# Hazel PHI: 10-modules

June 29, 2021

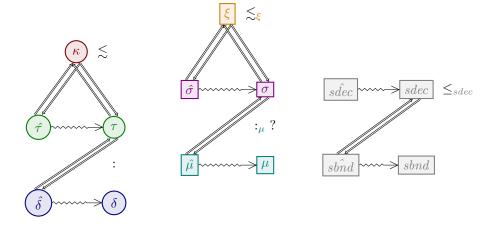
# prerequisites

- Hazel PHI: 9-type-aliases-redux
  - github
  - $\ current \ commit: \ 4410cd565ce717707e580e44f64868d3175fe2a6$
- (optional) Hazel PHI: 1-labeled-tuples
  - github
  - $\ current \ commit: \ 0a7d0b53ee7286d03ea3be13a7ac91a86f1c90b1$

### how to read

800000	kinds	D08000	signature kind
008000	types (constructors)	800080	signatures
000080	terms	008080	modules

#### notes



external typ/sig/mod syntax not written out yet (waiting for construction dust to settle); patterns not handled yet—will be left till end.

# syntax



```
HTyp
                                                                                                                       type variable
                                              t
                                              bse
                                                                                                                           base type
                                                                                                                          type binop
                                              	au_1 \oplus 	au_2
                                                                                                                             list type
                                              [\tau]
                                                                                                                       type function
                                              \lambda t :: \kappa.\tau
                                                                                                                   type application
                                                                                                 labelled product type (record)
                                              \{lab_1 \hookrightarrow \tau_1, \dots \, lab_n \hookrightarrow \tau_n\}
                                                                                                          module type projection
                                                                                                                   empty type hole
                                              (|\tau|)
                                                                                                              nonempty type hole
               base type
                               bse
                                              Int
                                              Float
                                              Bool
          HTyp BinOp
                                \oplus
   external expression
                                              signature s = \hat{\sigma} in \hat{\delta}
                                              module m = \hat{\mu} in \hat{\delta}
                                              module m:_{\mu}s=\hat{\mu} in \hat{\delta}
                                              functor something = something in \hat{\delta}
                                              \hat{\mu}.lab
                                                                                                          module term projection
   internal expression
                                \delta
                                        ::=
                                              signature s=\sigma in \delta
                                              module m:_{\mu} s = \mu in \delta
                                              functor something = something in \delta
                                              \mu.lab
                                                                                                          module term projection
         signature kind
                                              SSigKind(\sigma)
                                              SigKHole
               signature
                                                                                                                 signature variable
                                              \{sdecs\}
                                                                                                                structure signature
                                              \Pi_{m:_{\mu}\sigma_{1}}.\sigma_{2}
                                                                                                                  functor signature
                                                                                                             empty signature hole
                                              (s)
                                                                                                         nonempty signature hole
                  module
                                              m
                                                                                                                   module variable
                                              \{sbnds\}
                                                                                                                            structure
                                                                                                                              functor
                                              \lambda m:_{\mu} \sigma.\mu
                                                                                                                functor application
                                              \mu_1 \mu_2
                                                                                                            submodule projection
                                              \mu.lab
                                                                                                                empty module hole
                                                                                                           nonempty module hole
                                              (\mu)
signature declarations
                              sdecs
                                              sdec, sdecs
 signature declaration
                              sdec
                                              type lab
                                              type lab = \tau
                                              {\tt val}\ lab{:}\tau
                                              module lab:_{\mu}\sigma
                                              functor lab:_{\mu}\sigma
    structure bindings
                             sbnds ::=
                                              sbnd, sbnds
     structure binding
                              sbnd ::= type t = \tau
                                              \mathtt{let}\ x{:}\tau = \delta
                                              {\tt module}\ m=\mu
                                              module m:_{\mu} s = \mu
```

