2
$$\frac{\tau \not\blacktriangleright_{\times} \qquad \Gamma \vdash_M \check{p}_1 \Leftarrow ? \dashv \Gamma_1 \qquad \Gamma_1 \vdash_M \check{p}_2 \Leftarrow ? \dashv \Gamma_2}{\Gamma \vdash_M (\!|(\check{p}_1, \check{p}_2)|\!)^{\Leftarrow}_{\not\blacktriangleright_{\times}} \Leftarrow \tau \dashv \Gamma_2}$$

MAPAnn1
$$\frac{\tau \sim \tau' \qquad \Gamma \vdash_M \check{p} \Leftarrow \tau' \dashv \Gamma'}{\Gamma \vdash_M \check{p} : \tau' \Leftarrow \tau \dashv \Gamma'}$$

MAPAnn2
$$\frac{\tau \nsim \tau' \qquad \Gamma \vdash_M \check{p} \Leftarrow \tau' \dashv \Gamma'}{\Gamma \vdash_M (\!|\check{p} : \tau'|\!)_{\nsim} \Leftarrow \tau \dashv \Gamma'}$$

$\boxed{\check{p} \text{ markless}}$ $\check{p}$ has no marks

MLPWild
$$\frac{}{\_ \text{ markless}}$$

MLPVar
$$\frac{}{x \text{ markless}}$$

MLPPair
$$\frac{\check{p}_1 \text{ markless} \qquad \check{p}_2 \text{ markless}}{(\check{p}_1, \check{p}_2) \text{ markless}}$$

MLPAnn
$$\frac{\check{p} \text{ markless}}{\check{p} : \tau \text{ markless}}$$

C.6 Pattern mark erasure

$\boxed{\check{p}^{\square}}$ is a metafunction MPat → UPat defined as follows:

$$\begin{aligned}
_^{\square} &= _ \\
x^{\square} &= x \\
(\check{p}_1, \check{p}_2)^{\square} &= (\check{p}_1^{\square}, \check{p}_2^{\square}) \\
(\!|(\check{p}_1, \check{p}_2)|\!)^{\Leftarrow\,\square}_{\not\blacktriangleright_{\times}} &= (\check{p}_1^{\square}, \check{p}_2^{\square}) \\
(\check{p} : \tau)^{\square} &= (\check{p}^{\square}) : \tau \\
(\!|\check{p} : \tau|\!)_{\nsim}^{\square} &= (\check{p}^{\square}) : \tau
\end{aligned}$$

C.7 Unmarked expressions

$\boxed{\Gamma \vdash_U e \Rightarrow \tau}$ e synthesizes type τ

USLetPat
$$\frac{\Gamma \vdash_U e_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_U p \Leftarrow \tau_1 \dashv \Gamma' \qquad \Gamma' \vdash_U e_2 \Rightarrow \tau_2}{\Gamma \vdash_U \mathsf{let}\ p = e_1\ \mathsf{in}\ e_2 \Rightarrow \tau_2}$$

$\boxed{\Gamma \vdash_U e \Leftarrow \tau}$ e analyzes against type τ

UASynSwitch
$$\frac{\Gamma \vdash_U e \Rightarrow \tau}{\Gamma \vdash_U e \Leftarrow ?^{\Rightarrow}}$$

UALetPat
$$\frac{\Gamma \vdash_U e_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_U p \Leftarrow \tau_1 \dashv \Gamma' \qquad \Gamma' \vdash_U e_2 \Leftarrow \tau_2}{\Gamma \vdash_U \mathsf{let}\ p = e_1\ \mathsf{in}\ e_2 \Leftarrow \tau_2}$$

$\boxed{e \text{ subsumable}}$ e is subsumable

USuLetPat
$$\frac{}{\mathsf{let}\ p = e_1\ \mathsf{in}\ e_2 \text{ subsumable}}$$

C.8 Marking

$\boxed{\Gamma \vdash e \looparrowright \check{e} \Rightarrow \tau}$ e is marked into $\check{e}$ and synthesizes type τ

$$\textsc{MKSLetPat}\quad \dfrac{\begin{array}{c}\Gamma \vdash p \looparrowright \check{p} \Rightarrow \tau_p \qquad \Gamma \vdash e_1 \looparrowright \check{e}_1 \Leftarrow \tau_p \\ \Gamma \vdash_U e_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_U p \Leftarrow \tau_1 \dashv \Gamma' \qquad \Gamma' \vdash e_2 \looparrowright \check{e}_2 \Rightarrow \tau_2\end{array}}{\Gamma \vdash \mathsf{let}\ p = e_1\ \mathsf{in}\ e_2 \looparrowright \mathsf{let}\ \check{p} = \check{e}_1\ \mathsf{in}\ \check{e}_2 \Rightarrow \tau_2}$$

$\boxed{\Gamma \vdash e \looparrowright \check{e} \Leftarrow \tau}$ e is marked into $\check{e}$ and analyzes against type τ

$$\textsc{MKALetPat}\quad \dfrac{\begin{array}{c}\Gamma \vdash p \looparrowright \check{p} \Rightarrow \tau_p \qquad \Gamma \vdash e_1 \looparrowright \check{e}_1 \Leftarrow \tau_p \\ \Gamma \vdash_U e_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_U p \Leftarrow \tau_1 \dashv \Gamma' \qquad \Gamma' \vdash e_2 \looparrowright \check{e}_2 \Leftarrow \tau_2\end{array}}{\Gamma \vdash \mathsf{let}\ p = e_1\ \mathsf{in}\ e_2 \looparrowright \mathsf{let}\ \check{p} = \check{e}_1\ \mathsf{in}\ \check{e}_2 \Leftarrow \tau_2}$$

C.9 Marked expressions

$\boxed{\Gamma \vdash_M \check{e} \Rightarrow \tau}$ $\check{e}$ synthesizes type τ

$$\textsc{MSLetPat}\quad \dfrac{\begin{array}{c}\Gamma \vdash_M \check{p} \Rightarrow \tau_p \qquad \Gamma \vdash_M \check{e}_1 \Leftarrow \tau_p \qquad \Gamma \vdash_M \check{e}_1 \Rightarrow \tau_1 \\ \Gamma \vdash_M \check{p} \Leftarrow \tau_1 \dashv \Gamma' \qquad \Gamma' \vdash_M \check{e}_2 \Rightarrow \tau_2\end{array}}{\Gamma \vdash_M \mathsf{let}\ \check{p} = \check{e}_1\ \mathsf{in}\ \check{e}_2 \Rightarrow \tau_2}$$

$\boxed{\Gamma \vdash_M \check{e} \Leftarrow \tau}$ $\check{e}$ analyzes against type τ

$$\textsc{MALetPat}\quad \dfrac{\begin{array}{c}\Gamma \vdash_M \check{p} \Rightarrow \tau_p \qquad \Gamma \vdash_M \check{e}_1 \Leftarrow \tau_p \qquad \Gamma \vdash_M \check{e}_1 \Rightarrow \tau_1 \\ \Gamma \vdash_M \check{p} \Leftarrow \tau_1 \dashv \Gamma' \qquad \Gamma' \vdash_M \check{e}_2 \Leftarrow \tau_2\end{array}}{\Gamma \vdash_M \mathsf{let}\ \check{p} = \check{e}_1\ \mathsf{in}\ \check{e}_2 \Leftarrow \tau_2}$$

$\boxed{\check{e}\ \mathsf{subsumable}}$ $\check{e}$ is subsumable

$$\textsc{MSuLetPat}\quad \dfrac{}{\mathsf{let}\ p = \check{e}_1\ \mathsf{in}\ \check{e}_2\ \mathsf{subsumable}}$$

$\boxed{\check{e}\ \mathsf{markless}}$ $\check{e}$ has no marks

$$\textsc{MLLetPat}\quad \dfrac{\check{p}\ \mathsf{markless} \qquad \check{e}_1\ \mathsf{markless} \qquad \check{e}_2\ \mathsf{markless}}{\mathsf{let}\ \check{p} = \check{e}_1\ \mathsf{in}\ \check{e}_2\ \mathsf{markless}}$$

C.10 Mark erasure

$\boxed{\check{e}^{\square}}$ is a metafunction MExp $\to$ UExp defined as follows:

$$\begin{array}{rcl} & \vdots & \\ (\mathsf{let}\ \check{p} = \check{e}_1\ \mathsf{in}\ \check{e}_2)^{\square} & = & \mathsf{let}\ (\check{p}^{\square}) = (\check{e}_1^{\square})\ \mathsf{in}\ (\check{e}_2^{\square}) \end{array}$$

C.11 Metatheorems

In addition to the original metatheorems above (see Section B.7), the following ones governing patterns additionally hold.

Theorem C.1 (Pattern Marking Totality).

1. *For all* Γ *and* p, *there exist* $\check{p}$ *and* τ *such that* $\Gamma \vdash p \looparrowright \check{p} \Rightarrow \tau$.
2. *For all* Γ, p, *and* τ, *there exists* $\check{p}$ *and* Γ' *such that* $\Gamma \vdash p \looparrowright \check{p} \Leftarrow \tau \dashv \Gamma'$.

Theorem C.2 (Pattern Marking Well-Formedness).

1. *If* $\Gamma \vdash p \looparrowright \check{p} \Rightarrow \tau$, *then* $\Gamma \vdash_{M} \check{p} \Rightarrow \tau$ *and* $\check{p}^{\square} = p$.
2. *If* $\Gamma \vdash p \looparrowright \check{p} \Leftarrow \tau \dashv \Gamma'$, *then* $\Gamma \vdash_{M} \check{p} \Leftarrow \tau \dashv \Gamma'$ *and* $\check{p}^{\square} = p$.

Theorem C.3 (Pattern Marking of Well-Typed/Ill-Typed Patterns).

1. (a) *If* $\Gamma \vdash_{U} p \Rightarrow \tau$ *and* $\Gamma \vdash p \looparrowright \check{p} \Rightarrow \tau$, *then* $\check{p}$ markless.
   (b) *If* $\Gamma \vdash_{U} p \Leftarrow \tau \dashv \Gamma'$ *and* $\Gamma \vdash p \looparrowright \check{p} \Leftarrow \tau \dashv \Gamma'$, *then* $\check{p}$ markless.
2. (a) *If there does not exist* τ *such that* $\Gamma \vdash_{U} p \Rightarrow \tau$, *then for all* $\check{p}$ *and* τ' *such that* $\Gamma \vdash p \looparrowright \check{p} \Rightarrow \tau'$, *it is not the case that* $\check{p}$ markless.
   (b) *If there does not exist* τ *and* Γ' *such that* $\Gamma \vdash_{U} p \Leftarrow \tau \dashv \Gamma'$, *then for all* $\check{p}, \tau'$, *and* Γ' *such that* $\Gamma \vdash p \looparrowright \check{p} \Leftarrow \tau' \dashv \Gamma'$, *it is not the case that* $\check{p}$ markless.

Theorem C.4 (Pattern Marking Unicity).

1. *If* $\Gamma \vdash p \looparrowright \check{p}_1 \Rightarrow \tau_1$ *and* $\Gamma \vdash p \looparrowright \check{p}_2 \Rightarrow \tau_2$, *then* $\check{p}_1 = \check{p}_2$ *and* $\tau_1 = \tau_2$.
2. *If* $\Gamma \vdash p \looparrowright \check{p}_1 \Leftarrow \tau \dashv \Gamma_1$ *and* $\Gamma \vdash p \looparrowright \check{p}_2 \Leftarrow \tau \dashv \Gamma_2$, *then* $\check{p}_1 = \check{p}_2$ *and* $\Gamma_1 = \Gamma_2$.

D Extension: System F-style polymorphism

In this section, we describe an extension of the marked lambda calculus for System F-style parametric polymorphism, as sketched out in Section 2.4 of the paper.

MECHANIZATION ×

D.1 Syntax

$$
\begin{array}{rclll}
\mathsf{Type} & \tau & ::= & \cdots \mid \forall\alpha.\,\tau \mid \alpha \\
\mathsf{MType} & \check{\tau} & ::= & \cdots \mid \forall\alpha.\,\check{\tau} \mid \alpha \mid (\!|\alpha|\!)_{\square} \\
\mathsf{UExp} & e & ::= & \cdots \mid \Lambda\alpha.\,e \mid e\,[\tau] \\
\mathsf{MExp} & \check{e} & ::= & \cdots \mid \Lambda\alpha.\,\check{e} \mid \check{e}\,[\check{\tau}] \\
& & \mid & (\!|\Lambda\alpha.\,\check{e}|\!)^{\Leftarrow}_{\blacktriangleright_\forall} \mid (\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_\forall}\,[\check{\tau}]
\end{array}
$$

D.2 Unmarked types

$\boxed{\Sigma \vdash_U \tau_1 \sim \tau_2}$ τ_1 and τ_2 are consistent

$$
\cdots \qquad
\frac{\Sigma, \alpha \vdash_U \tau \sim \tau'}{\Sigma \vdash_U \forall\alpha.\,\tau \sim \forall\alpha.\,\tau'}\;\textsc{TCForall}
\qquad
\frac{\alpha \in \Sigma}{\Sigma \vdash_U \alpha \sim \alpha}\;\textsc{TCVar}
$$

$\boxed{\Sigma \vdash_U \tau}$ τ is well-formed

$$
\frac{}{\Sigma \vdash_U ?}\;\textsc{TWFUnknown}
\qquad
\frac{}{\Sigma \vdash_U \mathsf{num}}\;\textsc{TWFNum}
\qquad
\frac{}{\Sigma \vdash_U \mathsf{bool}}\;\textsc{TWFBool}
\qquad
\frac{\Sigma \vdash_U \check{\tau}_1 \qquad \Sigma \vdash_U \check{\tau}_2}{\Sigma \vdash_U \check{\tau}_1 \to \check{\tau}_2}\;\textsc{TWFArr}
$$

$$
\frac{\Sigma \vdash_U \check{\tau}_1 \qquad \Sigma \vdash_U \check{\tau}_2}{\Sigma \vdash_U \check{\tau}_1 \times \check{\tau}_2}\;\textsc{TWFProd}
\qquad
\frac{\Sigma, \alpha \vdash_U \check{\tau}}{\Sigma \vdash_U \forall\alpha.\,\check{\tau}}\;\textsc{TWFForall}
\qquad
\frac{\alpha \in \Sigma}{\Sigma \vdash_U \alpha}\;\textsc{TWFVar}
$$

$\boxed{\tau \blacktriangleright_\forall \forall\alpha.\,\tau'}$ τ has matched forall type $\forall\alpha.\,\tau'$

$$
\frac{}{? \blacktriangleright_\forall \forall\alpha.\,?}\;\textsc{TMFUnknown}
\qquad
\frac{}{\forall\alpha.\,\tau \blacktriangleright_\forall \forall\alpha.\,\tau}\;\textsc{TMFForall}
$$

$\boxed{\tau_1 \sqcup \tau_2}$ is a *partial* metafunction $\mathsf{Type} \times \mathsf{Type} \rightharpoonup \mathsf{Type}$ defined as follows:

$$
\begin{array}{rcl}
& \vdots & \\
(\forall\alpha.\,\tau) \sqcup (\forall\alpha.\,\tau') & = & \forall\alpha.\,(\tau \sqcup \tau') \\
\alpha \sqcup \alpha & = & \alpha
\end{array}
$$

$\boxed{\tau_1[\tau_2/\alpha]}$ is a metafunction $\mathsf{Type} \times \mathsf{Type} \times \mathsf{TypeVar} \to \mathsf{Type}$ defined as follows:

$$
\begin{array}{rcll}
?[\tau/\alpha] & = & ? & \\
\mathsf{num}[\tau/\alpha] & = & \mathsf{num} & \\
\mathsf{bool}[\tau/\alpha] & = & \mathsf{bool} & \\
(\tau_1 \to \tau_2)[\tau/\alpha] & = & (\tau_1[\tau/\alpha]) \to (\tau_2[\tau/\alpha]) & \\
(\tau_1 \times \tau_2)[\tau/\alpha] & = & (\tau_1[\tau/\alpha]) \times (\tau_2[\tau/\alpha]) & \\
(\forall\alpha'.\,\tau')[\tau/\alpha] & = & \forall\alpha'.\,\tau' & \alpha = \alpha' \\
(\forall\alpha'.\,\tau')[\tau/\alpha] & = & \forall\alpha'.\,(\tau'[\tau/\alpha]) & \alpha \neq \alpha' \\
\alpha'[\tau/\alpha] & = & \tau & \alpha = \alpha' \\
\alpha'[\tau/\alpha] & = & \alpha' & \alpha \neq \alpha'
\end{array}
$$

D.3 Type marking

$\boxed{\Sigma \vdash \tau \looparrowright \check{\tau}}$ τ is marked into $\check{\tau}$

$$\textsc{MKTUnknown}\ \frac{}{\Sigma \vdash ? \looparrowright ?} \qquad \textsc{MKTNum}\ \frac{}{\Sigma \vdash \mathsf{num} \looparrowright \mathsf{num}} \qquad \textsc{MKTBool}\ \frac{}{\Sigma \vdash \mathsf{bool} \looparrowright \mathsf{bool}}$$

$$\textsc{MKTArr}\ \frac{\Sigma \vdash \tau_1 \looparrowright \check{\tau}_1 \qquad \Sigma \vdash \tau_2 \looparrowright \check{\tau}_2}{\Sigma \vdash \tau_1 \to \tau_2 \looparrowright \check{\tau}_1 \to \check{\tau}_2}$$

$$\textsc{MKTProd}\ \frac{\Sigma \vdash \tau_1 \looparrowright \check{\tau}_1 \qquad \Sigma \vdash \tau_2 \looparrowright \check{\tau}_2}{\Sigma \vdash \tau_1 \times \tau_2 \looparrowright \check{\tau}_1 \times \check{\tau}_2} \qquad \textsc{MKTForall}\ \frac{\Sigma, \alpha \vdash \tau \looparrowright \check{\tau}}{\Sigma \vdash \forall\alpha.\, \tau \looparrowright \forall\alpha.\, \check{\tau}}$$

$$\textsc{MKTVar}\ \frac{\alpha \in \Sigma}{\Sigma \vdash \alpha \looparrowright \alpha} \qquad \textsc{MKTFree}\ \frac{\alpha \notin \Sigma}{\Sigma \vdash \alpha \looparrowright (\!|\alpha|\!)_{\square}}$$

D.4 Marked types

$\boxed{\Sigma \vdash_{M} \check{\tau}_1 \sim \check{\tau}_2}$ $\check{\tau}_1$ and $\check{\tau}_2$ are consistent

$$\ldots \qquad \textsc{MTCForall}\ \frac{\Sigma, \alpha \vdash_{M} \check{\tau} \sim \check{\tau}'}{\Sigma \vdash_{M} \forall\alpha.\, \check{\tau} \sim \forall\alpha.\, \check{\tau}'} \qquad \textsc{MTCVar}\ \frac{\alpha \in \Sigma}{\Sigma \vdash_{M} \alpha \sim \alpha}$$

$$\textsc{MTCFree1}\ \frac{\alpha \notin \Sigma}{\Sigma \vdash_{M} (\!|\alpha|\!)_{\square} \sim \check{\tau}} \qquad \textsc{MTCFree2}\ \frac{\alpha \notin \Sigma}{\Sigma \vdash_{M} \check{\tau} \sim (\!|\alpha|\!)_{\square}}$$

$\boxed{\Sigma \vdash_{M} \check{\tau}}$ $\check{\tau}$ is well-formed

$$\textsc{MTWFUnknown}\ \frac{}{\Sigma \vdash_{M} ?} \qquad \textsc{MTWFNum}\ \frac{}{\Sigma \vdash_{M} \mathsf{num}} \qquad \textsc{MTWFBool}\ \frac{}{\Sigma \vdash_{M} \mathsf{bool}}$$

$$\textsc{MTWFArr}\ \frac{\Sigma \vdash_{M} \check{\tau}_1 \qquad \Sigma \vdash_{M} \check{\tau}_2}{\Sigma \vdash_{M} \check{\tau}_1 \to \check{\tau}_2} \qquad \textsc{MTWFProd}\ \frac{\Sigma \vdash_{M} \check{\tau}_1 \qquad \Sigma \vdash_{M} \check{\tau}_2}{\Sigma \vdash_{M} \check{\tau}_1 \times \check{\tau}_2}$$

$$\textsc{MTWFForall}\ \frac{\Sigma, \alpha \vdash_{M} \check{\tau}}{\Sigma \vdash_{M} \forall\alpha.\, \check{\tau}} \qquad \textsc{MTWFVar}\ \frac{\alpha \in \Sigma}{\Sigma \vdash_{M} \alpha} \qquad \textsc{MTWFFree}\ \frac{\alpha \notin \Sigma}{\Sigma \vdash_{M} (\!|\alpha|\!)_{\square}}$$

