$\dot{e} ::= \ldots \mid \lambda x.\dot{e} \mid \dot{e} : \tau \mid (\!|\,|\!)^u \mid (\!|\dot{e}|\!)^u$

$e ::= \ldots \mid \lambda x:\tau.e \mid (\!|\,|\!)^{u:\tau}_{\sigma} \mid (\!|e|\!)^{u:\tau}_{\sigma}$

$\tau ::= \ldots$

If $\Gamma \vdash \dot{e} \Rightarrow \tau \rightsquigarrow e \dashv \Delta$ then $\Gamma \vdash e : \tau \dashv \Delta$.

If $\Gamma \vdash \dot{e} \Leftarrow \tau \rightsquigarrow e : \tau' \dashv \Delta$ then $\Gamma \vdash e : \tau' \dashv \Delta$ and $\tau \sim \tau'$.

$$\dfrac{\dot{e} \neq (\!|\,|\!)^u \quad \dot{e} \neq (\!|\dot{e}'|\!)^u \quad \Gamma \vdash \dot{e} \rightsquigarrow e \Rightarrow \tau' \dashv \Delta \qquad \tau \sim \tau'}{\Gamma \vdash \dot{e} \Leftarrow \tau \rightsquigarrow e : \tau' \dashv \Delta}$$

$$\dfrac{}{\Gamma \vdash (\!|\,|\!)^u \Leftarrow \tau \rightsquigarrow (\!|\,|\!)^u_{id(\Gamma)} : \tau \dashv u :: [\Gamma]\tau}$$

$$\dfrac{\tau \blacktriangleright_{\to} \tau_1 \to \tau_2 \qquad \Gamma, x:\tau_1 \vdash \dot{e} \Leftarrow \tau_2 \rightsquigarrow e : \tau_2' \dashv \Delta}{\Gamma \vdash \lambda x.\dot{e} \Leftarrow \tau \rightsquigarrow \lambda x:\tau_1.e : \tau_1 \to \tau_2'}$$

$$\dfrac{}{\Gamma \vdash (\!|\,|\!)^u \Rightarrow (\!|\,|\!) \rightsquigarrow (\!|\,|\!)^u_{id(\Gamma)} \dashv u :: [\Gamma](\!|\,|\!)}$$

Δ is fin map from metavariables to assumptions $u :: [\Gamma]\tau$.

$\langle num \to (\!|\,|\!)\rangle$

$(\!|\,|\!)^u(3) \rightsquigarrow ((\!|\,|\!)^u_{id(\Gamma)})(3) \dashv u : [\Gamma](\!|\,|\!)$

$num \to (\!|\,|\!)$

$$\dfrac{\Gamma \vdash \dot{e}_1 \Rightarrow (\!|\,|\!) \rightsquigarrow e_1 \dashv \Delta_1 \qquad \Gamma \vdash \dot{e}_2 \Leftarrow (\!|\,|\!) \rightsquigarrow e_2 : \tau_2 \dashv \Delta_2}{\Gamma \vdash \dot{e}_1(\dot{e}_2) \Rightarrow \tau \rightsquigarrow (\langle \tau_2 \to (\!|\,|\!) \rangle e_1)\, e_2 \dashv \Delta_1 \cup \Delta_2}$$

$(dom(\Delta_1) \cap dom(\Delta_2) = \emptyset)$

Hammer premise

$$\dfrac{\Gamma \vdash \dot{e}_1 \Rightarrow (\!|\,|\!) \rightsquigarrow e_1 \dashv \Delta_1 \qquad \Gamma \vdash \dot{e}_2 \Leftarrow (\!|\,|\!) \rightsquigarrow e_2 : \tau_2 \dashv \Delta_2 \qquad \Gamma \vdash e_1 \Leftarrow \tau_2 \to (\!|\,|\!) \rightsquigarrow e_1' \dashv \Delta_1'}{\Gamma \vdash \dot{e}_1(\dot{e}_2) \Rightarrow (\!|\,|\!) \rightsquigarrow (\langle \tau_2 \to (\!|\,|\!) \rangle e_1')\, e_2 \dashv \Delta_1' \cup \Delta_2}$$

$$\dfrac{\Gamma \vdash \dot{e}_1 \Rightarrow \tau_2 \to \tau \rightsquigarrow e_1 \dashv \Delta_1 \qquad \Gamma \vdash \dot{e}_2 \Leftarrow \tau_2 \rightsquigarrow e_2 : \tau_2' \dashv \Delta_2 \qquad \tau_2 \sim \tau_2' \qquad \tau_2 \neq \tau_2'}{\Gamma \vdash \dot{e}_1(\dot{e}_2) \Rightarrow \tau \rightsquigarrow e_1(\langle \tau_2 \rangle e_2) \dashv \Delta_1 \cup \Delta_2}$$

$$\dfrac{\Gamma \vdash e : \tau' \dashv \Delta \qquad \tau \sim \tau'}{\Gamma \vdash \langle \tau \rangle (e) : \tau \dashv \Delta}$$

$$\dfrac{\Gamma \vdash \dot{e}_1 \Rightarrow \tau_2 \to \tau \rightsquigarrow e_1 \dashv \Delta_1 \qquad \Gamma \vdash \dot{e}_2 \Leftarrow \tau_2 \rightsquigarrow e_2 : \tau_2 \dashv \Delta_2}{\Gamma \vdash \dot{e}_1(\dot{e}_2) \Rightarrow \tau \rightsquigarrow e_1(e_2) \dashv \Delta_1 \cup \Delta_2}$$

$$\frac{e\ \mathsf{val} \qquad \emptyset \vdash e : \tau}{\langle \tau \rangle (e) \mapsto e}$$