# contexts

```
\Delta, ?; \Gamma, x:\tau; \Phi, t::\kappa; \Xi, m:_{\mu}\sigma; \Psi, s::_{\sigma}\xi
```

#### statics

```
scratch
      \Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \mid \kappa_1 \text{ is a consistent subkind of } \kappa_2
                                                                                                      KCSubsumption
                                                                                                       test
                                                                                                       test
\Delta; \Phi; \Xi; \Psi \vdash \xi_1 \lesssim_{\xi} \xi_2 \mid \xi_1 \text{ is a consistent sub signature kind of } \xi_2
                               nameMe
                                           \begin{array}{l} \exists sdec_x \in sdecs_1 \ st \ \Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(\{sdec_x\}) \lesssim_{\mathbf{\xi}} \texttt{SSigKind}(\{sdec_2\}) \\ \Delta; \Phi, \mathsf{type}(\Delta; \Phi; \Xi; \Psi, sdec_2); \Xi, \mathsf{submodule}(sdec_2); \Psi \vdash \{sdecs_1\} \lesssim_{\mathbf{\xi}} \{sdecs_2\} \end{array}
                                \Delta; \Phi; \Xi; \Psi \vdash \underline{\mathsf{SSigKind}}(\{sdec_{11}, s\overline{dec_{12}}, sdecs_{13} \text{ as } sdecs_1\}) \lesssim_{\xi} \underline{\mathsf{SSigKind}}(\{sdec_2, sdecs_2\})
          single
          \frac{\Delta; \Phi; \Xi; \Psi \vdash sdec_1 \leq_{sdec} sdec_2}{\Delta; \Phi; \Xi; \Psi \vdash \mathtt{SSigKind}(\{sdec_1\}) \lesssim_{\xi} \mathtt{SSigKind}(\{sdec_2\})} \qquad \frac{\mathtt{nll}}{\Delta; \Phi; \Xi; \Psi \vdash \mathtt{SSigKind}(\{sdec_3\}) \lesssim_{\xi} \mathtt{SSigKind}(\{\cdot\})}
                              nameMe?delete?
                funct
                \frac{\Delta; \Phi; \Xi; \Psi \vdash \operatorname{SSigKind}(\sigma_{21}) \lesssim_{\xi} \operatorname{SSigKind}(\sigma_{11})}{\Delta; \Phi; \Xi; \Psi \vdash \operatorname{SSigKind}(\Pi_{m:_{\mu}\sigma_{11}}, \sigma_{12})} \lesssim_{\xi} \operatorname{SSigKind}(\Pi_{m:_{\mu}\sigma_{21}}, \sigma_{22})}
             CSubSigKindHoleR
                                                                                     \Delta; \Phi; \Xi; \Psi \vdash \xi \lesssim_{\xi} SigKHole
\Delta; \Phi; \Xi; \Psi \vdash \sigma \Rightarrow \xi \mid \sigma \text{ synthesizes signature kind } \xi
            SynSigKndVar
                                                                                 SynSigKndVarFail
            \frac{s::_{\sigma}\xi \in \Psi}{\Delta; \Phi; \Xi; \Psi \vdash s \Rightarrow \texttt{SSigKind}(s)} \qquad \frac{s \notin \mathsf{dom}(\Psi)}{\Delta; \Phi; \Xi; \Psi \vdash s \Rightarrow \texttt{SigKHole}} \qquad \frac{\{sdecs\}wellformed?}{\vdash \{sdecs\}} \Rightarrow \frac{\mathsf{SSigKind}(\{sdecs\})}{}
     \Delta; \Phi; \Xi; \Psi \vdash \sigma \Leftarrow \xi  \sigma analyzes against signature kind \xi
```

Sub
$$\Delta; \Phi; \Xi; \Psi \vdash \sigma \Rightarrow \xi_{1} \qquad \Delta; \Phi; \Xi; \Psi \vdash \xi_{1} \lesssim_{\xi} \xi$$