$\boxed{\check{\tau} \blacktriangleright_{\forall} \forall\alpha.\, \check{\tau}'}$ $\check{\tau}$ has matched forall type $\forall\alpha.\, \check{\tau}'$

$$\textsc{MTMFUnknown}\ \frac{}{? \blacktriangleright_{\forall} \forall\alpha.\ ?} \qquad \textsc{MTMFForall}\ \frac{}{\forall\alpha.\, \check{\tau} \blacktriangleright_{\forall} \forall\alpha.\, \check{\tau}} \qquad \textsc{MTMFFree}\ \frac{}{(\!|\alpha|\!)_{\square} \blacktriangleright_{\forall} \forall\alpha.\ ?}$$

$\boxed{\check{\tau}_1 \sqcup \check{\tau}_2}$ is a *partial* metafunction MType × MType ⇀ MType defined as follows:

$$\begin{array}{rcl} & \vdots & \\ (\forall\alpha.\, \check{\tau}) \sqcup (\forall\alpha.\, \check{\tau}') & = & \forall\alpha.\, (\check{\tau} \sqcup \check{\tau}') \\ \alpha \sqcup \alpha & = & \alpha \\ (\!|\alpha|\!)_{\square} \sqcup \check{\tau} & = & \check{\tau} \\ \check{\tau} \sqcup (\!|\alpha|\!)_{\square} & = & \check{\tau} \end{array}$$

$\boxed{\check\tau_1[\check\tau_2/\alpha]}$ is a metafunction MType × MType × MTypeVar → MType defined as follows:

$$
\begin{array}{rcll}
?[\check\tau/\alpha] & = & ? & \\
\mathsf{num}[\check\tau/\alpha] & = & \mathsf{num} & \\
\mathsf{bool}[\check\tau/\alpha] & = & \mathsf{bool} & \\
(\check\tau_1 \to \check\tau_2)[\check\tau/\alpha] & = & (\check\tau_1[\check\tau/\alpha]) \to (\check\tau_2[\check\tau/\alpha]) & \\
(\check\tau_1 \times \check\tau_2)[\check\tau/\alpha] & = & (\check\tau_1[\check\tau/\alpha]) \times (\check\tau_2[\check\tau/\alpha]) & \\
(\forall\alpha'.\,\check\tau')[\check\tau/\alpha] & = & \forall\alpha'.\,\check\tau' & \alpha = \alpha' \\
(\forall\alpha'.\,\check\tau')[\check\tau/\alpha] & = & \forall\alpha'.\,(\check\tau'[\check\tau/\alpha]) & \alpha \neq \alpha' \\
\alpha'[\check\tau/\alpha] & = & \check\tau & \alpha = \alpha' \\
\alpha'[\check\tau/\alpha] & = & \alpha' & \alpha \neq \alpha' \\
\llparenthesis\alpha'\rrparenthesis_{\square}[\check\tau/\alpha] & = & \llparenthesis\alpha'\rrparenthesis_{\square} &
\end{array}
$$

$\boxed{\check\tau\ \mathsf{markless}}$ $\check\tau$ has no marks

$$
\textsc{MLTUnknown}\ \frac{}{?\ \mathsf{markless}}
\qquad
\textsc{MLTNum}\ \frac{}{\mathsf{num}\ \mathsf{markless}}
\qquad
\textsc{MLTBool}\ \frac{}{\mathsf{bool}\ \mathsf{markless}}
\qquad
\textsc{MLTArr}\ \frac{\check\tau_1\ \mathsf{markless} \qquad \check\tau_2\ \mathsf{markless}}{\check\tau_1 \to \check\tau_2\ \mathsf{markless}}
$$

$$
\textsc{MLTProd}\ \frac{\check\tau_1\ \mathsf{markless} \qquad \check\tau_2\ \mathsf{markless}}{\check\tau_1 \times \check\tau_2\ \mathsf{markless}}
\qquad
\textsc{MLTForall}\ \frac{\check\tau\ \mathsf{markless}}{\forall\alpha.\,\check\tau\ \mathsf{markless}}
\qquad
\textsc{MLTVar}\ \frac{}{\alpha\ \mathsf{markless}}
$$

D.5 Type mark erasure

$\boxed{\check\tau^{\square}}$ is a metafunction MType → Type defined as follows:

$$
\begin{array}{rcl}
?^{\square} & = & ? \\
\mathsf{num}^{\square} & = & \mathsf{num} \\
\mathsf{bool}^{\square} & = & \mathsf{bool} \\
(\check\tau_1 \to \check\tau_2)^{\square} & = & (\check\tau_1^{\square}) \to (\check\tau_2^{\square}) \\
(\check\tau_1 \times \check\tau_2)^{\square} & = & (\check\tau_1^{\square}) \times (\check\tau_2^{\square}) \\
(\forall\alpha.\,\check\tau)^{\square} & = & \forall\alpha.\,(\check\tau^{\square}) \\
\alpha^{\square} & = & \alpha \\
\llparenthesis\alpha\rrparenthesis_{\square}^{\ \square} & = & \alpha
\end{array}
$$

D.6 Unmarked expressions

$\boxed{\Sigma;\Gamma \vdash_{\overline{U}} e \Rightarrow \tau}$ e synthesizes type τ

$$
\ldots
\qquad
\textsc{USTypeLam}\ \frac{\Sigma,\alpha;\Gamma \vdash_{\overline{U}} e \Rightarrow \tau}{\Sigma;\Gamma \vdash_{\overline{U}} \Lambda\alpha.\,e \Rightarrow \forall\alpha.\,\tau}
\qquad
\textsc{USTypeAp}\ \frac{\Sigma;\Gamma \vdash_{\overline{U}} e \Rightarrow \tau \qquad \Sigma \vdash_{\overline{U}} \tau_2 \qquad \tau \blacktriangleright_{\forall} \forall\alpha.\,\tau_1}{\Sigma;\Gamma \vdash_{\overline{U}} e\ [\tau_2] \Rightarrow \tau_1[\tau_2/\alpha]}
$$

$\boxed{\Sigma;\Gamma \vdash_{\overline{U}} e \Leftarrow \tau}$ e analyzes against type τ

$$
\ldots
\qquad
\textsc{UATypeLam}\ \frac{\tau \blacktriangleright_{\forall} \forall\alpha.\,\tau' \qquad \Sigma,\alpha;\Gamma \vdash_{\overline{U}} e \Leftarrow \tau'}{\Sigma;\Gamma \vdash_{\overline{U}} \Lambda\alpha.\,e \Leftarrow \tau}
$$

$\boxed{e\ \mathsf{subsumable}}$ e is subsumable

$$
\ldots
\qquad
\textsc{USuTypeAp}\ \frac{}{e\ [\tau]\ \mathsf{subsumable}}
$$

D.7 Marking

$\boxed{\Sigma; \Gamma \vdash e \looparrowright \check{e} \Rightarrow \check{\tau}}$ e is marked into $\check{e}$ and synthesizes type $\check{\tau}$

$$\cdots \qquad \overset{\textsc{MKSTypeLam}}{\dfrac{\Sigma, \alpha; \Gamma \vdash e \looparrowright \check{e} \Rightarrow \check{\tau}}{\Sigma; \Gamma \vdash \Lambda\alpha.\, e \looparrowright \Lambda\alpha.\, \check{e} \Rightarrow \forall\alpha.\, \check{\tau}}} \qquad \overset{\textsc{MKSTypeAp1}}{\dfrac{\Sigma; \Gamma \vdash e \looparrowright \check{e} \Rightarrow \check{\tau} \qquad \Sigma \vdash \tau_2 \looparrowright \check{\tau}_2 \qquad \check{\tau} \blacktriangleright_{\forall} \forall\alpha.\, \check{\tau}_1}{\Sigma; \Gamma \vdash e\, [\tau_2] \looparrowright \check{e}\, [\check{\tau}_2] \Rightarrow \check{\tau}_1[\check{\tau}_2/\alpha]}}$$

$$\overset{\textsc{MKSTypeAp2}}{\dfrac{\Sigma; \Gamma \vdash e \looparrowright \check{e} \Rightarrow \check{\tau} \qquad \Sigma \vdash \tau_2 \looparrowright \check{\tau}_2 \qquad \check{\tau} \blacktriangleright_{\not\forall}}{\Sigma; \Gamma \vdash e\, [\tau_2] \looparrowright \check{e}\, [\check{\tau}_2] \Rightarrow ?}}$$

$\boxed{\Sigma; \Gamma \vdash e \looparrowright \check{e} \Leftarrow \check{\tau}}$ e is marked into $\check{e}$ and analyzes against type $\check{\tau}$

$$\cdots \qquad \overset{\textsc{MKATypeLam1}}{\dfrac{\check{\tau} \blacktriangleright_{\forall} \forall\alpha.\, \check{\tau}' \qquad \Sigma, \alpha; \Gamma \vdash e \looparrowright \check{e} \Leftarrow \check{\tau}'}{\Sigma; \Gamma \vdash \Lambda\alpha.\, e \looparrowright \Lambda\alpha.\, \check{e} \Leftarrow \check{\tau}}} \qquad \overset{\textsc{MKATypeLam2}}{\dfrac{\check{\tau} \blacktriangleright_{\not\forall} \qquad \Sigma, \alpha; \Gamma \vdash e \looparrowright \check{e} \Leftarrow ?}{\Sigma; \Gamma \vdash \Lambda\alpha.\, e \looparrowright (\!|\Lambda\alpha.\, \check{e}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\forall}} \Leftarrow \check{\tau}}}$$

D.8 Marked expressions

$\boxed{\Sigma; \Gamma \vdash_{M} \check{e} \Rightarrow \check{\tau}}$ $\check{e}$ synthesizes type $\check{\tau}$

$$\cdots \qquad \overset{\textsc{MSTypeLam}}{\dfrac{\Sigma, \alpha; \Gamma \vdash_{M} \check{e} \Rightarrow \check{\tau}}{\Sigma; \Gamma \vdash_{M} \Lambda\alpha.\, \check{e} \Rightarrow \forall\alpha.\, \check{\tau}}} \qquad \overset{\textsc{MSTypeAp1}}{\dfrac{\Sigma; \Gamma \vdash_{M} \check{e} \Rightarrow \check{\tau} \qquad \Sigma \vdash_{M} \check{\tau}_2 \qquad \check{\tau} \blacktriangleright_{\forall} \forall\alpha.\, \check{\tau}_1}{\Sigma; \Gamma \vdash_{M} \check{e}\, [\check{\tau}_2] \Rightarrow \check{\tau}_1[\check{\tau}_2/\alpha]}} \qquad \overset{\textsc{MSTypeAp2}}{\dfrac{\Sigma; \Gamma \vdash_{M} \check{e} \Rightarrow \check{\tau} \qquad \Sigma \vdash_{M} \check{\tau}_2 \qquad \check{\tau} \blacktriangleright_{\not\forall}}{\Sigma; \Gamma \vdash_{M} (\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\forall}}\, [\check{\tau}_2] \Rightarrow ?}}$$

$\boxed{\Sigma; \Gamma \vdash_{M} \check{e} \Leftarrow \check{\tau}}$ $\check{e}$ analyzes against type $\check{\tau}$

$$\cdots \qquad \overset{\textsc{MATypeLam1}}{\dfrac{\check{\tau} \blacktriangleright_{\forall} \forall\alpha.\, \check{\tau}' \qquad \Sigma, \alpha; \Gamma \vdash_{M} \check{e} \Leftarrow \check{\tau}'}{\Sigma; \Gamma \vdash_{M} \Lambda\alpha.\, \check{e} \Leftarrow \check{\tau}}} \qquad \overset{\textsc{MATypeLam2}}{\dfrac{\check{\tau} \blacktriangleright_{\not\forall} \qquad \Sigma, \alpha; \Gamma \vdash_{M} \check{e} \Leftarrow ?}{\Sigma; \Gamma \vdash_{M} (\!|\Lambda\alpha.\, \check{e}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\forall}} \Leftarrow \check{\tau}}}$$

$\boxed{\check{e} \text{ subsumable}}$ $\check{e}$ is subsumable

$$\cdots \qquad \overset{\textsc{MSuTypeAp1}}{\dfrac{}{\check{e}\, [\check{\tau}] \text{ subsumable}}} \qquad \overset{\textsc{MSuTypeAp2}}{\dfrac{}{(\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\forall}}\, [\check{\tau}] \text{ subsumable}}}$$

$\boxed{\check{e} \text{ markless}}$ $\check{e}$ has no marks

$$\cdots \qquad \overset{\textsc{MLTypeLam}}{\dfrac{\check{e} \text{ markless}}{\Lambda\alpha.\, \check{e} \text{ markless}}} \qquad \overset{\textsc{MLTypeAp}}{\dfrac{\check{e} \text{ markless} \qquad \check{\tau} \text{ markless}}{\check{e}\, [\check{\tau}] \text{ markless}}}$$

D.9 Mark erasure

$\boxed{\check{e}^{\square}}$ is a metafunction MExp $\to$ UExp defined as follows:

$$\begin{array}{rcl} & \vdots & \\ (\Lambda\alpha.\, \check{e})^{\square} & = & \Lambda\alpha.\, (\check{e}^{\square}) \\ (\!|\Lambda\alpha.\, \check{e}|\!)^{\Leftarrow\,\square}_{\blacktriangleright_{\not\forall}} & = & \Lambda\alpha.\, (\check{e}^{\square}) \\ (\check{e}\, [\check{\tau}])^{\square} & = & \check{e}^{\square}\, [\check{\tau}^{\square}] \\ ((\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\forall}}\, [\check{\tau}])^{\square} & = & \check{e}^{\square}\, [\check{\tau}^{\square}] \end{array}$$

D.10 Metatheorems

With polymorphism, we have the following modified metatheorems which additionally account for type well-formedness and marking.

Lemma D.1 (Unmarked Synthesis). *If* $\Sigma; \Gamma \vdash_U e \Rightarrow \tau$ *, then* $\Sigma \vdash_U \tau$ *.*

Lemma D.2 (Marked Synthesis). *If* $\Sigma; \Gamma \vdash_M \check{e} \Rightarrow \check{\tau}$ *, then* $\Sigma \vdash_M \check{\tau}$ *.*

Theorem D.3 (Marking Totality).

1. *For all* Σ *and* τ*, there exists* $\check{\tau}$ *such that* $\Sigma \vdash \tau \looparrowright \check{\tau}$ *.*
2. *For all* Σ*,* Γ*, and* e*, there exist* $\check{e}$ *and* $\check{\tau}$ *such that* $\Sigma; \Gamma \vdash e \looparrowright \check{e} \Rightarrow \check{\tau}$ *.*
3. *For all* Σ*,* Γ*,* e*, and* $\check{\tau}$ *such that* $\Sigma \vdash_M \check{\tau}$ *, there exists* $\check{e}$ *such that* $\Sigma; \Gamma \vdash e \looparrowright \check{e} \Leftarrow \check{\tau}$ *.*

Theorem D.4 (Marking Well-Formedness).

1. *If* $\Sigma \vdash \tau \looparrowright \check{\tau}$ *, then* $\Sigma \vdash_M \check{\tau}$ *and* $\check{\tau}^{\square} = \tau$*.*
2. *If* $\Sigma; \Gamma \vdash e \looparrowright \check{e} \Rightarrow \check{\tau}$ *, then* $\Sigma \vdash_M \check{\tau}$ *and* $\Sigma; \Gamma \vdash_M \check{e} \Rightarrow \check{\tau}$ *and* $\check{e}^{\square} = e$*.*
3. *If* $\Sigma; \Gamma \vdash e \looparrowright \check{e} \Leftarrow \check{\tau}$ *and* $\Sigma \vdash_M \check{\tau}$ *, then* $\Sigma; \Gamma \vdash_M \check{e} \Leftarrow \check{\tau}$ *and* $\check{e}^{\square} = e$*.*

Theorem D.5 (Marking of Well-Typed/Ill-Typed Expressions).

1. (a) *If* $\Sigma \vdash_U \tau$ *and* $\Sigma \vdash \tau \looparrowright \check{\tau}$ *, then* $\check{\tau}$ markless.
   (b) *If* $\Sigma; \Gamma \vdash_U e \Rightarrow \tau$ *and* $\Sigma; \Gamma \vdash e \looparrowright \check{e} \Rightarrow \check{\tau}$ *, then* $\Sigma \vdash \tau \looparrowright \check{\tau}$ *and* $\check{e}$ markless.
   (c) *If* $\Sigma; \Gamma \vdash_U e \Leftarrow \tau$ *and* $\Sigma \vdash \tau \looparrowright \check{\tau}$ *and* $\Sigma; \Gamma \vdash e \looparrowright \check{e} \Leftarrow \check{\tau}$ *, then* $\check{e}$ markless.
2. (a) *If it is not the case that* $\Sigma \vdash_U \tau$ *, then for all* $\check{\tau}$ *such that* $\Sigma \vdash \tau \looparrowright \check{\tau}$ *, it is not the case that* $\check{\tau}$ markless.
   (b) *If there does not exist* τ *such that* $\Sigma; \Gamma \vdash_U e \Rightarrow \tau$ *, then for all* $\check{e}$ *and* $\check{\tau}$ *such that* $\Sigma; \Gamma \vdash e \looparrowright \check{e} \Rightarrow \check{\tau}$ *, it is not the case that* $\check{e}$ markless.
   (c) *If there does not exist* τ *such that* $\Sigma; \Gamma \vdash_U e \Leftarrow \tau$ *, then for all* $\check{e}$ *and* $\check{\tau}$ *such that* $\Sigma; \Gamma \vdash e \looparrowright \check{e} \Leftarrow \check{\tau}$ *, it is not the case that* $\check{e}$ markless.

Theorem D.6 (Marking Unicity).

1. *If* $\Sigma \vdash \tau \looparrowright \check{\tau}_1$ *, and* $\Sigma \vdash \tau \looparrowright \check{\tau}_2$ *, then* $\check{\tau}_1 = \check{\tau}_2$*.*
2. *If* $\Sigma; \Gamma \vdash e \looparrowright \check{e}_1 \Rightarrow \check{\tau}_1$ *and* $\Sigma; \Gamma \vdash e \looparrowright \check{e}_2 \Rightarrow \check{\tau}_2$ *, then* $\check{e}_1 = \check{e}_2$ *and* $\check{\tau}_1 = \check{\tau}_2$*.*
3. *If* $\Sigma; \Gamma \vdash e \looparrowright \check{e}_1 \Leftarrow \check{\tau}$ *and* $\Sigma; \Gamma \vdash e \looparrowright \check{e}_2 \Leftarrow \check{\tau}$ *, then* $\check{e}_1 = \check{e}_2$*.*

E Untyped hazelnut

In this section we describe an *untyped* version of the Hazelnut action calculus that might be layered with the marked lambda calculus to yield a structure editing calculus that supports non-local hole fixes. This is described in Section 3.2 of the paper.

Mechanization ○

- `hazelnut.agda`

E.1 Syntax

$$
\begin{array}{rrcl}
\text{ZType} & \underline{\tau} & ::= & \triangleright\tau\triangleleft \mid \underline{\tau} \to \tau \mid \tau \to \underline{\tau} \mid \underline{\tau} \times \tau \mid \tau \times \underline{\tau} \\
\text{ZExp} & \underline{e} & ::= & \triangleright e\triangleleft \mid \lambda x : \underline{\tau}.\, e \mid \lambda x : \tau.\, \underline{e} \mid \underline{e}\ e \mid e\ \underline{e} \\
 & & \mid & \mathsf{let}\ x = \underline{e}\ \mathsf{in}\ e \mid \mathsf{let}\ x = e\ \mathsf{in}\ \underline{e} \\
 & & \mid & \underline{e} + e \mid e + \underline{e} \\
 & & \mid & \mathsf{if}\ \underline{e}\ \mathsf{then}\ e\ \mathsf{else}\ e \mid \mathsf{if}\ e\ \mathsf{then}\ \underline{e}\ \mathsf{else}\ e \mid \mathsf{if}\ e\ \mathsf{then}\ e\ \mathsf{else}\ \underline{e} \\
 & & \mid & (\underline{e}, e) \mid (e, \underline{e}) \mid \pi_1\underline{e} \mid \pi_2\underline{e}
\end{array}
$$

E.2 Cursor erasure

E.2.1 Type cursor erasure

$\boxed{\underline{\tau}^\diamond}$ is a metafunction ZType → Type defined as follows:

$$
\begin{array}{rcl}
\triangleright\tau\triangleleft^\diamond & = & \tau \\
(\underline{\tau} \to \tau)^\diamond & = & (\underline{\tau}^\diamond) \to \tau \\
(\tau \to \underline{\tau})^\diamond & = & \tau \to (\underline{\tau}^\diamond) \\
(\underline{\tau} \times \tau)^\diamond & = & (\underline{\tau}^\diamond) \times \tau \\
(\tau \times \underline{\tau})^\diamond & = & \tau \times (\underline{\tau}^\diamond)
\end{array}
$$

E.2.2 Expression cursor erasure

$\boxed{\underline{e}^\diamond}$ is a metafunction ZExp → UExp defined as follows:

$$
\begin{array}{rcl}
\triangleright e\triangleleft^\diamond & = & e \\
(\lambda x : \underline{\tau}.\, e)^\diamond & = & \lambda x : (\underline{\tau}^\diamond).\, e \\
(\lambda x : \tau.\, \underline{e})^\diamond & = & \lambda x : \tau.\, (\underline{e}^\diamond) \\
(\underline{e}\ e)^\diamond & = & (\underline{e}^\diamond)\ e \\
(e\ \underline{e})^\diamond & = & e\ (\underline{e}^\diamond) \\
(\mathsf{let}\ x = \underline{e}\ \mathsf{in}\ e)^\diamond & = & \mathsf{let}\ x = (\underline{e}^\diamond)\ \mathsf{in}\ e \\
(\mathsf{let}\ x = e\ \mathsf{in}\ \underline{e})^\diamond & = & \mathsf{let}\ x = e\ \mathsf{in}\ (\underline{e}^\diamond) \\
(\underline{e} + e)^\diamond & = & (\underline{e}^\diamond) + e \\
(e + \underline{e})^\diamond & = & e + (\underline{e}^\diamond) \\
(\mathsf{if}\ \underline{e}\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2)^\diamond & = & \mathsf{if}\ (\underline{e}^\diamond)\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \\
(\mathsf{if}\ e_1\ \mathsf{then}\ \underline{e}\ \mathsf{else}\ e_2)^\diamond & = & \mathsf{if}\ e_1\ \mathsf{then}\ (\underline{e}^\diamond)\ \mathsf{else}\ e_2 \\
(\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ \underline{e})^\diamond & = & \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ (\underline{e}^\diamond) \\
(\underline{e}, e)^\diamond & = & (\underline{e}^\diamond, e) \\
(e, \underline{e})^\diamond & = & (e, \underline{e}^\diamond) \\
(\pi_1\underline{e})^\diamond & = & \pi_1(\underline{e}^\diamond) \\
(\pi_2\underline{e})^\diamond & = & \pi_2(\underline{e}^\diamond)
\end{array}
$$

E.3 Action model

$$
\begin{array}{rrcl}
\text{Action} & \alpha & ::= & \mathsf{move}\ \delta \mid \mathsf{construct}\ \psi \mid \mathsf{del} \\
\text{ActionList} & \overline{\alpha} & ::= & \cdot \mid \alpha;\overline{\alpha} \\
\text{Dir} & \delta & ::= & \mathsf{child}\ n \mid \mathsf{parent} \\
\text{Shape} & \psi & ::= & \mathsf{arrow}_\mathsf{L} \mid \mathsf{arrow}_\mathsf{R} \mid \mathsf{prod}_\mathsf{L} \mid \mathsf{prod}_\mathsf{R} \mid \mathsf{num} \mid \mathsf{bool} \\
 & & \mid & \mathsf{var}\ x \mid \mathsf{lam}\ x \mid \mathsf{ap}_\mathsf{L} \mid \mathsf{ap}_\mathsf{R} \\
 & & \mid & \mathsf{let}_\mathsf{L}\ x \mid \mathsf{let}_\mathsf{R}\ x \\
 & & \mid & \mathsf{lit}\ n \mid \mathsf{plus}_\mathsf{L} \mid \mathsf{plus}_\mathsf{R} \\
 & & \mid & \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}_\mathsf{C} \mid \mathsf{if}_\mathsf{L} \mid \mathsf{if}_\mathsf{R} \\
 & & \mid & \mathsf{pair}_\mathsf{L} \mid \mathsf{pair}_\mathsf{R} \mid \mathsf{proj}_\mathsf{L} \mid \mathsf{proj}_\mathsf{R}
\end{array}
$$

E.3.1 Shape sort

$\boxed{\psi\ \mathsf{tshape}}$ ψ is a shape on types

$$\textsc{ASortArrow1}\ \frac{}{\mathsf{arrow}_\mathsf{L}\ \mathsf{tshape}} \qquad \textsc{ASortArrow2}\ \frac{}{\mathsf{arrow}_\mathsf{R}\ \mathsf{tshape}} \qquad \textsc{ASortProd1}\ \frac{}{\mathsf{prod}_\mathsf{L}\ \mathsf{tshape}} \qquad \textsc{ASortProd2}\ \frac{}{\mathsf{prod}_\mathsf{R}\ \mathsf{tshape}} \qquad \textsc{ASortNum}\ \frac{}{\mathsf{num}\ \mathsf{tshape}} \qquad \textsc{ASortBool}\ \frac{}{\mathsf{bool}\ \mathsf{tshape}}$$

$\boxed{\psi\ \mathsf{eshape}}$ ψ is a shape on expressions

$$\textsc{ASortVar}\ \frac{}{\mathsf{var}\ x\ \mathsf{eshape}} \qquad \textsc{ASortLam}\ \frac{}{\mathsf{lam}\ x\ \mathsf{eshape}} \qquad \textsc{ASortAp1}\ \frac{}{\mathsf{ap}_\mathsf{L}\ \mathsf{eshape}} \qquad \textsc{ASortAp2}\ \frac{}{\mathsf{ap}_\mathsf{R}\ \mathsf{eshape}} \qquad \textsc{ASortLet1}\ \frac{}{\mathsf{let}_\mathsf{L}\ x\ \mathsf{eshape}} \qquad \textsc{ASortLet2}\ \frac{}{\mathsf{let}_\mathsf{R}\ x\ \mathsf{eshape}}$$

$$\textsc{ASortLit}\ \frac{}{\mathsf{lit}\ n\ \mathsf{eshape}} \qquad \textsc{ASortPlus1}\ \frac{}{\mathsf{plus}_\mathsf{L}\ \mathsf{eshape}} \qquad \textsc{ASortPlus2}\ \frac{}{\mathsf{plus}_\mathsf{R}\ \mathsf{eshape}} \qquad \textsc{ASortTrue}\ \frac{}{\mathsf{true}\ \mathsf{eshape}} \qquad \textsc{ASortFalse}\ \frac{}{\mathsf{false}\ \mathsf{eshape}} \qquad \textsc{ASortIf1}\ \frac{}{\mathsf{if}_\mathsf{C}\ \mathsf{eshape}} \qquad \textsc{ASortIf2}\ \frac{}{\mathsf{if}_\mathsf{L}\ \mathsf{eshape}}$$

$$\textsc{ASortIf3}\ \frac{}{\mathsf{if}_\mathsf{R}\ \mathsf{eshape}} \qquad \textsc{ASortPairL}\ \frac{}{\mathsf{pair}_\mathsf{L}\ \mathsf{eshape}} \qquad \textsc{ASortPairR}\ \frac{}{\mathsf{pair}_\mathsf{R}\ \mathsf{eshape}} \qquad \textsc{ASortProjL}\ \frac{}{\mathsf{proj}_\mathsf{L}\ \mathsf{eshape}} \qquad \textsc{ASortProjR}\ \frac{}{\mathsf{proj}_\mathsf{R}\ \mathsf{eshape}}$$

E.3.2 Type actions

$\boxed{\underline{\tau} \xrightarrow{\alpha} \underline{\tau}'}$

Movement

$$\textsc{ATMArrChild1}\ \frac{}{\triangleright\tau_1 \to \tau_2\triangleleft \xrightarrow{\mathsf{move\ child\ 1}} \triangleright\tau_1\triangleleft \to \tau_2} \qquad \textsc{ATMArrChild2}\ \frac{}{\triangleright\tau_1 \to \tau_2\triangleleft \xrightarrow{\mathsf{move\ child\ 2}} \tau_2 \to \triangleright\tau_1\triangleleft} \qquad \textsc{ATMArrParent1}\ \frac{}{\triangleright\tau_1\triangleleft \to \tau_2 \xrightarrow{\mathsf{move\ parent}} \triangleright\tau_1 \to \tau_2\triangleleft}$$

$$\textsc{ATMArrParent2}\ \frac{}{\tau_2 \to \triangleright\tau_1\triangleleft \xrightarrow{\mathsf{move\ parent}} \triangleright\tau_1 \to \tau_2\triangleleft} \qquad \textsc{ATMProdChild1}\ \frac{}{\triangleright\tau_1 \times \tau_2\triangleleft \xrightarrow{\mathsf{move\ child\ 1}} \triangleright\tau_1\triangleleft \times \tau_2} \qquad \textsc{ATMProdChild2}\ \frac{}{\triangleright\tau_1 \times \tau_2\triangleleft \xrightarrow{\mathsf{move\ child\ 2}} \tau_2 \times \triangleright\tau_1\triangleleft}$$

$$\textsc{ATMProdParent1}\ \frac{}{\triangleright\tau_1\triangleleft \times \tau_2 \xrightarrow{\mathsf{move\ parent}} \triangleright\tau_1 \times \tau_2\triangleleft} \qquad \textsc{ATMProdParent2}\ \frac{}{\tau_2 \times \triangleright\tau_1\triangleleft \xrightarrow{\mathsf{move\ parent}} \triangleright\tau_1 \times \tau_2\triangleleft}$$

Deletion

$$\textsc{ATDel}\ \frac{}{\triangleright\tau\triangleleft \xrightarrow{\mathsf{del}} \triangleright\,?\,\triangleleft}$$

Construction

ATConArrow1
$$\frac{}{\triangleright\tau\triangleleft \xrightarrow{\text{construct arrow}_\text{L}} \tau \to \triangleright ? \triangleleft}$$

ATConArrow2
$$\frac{}{\triangleright\tau\triangleleft \xrightarrow{\text{construct arrow}_\text{R}} \triangleright ? \triangleleft \to \tau}$$

ATConProd1
$$\frac{}{\triangleright\tau\triangleleft \xrightarrow{\text{construct prod}_\text{L}} \tau \times \triangleright ? \triangleleft}$$

ATConProd2
$$\frac{}{\triangleright\tau\triangleleft \xrightarrow{\text{construct prod}_\text{R}} \triangleright ? \triangleleft \times \tau}$$

ATConNum
$$\frac{}{\triangleright ? \triangleleft \xrightarrow{\text{construct num}} \triangleright\mathsf{num}\triangleleft}$$

ATConBool
$$\frac{}{\triangleright ? \triangleleft \xrightarrow{\text{construct bool}} \triangleright\mathsf{bool}\triangleleft}$$

Zipper Cases

ATZipArr1
$$\frac{\underline{\tau} \xrightarrow{\alpha} \underline{\tau'}}{\underline{\tau} \to \tau \xrightarrow{\alpha} \underline{\tau'} \to \tau}$$

ATZipArr2
$$\frac{\underline{\tau} \xrightarrow{\alpha} \underline{\tau'}}{\tau \to \underline{\tau} \xrightarrow{\alpha} \tau \to \underline{\tau'}}$$

ATZipProd1
$$\frac{\underline{\tau} \xrightarrow{\alpha} \underline{\tau'}}{\underline{\tau} \times \tau \xrightarrow{\alpha} \underline{\tau'} \times \tau}$$

ATZipProd2
$$\frac{\underline{\tau} \xrightarrow{\alpha} \underline{\tau'}}{\tau \times \underline{\tau} \xrightarrow{\alpha} \tau \times \underline{\tau'}}$$

E.3.3 Expression movement

$$\boxed{\underline{e} \xrightarrow{\text{move } \delta} \underline{e'}}$$

AEMLamChild1
$$\frac{}{\triangleright\lambda x : \tau.\, e\triangleleft \xrightarrow{\text{move child 1}} \lambda x : \triangleright\tau\triangleleft.\, e}$$

AEMLamChild2
$$\frac{}{\triangleright\lambda x : \tau.\, e\triangleleft \xrightarrow{\text{move child 2}} \lambda x : \tau.\, \triangleright e\triangleleft}$$

AEMLamParent1
$$\frac{}{\lambda x : \triangleright\tau\triangleleft.\, e \xrightarrow{\text{move parent}} \triangleright\lambda x : \tau.\, e\triangleleft}$$

AEMLamParent2
$$\frac{}{\lambda x : \tau.\, \triangleright e\triangleleft \xrightarrow{\text{move parent}} \triangleright\lambda x : \tau.\, e\triangleleft}$$

AEMApChild1
$$\frac{}{\triangleright e_1\ e_2\triangleleft \xrightarrow{\text{move child 1}} \triangleright e_1\triangleleft\ e_2}$$

AEMApChild2
$$\frac{}{\triangleright e_1\ e_2\triangleleft \xrightarrow{\text{move child 2}} e_1\ \triangleright e_2\triangleleft}$$

AEMApParent1
$$\frac{}{\triangleright e_1\triangleleft\ e_2 \xrightarrow{\text{move parent}} \triangleright e_1\ e_2\triangleleft}$$

AEMApParent2
$$\frac{}{e_1\ \triangleright e_2\triangleleft \xrightarrow{\text{move parent}} \triangleright e_1\ e_2\triangleleft}$$

AEMLetChild1
$$\frac{}{\triangleright\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2\triangleleft \xrightarrow{\text{move child 1}} \mathsf{let}\ x = \triangleright e_1\triangleleft\ \mathsf{in}\ e_2}$$

AEMLetChild2
$$\frac{}{\triangleright\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2\triangleleft \xrightarrow{\text{move child 2}} \mathsf{let}\ x = e_1\ \mathsf{in}\ \triangleright e_2\triangleleft}$$

AEMLetParent1
$$\frac{}{\mathsf{let}\ x = \triangleright e_1\triangleleft\ \mathsf{in}\ e_2 \xrightarrow{\text{move parent}} \triangleright\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2\triangleleft}$$

AEMLetParent2
$$\frac{}{\mathsf{let}\ x = e_1\ \mathsf{in}\ \triangleright e_2\triangleleft \xrightarrow{\text{move parent}} \triangleright\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2\triangleleft}$$

AEMPlusChild1
$$\frac{}{\triangleright e_1 + e_2\triangleleft \xrightarrow{\text{move child 1}} \triangleright e_1\triangleleft + e_2}$$

AEMPlusChild2
$$\frac{}{\triangleright e_1 + e_2\triangleleft \xrightarrow{\text{move child 2}} e_1 + \triangleright e_2\triangleleft}$$

AEMPlusParent1
$$\frac{}{\triangleright e_1\triangleleft + e_2 \xrightarrow{\text{move parent}} \triangleright e_1 + e_2\triangleleft}$$

AEMPlusParent2
$$\frac{}{e_1 + \triangleright e_2\triangleleft \xrightarrow{\text{move parent}} \triangleright e_1 + e_2\triangleleft}$$

AEMIfChild1
$$\frac{}{\triangleright\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3\triangleleft \xrightarrow{\text{move child 1}} \mathsf{if}\ \triangleright e_1\triangleleft\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3}$$

AEMIfChild2
$$\frac{}{\triangleright\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3\triangleleft \xrightarrow{\text{move child 2}} \mathsf{if}\ e_1\ \mathsf{then}\ \triangleright e_2\triangleleft\ \mathsf{else}\ e_3}$$

AEMIfChild3
$$\frac{}{\triangleright\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3\triangleleft \xrightarrow{\text{move child 3}} \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ \triangleright e_3\triangleleft}$$

AEMIfParent1
$$\frac{}{\mathsf{if}\ \triangleright e_1\triangleleft\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \xrightarrow{\text{move parent}} \triangleright\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3\triangleleft}$$

$$\textsc{AEMIfParent2}\quad \frac{}{\mathsf{if}\ e_1\ \mathsf{then}\ \triangleright e_2 \triangleleft\ \mathsf{else}\ e_3 \xrightarrow{\mathsf{move\ parent}} \triangleright \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \triangleleft}$$

$$\textsc{AEMIfParent3}\quad \frac{}{\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ \triangleright e_3 \triangleleft \xrightarrow{\mathsf{move\ parent}} \triangleright \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \triangleleft}$$

$$\textsc{AEMPairChild1}\quad \frac{}{\triangleright (e_1, e_2) \triangleleft \xrightarrow{\mathsf{move\ child}\ 1} (\triangleright e_1 \triangleleft, e_2)}$$

$$\textsc{AEMPairChild2}\quad \frac{}{\triangleright (e_1, e_2) \triangleleft \xrightarrow{\mathsf{move\ child}\ 2} (e_1, \triangleright e_2 \triangleleft)}$$

$$\textsc{AEMPairParent1}\quad \frac{}{(\triangleright e_1 \triangleleft, e_2) \xrightarrow{\mathsf{move\ parent}} \triangleright (e_1, e_2) \triangleleft}$$

$$\textsc{AEMPairParent2}\quad \frac{}{(e_1, \triangleright e_2 \triangleleft) \xrightarrow{\mathsf{move\ parent}} \triangleright (e_1, e_2) \triangleleft}$$

$$\textsc{AEMProjLChild}\quad \frac{}{\triangleright \pi_1 e_1 \triangleleft \xrightarrow{\mathsf{move\ child}\ 1} \pi_1 \triangleright e_1 \triangleleft}$$

$$\textsc{AEMProjLParent}\quad \frac{}{\pi_1 \triangleright e_1 \triangleleft \xrightarrow{\mathsf{move\ parent}} \triangleright \pi_1 e_1 \triangleleft}$$

$$\textsc{AEMProjRChild}\quad \frac{}{\triangleright \pi_2 e_1 \triangleleft \xrightarrow{\mathsf{move\ child}\ 1} \pi_2 \triangleright e_1 \triangleleft}$$

$$\textsc{AEMProjRParent}\quad \frac{}{\pi_2 \triangleright e_1 \triangleleft \xrightarrow{\mathsf{move\ parent}} \triangleright \pi_2 e_1 \triangleleft}$$

E.3.4 Expression actions

$$\boxed{\underline{e} \xrightarrow{\alpha} \underline{e}'}$$

Movement

$$\textsc{AEMove}\quad \frac{\underline{e} \xrightarrow{\mathsf{move}\ \delta} \underline{e}'}{\underline{e} \xrightarrow{\mathsf{move}\ \delta} \underline{e}'}$$

Deletion

$$\textsc{AEDel}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{del}} \triangleright (\!|\,|\!) \triangleleft}$$

Construction

$$\textsc{AEConVar}\quad \frac{}{\triangleright (\!|\,|\!) \triangleleft \xrightarrow{\mathsf{construct\ var}\ x} \triangleright x \triangleleft}$$

$$\textsc{AEConLam}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{construct\ lam}\ x} \lambda x : \triangleright ? \triangleleft .\ e}$$

$$\textsc{AEConAp1}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{construct\ ap}_{\mathsf{L}}} e\ \triangleright (\!|\,|\!) \triangleleft}$$

$$\textsc{AEConAp2}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{construct\ ap}_{\mathsf{R}}} \triangleright (\!|\,|\!) \triangleleft\ e}$$

$$\textsc{AEConLet1}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{construct\ let}_{\mathsf{L}}\ x} \mathsf{let}\ x = e\ \mathsf{in}\ \triangleright (\!|\,|\!) \triangleleft}$$

$$\textsc{AEConLet2}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{construct\ let}_{\mathsf{R}}\ x} \mathsf{let}\ x = \triangleright (\!|\,|\!) \triangleleft\ \mathsf{in}\ e}$$

$$\textsc{AEConNum}\quad \frac{}{\triangleright (\!|\,|\!) \triangleleft \xrightarrow{\mathsf{construct\ lit}\ \underline{n}} \triangleright \underline{n} \triangleleft}$$

$$\textsc{AEConPlus1}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{construct\ plus}_{\mathsf{L}}} e + \triangleright (\!|\,|\!) \triangleleft}$$

$$\textsc{AEConPlus2}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{construct\ plus}_{\mathsf{R}}} \triangleright (\!|\,|\!) \triangleleft + e}$$

$$\textsc{AEConTrue}\quad \frac{}{\triangleright (\!|\,|\!) \triangleleft \xrightarrow{\mathsf{construct\ true}} \triangleright \mathsf{tt} \triangleleft}$$

$$\textsc{AEConFalse}\quad \frac{}{\triangleright (\!|\,|\!) \triangleleft \xrightarrow{\mathsf{construct\ false}} \triangleright \mathsf{ff} \triangleleft}$$

$$\textsc{AEConIf1}\quad \frac{}{\triangleright e \triangleleft \xrightarrow{\mathsf{construct\ if}_{\mathsf{C}}} \mathsf{if}\ e\ \mathsf{then}\ \triangleright (\!|\,|\!) \triangleleft\ \mathsf{else}\ (\!|\,|\!)}$$