$$\Delta; \Phi; \Xi; \Psi \vdash \sigma \Leftarrow \xi$$

 $\Delta; \Phi; \Xi; \Psi \vdash sdec_1 \leq_{sdec} sdec_2$  $sdec_1$  is a subsdec of  $sdec_2$ singleType2 singleType  $\frac{\Delta;\Phi;\Xi;\Psi\vdash\tau_{\mathit{1}}\equiv\tau_{\mathit{2}}}{\Delta;\Phi;\Xi;\Psi\vdash\mathsf{type}\ lab=\tau_{\mathit{1}}\leq_{sdec}\mathsf{type}\ lab=\tau_{\mathit{2}}}$  $\overline{\Delta ; \Phi ; \Xi ; \Psi \vdash \mathsf{type} \; lab = \tau <_{sdec} \; \mathsf{type} \; lab}$ singleVasingleType3  $\Delta; \Phi; \Xi; \Psi \vdash \tau_1 \equiv \tau_2$  $\frac{1}{\Delta;\Phi;\Xi;\Psi\vdash \mathtt{val}} \frac{1}{lab:\tau_1} \leq_{sdec} \mathtt{val} \frac{lab:\tau_2}{lab:\tau_2}$  $\overline{\Delta; \Phi; \Xi; \Psi \vdash \text{type } lab \leq_{sdec} \text{type } lab}$  $\Delta; \Phi; \Xi; \Psi \vdash \sigma_1 \Leftarrow SSigKind(\sigma_2)$  $\Delta; \Phi; \Xi; \Psi \vdash \text{module } lab:_{\mu}\sigma_{1} <_{sdec} \text{module } lab:_{\mu}\sigma_{2}$ elab  $\Gamma; \Phi; \Xi \vdash \hat{\delta} \Rightarrow \tau \leadsto \delta \dashv \Delta$  $\hat{\delta}$  synthesizes type  $\tau$  and elaborates to  $\delta$  with hole context  $\Delta$ SynElabLetMod SynElabLetModAnn  $\frac{\Phi;\Xi\vdash\hat{\sigma}\ \Rightarrow\ \xi\leadsto\sigma\dashv\Delta_1\qquad \Gamma;\Phi;\Xi\vdash\hat{\mu}\ \Leftarrow\ \sigma\leadsto\mu\dashv\Delta_2\qquad \Gamma;\Phi;\Xi,m:_{\mu}\sigma\vdash\hat{\delta}\ \Rightarrow\ \tau\leadsto\delta\dashv\Delta_3}{\Gamma;\Phi;\Xi\vdash\operatorname{module}\ m:_{\mu}\hat{\sigma}=\hat{\mu}\ \operatorname{in}\ \hat{\delta}\ \Rightarrow\ \tau\leadsto\operatorname{module}\ m:_{\mu}\sigma=\mu\ \operatorname{in}\ \delta\dashv\Delta_1\cup\Delta_2\cup\Delta_3}$ SynElabModTermPrj  $\frac{\Gamma; \Phi; \Xi \vdash \hat{\mu} \ \Rightarrow \ \sigma \leadsto \mu \dashv \Delta \qquad \Phi; \Xi \vdash \sigma \ \Rightarrow \ \xi}{\Gamma; \Phi; \Xi \vdash \hat{\mu}.lab \ \Rightarrow \ \tau \leadsto \mu.lab \dashv \Delta}$  $\hat{\tau}$  synthesizes kind  $\kappa$  and elaborates to  $\tau$  with hole context  $\Delta$ SynElabModTypPrj  $\Phi;\Xi \vdash m \;\Rightarrow\; \sigma \leadsto m \dashv \Delta \qquad something \sigma \kappa$  $\Phi:\Xi \vdash m.lab \Rightarrow \kappa \leadsto m.lab \dashv \Delta$  $\Phi;\Xi \vdash \hat{\tau} \Leftarrow \kappa \leadsto \tau \dashv \Delta \mid \hat{\tau}$  analyzes against kind  $\kappa$  and elaborates to  $\tau$  with hole context  $\Delta$  $\hat{\mu}$  synthesizes signature  $\sigma$  and elaborates to  $\mu$  with hole context  $\Delta$  $\Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta$ SynElabModVar SynElabModVarFail  $\frac{m:_{\mu}\sigma\in\Xi}{\Gamma;\Phi;\Xi\vdash m\ \Rightarrow\ \sigma\leadsto m\dashv\cdot}$  $\frac{m \notin \mathsf{dom}(\Xi)}{\Gamma; \Phi; \Xi \vdash m \Rightarrow (||) \leadsto (|m|)^{\mathsf{u}} \dashv u_{:u}(||)}$ SynElabConsStruct  $\Gamma; \Phi; \Xi \vdash s\hat{bnd} \Rightarrow sdec \leadsto sbnd \dashv \Delta_1$  $\Gamma, \mathsf{val}(\mathit{sdec}); \Phi, \mathsf{type}(\Delta_1; \Phi; \Xi; \Psi, \mathit{sdec}); \Xi, \mathsf{submodule}(\mathit{sdec}) \vdash \{\mathit{sbnds}\} \ \Rightarrow \ \{\mathit{sdecs}\} \leadsto \{\mathit{sbnds}\} \dashv \Delta_2 \bowtie \{\mathit{sbnds}\} \vdash \{\mathit{sbnds}\} \bowtie \{\mathit{sdecs}\} \bowtie \{\mathit{sbnds}\} \vdash \{\mathit{sbnds}\} \bowtie \{\mathit{sdecs}\} \bowtie \{\mathit{sbnds}\} \vdash \{\mathit{sbnds}\} \{\mathit$  $\Gamma; \Phi; \Xi \vdash \{\hat{sbnd}, \hat{sbnds}\} \implies \{\hat{sdec}, \hat{sdecs}\} \leadsto \{\hat{sbnd}, \hat{sbnds}\} \dashv \Delta_1 \cup \Delta_2$ SynElabNilStruct SynElabEmptyModHole SynElabNonemptyModHole functor stuff