AEConIf2
$$\frac{}{\triangleright e \triangleleft \xrightarrow{\text{construct if}_C} \text{if } \triangleright (\!|\,|\!) \triangleleft \text{ then } e \text{ else } (\!|\,|\!)}$$

AEConIf3
$$\frac{}{\triangleright e \triangleleft \xrightarrow{\text{construct if}_C} \text{if } \triangleright (\!|\,|\!) \triangleleft \text{ then } (\!|\,|\!) \text{ else } e}$$

AEConPair1
$$\frac{}{\triangleright e \triangleleft \xrightarrow{\text{construct pair}_L} (e, \triangleright (\!|\,|\!) \triangleleft)}$$

AEConPair2
$$\frac{}{\triangleright e \triangleleft \xrightarrow{\text{construct pair}_R} (\triangleright (\!|\,|\!) \triangleleft, e)}$$

AEConProjL
$$\frac{}{\triangleright e \triangleleft \xrightarrow{\text{construct proj}_L} \triangleright \pi_1 e \triangleleft}$$

AEConProjR
$$\frac{}{\triangleright e \triangleleft \xrightarrow{\text{construct proj}_R} \triangleright \pi_2 e \triangleleft}$$

Zipper Cases

AEZipLam1
$$\frac{\underline{\tau} \xrightarrow{\alpha} \underline{\tau}'}{\lambda x : \underline{\tau}.\, e \xrightarrow{\alpha} \lambda x : \underline{\tau}'.\, e}$$

AEZipLam2
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\lambda x : \tau.\, \underline{e} \xrightarrow{\alpha} \lambda x : \tau.\, \underline{e}'}$$

AEZipAp1
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\underline{e}\ e \xrightarrow{\alpha} \underline{e}'\ e}$$

AEZipAp2
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{e\ \underline{e} \xrightarrow{\alpha} e\ \underline{e}'}$$

AEZipLet1
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\text{let } x = \underline{e} \text{ in } e \xrightarrow{\alpha} \text{let } x = \underline{e}' \text{ in } e}$$

AEZipLet2
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\text{let } x = e \text{ in } \underline{e} \xrightarrow{\alpha} \text{let } x = e \text{ in } \underline{e}'}$$

AEZipPlus1
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\underline{e} + e \xrightarrow{\alpha} \underline{e}' + e}$$

AEZipPlus2
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{e + \underline{e} \xrightarrow{\alpha} e + \underline{e}'}$$

AEZipIf1
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\text{if } \underline{e} \text{ then } e_1 \text{ else } e_2 \xrightarrow{\alpha} \text{if } \underline{e}' \text{ then } e_1 \text{ else } e_2}$$

AEZipIf2
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\text{if } e_1 \text{ then } \underline{e} \text{ else } e_2 \xrightarrow{\alpha} \text{if } e_1 \text{ then } \underline{e}' \text{ else } e_2}$$

AEZipIf3
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\text{if } e_1 \text{ then } e_2 \text{ else } \underline{e} \xrightarrow{\alpha} \text{if } e_1 \text{ then } e_2 \text{ else } \underline{e}'}$$

AEZipPair1
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{(\underline{e}, e) \xrightarrow{\alpha} (\underline{e}', e)}$$

AEZipPair2
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{(e, \underline{e}) \xrightarrow{\alpha} (e, \underline{e}')}$$

AEZipProjL
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\pi_1 \underline{e} \xrightarrow{\alpha} \pi_1 \underline{e}'}$$

AEZipProjR
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}'}{\pi_2 \underline{e} \xrightarrow{\alpha} \pi_2 \underline{e}'}$$

E.3.5 Iterated actions

$\boxed{\underline{\tau} \xrightarrow{\overline{\alpha}}{}^{*} \underline{\tau}'}$

ATIRefl
$$\frac{}{\underline{\tau} \xrightarrow{\cdot}{}^{*} \underline{\tau}}$$

ATITyp
$$\frac{\underline{\tau} \xrightarrow{\alpha} \underline{\tau}' \qquad \underline{\tau}' \xrightarrow{\overline{\alpha}}{}^{*} \underline{\tau}''}{\underline{\tau} \xrightarrow{\alpha;\overline{\alpha}}{}^{*} \underline{\tau}''}$$

$\boxed{\underline{e} \xrightarrow{\overline{\alpha}}{}^{*} \underline{e}'}$

AEIRefl
$$\frac{}{\underline{e} \xrightarrow{\cdot}{}^{*} \underline{e}}$$

AEIExp
$$\frac{\underline{e} \xrightarrow{\alpha} \underline{e}' \qquad \underline{e}' \xrightarrow{\overline{\alpha}}{}^{*} \underline{e}''}{\underline{e} \xrightarrow{\alpha;\overline{\alpha}}{}^{*} \underline{e}''}$$

$\boxed{\overline{\alpha} \text{ movements}}$

AMINil
$$\frac{}{\cdot \text{ movements}}$$

AMICons
$$\frac{\overline{\alpha} \text{ movements}}{\text{move } \delta; \overline{\alpha} \text{ movements}}$$

E.4 Metatheorems

Theorem E.1 (Movement Erasure Invariance)**.**

1. *If* $\underline{\tau} \xrightarrow{\text{move } \delta} \underline{\tau}'$*, then* $\underline{\tau}^{\diamond} = \underline{\tau}'^{\diamond}$.
2. *If* $\underline{e} \xrightarrow{\text{move } \delta} \underline{e}'$*, then* $\underline{e}^{\diamond} = \underline{e}'^{\diamond}$.

Theorem E.2 (Reachability)**.**

1. *If* $\underline{\tau}^{\diamond} = \underline{\tau}'^{\diamond}$*, then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\underline{\tau} \overset{\overline{\alpha}}{\to}{}^{*} \underline{\tau}'$.
2. *If* $\underline{e}^{\diamond} = \underline{e}'^{\diamond}$*, then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\underline{e} \overset{\overline{\alpha}}{\to}{}^{*} \underline{e}'$.

Lemma E.2.1 (Reach Up)**.**

1. *If* $\underline{\tau}^{\diamond} = \tau$*, then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\underline{\tau} \overset{\overline{\alpha}}{\to}{}^{*} \triangleright\tau\triangleleft$.
2. *If* $\underline{e}^{\diamond} = e$*, then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\underline{e} \overset{\overline{\alpha}}{\to}{}^{*} \triangleright e\triangleleft$.

Lemma E.2.2 (Reach Down)**.**

1. *If* $\underline{\tau}^{\diamond} = \tau$*, then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\triangleright\tau\triangleleft \overset{\overline{\alpha}}{\to}{}^{*} \underline{\tau}$.
2. *If* $\underline{e}^{\diamond} = e$*, then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\triangleright e\triangleleft \overset{\overline{\alpha}}{\to}{}^{*} \underline{e}$.

Theorem E.3 (Constructability)**.**

1. *For every* τ*, there exists* $\overline{\alpha}$ *such that* $\triangleright ? \triangleleft \overset{\overline{\alpha}}{\to}{}^{*} \triangleright\tau\triangleleft$.
2. *For every* e*, there exists* $\overline{\alpha}$ *such that* $\triangleright\llparenthesis\rrparenthesis\triangleleft \overset{\overline{\alpha}}{\to}{}^{*} \triangleright e\triangleleft$.

Theorem E.4 (Determinism)**.**

1. *If* $\underline{\tau} \overset{\alpha}{\to}{}^{*} \underline{\tau}'$ *and* $\underline{\tau} \overset{\alpha}{\to}{}^{*} \underline{\tau}''$*, then* $\underline{\tau}' = \underline{\tau}''$.
2. *If* $\underline{e} \overset{\alpha}{\to}{}^{*} \underline{e}'$ *and* $\underline{e} \overset{\alpha}{\to}{}^{*} \underline{e}''$*, then* $\underline{e}' = \underline{e}''$.

F Typed hazelnut

We now give a description of a *typed* version of the Hazelnut action calculus that incorporates the marked lambda calculus to solve the problem of non-local hole fixes. Here, unlike in the integration of the untyped version and the marked lambda calculus given in Section E, remarking is performed only when necessary instead of after every action. This system is sketched out in Section 3.2 of the paper.

Mechanization ×

F.1 Syntax

Zippered types are the same as in the untyped model.

$$
\begin{array}{llcl}
\mathsf{ZMExp} & \underline{\check{e}} & ::= & \triangleright\check{e}\triangleleft \mid \lambda x : \underline{\tau}.\,\check{e} \mid \lambda x : \tau.\,\underline{\check{e}} \mid \underline{\check{e}}\ \check{e} \mid \check{e}\ \underline{\check{e}} \\
& & \mid & \mathsf{let}\ x = \underline{\check{e}}\ \mathsf{in}\ \check{e} \mid \mathsf{let}\ x = \check{e}\ \mathsf{in}\ \underline{\check{e}} \\
& & \mid & \underline{\check{e}} + \check{e} \mid \check{e} + \underline{\check{e}} \\
& & \mid & \mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}\ \mathsf{else}\ \check{e} \mid \mathsf{if}\ \check{e}\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e} \mid \mathsf{if}\ \check{e}\ \mathsf{then}\ \check{e}\ \mathsf{else}\ \underline{\check{e}} \\
& & \mid & (\underline{\check{e}}, e) \mid (e, \underline{\check{e}}) \mid \pi_1\underline{\check{e}} \mid \pi_2\underline{\check{e}} \\
& & \mid & (\!|\underline{\check{e}}|\!)_{\not\sim} \\
& & \mid & (\!|\lambda x : \underline{\tau}.\,\check{e}|\!)_{:} \mid (\!|\lambda x : \tau.\,\underline{\check{e}}|\!)_{:} \mid (\!|\lambda x : \underline{\tau}.\,\check{e}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\to}} \mid (\!|\lambda x : \tau.\,\underline{\check{e}}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\to}} \mid (\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \check{e} \mid (\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \underline{\check{e}} \\
& & \mid & (\!|\mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}\ \mathsf{else}\ \check{e}|\!)_{\not\sqcup} \mid (\!|\mathsf{if}\ \check{e}\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}|\!)_{\not\sqcup} \mid (\!|\mathsf{if}\ \check{e}\ \mathsf{then}\ \check{e}\ \mathsf{else}\ \underline{\check{e}}|\!)_{\not\sqcup} \\
& & \mid & (\!|(\underline{\check{e}}, \check{e})|\!)^{\Leftarrow}_{\blacktriangleright_{\not\times}} \mid (\!|(\check{e}, \underline{\check{e}})|\!)^{\Leftarrow}_{\blacktriangleright_{\not\times}} \mid \pi_1(\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\times}} \mid \pi_2(\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\times}}
\end{array}
$$

F.1.1 Well-formedness

$\boxed{\underline{\check{e}}\ \mathsf{WF}}$ $\underline{\check{e}}$ is well-formed

$$\textsc{WFCursor}\quad \frac{}{\triangleright\check{e}\triangleleft\ \mathsf{WF}}
\qquad
\textsc{WFLam1}\quad \frac{}{\lambda x : \underline{\tau}.\,\check{e}\ \mathsf{WF}}
\qquad
\textsc{WFLam2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\lambda x : \tau.\,\underline{\check{e}}\ \mathsf{WF}}$$

$$\textsc{WFLam3}\quad \frac{}{(\!|\lambda x : \underline{\tau}.\,\check{e}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\to}}\ \mathsf{WF}}
\qquad
\textsc{WFLam4}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|\lambda x : \tau.\,\underline{\check{e}}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\to}}\ \mathsf{WF}}
\qquad
\textsc{WFLam5}\quad \frac{}{(\!|\lambda x : \underline{\tau}.\,\check{e}|\!)_{:}\ \mathsf{WF}}$$

$$\textsc{WFLam6}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|\lambda x : \tau.\,\underline{\check{e}}|\!)_{:}\ \mathsf{WF}}
\qquad
\textsc{WFAp1}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\underline{\check{e}}\ \check{e}\ \mathsf{WF}}
\qquad
\textsc{WFAp2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\check{e}\ \underline{\check{e}}\ \mathsf{WF}}
\qquad
\textsc{WFAp3}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \check{e}\ \mathsf{WF}}
\qquad
\textsc{WFAp4}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \underline{\check{e}}\ \mathsf{WF}}$$

$$\textsc{WFLet1}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\mathsf{let}\ x = \underline{\check{e}}\ \mathsf{in}\ \check{e}\ \mathsf{WF}}
\qquad
\textsc{WFLet2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\mathsf{let}\ x = \check{e}\ \mathsf{in}\ \underline{\check{e}}\ \mathsf{WF}}
\qquad
\textsc{WFPlus1}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\underline{\check{e}} + \check{e}\ \mathsf{WF}}
\qquad
\textsc{WFPlus2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\check{e} + \underline{\check{e}}\ \mathsf{WF}}$$

$$\textsc{WFIf1}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2\ \mathsf{WF}}
\qquad
\textsc{WFIf2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2\ \mathsf{WF}}
\qquad
\textsc{WFIf3}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}}\ \mathsf{WF}}$$

$$\textsc{WFInconsistentBranches1}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|\mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup}\ \mathsf{WF}}
\qquad
\textsc{WFInconsistentBranches2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup}\ \mathsf{WF}}
\qquad
\textsc{WFInconsistentBranches3}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}}|\!)_{\not\sqcup}\ \mathsf{WF}}$$

$$\textsc{WFPair1}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\underline{\check{e}}, \check{e})\ \mathsf{WF}}
\qquad
\textsc{WFPair2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\check{e}, \underline{\check{e}})\ \mathsf{WF}}
\qquad
\textsc{WFPair3}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|(\underline{\check{e}}, \check{e})|\!)^{\Leftarrow}_{\blacktriangleright_{\not\times}}\ \mathsf{WF}}
\qquad
\textsc{WFPair4}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{(\!|(\check{e}, \underline{\check{e}})|\!)^{\Leftarrow}_{\blacktriangleright_{\not\times}}\ \mathsf{WF}}$$

$$\textsc{WFProjL1}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\pi_1\underline{\check{e}}\ \mathsf{WF}}
\qquad
\textsc{WFProjL2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\pi_1(\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\times}}\ \mathsf{WF}}
\qquad
\textsc{WFProjR1}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\pi_2\underline{\check{e}}\ \mathsf{WF}}
\qquad
\textsc{WFProjR2}\quad \frac{\underline{\check{e}}\ \mathsf{WF}}{\pi_2(\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\times}}\ \mathsf{WF}}$$

$$\textsc{WFInconsistentTypes}\quad \frac{\underline{\check{e}} \neq \triangleright\check{e}\triangleleft \qquad \underline{\check{e}}\ \mathsf{WF}}{(\!|\underline{\check{e}}|\!)_{\not\sim}\ \mathsf{WF}}
\qquad
\textsc{WFLam3}\quad \frac{}{(\!|\lambda x : \underline{\tau}.\,\check{e}|\!)_{:}\ \mathsf{WF}}
\qquad
\textsc{WFLam4}\quad \frac{\underline{e}\ \mathsf{WF}}{(\!|\lambda x : \tau.\,\underline{e}|\!)_{:}\ \mathsf{WF}}
\qquad
\textsc{WFLam5}\quad \frac{}{(\!|\lambda x : \underline{\tau}.\,\check{e}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\to}}\ \mathsf{WF}}$$

$$\textsc{WFLam6}\quad \frac{\underline{e}\ \mathsf{WF}}{(\!|\lambda x : \tau.\,\underline{e}|\!)^{\Leftarrow}_{\blacktriangleright_{\not\to}}\ \mathsf{WF}}
\qquad
\textsc{WFAp3}\quad \frac{\underline{e}\ \mathsf{WF}}{(\!|\underline{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \check{e}\ \mathsf{WF}}
\qquad
\textsc{WFAp4}\quad \frac{\underline{e}\ \mathsf{WF}}{(\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \underline{e}\ \mathsf{WF}}$$

$$\textsc{WFInconsistentBranches1}\quad \frac{\underline{e}\ \mathsf{WF}}{(\!|\mathsf{if}\ \underline{e}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup}\ \mathsf{WF}}
\qquad
\textsc{WFInconsistentBranches2}\quad \frac{\underline{e}\ \mathsf{WF}}{(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{e}\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup}\ \mathsf{WF}}$$

$$\frac{\underline{e}\ \mathsf{WF}}{\llparenthesis \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{e} \rrparenthesis_{\not\sqcup}\ \mathsf{WF}}\ \text{WFInconsistentBranches3}
\qquad
\frac{\underline{e}\ \mathsf{WF}}{\llparenthesis (\underline{e}, \check{e}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright\not\times}\ \mathsf{WF}}\ \text{WFPair3}
\qquad
\frac{\underline{e}\ \mathsf{WF}}{\llparenthesis (\check{e}, \underline{e}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright\not\times}\ \mathsf{WF}}\ \text{WFPair4}$$

$$\frac{\underline{e}\ \mathsf{WF}}{\pi_1 \llparenthesis \underline{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\times}\ \mathsf{WF}}\ \text{WFProjL2}
\qquad
\frac{\underline{e}\ \mathsf{WF}}{\pi_2 \llparenthesis \underline{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\times}\ \mathsf{WF}}\ \text{WFProjR2}$$

F.2 Cursor erasure

F.2.1 Type cursor erasure

Type cursor erasure is the same as in the untyped model.

F.2.2 Expression cursor erasure

$\boxed{\underline{\check{e}}^{\diamond}}$ is a metafunction ZMExp → MExp defined as follows:

$$\begin{array}{rcl}
\triangleright\check{e}\triangleleft^{\diamond} & = & \check{e} \\
(\lambda x : \underline{\tau}.\, \check{e})^{\diamond} & = & \lambda x : (\underline{\tau}^{\diamond}).\, \check{e} \\
(\lambda x : \tau.\, \underline{\check{e}})^{\diamond} & = & \lambda x : \tau.\, (\underline{\check{e}}^{\diamond}) \\
\llparenthesis \lambda x : \underline{\tau}.\, \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow}{}^{\diamond} & = & \llparenthesis \lambda x : (\underline{\tau}^{\diamond}).\, \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow} \\
\llparenthesis \lambda x : \tau.\, \underline{\check{e}} \rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow}{}^{\diamond} & = & \llparenthesis \lambda x : \tau.\, (\underline{\check{e}}^{\diamond}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow} \\
\llparenthesis \lambda x : \underline{\tau}.\, \check{e} \rrparenthesis_{:}{}^{\diamond} & = & \llparenthesis \lambda x : (\underline{\tau}^{\diamond}).\, \check{e} \rrparenthesis_{:} \\
\llparenthesis \lambda x : \tau.\, \underline{\check{e}} \rrparenthesis_{:}{}^{\diamond} & = & \llparenthesis \lambda x : \tau.\, (\underline{\check{e}}^{\diamond}) \rrparenthesis_{:} \\
(\underline{\check{e}}\ \check{e})^{\diamond} & = & (\underline{\check{e}}^{\diamond})\ \check{e} \\
(\check{e}\ \underline{\check{e}})^{\diamond} & = & \check{e}\ (\underline{\check{e}}^{\diamond}) \\
(\llparenthesis \underline{\check{e}} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \check{e})^{\diamond} & = & \llparenthesis \underline{\check{e}}^{\diamond} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \check{e} \\
(\llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \underline{\check{e}})^{\diamond} & = & \llparenthesis \check{e} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ (\underline{\check{e}}^{\diamond}) \\
(\mathsf{let}\ x = \underline{\check{e}}\ \mathsf{in}\ \check{e})^{\diamond} & = & \mathsf{let}\ x = (\underline{\check{e}}^{\diamond})\ \mathsf{in}\ \check{e} \\
(\mathsf{let}\ x = \check{e}\ \mathsf{in}\ \underline{\check{e}})^{\diamond} & = & \mathsf{let}\ x = \check{e}\ \mathsf{in}\ (\underline{\check{e}}^{\diamond}) \\
(\underline{\check{e}} + \check{e})^{\diamond} & = & (\underline{\check{e}}^{\diamond}) + \check{e} \\
(\check{e} + \underline{\check{e}})^{\diamond} & = & \check{e} + (\underline{\check{e}}^{\diamond}) \\
(\mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2)^{\diamond} & = & \mathsf{if}\ (\underline{\check{e}}^{\diamond})\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2 \\
(\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2)^{\diamond} & = & \mathsf{if}\ \check{e}_1\ \mathsf{then}\ (\underline{\check{e}}^{\diamond})\ \mathsf{else}\ \check{e}_2 \\
(\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}})^{\diamond} & = & \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ (\underline{\check{e}}^{\diamond}) \\
\llparenthesis \mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2 \rrparenthesis_{\not\sqcup}{}^{\diamond} & = & \llparenthesis \mathsf{if}\ (\underline{\check{e}}^{\diamond})\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2 \rrparenthesis_{\not\sqcup} \\
\llparenthesis \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2 \rrparenthesis_{\not\sqcup}{}^{\diamond} & = & \llparenthesis \mathsf{if}\ \check{e}_1\ \mathsf{then}\ (\underline{\check{e}}^{\diamond})\ \mathsf{else}\ \check{e}_2 \rrparenthesis_{\not\sqcup} \\
\llparenthesis \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}} \rrparenthesis_{\not\sqcup}{}^{\diamond} & = & \llparenthesis \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ (\underline{\check{e}}^{\diamond}) \rrparenthesis_{\not\sqcup} \\
(\underline{\check{e}}, \check{e})^{\diamond} & = & (\underline{\check{e}}^{\diamond}, \check{e}) \\
(\check{e}, \underline{\check{e}})^{\diamond} & = & (\check{e}, \underline{\check{e}}^{\diamond}) \\
\llparenthesis (\underline{\check{e}}, \check{e}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright\not\times}{}^{\diamond} & = & \llparenthesis (\underline{\check{e}}^{\diamond}, \check{e}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright\not\times} \\
\llparenthesis (\check{e}, \underline{\check{e}}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright\not\times}{}^{\diamond} & = & \llparenthesis (\check{e}, \underline{\check{e}}^{\diamond}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright\not\times} \\
(\pi_1 \underline{\check{e}})^{\diamond} & = & \pi_1 (\underline{\check{e}}^{\diamond}) \\
(\pi_1 \llparenthesis \underline{\check{e}} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\times})^{\diamond} & = & \pi_1 \llparenthesis \underline{\check{e}}^{\diamond} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\times} \\
(\pi_2 \underline{\check{e}})^{\diamond} & = & \pi_2 (\underline{\check{e}}^{\diamond}) \\
(\pi_2 \llparenthesis \underline{\check{e}} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\times})^{\diamond} & = & \pi_2 \llparenthesis \underline{\check{e}}^{\diamond} \rrparenthesis^{\Rightarrow}_{\blacktriangleright\not\times} \\
\llparenthesis \underline{\check{e}} \rrparenthesis_{\nrightarrow}{}^{\diamond} & = & \llparenthesis \underline{\check{e}}^{\diamond} \rrparenthesis_{\nrightarrow}
\end{array}$$

F.3 Action model

The action syntax is the same in the untyped model.

F.3.1 Shape sort

The shape sort judgments are the same as in the untyped model.

F.3.2 Type actions

Type actions are the same as in the untyped model.

F.3.3 Expression movement

$$\boxed{\underline{\check{e}} \xrightarrow{\text{move } \delta} \underline{\check{e}'}}$$

$$\textsc{AEMLamChild1}\quad \frac{}{\triangleright\lambda x : \tau.\, \check{e}\triangleleft \xrightarrow{\text{move child 1}} \lambda x : \triangleright\tau\triangleleft.\, \check{e}}$$

$$\textsc{AEMLamChild2}\quad \frac{}{\triangleright\lambda x : \tau.\, \check{e}\triangleleft \xrightarrow{\text{move child 2}} \lambda x : \tau.\, \triangleright\check{e}\triangleleft}$$

$$\textsc{AEMLamChild3}\quad \frac{}{\triangleright\llparenthesis\lambda x : \tau.\, \check{e}\rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow}\triangleleft \xrightarrow{\text{move child 1}} \llparenthesis\lambda x : \triangleright\tau\triangleleft.\, \check{e}\rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow}}$$

$$\textsc{AEMLamChild4}\quad \frac{}{\triangleright\llparenthesis\lambda x : \tau.\, \check{e}\rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow}\triangleleft \xrightarrow{\text{move child 2}} \llparenthesis\lambda x : \tau.\, \triangleright\check{e}\triangleleft\rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow}}$$

$$\textsc{AEMLamChild5}\quad \frac{}{\triangleright\llparenthesis\lambda x : \tau.\, \check{e}\rrparenthesis_{:}\triangleleft \xrightarrow{\text{move child 1}} \llparenthesis\lambda x : \triangleright\tau\triangleleft.\, \check{e}\rrparenthesis_{:}}$$

$$\textsc{AEMLamChild6}\quad \frac{}{\triangleright\llparenthesis\lambda x : \tau.\, \check{e}\rrparenthesis_{:}\triangleleft \xrightarrow{\text{move child 2}} \llparenthesis\lambda x : \tau.\, \triangleright\check{e}\triangleleft\rrparenthesis_{:}}$$

$$\textsc{AEMLamParent1}\quad \frac{}{\lambda x : \triangleright\tau\triangleleft.\, \check{e} \xrightarrow{\text{move parent}} \triangleright\lambda x : \tau.\, \check{e}\triangleleft}$$

$$\textsc{AEMLamParent2}\quad \frac{}{\lambda x : \tau.\, \triangleright\check{e}\triangleleft \xrightarrow{\text{move parent}} \triangleright\lambda x : \tau.\, \check{e}\triangleleft}$$

$$\textsc{AEMLamParent3}\quad \frac{}{\llparenthesis\lambda x : \triangleright\tau\triangleleft.\, \check{e}\rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow} \xrightarrow{\text{move parent}} \triangleright\llparenthesis\lambda x : \tau.\, \check{e}\rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow}\triangleleft}$$

$$\textsc{AEMLamParent4}\quad \frac{}{\llparenthesis\lambda x : \tau.\, \triangleright\check{e}\triangleleft\rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow} \xrightarrow{\text{move parent}} \triangleright\llparenthesis\lambda x : \tau.\, \check{e}\rrparenthesis^{\Leftarrow}_{\blacktriangleright\nrightarrow}\triangleleft}$$

$$\textsc{AEMLamParent5}\quad \frac{}{\llparenthesis\lambda x : \triangleright\tau\triangleleft.\, \check{e}\rrparenthesis_{:} \xrightarrow{\text{move parent}} \triangleright\llparenthesis\lambda x : \tau.\, \check{e}\rrparenthesis_{:}\triangleleft}$$

$$\textsc{AEMLamParent6}\quad \frac{}{\llparenthesis\lambda x : \tau.\, \triangleright\check{e}\triangleleft\rrparenthesis_{:} \xrightarrow{\text{move parent}} \triangleright\llparenthesis\lambda x : \tau.\, \check{e}\rrparenthesis_{:}\triangleleft}$$

$$\textsc{AEMApChild1}\quad \frac{}{\triangleright\check{e}_1\ \check{e}_2\triangleleft \xrightarrow{\text{move child 1}} \triangleright\check{e}_1\triangleleft\ \check{e}_2}$$

$$\textsc{AEMApChild2}\quad \frac{}{\triangleright\check{e}_1\ \check{e}_2\triangleleft \xrightarrow{\text{move child 2}} \check{e}_1\ \triangleright\check{e}_2\triangleleft}$$

$$\textsc{AEMApChild3}\quad \frac{}{\triangleright\llparenthesis\check{e}_1\rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \check{e}_2\triangleleft \xrightarrow{\text{move child 1}} \llparenthesis\triangleright\check{e}_1\triangleleft\rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \check{e}_2}$$

$$\textsc{AEMApChild4}\quad \frac{}{\triangleright\llparenthesis\check{e}_1\rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \check{e}_2\triangleleft \xrightarrow{\text{move child 2}} \llparenthesis\check{e}_1\rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \triangleright\check{e}_2\triangleleft}$$

$$\textsc{AEMApParent1}\quad \frac{}{\triangleright\check{e}_1\triangleleft\ \check{e}_2 \xrightarrow{\text{move parent}} \triangleright\check{e}_1\ \check{e}_2\triangleleft}$$

$$\textsc{AEMApParent2}\quad \frac{}{\check{e}_1\ \triangleright\check{e}_2\triangleleft \xrightarrow{\text{move parent}} \triangleright\check{e}_1\ \check{e}_2\triangleleft}$$

$$\textsc{AEMApParent3}\quad \frac{}{\llparenthesis\triangleright\check{e}_1\triangleleft\rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \check{e}_2 \xrightarrow{\text{move parent}} \triangleright\llparenthesis\check{e}_1\rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \check{e}_2\triangleleft}$$

$$\textsc{AEMApParent4}\quad \frac{}{\llparenthesis\check{e}_1\rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \triangleright\check{e}_2\triangleleft \xrightarrow{\text{move parent}} \triangleright\llparenthesis\check{e}_1\rrparenthesis^{\Rightarrow}_{\blacktriangleright\nrightarrow}\ \check{e}_2\triangleleft}$$

$$\textsc{AEMLetChild1}\quad \frac{}{\triangleright\mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2\triangleleft \xrightarrow{\text{move child 1}} \mathsf{let}\ x = \triangleright\check{e}_1\triangleleft\ \mathsf{in}\ \check{e}_2}$$

$$\textsc{AEMLetChild2}\quad \frac{}{\triangleright\mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2\triangleleft \xrightarrow{\text{move child 2}} \mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \triangleright\check{e}_2\triangleleft}$$

$$\textsc{AEMLetParent1}\quad \frac{}{\mathsf{let}\ x = \triangleright\check{e}_1\triangleleft\ \mathsf{in}\ \check{e}_2 \xrightarrow{\text{move parent}} \triangleright\mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2\triangleleft}$$

$$\textsc{AEMLetParent2}\quad \frac{}{\mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \triangleright\check{e}_2\triangleleft \xrightarrow{\text{move parent}} \triangleright\mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \check{e}_2\triangleleft}$$

$$\textsc{AEMPlusChild1}\quad \frac{}{\triangleright\check{e}_1 + \check{e}_2\triangleleft \xrightarrow{\text{move child 1}} \triangleright\check{e}_1\triangleleft + \check{e}_2}$$

$$\textsc{AEMPlusChild2}\quad \frac{}{\triangleright\check{e}_1 + \check{e}_2\triangleleft \xrightarrow{\text{move child 2}} \check{e}_1 + \triangleright\check{e}_2\triangleleft}$$

$$\textsc{AEMPlusParent1}\quad \frac{}{\triangleright\check{e}_1\triangleleft + \check{e}_2 \xrightarrow{\text{move parent}} \triangleright\check{e}_1 + \check{e}_2\triangleleft}$$

AEMPlusParent2
$$\frac{}{\check{e}_1 + \triangleright\check{e}_2\triangleleft \xrightarrow{\text{move parent}} \triangleright\check{e}_1 + \check{e}_2\triangleleft}$$

AEMIfChild1
$$\frac{}{\triangleright\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3\triangleleft \xrightarrow{\text{move child 1}} \mathsf{if}\ \triangleright\check{e}_1\triangleleft\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3}$$

AEMIfChild2
$$\frac{}{\triangleright\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3\triangleleft \xrightarrow{\text{move child 2}} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \triangleright\check{e}_2\triangleleft\ \mathsf{else}\ \check{e}_3}$$

AEMIfChild3
$$\frac{}{\triangleright\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3\triangleleft \xrightarrow{\text{move child 3}} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \triangleright\check{e}_3\triangleleft}$$

AEMIfParent1
$$\frac{}{\mathsf{if}\ \triangleright\check{e}_1\triangleleft\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \xrightarrow{\text{move parent}} \triangleright\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3\triangleleft}$$

AEMIfParent2
$$\frac{}{\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \triangleright\check{e}_2\triangleleft\ \mathsf{else}\ \check{e}_3 \xrightarrow{\text{move parent}} \triangleright\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3\triangleleft}$$

AEMIfParent3
$$\frac{}{\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \triangleright\check{e}_3\triangleleft \xrightarrow{\text{move parent}} \triangleright\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3\triangleleft}$$

AEMInconsistentBranchesChild1
$$\frac{}{\triangleright(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup}\triangleleft \xrightarrow{\text{move child 1}} (\!|\mathsf{if}\ \triangleright\check{e}_1\triangleleft\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup}}$$

AEMInconsistentBranchesChild2
$$\frac{}{\triangleright(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup}\triangleleft \xrightarrow{\text{move child 2}} (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \triangleright\check{e}_2\triangleleft\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup}}$$

AEMInconsistentBranchesChild3
$$\frac{}{\triangleright(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup}\triangleleft \xrightarrow{\text{move child 3}} (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \triangleright\check{e}_3\triangleleft|\!)_{\sqcup}}$$

AEMInconsistentBranchesParent1
$$\frac{}{(\!|\mathsf{if}\ \triangleright\check{e}_1\triangleleft\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup} \xrightarrow{\text{move parent}} \triangleright(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup}\triangleleft}$$

AEMInconsistentBranchesParent2
$$\frac{}{(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \triangleright\check{e}_2\triangleleft\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup} \xrightarrow{\text{move parent}} \triangleright(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup}\triangleleft}$$

AEMInconsistentBranchesParent3
$$\frac{}{(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \triangleright\check{e}_3\triangleleft|\!)_{\sqcup} \xrightarrow{\text{move parent}} \triangleright(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)_{\sqcup}\triangleleft}$$

AEMPairChild1
$$\frac{}{\triangleright(\check{e}_1, \check{e}_2)\triangleleft \xrightarrow{\text{move child 1}} (\triangleright\check{e}_1\triangleleft, \check{e}_2)}$$

AEMPairChild2
$$\frac{}{\triangleright(\check{e}_1, \check{e}_2)\triangleleft \xrightarrow{\text{move child 2}} (\check{e}_1, \triangleright\check{e}_2\triangleleft)}$$

AEMPairChild3
$$\frac{}{\triangleright(\!|(\check{e}_1, \check{e}_2)|\!)^{\Leftarrow}_{\blacktriangleright_\times}\triangleleft \xrightarrow{\text{move child 1}} (\!|(\triangleright\check{e}_1\triangleleft, \check{e}_2)|\!)^{\Leftarrow}_{\blacktriangleright_\times}}$$

AEMPairChild4
$$\frac{}{\triangleright(\!|(\check{e}_1, \check{e}_2)|\!)^{\Leftarrow}_{\blacktriangleright_\times}\triangleleft \xrightarrow{\text{move child 2}} (\!|(\check{e}_1, \triangleright\check{e}_2\triangleleft)|\!)^{\Leftarrow}_{\blacktriangleright_\times}}$$

AEMPairParent1
$$\frac{}{(\triangleright\check{e}_1\triangleleft, \check{e}_2) \xrightarrow{\text{move parent}} \triangleright(\check{e}_1, \check{e}_2)\triangleleft}$$

AEMPairParent2
$$\frac{}{(\check{e}_1, \triangleright\check{e}_2\triangleleft) \xrightarrow{\text{move parent}} \triangleright(\check{e}_1, \check{e}_2)\triangleleft}$$

AEMPairParent3

$$\frac{}{\llparenthesis(\triangleright\check{e}_1\triangleleft, \check{e}_2)\rrparenthesis^{\Leftarrow}_{\blacktriangleright\times} \xrightarrow{\text{move parent}} \triangleright\llparenthesis(\check{e}_1, \check{e}_2)\rrparenthesis^{\Leftarrow}_{\blacktriangleright\times}\triangleleft}$$

AEMPairParent4

$$\frac{}{\llparenthesis(\check{e}_1, \triangleright\check{e}_2\triangleleft)\rrparenthesis^{\Leftarrow}_{\blacktriangleright\times} \xrightarrow{\text{move parent}} \triangleright\llparenthesis(\check{e}_1, \check{e}_2)\rrparenthesis^{\Leftarrow}_{\blacktriangleright\times}\triangleleft}$$

AEMProjLChild1

$$\frac{}{\triangleright\pi_1\check{e}\triangleleft \xrightarrow{\text{move child 1}} \pi_1\triangleright\check{e}\triangleleft}$$

AEMProjLChild2

$$\frac{}{\triangleright\pi_1\llparenthesis\check{e}\rrparenthesis^{\Rightarrow}_{\blacktriangleright\times}\triangleleft \xrightarrow{\text{move child 1}} \pi_1\llparenthesis\triangleright\check{e}\triangleleft\rrparenthesis^{\Rightarrow}_{\blacktriangleright\times}}$$

AEMProjLParent1

$$\frac{}{\pi_1\triangleright\check{e}\triangleleft \xrightarrow{\text{move parent}} \triangleright\pi_1\check{e}\triangleleft}$$

AEMProjLParent2

$$\frac{}{\pi_1\llparenthesis\triangleright\check{e}\triangleleft\rrparenthesis^{\Rightarrow}_{\blacktriangleright\times} \xrightarrow{\text{move parent}} \triangleright\pi_1\llparenthesis\check{e}\rrparenthesis^{\Rightarrow}_{\blacktriangleright\times}\triangleleft}$$

AEMProjRChild1

$$\frac{}{\triangleright\pi_2\check{e}\triangleleft \xrightarrow{\text{move child 1}} \pi_2\triangleright\check{e}\triangleleft}$$

AEMProjRChild2

$$\frac{}{\triangleright\pi_2\llparenthesis\check{e}\rrparenthesis^{\Rightarrow}_{\blacktriangleright\times}\triangleleft \xrightarrow{\text{move child 1}} \pi_2\llparenthesis\triangleright\check{e}\triangleleft\rrparenthesis^{\Rightarrow}_{\blacktriangleright\times}}$$

AEMProjRParent1

$$\frac{}{\pi_2\triangleright\check{e}\triangleleft \xrightarrow{\text{move parent}} \triangleright\pi_2\check{e}\triangleleft}$$

AEMProjRParent2

$$\frac{}{\pi_2\llparenthesis\triangleright\check{e}\triangleleft\rrparenthesis^{\Rightarrow}_{\blacktriangleright\times} \xrightarrow{\text{move parent}} \triangleright\pi_2\llparenthesis\check{e}\rrparenthesis^{\Rightarrow}_{\blacktriangleright\times}\triangleleft}$$

AEMInconsistentTypesChild

$$\frac{\triangleright\underline{\check{e}}\triangleleft \xrightarrow{\text{move child } n} \underline{\check{e}}'}{\triangleright\llparenthesis\underline{\check{e}}\rrparenthesis_{\nsim}\triangleleft \xrightarrow{\text{move child } n} \llparenthesis\underline{\check{e}}'\rrparenthesis_{\nsim}}$$

AEMInconsistentTypesParent

$$\frac{\underline{\check{e}} \xrightarrow{\text{move parent}} \triangleright\underline{\check{e}}'\triangleleft}{\llparenthesis\underline{\check{e}}\rrparenthesis_{\nsim} \xrightarrow{\text{move parent}} \triangleright\llparenthesis\underline{\check{e}}'\rrparenthesis_{\nsim}\triangleleft}$$

F.3.4 Synthetic expression actions

$$\boxed{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}}' \Rightarrow \tau'}$$

Movement

ASEMove

$$\frac{\underline{\check{e}} \xrightarrow{\text{move } \delta} \underline{\check{e}}'}{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\text{move } \delta} \underline{\check{e}}' \Rightarrow \tau}$$

Deletion

ASEDel

$$\frac{}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{del}} \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow ?}$$

Construction

ASEConVar

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow ? \xrightarrow{\text{construct var } x} \triangleright x\triangleleft \Rightarrow \tau}$$

ASEConFree

$$\frac{x \notin \mathrm{dom}(\Gamma)}{\Gamma \vdash \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow ? \xrightarrow{\text{construct var } x} \triangleright\llparenthesis x\rrparenthesis_{\square}\triangleleft \Rightarrow ?}$$

ASEConLam

$$\frac{\Gamma, x : ? \vdash \check{e}^{\square} \looparrowright \check{e}' \Rightarrow \tau'}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct lam } x} \lambda x : \triangleright ? \triangleleft.\, \check{e}' \Rightarrow ? \to \tau'}$$

ASEConApL1

$$\frac{\tau \blacktriangleright_{\to} \tau_1 \to \tau_2}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct ap}_{\mathrm{L}}} \llparenthesis\check{e}\rrparenthesis_{\nsim}\ \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \tau_2}$$

ASEConApL2

$$\frac{\tau \blacktriangleright_{\nrightarrow}}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct ap}_{\mathrm{L}}} \llparenthesis\check{e}\rrparenthesis_{\nsim}\ \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow ?}$$

ASEConApR

$$\frac{\Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow ?}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct ap}_{\mathrm{R}}} \triangleright\llparenthesis\rrparenthesis\triangleleft\ \check{e}' \Rightarrow ?}$$

ASEConLet1

$$\frac{}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct let}_{\mathrm{L}}\ x} \mathsf{let}\ x = \check{e}\ \mathsf{in}\ \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow ?}$$

ASEConLet2

$$\frac{\Gamma, x : ? \vdash \check{e}^{\square} \looparrowright \check{e}' \Rightarrow \tau'}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct let}_{\mathrm{L}}\ x} \mathsf{let}\ x = \triangleright\llparenthesis\rrparenthesis\triangleleft\ \mathsf{in}\ \check{e}' \Rightarrow \tau'}$$

$$\textsc{ASEConNum}\quad \frac{}{\Gamma \vdash \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow ? \xrightarrow{\text{construct lit } \underline{n}} \triangleright\underline{n}\triangleleft \Rightarrow \mathsf{num}}$$

$$\textsc{ASEConPlusL}\quad \frac{\Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \mathsf{num}}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct plus}_{\mathsf{L}}} \check{e}' + \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \mathsf{num}}$$

$$\textsc{ASEConPlusR}\quad \frac{\Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \mathsf{num}}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct plus}_{\mathsf{R}}} \triangleright\llparenthesis\rrparenthesis\triangleleft + \check{e}' \Rightarrow \mathsf{num}}$$

$$\textsc{ASEConIfC}\quad \frac{\Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \mathsf{bool}}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct if}_{\mathsf{C}}} \mathsf{if}\ \check{e}'\ \mathsf{then}\ \triangleright\llparenthesis\rrparenthesis\triangleleft\ \mathsf{else}\ \llparenthesis\rrparenthesis \Rightarrow ?}$$

$$\textsc{ASEConIfL}\quad \frac{}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct if}_{\mathsf{L}}} \mathsf{if}\ \triangleright\llparenthesis\rrparenthesis\triangleleft\ \mathsf{then}\ \check{e}\ \mathsf{else}\ \llparenthesis\rrparenthesis \Rightarrow \tau}$$

$$\textsc{ASEConIfR}\quad \frac{}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct if}_{\mathsf{R}}} \mathsf{if}\ \triangleright\llparenthesis\rrparenthesis\triangleleft\ \mathsf{then}\ \llparenthesis\rrparenthesis\ \mathsf{else}\ \check{e} \Rightarrow \tau}$$

$$\textsc{ASEConPairL}\quad \frac{}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct pair}_{\mathsf{L}}} (\triangleright\check{e}\triangleleft, \llparenthesis\rrparenthesis) \Rightarrow \tau \times ?}$$

$$\textsc{ASEConPairR}\quad \frac{}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct pair}_{\mathsf{R}}} (\llparenthesis\rrparenthesis, \triangleright\check{e}\triangleleft) \Rightarrow ? \times \tau}$$

$$\textsc{ASEConProjL}\quad \frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct proj}_{\mathsf{L}}} \pi_1 \triangleright\check{e}\triangleleft \Rightarrow \tau_1}$$

$$\textsc{ASEConProjL2}\quad \frac{\tau \blacktriangleright_{\not\times}}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct proj}_{\mathsf{L}}} \pi_1 \llparenthesis \triangleright\check{e}\triangleleft \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\times}} \Rightarrow ?}$$

$$\textsc{ASEConProjR1}\quad \frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct proj}_{\mathsf{R}}} \pi_2 \triangleright\check{e}\triangleleft \Rightarrow \tau_2}$$

$$\textsc{ASEConProjR2}\quad \frac{\tau \blacktriangleright_{\not\times}}{\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\text{construct proj}_{\mathsf{R}}} \pi_2 \llparenthesis \triangleright\check{e}\triangleleft \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\times}} \Rightarrow ?}$$

Zipper Cases

$$\textsc{ASEZipLamT1}\quad \frac{\underline{\tau_1} \xrightarrow{\alpha} \underline{\tau_1'} \qquad \underline{\tau_1}^{\diamond} = \underline{\tau_1'}^{\diamond}}{\Gamma \vdash \lambda x : \underline{\tau_1}.\, \check{e} \Rightarrow \underline{\tau_1}^{\diamond} \to \tau_2 \xrightarrow{\alpha} \lambda x : \underline{\tau_1'}.\, \check{e} \Rightarrow \underline{\tau_1}^{\diamond} \to \tau_2}$$

$$\textsc{ASEZipLamT2}\quad \frac{\underline{\tau_1} \xrightarrow{\alpha} \underline{\tau_1'} \qquad \underline{\tau_1}^{\diamond} \neq \underline{\tau_1'}^{\diamond} \qquad \Gamma, x : \underline{\tau_1'}^{\diamond} \vdash \check{e}^{\square} \looparrowright \check{e}' \Rightarrow \tau_2'}{\Gamma \vdash \lambda x : \underline{\tau_1}.\, \check{e} \Rightarrow \underline{\tau_1}^{\diamond} \to \tau_2 \xrightarrow{\alpha} \lambda x : \underline{\tau_1'}.\, \check{e}' \Rightarrow \underline{\tau_1'}^{\diamond} \to \tau_2'}$$

$$\textsc{ASEZipLamE}\quad \frac{\Gamma, x : \tau_1 \vdash \underline{\check{e}} \Rightarrow \tau_2 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_2'}{\Gamma \vdash \lambda x : \tau_1.\, \underline{\check{e}} \Rightarrow \tau_1 \to \tau_2 \xrightarrow{\alpha} \lambda x : \tau_1.\, \underline{\check{e}'} \Rightarrow \tau_1 \to \tau_2'}$$

$$\textsc{ASEZipApL1}\quad \frac{\Gamma \models_{M} \underline{\check{e}_1}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}_1} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}_1'} \Rightarrow \tau_1' \qquad \tau_1' \blacktriangleright_{\to} \tau_2 \to \tau_3 \qquad \Gamma \vdash \check{e}_2 \Leftarrow \tau_2}{\Gamma \vdash \underline{\check{e}_1}\ \check{e}_2 \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}_1'}\ \check{e}_2 \Rightarrow \tau_3}$$

$$\textsc{ASEZipApL2}\quad \frac{\Gamma \models_{M} \underline{\check{e}_1}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}_1} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}_1'} \Rightarrow \tau_1' \qquad \tau_1' \blacktriangleright_{\to} \tau_2 \to \tau_3 \qquad \Gamma \vdash \check{e}_2 \not\Leftarrow \tau_2}{\Gamma \vdash \underline{\check{e}_1}\ \check{e}_2 \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}_1'}\ \llparenthesis \check{e}_2 \rrparenthesis_{\not\sim} \Rightarrow \tau_3}$$

$$\textsc{ASEZipApL3}\quad \frac{\Gamma \models_{M} \underline{\check{e}_1}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}_1} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}_1'} \Rightarrow \tau_1' \qquad \tau_1' \blacktriangleright_{\not\to}}{\Gamma \vdash \underline{\check{e}_1}\ \check{e}_2 \Rightarrow \tau \xrightarrow{\alpha} \llparenthesis \underline{\check{e}_1'} \rrparenthesis^{\Rightarrow}_{\blacktriangleright_{\not\to}}\ \check{e}_2 \Rightarrow ?}$$

$$\textsc{ASEZipApL4}\quad \frac{\Gamma \models_{M} \underline{\check{e}_1}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}_1} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}_1'} \Rightarrow \tau_1' \qquad \tau_1' \blacktriangleright_{\to} \tau_2 \to \tau_3 \qquad \Gamma \vdash \check{e}_2 \Leftarrow \tau_2}{\Gamma \vdash \llparenthesis \underline{\check{e}_1} \rrparenthesis_{\not\sim}\ \check{e}_2 \Rightarrow ? \xrightarrow{\alpha} \underline{\check{e}_1'}\ \check{e}_2 \Rightarrow \tau_3}$$

$$\textsc{ASEZipApL5}\quad \frac{\Gamma \models_{M} \underline{\check{e}_1}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}_1} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}_1'} \Rightarrow \tau_1' \qquad \tau_1' \blacktriangleright_{\to} \tau_2 \to \tau_3 \qquad \Gamma \vdash \check{e}_2 \not\Leftarrow \tau_2}{\Gamma \vdash \llparenthesis \underline{\check{e}_1} \rrparenthesis_{\not\sim}\ \check{e}_2 \Rightarrow ? \xrightarrow{\alpha} \underline{\check{e}_1'}\ \llparenthesis \check{e}_2 \rrparenthesis_{\not\sim} \Rightarrow \tau_3}$$

$$\textsc{ASEZipApL6}\quad \frac{\Gamma \models_{M} \underline{\check{e}_1}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}_1} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}_1'} \Rightarrow \tau_1' \qquad \tau_1' \blacktriangleright_{\not\to}}{\Gamma \vdash \llparenthesis \underline{\check{e}_1} \rrparenthesis_{\not\sim}\ \check{e}_2 \Rightarrow ? \xrightarrow{\alpha} \llparenthesis \underline{\check{e}_1'} \rrparenthesis_{\not\sim}\ \check{e}_2 \Rightarrow ?}$$

$$\textsc{ASEZipApR1}\quad \frac{\Gamma \models_{M} \check{e}_1 \Rightarrow \tau_1 \qquad \tau_1 \blacktriangleright_{\to} \tau_2 \to \tau_3 \qquad \Gamma \vdash \underline{\check{e}_2} \xrightarrow{\alpha} \underline{\check{e}_2'} \Leftarrow \tau_2}{\Gamma \vdash \check{e}_1\ \underline{\check{e}_2} \Rightarrow \tau \xrightarrow{\alpha} \check{e}_1\ \underline{\check{e}_2'} \Rightarrow \tau_3}$$

ASEZipApR2
$$\frac{\Gamma \vdash \underline{\check{e}_2} \xrightarrow{\alpha} \underline{\check{e}_2'} \Leftarrow ?}{\Gamma \vdash (\!|\check{e}_1|\!)^{\rightarrow}_{\blacktriangleright\nrightarrow}\ \underline{\check{e}_2} \Rightarrow ? \xrightarrow{\alpha} (\!|\check{e}_1|\!)^{\rightarrow}_{\blacktriangleright\nrightarrow}\ \underline{\check{e}_2'} \Rightarrow ?}$$

ASEZipLetL1
$$\frac{\Gamma \vdash_M \check{e}_1^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}_1} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}_1'} \Rightarrow \tau_1' \qquad \tau_1 = \tau_1'}{\Gamma \vdash \mathsf{let}\ x = \underline{\check{e}_1}\ \mathsf{in}\ \check{e}_2 \Rightarrow \tau_2 \xrightarrow{\alpha} \mathsf{let}\ x = \underline{\check{e}_1'}\ \mathsf{in}\ \check{e}_2 \Rightarrow \tau_2}$$

ASEZipLetL2
$$\frac{\begin{array}{c}\Gamma \vdash_M \check{e}_1^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}_1} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}_1'} \Rightarrow \tau_1' \\ \tau_1 \neq \tau_1' \qquad \Gamma,\ x : \tau_1' \vdash \check{e}_2^{\square} \looparrowright \check{e}_2' \Rightarrow \tau_2'\end{array}}{\Gamma \vdash \mathsf{let}\ x = \underline{\check{e}_1}\ \mathsf{in}\ \check{e}_2 \Rightarrow \tau_2 \xrightarrow{\alpha} \mathsf{let}\ x = \underline{\check{e}_1'}\ \mathsf{in}\ \check{e}_2' \Rightarrow \tau_2'}$$

ASEZipLetR
$$\frac{\begin{array}{c}\Gamma \vdash_M \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma,\ x : \tau_1 \vdash_M \check{e}_2^{\diamond} \Rightarrow \tau_2 \\ \Gamma,\ x : \tau_1 \vdash \underline{\check{e}_2} \Rightarrow \tau_2 \xrightarrow{\alpha} \underline{\check{e}_2'} \Rightarrow \tau_2'\end{array}}{\Gamma \vdash \mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \underline{\check{e}_2} \Rightarrow \tau_2 \xrightarrow{\alpha} \mathsf{let}\ x = \check{e}_1\ \mathsf{in}\ \underline{\check{e}_2'} \Rightarrow \tau_2'}$$

ASEZipPlusL
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}'} \Leftarrow \mathsf{num}}{\Gamma \vdash \underline{\check{e}} + \check{e} \Rightarrow \mathsf{num} \xrightarrow{\alpha} \underline{\check{e}'} + \check{e} \Rightarrow \mathsf{num}}$$

ASEZipPlusR
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}'} \Leftarrow \mathsf{num}}{\Gamma \vdash \check{e} + \underline{\check{e}} \Rightarrow \mathsf{num} \xrightarrow{\alpha} \check{e} + \underline{\check{e}'} \Rightarrow \mathsf{num}}$$

ASEZipIfC
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}'} \Leftarrow \mathsf{bool}}{\Gamma \vdash \mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2 \Rightarrow \tau \xrightarrow{\alpha} \mathsf{if}\ \underline{\check{e}'}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2 \Rightarrow \tau}$$

ASEZipIfL1
$$\frac{\begin{array}{c}\Gamma \vdash_M \check{e}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash_M \check{e}_2 \Rightarrow \tau_2 \\ \Gamma \vdash \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_1' \qquad \tau_1' \sim \tau_2 \qquad \tau' = \tau_1' \sqcup \tau_2\end{array}}{\Gamma \vdash \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2 \Rightarrow \tau \xrightarrow{\alpha} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}'}\ \mathsf{else}\ \check{e}_2 \Rightarrow \tau'}$$

ASEZipIfL2
$$\frac{\begin{array}{c}\Gamma \vdash_M \check{e}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash_M \check{e}_2 \Rightarrow \tau_2 \\ \Gamma \vdash \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_1' \qquad \tau_1' \nsim \tau_2\end{array}}{\Gamma \vdash \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2 \Rightarrow \tau \xrightarrow{\alpha} (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}'}\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup} \Rightarrow ?}$$

ASEZipIfR1
$$\frac{\begin{array}{c}\Gamma \vdash_M \check{e}_2 \Rightarrow \tau_1 \qquad \Gamma \vdash_M \check{e}^{\diamond} \Rightarrow \tau_2 \\ \Gamma \vdash \underline{\check{e}} \Rightarrow \tau_2 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_2' \qquad \tau_1 \sim \tau_2' \qquad \tau' = \tau_1 \sqcup \tau_2'\end{array}}{\Gamma \vdash \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}'} \Rightarrow \tau'}$$

ASEZipIfR2
$$\frac{\begin{array}{c}\Gamma \vdash_M \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma \vdash_M \check{e}^{\diamond} \Rightarrow \tau_2 \\ \Gamma \vdash \underline{\check{e}} \Rightarrow \tau_2 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_2' \qquad \tau_1 \nsim \tau_2'\end{array}}{\Gamma \vdash \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}'}|\!)_{\not\sqcup} \Rightarrow ?}$$

ASEZipInconsistentBranchesC
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}'} \Leftarrow \mathsf{bool}}{\Gamma \vdash (\!|\mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup} \Rightarrow \tau \xrightarrow{\alpha} (\!|\mathsf{if}\ \underline{\check{e}'}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup} \Rightarrow \tau}$$

ASEZipInconsistentBranchesL1
$$\frac{\begin{array}{c}\Gamma \vdash \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_1' \\ \Gamma \vdash_M \check{e}_2 \Rightarrow \tau_2 \qquad \tau_1' \sim \tau_2 \qquad \tau' = \tau_1' \sqcup \tau_2\end{array}}{\Gamma \vdash (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup} \Rightarrow \tau \xrightarrow{\alpha} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}'}\ \mathsf{else}\ \check{e}_2 \Rightarrow \tau'}$$

ASEZipInconsistentBranchesL2
$$\frac{\begin{array}{c}\Gamma \vdash \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_1' \\ \Gamma \vdash_M \check{e}_2 \Rightarrow \tau_2 \qquad \tau_1' \nsim \tau_2\end{array}}{\Gamma \vdash (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup} \Rightarrow \tau \xrightarrow{\alpha} (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}'}\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup} \Rightarrow ?}$$

ASEZipInconsistentBranchesR1

$$\frac{\Gamma \vdash_{\overline{M}} \check{e}_1 \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}} \Rightarrow \tau_2 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_2' \qquad \tau_1 \sim \tau_2' \qquad \tau' = \tau_1 \sqcup \tau_2'}{\Gamma \vdash (\!|\text{if } \check{e}_1 \text{ then } \check{e}_2 \text{ else } \underline{\check{e}}|\!)_{\not\sqcup} \Rightarrow \tau \xrightarrow{\alpha} \text{if } \check{e}_1 \text{ then } \check{e}_2 \text{ else } \underline{\check{e}'} \Rightarrow \tau'}$$

ASEZipInconsistentBranchesR2

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau_2 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_2' \qquad \Gamma \vdash_{\overline{M}} \check{e}_2 \Rightarrow \tau_1 \qquad \tau_1 \nsim \tau_2'}{\Gamma \vdash (\!|\text{if } \check{e}_1 \text{ then } \check{e}_2 \text{ else } \underline{\check{e}}|\!)_{\not\sqcup} \Rightarrow \tau \xrightarrow{\alpha} (\!|\text{if } \check{e}_1 \text{ then } \check{e}_2 \text{ else } \underline{\check{e}'}|\!)_{\not\sqcup} \Rightarrow ?}$$

ASEZipPairL

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_1'}{\Gamma \vdash (\underline{\check{e}}, \check{e}) \Rightarrow \tau_1 \times \tau_2 \xrightarrow{\alpha} (\underline{\check{e}'}, \check{e}) \Rightarrow \tau_1' \times \tau_2}$$

ASEZipPairR

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau_2 \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau_2'}{\Gamma \vdash (\check{e}, \underline{\check{e}}) \Rightarrow \tau_1 \times \tau_2 \xrightarrow{\alpha} (\check{e}, \underline{\check{e}'}) \Rightarrow \tau_1' \times \tau_2}$$

ASEZipProjL1

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau' \qquad \tau' \blacktriangleright_{\times} \tau_1' \times \tau_2'}{\Gamma \vdash \pi_1 \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \pi_1 \underline{\check{e}'} \Rightarrow \tau_1'}$$

ASEZipProjL2

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau' \qquad \tau' \not\blacktriangleright_{\times}}{\Gamma \vdash \pi_1 \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \pi_1 (\!|\underline{\check{e}'}|\!)^{\Rightarrow}_{\not\blacktriangleright_{\times}} \Rightarrow ?}$$

ASEZipProjL3

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau' \qquad \tau' \blacktriangleright_{\times} \tau_1' \times \tau_2'}{\Gamma \vdash \pi_1 (\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\not\blacktriangleright_{\times}} \Rightarrow ? \xrightarrow{\alpha} \pi_1 \underline{\check{e}'} \Rightarrow \tau_1'}$$

ASEZipProjL4

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau' \qquad \tau' \not\blacktriangleright_{\times}}{\Gamma \vdash \pi_1 (\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\not\blacktriangleright_{\times}} \Rightarrow ? \xrightarrow{\alpha} \pi_1 (\!|\underline{\check{e}'}|\!)^{\Rightarrow}_{\not\blacktriangleright_{\times}} \Rightarrow ?}$$

ASEZipProjR1

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau' \qquad \tau' \blacktriangleright_{\times} \tau_1' \times \tau_2'}{\Gamma \vdash \pi_2 \underline{\check{e}} \Rightarrow \tau_2 \xrightarrow{\alpha} \pi_2 \underline{\check{e}'} \Rightarrow \tau_2'}$$

ASEZipProjR2

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau' \qquad \tau' \not\blacktriangleright_{\times}}{\Gamma \vdash \pi_2 \underline{\check{e}} \Rightarrow \tau_2 \xrightarrow{\alpha} \pi_2 (\!|\underline{\check{e}'}|\!)^{\Rightarrow}_{\not\blacktriangleright_{\times}} \Rightarrow ?}$$

ASEZipProjL3

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau' \qquad \tau' \blacktriangleright_{\times} \tau_1' \times \tau_2'}{\Gamma \vdash \pi_2 (\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\not\blacktriangleright_{\times}} \Rightarrow ? \xrightarrow{\alpha} \pi_2 \underline{\check{e}'} \Rightarrow \tau_2'}$$

ASEZipProjR4

$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau' \qquad \tau' \not\blacktriangleright_{\times}}{\Gamma \vdash \pi_2 (\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\not\blacktriangleright_{\times}} \Rightarrow ? \xrightarrow{\alpha} \pi_2 (\!|\underline{\check{e}'}|\!)^{\Rightarrow}_{\not\blacktriangleright_{\times}} \Rightarrow ?}$$

F.3.5 Analytic expression actions

$$\boxed{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}'} \Leftarrow \tau'}$$

Subsumption

AAESubsume1

$$\frac{\Gamma \vdash_{\overline{M}} \underline{\check{e}}^{\diamond} \Rightarrow \tau' \qquad \Gamma \vdash \underline{\check{e}} \Rightarrow \tau' \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau'' \qquad \tau \sim \tau'' \qquad \underline{\check{e}}^{\diamond} \text{ subsumable}}{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}'} \Leftarrow \tau}$$

AAESubsume2

$$\frac{\Gamma \vdash_{\overline{M}} \underline{\check{e}}^{\diamond} \Rightarrow \tau' \qquad \Gamma \vdash \underline{\check{e}} \Rightarrow \tau' \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau'' \qquad \tau \nsim \tau'' \qquad \underline{\check{e}}^{\diamond} \text{ subsumable}}{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} (\!|\underline{\check{e}'}|\!)_{\nsim} \Leftarrow \tau}$$

AAEInconsistentTypes1

$$\frac{\Gamma \vdash_{\overline{M}} \underline{\check{e}}^{\diamond} \Rightarrow \tau' \qquad \Gamma \vdash \underline{\check{e}} \Rightarrow \tau' \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau'' \qquad \tau \sim \tau'' \qquad \underline{\check{e}}^{\diamond} \text{ subsumable}}{\Gamma \vdash (\!|\underline{\check{e}}|\!)_{\nsim} \xrightarrow{\alpha} \underline{\check{e}'} \Leftarrow \tau}$$

AAEInconsistentTypes2

$$\frac{\Gamma \vdash_{\overline{M}} \underline{\check{e}}^{\diamond} \Rightarrow \tau' \qquad \Gamma \vdash \underline{\check{e}} \Rightarrow \tau' \xrightarrow{\alpha} \underline{\check{e}'} \Rightarrow \tau'' \qquad \tau \nsim \tau'' \qquad \underline{\check{e}}^{\diamond} \text{ subsumable}}{\Gamma \vdash (\!|\underline{\check{e}}|\!)_{\nsim} \xrightarrow{\alpha} (\!|\underline{\check{e}'}|\!)_{\nsim} \Leftarrow \tau}$$

Movement

AAEMove

$$\frac{\underline{\check{e}} \xrightarrow{\text{move } \delta} \underline{\check{e}'}}{\Gamma \vdash \underline{\check{e}} \xrightarrow{\text{move } \delta} \underline{\check{e}'} \Leftarrow \tau}$$

Deletion

AAEDel

$$\frac{}{\Gamma \vdash \triangleright \check{e} \triangleleft \xrightarrow{\text{del}} \triangleright (\!|\,|\!) \triangleleft \Leftarrow \tau}$$

Construction

AAEConLam1

$$\frac{\tau \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \Gamma,\ x : \tau_1 \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau_2}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct lam } x} \lambda x : \triangleright\tau_1\triangleleft.\ \check{e}' \Leftarrow \tau}$$

AAEConLam2

$$\frac{\tau \blacktriangleright_{\not\rightarrow} \qquad \Gamma,\ x : ? \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow ?}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct lam } x} \llparenthesis \lambda x : \triangleright ? \triangleleft.\ \check{e}' \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\rightarrow}} \Leftarrow \tau}$$

AAEConLetL

$$\frac{\Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Rightarrow \tau}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct let}_{\mathsf{L}}\ x} \text{let } x = \check{e}' \text{ in } \triangleright\llparenthesis\rrparenthesis\triangleleft \Leftarrow \tau}$$

AAEConLetR

$$\frac{\Gamma,\ x : ? \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct let}_{\mathsf{R}}\ x} \text{let } x = \triangleright\llparenthesis\rrparenthesis\triangleleft \text{ in } \check{e}' \Leftarrow \tau}$$

AAEConIfC

$$\frac{\Gamma \vdash \check{e} \looparrowright \check{e}' \Leftarrow \text{bool}}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct if}_{\mathsf{C}}} \text{if } \check{e}' \text{ then } \triangleright\llparenthesis\rrparenthesis\triangleleft \text{ else } \llparenthesis\rrparenthesis \Leftarrow \tau}$$

AAEConIfL

$$\frac{}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct if}_{\mathsf{L}}} \text{if } \triangleright\llparenthesis\rrparenthesis\triangleleft \text{ then } \check{e}' \text{ else } \llparenthesis\rrparenthesis \Leftarrow \tau}$$

AAEConIfR

$$\frac{}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct if}_{\mathsf{R}}} \text{if } \triangleright\llparenthesis\rrparenthesis\triangleleft \text{ then } \llparenthesis\rrparenthesis \text{ else } \check{e}' \Leftarrow \tau}$$

AAEConPairL1

$$\frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau_1}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct pair}_{\mathsf{L}}} (\check{e}', \triangleright\llparenthesis\rrparenthesis\triangleleft) \Leftarrow \tau}$$

AAEConPairL2

$$\frac{\tau \blacktriangleright_{\not\times} \qquad \Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow ?}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct pair}_{\mathsf{L}}} \llparenthesis(\check{e}', \triangleright\llparenthesis\rrparenthesis\triangleleft)\rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\times}} \Leftarrow \tau}$$

AAEConPairR1

$$\frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau_2}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct pair}_{\mathsf{R}}} (\triangleright\llparenthesis\rrparenthesis\triangleleft, \check{e}') \Leftarrow \tau}$$

AAEConPairR2

$$\frac{\tau \blacktriangleright_{\not\times} \qquad \Gamma \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow ?}{\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\text{construct pair}_{\mathsf{R}}} \llparenthesis(\triangleright\llparenthesis\rrparenthesis\triangleleft, \check{e}')\rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\times}} \Leftarrow \tau}$$

Zipper Cases

AAEZipLamT1

$$\frac{\underline{\tau_3} \xrightarrow{\alpha} \underline{\tau_3'} \qquad \underline{\tau_3}^{\diamond} = \underline{\tau_3'}^{\diamond}}{\Gamma \vdash \lambda x : \underline{\tau_3}.\ \check{e} \xrightarrow{\alpha} \lambda x : \underline{\tau_3'}.\ \check{e} \Leftarrow \tau}$$

AAEZipLamT2

$$\frac{\underline{\tau_3} \xrightarrow{\alpha} \underline{\tau_3'} \qquad \underline{\tau_3}^{\diamond} \neq \underline{\tau_3'}^{\diamond} \qquad \tau \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \underline{\tau_3'}^{\diamond} \sim \tau_1 \qquad \Gamma,\ x : \underline{\tau_3'}^{\diamond} \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau_2}{\Gamma \vdash \lambda x : \underline{\tau_3}.\ \check{e} \xrightarrow{\alpha} \lambda x : \underline{\tau_3'}.\ \check{e}' \Leftarrow \tau}$$

AAEZipLamT3

$$\frac{\underline{\tau_3} \xrightarrow{\alpha} \underline{\tau_3'} \qquad \underline{\tau_3}^{\diamond} \neq \underline{\tau'}^{\diamond} \qquad \tau \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \underline{\tau_3'}^{\diamond} \nsim \tau_1 \qquad \Gamma,\ x : \underline{\tau_3'}^{\diamond} \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau_2}{\Gamma \vdash \lambda x : \underline{\tau_3}.\ \check{e} \xrightarrow{\alpha} \llparenthesis \lambda x : \underline{\tau_3'}.\ \check{e}' \rrparenthesis_{:} \Leftarrow \tau}$$

AAEZipLamT4

$$\frac{\underline{\tau_3} \xrightarrow{\alpha} \underline{\tau_3'} \qquad \underline{\tau_3}^{\diamond} = \underline{\tau_3'}^{\diamond}}{\Gamma \vdash \llparenthesis \lambda x : \underline{\tau_3}.\ \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\rightarrow}} \xrightarrow{\alpha} \llparenthesis \lambda x : \underline{\tau_3'}.\ \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\rightarrow}} \Leftarrow \tau}$$

AAEZipLamT5

$$\frac{\underline{\tau_3} \xrightarrow{\alpha} \underline{\tau_3'} \qquad \underline{\tau_3}^{\diamond} \neq \underline{\tau_3'}^{\diamond} \qquad \Gamma,\ x : \underline{\tau_3'}^{\diamond} \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow ?}{\Gamma \vdash \llparenthesis \lambda x : \underline{\tau_3}.\ \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\rightarrow}} \xrightarrow{\alpha} \llparenthesis \lambda x : \underline{\tau_3'}.\ \check{e} \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\rightarrow}} \Leftarrow \tau}$$

AAEZipLamT6

$$\frac{\underline{\tau_3} \xrightarrow{\alpha} \underline{\tau_3'} \qquad \underline{\tau_3}^{\diamond} = \underline{\tau_3'}^{\diamond}}{\Gamma \vdash \llparenthesis \lambda x : \underline{\tau_3}.\ \check{e} \rrparenthesis_{:} \xrightarrow{\alpha} \llparenthesis \lambda x : \underline{\tau_3'}.\ \check{e} \rrparenthesis_{:} \Leftarrow \tau}$$

AAEZipLamT7

$$\frac{\underline{\tau_3} \xrightarrow{\alpha} \underline{\tau_3'} \qquad \underline{\tau_3}^{\diamond} \neq \underline{\tau_3'}^{\diamond} \qquad \tau \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \underline{\tau_3'}^{\diamond} \sim \tau_1 \qquad \Gamma,\ x : \underline{\tau_3'}^{\diamond} \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau_2}{\Gamma \vdash \llparenthesis \lambda x : \underline{\tau_3}.\ \check{e} \rrparenthesis_{:} \xrightarrow{\alpha} \lambda x : \underline{\tau_3'}.\ \check{e}' \Leftarrow \tau}$$

AAEZipLamT8

$$\frac{\underline{\tau_3} \xrightarrow{\alpha} \underline{\tau_3'} \qquad \underline{\tau_3}^{\diamond} \neq \underline{\tau_3'}^{\diamond} \qquad \tau \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \underline{\tau_3'}^{\diamond} \nsim \tau_1 \qquad \Gamma,\ x : \underline{\tau_3'}^{\diamond} \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau_2}{\Gamma \vdash \llparenthesis \lambda x : \underline{\tau_3}.\ \check{e} \rrparenthesis_{:} \xrightarrow{\alpha} \llparenthesis \lambda x : \underline{\tau_3'}.\ \check{e}' \rrparenthesis_{:} \Leftarrow \tau}$$

AAEZipLamE1
$$\frac{\tau \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \Gamma,\ x : \tau_3 \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau_2}{\Gamma \vdash \lambda x : \tau_3 .\ \underline{\check{e}} \xrightarrow{\alpha} \lambda x : \tau_3 .\ \underline{\check{e}}' \Leftarrow \tau}$$

AAEZipLamE2
$$\frac{\Gamma,\ x : \tau_3 \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow ?}{\Gamma \vdash \llparenthesis \lambda x : \tau_3 .\ \underline{\check{e}} \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\rightarrow}} \xrightarrow{\alpha} \llparenthesis \lambda x : \tau_3 .\ \underline{\check{e}}' \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\not\rightarrow}} \Leftarrow \tau}$$

AAEZipLamE3
$$\frac{\tau \blacktriangleright_{\rightarrow} \tau_1 \to \tau_2 \qquad \Gamma,\ x : \tau_3 \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau_2}{\Gamma \vdash \llparenthesis \lambda x : \tau_3 .\ \underline{\check{e}} \rrparenthesis_{:} \xrightarrow{\alpha} \llparenthesis \lambda x : \tau_3 .\ \underline{\check{e}}' \rrparenthesis_{:} \Leftarrow \tau}$$

AAEZipLetL1
$$\frac{\Gamma \vdash_{M} \check{e}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}}' \Rightarrow \tau_1' \qquad \tau_1 = \tau_1'}{\Gamma \vdash \mathsf{let}\ x = \underline{\check{e}}\ \mathsf{in}\ \check{e} \xrightarrow{\alpha} \mathsf{let}\ x = \underline{\check{e}}'\ \mathsf{in}\ \check{e} \Leftarrow \tau}$$

AAEZipLetL2
$$\frac{\Gamma \vdash_{M} \check{e}^{\diamond} \Rightarrow \tau_1 \qquad \Gamma \vdash \underline{\check{e}} \Rightarrow \tau_1 \xrightarrow{\alpha} \underline{\check{e}}' \Rightarrow \tau_1' \qquad \tau_1 \neq \tau_1' \qquad \Gamma,\ x : \tau_1' \vdash \check{e}^{\square} \looparrowright \check{e}' \Leftarrow \tau}{\Gamma \vdash \mathsf{let}\ x = \underline{\check{e}}\ \mathsf{in}\ \check{e} \xrightarrow{\alpha} \mathsf{let}\ x = \underline{\check{e}}'\ \mathsf{in}\ \check{e}' \Leftarrow \tau}$$

AAEZipLetR
$$\frac{\Gamma \vdash_{M} \check{e} \Rightarrow \tau_1 \qquad \Gamma,\ x : \tau_1 \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau}{\Gamma \vdash \mathsf{let}\ x = \check{e}\ \mathsf{in}\ \underline{\check{e}} \xrightarrow{\alpha} \mathsf{let}\ x = \check{e}\ \mathsf{in}\ \underline{\check{e}}' \Leftarrow \tau}$$

AAEZipIfC
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \mathsf{bool}}{\Gamma \vdash \mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2 \xrightarrow{\alpha} \mathsf{if}\ \underline{\check{e}}'\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2 \Leftarrow \tau}$$

AAEZipIfL
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau}{\Gamma \vdash \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2 \xrightarrow{\alpha} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}'\ \mathsf{else}\ \check{e}_2 \Leftarrow \tau}$$

AAEZipIfR
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau}{\Gamma \vdash \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}} \xrightarrow{\alpha} \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}}' \Leftarrow \tau}$$

AAEZipPairL1
$$\frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau_1}{\Gamma \vdash (\underline{\check{e}}, \check{e}) \xrightarrow{\alpha} (\underline{\check{e}}', \check{e}) \Leftarrow \tau}$$

AAEZipPairL2
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow ?}{\Gamma \vdash \llparenthesis (\underline{\check{e}}, \check{e}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\times}} \xrightarrow{\alpha} \llparenthesis (\underline{\check{e}}', \check{e}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\times}} \Leftarrow \tau}$$

AAEZipPairR1
$$\frac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \qquad \Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau_2}{\Gamma \vdash (\check{e}, \underline{\check{e}}) \xrightarrow{\alpha} (\check{e}, \underline{\check{e}}') \Leftarrow \tau}$$

AAEZipPairR2
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow ?}{\Gamma \vdash \llparenthesis (\check{e}, \underline{\check{e}}) \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\times}} \xrightarrow{\alpha} \llparenthesis (\check{e}, \underline{\check{e}}') \rrparenthesis^{\Leftarrow}_{\blacktriangleright_{\times}} \Leftarrow \tau}$$

F.3.6 Iterated actions

The iterated type action and movements judgments are the same as in the untyped model.

$$\boxed{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\overline{\alpha}}{}^{*} \underline{\check{e}}' \Rightarrow \tau'}$$

ASEIRefl
$$\frac{}{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\cdot}{}^{*} \underline{\check{e}} \Rightarrow \tau}$$

ASEIExp
$$\frac{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}}' \Rightarrow \tau' \qquad \Gamma \vdash \underline{\check{e}}' \Rightarrow \tau' \xrightarrow{\overline{\alpha}}{}^{*} \underline{\check{e}}'' \Rightarrow \tau''}{\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha;\overline{\alpha}}{}^{*} \underline{\check{e}}'' \Rightarrow \tau''}$$

$$\boxed{\Gamma \vdash \underline{\check{e}} \xrightarrow{\overline{\alpha}}{}^{*} \underline{\check{e}}' \Leftarrow \tau}$$

AAEIRefl
$$\frac{}{\Gamma \vdash \underline{\check{e}} \xrightarrow{\cdot}{}^{*} \underline{\check{e}} \Leftarrow \tau}$$

AAEIExp
$$\frac{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau' \qquad \Gamma \vdash \underline{\check{e}}' \xrightarrow{\overline{\alpha}}{}^{*} \underline{\check{e}}'' \Leftarrow \tau''}{\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha;\overline{\alpha}}{}^{*} \underline{\check{e}}'' \Leftarrow \tau''}$$

F.4 Mark erasure

$\boxed{\underline{\check{e}}^{\Box}}$ is a metafunction ZMExp → ZExp defined as follows:

$$
\begin{aligned}
{\triangleright\check{e}\triangleleft}^{\Box} &= \triangleright\check{e}^{\Box}\triangleleft \\
(\lambda x : \underline{\tau}.\, \check{e})^{\Box} &= \lambda x : \underline{\tau}.\, (\check{e}^{\Box}) \\
(\lambda x : \tau.\, \underline{\check{e}})^{\Box} &= \lambda x : \tau.\, (\underline{\check{e}}^{\Box}) \\
{(\!|\lambda x : \underline{\tau}.\, \check{e}|\!)^{\Leftarrow}_{\blacktriangleright\not\to}}^{\Box} &= \lambda x : \underline{\tau}.\, (\check{e}^{\Box}) \\
{(\!|\lambda x : \tau.\, \underline{\check{e}}|\!)^{\Leftarrow}_{\blacktriangleright\not\to}}^{\Box} &= \lambda x : \tau.\, (\underline{\check{e}}^{\Box}) \\
{(\!|\lambda x : \underline{\tau}.\, \check{e}|\!)_{:}}^{\Box} &= \lambda x : \underline{\tau}.\, (\check{e}^{\Box}) \\
{(\!|\lambda x : \tau.\, \underline{\check{e}}|\!)_{:}}^{\Box} &= \lambda x : \tau.\, (\underline{\check{e}}^{\Box}) \\
(\underline{\check{e}}\ \check{e})^{\Box} &= (\underline{\check{e}}^{\Box})\ (\check{e}^{\Box}) \\
(\check{e}\ \underline{\check{e}})^{\Box} &= (\check{e}^{\Box})\ (\underline{\check{e}}^{\Box}) \\
((\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright\not\to}\ \check{e})^{\Box} &= \underline{\check{e}}^{\Box}\ (\check{e}^{\Box}) \\
((\!|\check{e}|\!)^{\Rightarrow}_{\blacktriangleright\not\to}\ \underline{\check{e}})^{\Box} &= \check{e}^{\Box}\ (\underline{\check{e}}^{\Box}) \\
(\mathsf{let}\ x = \underline{\check{e}}\ \mathsf{in}\ \check{e})^{\Box} &= \mathsf{let}\ x = (\underline{\check{e}}^{\Box})\ \mathsf{in}\ (\check{e}^{\Box}) \\
(\mathsf{let}\ x = \check{e}\ \mathsf{in}\ \underline{\check{e}})^{\Box} &= \mathsf{let}\ x = (\check{e}^{\Box})\ \mathsf{in}\ (\underline{\check{e}}^{\Box}) \\
(\underline{\check{e}} + \check{e})^{\Box} &= (\underline{\check{e}}^{\Box}) + (\check{e}^{\Box}) \\
(\check{e} + \underline{\check{e}})^{\Box} &= (\check{e}^{\Box}) + (\underline{\check{e}}^{\Box}) \\
(\mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2)^{\Box} &= \mathsf{if}\ (\underline{\check{e}}^{\Box})\ \mathsf{then}\ (\check{e}_1^{\Box})\ \mathsf{else}\ (\check{e}_2^{\Box}) \\
(\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2)^{\Box} &= \mathsf{if}\ (\check{e}_1^{\Box})\ \mathsf{then}\ (\underline{\check{e}}^{\Box})\ \mathsf{else}\ (\check{e}_2^{\Box}) \\
(\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}})^{\Box} &= \mathsf{if}\ (\check{e}_1^{\Box})\ \mathsf{then}\ (\check{e}_2^{\Box})\ \mathsf{else}\ (\underline{\check{e}}^{\Box}) \\
{(\!|\mathsf{if}\ \underline{\check{e}}\ \mathsf{then}\ \check{e}_1\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup}}^{\Box} &= \mathsf{if}\ (\underline{\check{e}}^{\Box})\ \mathsf{then}\ (\check{e}_1^{\Box})\ \mathsf{else}\ (\check{e}_2^{\Box}) \\
{(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \underline{\check{e}}\ \mathsf{else}\ \check{e}_2|\!)_{\not\sqcup}}^{\Box} &= \mathsf{if}\ (\check{e}_1^{\Box})\ \mathsf{then}\ (\underline{\check{e}}^{\Box})\ \mathsf{else}\ (\check{e}_2^{\Box}) \\
{(\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \underline{\check{e}}|\!)_{\not\sqcup}}^{\Box} &= \mathsf{if}\ (\check{e}_1^{\Box})\ \mathsf{then}\ (\check{e}_2^{\Box})\ \mathsf{else}\ (\underline{\check{e}}^{\Box}) \\
(\underline{\check{e}}, \check{e})^{\Box} &= (\underline{\check{e}}^{\Box}, \check{e}^{\Box}) \\
(\check{e}, \underline{\check{e}})^{\Box} &= (\check{e}^{\Box}, \underline{\check{e}}^{\Box}) \\
{(\!|(\underline{\check{e}}, \check{e})|\!)^{\Leftarrow}_{\blacktriangleright\not\times}}^{\Box} &= (\underline{\check{e}}^{\Box}, \check{e}^{\Box}) \\
{(\!|(\check{e}, \underline{\check{e}})|\!)^{\Leftarrow}_{\blacktriangleright\not\times}}^{\Box} &= (\check{e}^{\Box}, \underline{\check{e}}^{\Box}) \\
(\pi_1\underline{\check{e}})^{\Box} &= \pi_1(\underline{\check{e}}^{\Box}) \\
(\pi_1\, (\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright\not\times})^{\Box} &= \pi_1\underline{\check{e}}^{\Box} \\
(\pi_2\underline{\check{e}})^{\Box} &= \pi_2(\underline{\check{e}}^{\Box}) \\
(\pi_2\, (\!|\underline{\check{e}}|\!)^{\Rightarrow}_{\blacktriangleright\not\times})^{\Box} &= \pi_2\underline{\check{e}}^{\Box} \\
{(\!|\underline{\check{e}}|\!)_{\not\sim}}^{\Box} &= \underline{\check{e}}^{\Box}
\end{aligned}
$$

F.5 Metatheorems

Theorem F.1 (Erasure Commutativity). *For all* $\underline{\check{e}}$, $(\underline{\check{e}}^{\Box})^{\diamond} = (\underline{\check{e}}^{\diamond})^{\Box}$.

$$
\begin{tikzcd}
\underline{\check{e}} \in \mathsf{ZMExp} \arrow[r, "\_^{\Box}"] \arrow[d, "\_^{\diamond}"'] & \underline{e} \in \mathsf{ZExp} \arrow[d, "\_^{\diamond}"] \\
\check{e} \in \mathsf{MExp} \arrow[r, "\_^{\Box}"'] & e \in \mathsf{UExp}
\end{tikzcd}
$$

Theorem F.2 (Correctness).

1. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vdash_{M} \underline{\check{e}}^{\diamond} \Rightarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}}' \Rightarrow \tau'$ *and* $\underline{\check{e}}^{\Box} \xrightarrow{\alpha} \underline{e}'$, *then* $\underline{\check{e}}'^{\Box} = \underline{e}'$.

2. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vdash_{M} \underline{\check{e}}^{\diamond} \Leftarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau$ *and* $\underline{\check{e}}^{\Box} \xrightarrow{\alpha} \underline{e}'$, *then* $\underline{\check{e}}'^{\Box} = \underline{e}'$.

Theorem F.3 (Sensibility).

1. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vdash_{M} \underline{\check{e}}^{\diamond} \Rightarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha} \underline{\check{e}}' \Rightarrow \tau'$, *then* $\underline{\check{e}}'$ WF *and* $\Gamma \vdash_{M} \underline{\check{e}}'^{\diamond} \Rightarrow \tau'$.

2. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Leftarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha} \underline{\check{e}}' \Leftarrow \tau$, *then* $\underline{\check{e}}'$ WF *and* $\Gamma \vdash \underline{\check{e}}'^{\diamond} \Leftarrow \tau$.

Theorem F.4 (Movement Erasure Invariance).

1. *If* $\underline{\tau} \xrightarrow{\text{move } \delta} \underline{\tau}'$, *then* $\underline{\tau}^{\diamond} = \underline{\tau}'^{\diamond}$.
2. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Rightarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\text{move } \delta} \underline{\check{e}}' \Rightarrow \tau'$, *then* $\underline{\check{e}}'$ WF *and* $\underline{\check{e}}^{\diamond} = \underline{\check{e}}'^{\diamond}$ *and* $\tau = \tau'$.
3. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Leftarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \xrightarrow{\text{move } \delta} \underline{\check{e}}' \Leftarrow \tau$, *then* $\underline{\check{e}}'$ WF *and* $\underline{\check{e}}^{\diamond} = \underline{\check{e}}'^{\diamond}$.

Theorem F.5 (Reachability).

1. *If* $\underline{\tau}^{\diamond} = \underline{\tau}'^{\diamond}$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\underline{\tau} \xrightarrow{\overline{\alpha}}{}^{*} \underline{\tau}'$.
2. *If* $\underline{\check{e}}$ WF *and* $\underline{\check{e}}'$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Rightarrow \tau$ *and* $\underline{\check{e}}^{\diamond} = \underline{\check{e}}'^{\diamond}$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\overline{\alpha}}{}^{*} \underline{\check{e}}' \Rightarrow \tau$.
3. *If* $\underline{\check{e}}$ WF *and* $\underline{\check{e}}'$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Leftarrow \tau$ *and* $\underline{\check{e}}^{\diamond} = \underline{\check{e}}'^{\diamond}$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\Gamma \vdash \underline{\check{e}} \xrightarrow{\overline{\alpha}}{}^{*} \underline{\check{e}}' \Leftarrow \tau$.

Lemma F.5.1 (Reach Up).

1. *If* $\underline{\tau}^{\diamond} = \tau$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\underline{\tau} \xrightarrow{\overline{\alpha}}{}^{*} \triangleright\tau\triangleleft$.
2. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Rightarrow \tau$ *and* $\underline{\check{e}}^{\diamond} = \check{e}$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\overline{\alpha}}{}^{*} \triangleright\check{e}\triangleleft \Rightarrow \tau$.
3. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Leftarrow \tau$ *and* $\underline{\check{e}}^{\diamond} = \check{e}$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\Gamma \vdash \underline{\check{e}} \xrightarrow{\overline{\alpha}}{}^{*} \triangleright\check{e}\triangleleft \Leftarrow \tau$.

Lemma F.5.2 (Reach Down).

1. *If* $\underline{\tau}^{\diamond} = \tau$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\triangleright\tau\triangleleft \xrightarrow{\overline{\alpha}}{}^{*} \underline{\tau}$.
2. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Rightarrow \tau$ *and* $\underline{\check{e}}^{\diamond} = \check{e}$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\Gamma \vdash \triangleright\check{e}\triangleleft \Rightarrow \tau \xrightarrow{\overline{\alpha}}{}^{*} \underline{\check{e}} \Rightarrow \tau$.
3. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Leftarrow \tau$ *and* $\underline{\check{e}}^{\diamond} = \check{e}$, *then there exists* $\overline{\alpha}$ *such that* $\overline{\alpha}$ movements *and* $\Gamma \vdash \triangleright\check{e}\triangleleft \xrightarrow{\overline{\alpha}}{}^{*} \underline{\check{e}} \Leftarrow \tau$.

Theorem F.6 (Constructability).

1. *For every* τ, *there exists* $\overline{\alpha}$ *such that* $\triangleright ? \triangleleft \xrightarrow{\overline{\alpha}}{}^{*} \triangleright\tau\triangleleft$.
2. *If* $\Gamma \vDash_M \check{e} \Rightarrow \tau$, *then there exists* $\overline{\alpha}$ *such that* $\Gamma \vdash \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow ? \xrightarrow{\overline{\alpha}}{}^{*} \triangleright\check{e}\triangleleft \Rightarrow \tau$.
3. *If* $\Gamma \vDash_M \check{e} \Leftarrow \tau$, *then there exists* $\overline{\alpha}$ *such that* $\Gamma \vdash \triangleright\llparenthesis\rrparenthesis\triangleleft \xrightarrow{\overline{\alpha}}{}^{*} \triangleright\check{e}\triangleleft \Leftarrow \tau$.

Theorem F.7 (Determinism).

1. *If* $\underline{\tau} \xrightarrow{\alpha}{}^{*} \underline{\tau}'$ *and* $\underline{\tau} \xrightarrow{\alpha}{}^{*} \underline{\tau}''$ *then* $\underline{\tau}' = \underline{\tau}''$.
2. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Rightarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha}{}^{*} \underline{\check{e}}' \Rightarrow \tau'$ *and* $\Gamma \vdash \underline{\check{e}} \Rightarrow \tau \xrightarrow{\alpha}{}^{*} \underline{\check{e}}'' \Rightarrow \tau''$, *then* $\underline{\check{e}}' = \underline{\check{e}}''$ *and* $\tau' = \tau''$.
3. *If* $\underline{\check{e}}$ WF *and* $\Gamma \vDash_M \underline{\check{e}}^{\diamond} \Leftarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha}{}^{*} \underline{\check{e}}' \Leftarrow \tau$ *and* $\Gamma \vdash \underline{\check{e}} \xrightarrow{\alpha}{}^{*} \underline{\check{e}}'' \Leftarrow \tau$, *then* $\underline{\check{e}}' = \underline{\check{e}}''$.

G Constraint generation

Here, we give the list of constraint-generating bidirectional typing rules under the marked lambda calculus for type hole inference, described in Section 4 of the paper.

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$\boxed{\tau \blacktriangleright_{\rightarrow} \tau_1 \rightarrow \tau_2 \mid C}$ τ has matched arrow type $\tau_1 \rightarrow \tau_2$ and generates constraints C

$$\text{TMAHole-C}\quad \frac{}{?^p \blacktriangleright_{\rightarrow} ?^{\rightarrow_L(p)} \rightarrow ?^{\rightarrow_R(p)} \mid \{?^p \approx ?^{\rightarrow_L(p)} \rightarrow ?^{\rightarrow_R(p)}\}}$$

$$\text{TMAArr-C}\quad \frac{}{\tau_1 \rightarrow \tau_2 \blacktriangleright_{\rightarrow} \tau_1 \rightarrow \tau_2 \mid \{\}}$$

$\boxed{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \mid C}$ τ has matched binary product type $\tau_1 \times \tau_2$ and generates constraints C

$$\text{TMPHole-C}\quad \frac{}{?^p \blacktriangleright_{\times} ? \times ? \mid \{?^p \approx ?^{\times_L(p)} \times ?^{\times_R(p)}\}}$$

$$\text{TMPProd-C}\quad \frac{}{\tau_1 \times \tau_2 \blacktriangleright_{\times} \tau_1 \times \tau_2 \mid \{\}}$$

$\boxed{\Gamma \vdash \check{e} \Rightarrow \tau \mid C}$ $\check{e}$ synthesizes type τ and generates constraints C

$$\text{MSEHole-C}\quad \frac{}{\Gamma \vdash (\!|\,|\!)^u \Rightarrow ?^{exp(u)} \mid \{?^{exp(u)} \approx \mathsf{etc}\}}$$

$$\text{MSVar-C}\quad \frac{x : \tau \in \Gamma}{\Gamma \vdash x \Rightarrow \tau \mid \{\}}$$

$$\text{MSFree-C}\quad \frac{x \notin \mathrm{dom}(\Gamma)}{\Gamma \vdash (\!|x|\!)^u_{\square} \Rightarrow ?^{exp(u)} \mid \{?^{exp(u)} \approx \mathsf{etc}\}}$$

$$\text{MSLam-C}\quad \frac{\Gamma, x : \tau \vdash \check{e} \Rightarrow \tau_2 \mid C}{\Gamma \vdash \lambda x : \tau_1 . \check{e} \Rightarrow \tau_1 \rightarrow \tau_2 \mid C}$$

$$\text{MSAp1-C}\quad \frac{\Gamma \vdash \check{e}_1 \Rightarrow \tau \mid C_1 \qquad \tau \blacktriangleright_{\rightarrow} \tau_1 \rightarrow \tau_2 \mid C_2 \qquad \Gamma \vdash \check{e}_2 \Leftarrow \tau_1 \mid C_3}{\Gamma \vdash \check{e}_1\ \check{e}_2 \Rightarrow \tau_2 \mid C_1 \cup C_2 \cup C_3}$$

$$\text{MSAp2-C}\quad \frac{\Gamma \vdash \check{e}_1 \Rightarrow \tau \mid C_1 \qquad \tau \blacktriangleright_{\not\rightarrow} \qquad \Gamma \vdash \check{e}_2 \Leftarrow ?^{\rightarrow_L(exp(u))} \mid C_2}{\Gamma \vdash (\!|\check{e}_1|\!)^{\Rightarrow, u}_{\blacktriangleright_{\not\rightarrow}}\ \check{e}_2 \Rightarrow ?^{\rightarrow_R(exp(u))} \mid C_1 \cup C_2 \cup \{?^{exp(u)} \approx ?^{\rightarrow_L(exp(u))} \rightarrow ?^{\rightarrow_R(exp(u))}\}}$$

$$\text{MSNum-C}\quad \frac{}{\Gamma \vdash \underline{n} \Rightarrow \mathsf{num} \mid \{\}}$$

$$\text{MSPlus-C}\quad \frac{\Gamma \vdash \check{e}_1 \Leftarrow \mathsf{num} \mid C_1 \qquad \Gamma \vdash \check{e}_2 \Leftarrow \mathsf{num} \mid C_2}{\Gamma \vdash \check{e}_1 + \check{e}_2 \Rightarrow \mathsf{num} \mid C_1 \cup C_2}$$

$$\text{MSTrue-C}\quad \frac{}{\Gamma \vdash \mathsf{tt} \Rightarrow \mathsf{bool} \mid \{\}}$$

$$\text{MSFalse-C}\quad \frac{}{\Gamma \vdash \mathsf{ff} \Rightarrow \mathsf{bool} \mid \{\}}$$

$$\text{MSIf-C}\quad \frac{\Gamma \vdash \check{e}_1 \Leftarrow \mathsf{bool} \mid C_1 \qquad \Gamma \vdash \check{e}_2 \Rightarrow \tau_1 \mid C_2 \qquad \Gamma \vdash \check{e}_3 \Rightarrow \tau_2 \mid C_3}{\Gamma \vdash \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Rightarrow \tau_1 \sqcup \tau_2 \mid C_1 \cup C_2 \cup C_3 \cup \{\tau_1 \approx \tau_2\}}$$

$$\text{MSInconsistentBranches-C}\quad \frac{\Gamma \vdash \check{e}_1 \Leftarrow \mathsf{bool} \mid C_1 \qquad \Gamma \vdash \check{e}_2 \Rightarrow \tau_1 \mid C_2 \qquad \Gamma \vdash \check{e}_3 \Rightarrow \tau_2 \mid C_3 \qquad \tau_1 \nsim \tau_2}{\Gamma \vdash (\!|\mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3|\!)^u_{\sqcup} \Rightarrow ?^{exp(u)} \mid C_1 \cup C_2 \cup C_3 \cup \{\tau_1 \approx \tau_2, ?^{exp(u)} \approx \mathsf{etc}\}}$$

$$\text{MSPair-C}\quad \frac{\Gamma \vdash \check{e}_1 \Rightarrow \tau_1 \mid C_1 \qquad \Gamma \vdash \check{e}_2 \Rightarrow \tau_2 \mid C_2}{\Gamma \vdash (\check{e}_1, \check{e}_2) \Rightarrow \tau_1 \times \tau_2 \mid C_1 \cup C_2}$$

$$\text{MSProjL1-C}\quad \frac{\Gamma \vdash \check{e} \Rightarrow \tau \mid C_1 \qquad \tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \mid C_2}{\Gamma \vdash \pi_1 \check{e} \Rightarrow \tau_1 \mid C_1 \cup C_3}$$

MSProjR1-C

$$\dfrac{\Gamma \vdash \check{e} \Rightarrow \tau \mid C_1 \qquad \tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \mid C_2}{\Gamma \vdash \pi_2 \check{e} \Rightarrow \tau_2 \mid C_1 \cup C_2}$$

MSProjL2-C

$$\dfrac{\Gamma \vdash \check{e} \Rightarrow \tau \mid C \qquad \tau \blacktriangleright_{\not\times}}{\Gamma \vdash \pi_1 \llparenthesis \check{e} \rrparenthesis^{\Rightarrow,\ u}_{\blacktriangleright_{\not\times}} \Rightarrow ?^{\times_L(exp(u))} \mid C \cup \{?^{exp(u)} \approx ?^{\times_L(exp(u))} \times ?^{\times_R(exp(u))}, ?^{exp(u)} \approx \mathtt{etc}\}}$$

MSProjR2-C

$$\dfrac{\Gamma \vdash \check{e} \Rightarrow \tau \mid C \qquad \tau \blacktriangleright_{\not\times}}{\Gamma \vdash \pi_2 \llparenthesis \check{e} \rrparenthesis^{\Rightarrow,\ u}_{\blacktriangleright_{\not\times}} \Rightarrow ?^{\times_R(exp(u))} \mid C \cup \{?^{exp(u)} \approx ?^{\times_L(exp(u))} \times ?^{\times_R(exp(u))}, ?^{exp(u)} \approx \mathtt{etc}\}}$$

$\boxed{\Gamma \vdash \check{e} \Leftarrow \tau \mid C}$ $\check{e}$ analyzes against type τ and generates constraints C

MALam1-C

$$\dfrac{\tau_3 \blacktriangleright_{\rightarrow} \tau_1 \rightarrow \tau_2 \mid C_1 \qquad \tau \sim \tau_1 \qquad \Gamma,\ x : \tau \vdash \check{e} \Leftarrow \tau_2 \mid C_2}{\Gamma \vdash \lambda x : \tau .\, \check{e} \Leftarrow \tau_3 \mid C_1 \cup C_2 \cup \{\tau \approx \tau_1\}}$$

MALam2-C

$$\dfrac{\tau_3 \blacktriangleright_{\not\rightarrow} \qquad \Gamma,\ x : \tau \vdash \check{e} \Leftarrow ?^{anon} \mid C}{\Gamma \vdash \llparenthesis \lambda x : \tau .\, \check{e} \rrparenthesis^{\Leftarrow,\ u}_{\blacktriangleright_{\not\rightarrow}} \Leftarrow \tau_3 \mid C \cup \{?^{exp(u)} \approx \tau_3\}}$$

MALam3-C

$$\dfrac{\tau_3 \blacktriangleright_{\rightarrow} \tau_1 \rightarrow \tau_2 \mid C_1 \qquad \tau \nsim \tau_1 \qquad \Gamma,\ x : \tau \vdash \check{e} \Leftarrow \tau_2 \mid C_2}{\Gamma \vdash \llparenthesis \lambda x : \tau .\, \check{e} \rrparenthesis^{u}_{:} \Leftarrow \tau_3 \mid C_1 \cup C_2 \cup \{?^{exp(u)} \approx \tau_3\}}$$

MAIf

$$\dfrac{\Gamma \vdash \check{e}_1 \Leftarrow \mathsf{bool} \mid C_1 \qquad \Gamma \vdash \check{e}_1 \Leftarrow \tau \mid C_2 \qquad \Gamma \vdash \check{e}_2 \Leftarrow \tau \mid C_3}{\Gamma \vdash \mathsf{if}\ \check{e}_1\ \mathsf{then}\ \check{e}_2\ \mathsf{else}\ \check{e}_3 \Leftarrow \tau \mid C_1 \cup C_2 \cup C_3}$$

MAPair1-C

$$\dfrac{\tau \blacktriangleright_{\times} \tau_1 \times \tau_2 \mid C_1 \qquad \Gamma \vdash \check{e}_1 \Leftarrow \tau_1 \mid C_2 \qquad \Gamma \vdash \check{e}_2 \Leftarrow \tau_2 \mid C_3}{\Gamma \vdash (\check{e}_1, \check{e}_2) \Leftarrow \tau \mid C_1 \cup C_2 \cup C_3}$$

MAPair2-C

$$\dfrac{\tau \blacktriangleright_{\not\times} \qquad \Gamma \vdash \check{e}_1 \Leftarrow ?^{anon} \mid C_1 \qquad \Gamma \vdash \check{e}_2 \Leftarrow ?^{anon} \mid C_2}{\Gamma \vdash \llparenthesis (\check{e}_1, \check{e}_2) \rrparenthesis^{\Leftarrow,\ u}_{\blacktriangleright_{\not\times}} \Leftarrow \tau \mid C_1 \cup C_2 \cup \{?^{exp(u)} \approx \tau\}}$$

MAInconsistentTypes-C

$$\dfrac{\Gamma \vdash \check{e} \Rightarrow \tau' \mid C \qquad \tau \nsim \tau' \qquad \check{e}\ \mathsf{subsumable}}{\Gamma \vdash \llparenthesis \check{e} \rrparenthesis^{u}_{:} \Leftarrow \tau \mid C \cup \{\tau \approx ?^{exp(u)}\}}$$

MASubsume-C

$$\dfrac{\Gamma \vdash \check{e} \Rightarrow \tau' \mid C \qquad \tau \sim \tau' \qquad \check{e}\ \mathsf{subsumable}}{\Gamma \vdash \check{e} \Leftarrow \tau \mid C \cup \{\tau \approx \tau'\}}$$