 ${\tt AnaElabModSubsumption}$ 

 $\Gamma; \Phi; \Xi \vdash \hat{\mu} \Leftarrow \sigma \leadsto \mu \dashv \Delta$   $\hat{\mu}$  analyzes against signature  $\sigma$  and elaborates to  $\mu$  with hole context  $\Delta$ 

 $\overline{\Gamma; \Phi; \Xi \vdash \{\cdot\} \Rightarrow \{\cdot\} \leadsto \{\cdot\} \dashv \cdot}$ 

$$\frac{\Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta}{\Gamma; \Phi; \Xi \vdash \hat{\mu} \Leftarrow \sigma \leadsto \mu \dashv \Delta}$$

 $\overline{\Gamma; \Phi; \Xi \vdash (\!\!|)^{\mathbf{u}} \Rightarrow (\!\!|) \rightsquigarrow (\!\!|)^{\mathbf{u}} \dashv u_{:\mu}(\!\!|)} \qquad \overline{\Gamma; \Phi; \Xi \vdash (\!\!|m|)^{\mathbf{u}} \Rightarrow (\!\!|) \rightsquigarrow (\!\!|m|)^{\mathbf{u}} \dashv u_{:\mu}(\!\!|)}$ 

 $\Gamma; \Phi; \Xi \vdash s\hat{bnd} \Rightarrow sdec \leadsto sbnd \dashv \Delta$  |  $s\hat{bnd}$  synthesizes declaration sdec and elaborates to sbnd with hole context  $\Delta$ 

 ${\tt SynElabTypeSbnd}$ 

$$\Phi;\Xi\vdash\hat{\tau} \Rightarrow \kappa\leadsto\tau\dashv\Delta$$

SynElabValSbnd

 $\frac{\Phi;\Xi\vdash\hat{\tau}\ \Rightarrow\ \kappa\leadsto\tau\dashv\Delta}{\Gamma;\Phi;\Xi\vdash\mathsf{type}\ t=\hat{\tau}\ \Rightarrow\ \mathsf{type}\ t=\tau\leadsto\mathsf{type}\ t=\tau\dashv\Delta} \qquad \frac{\Phi;\Xi\vdash\hat{\tau}\ \Rightarrow\ \kappa\leadsto\tau\dashv\Delta_1\qquad \Gamma;\Phi;\Xi\vdash\hat{\delta}\ \Leftarrow\ \tau\leadsto\delta\dashv\Delta_2}{\Gamma;\Phi;\Xi\vdash\mathsf{let}\ x:\hat{\tau}=\hat{\delta}\ \Rightarrow\ \mathsf{val}\ x:\tau\leadsto\mathsf{let}\ x:\tau=\delta\dashv\Delta_1\cup\Delta_2}$ 

$$\Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta$$

 $\frac{\Gamma;\Phi;\Xi\vdash\hat{\mu}\ \Rightarrow\ \sigma\leadsto\mu\dashv\Delta}{\Gamma;\Phi;\Xi\vdash \mathtt{module}\ m=\hat{\mu}\ \Rightarrow\ \mathtt{module}\ m:_{\mu}\sigma\leadsto\mathtt{module}\ m:_{\mu}\sigma=\mu\dashv\Delta}$ 

SynElabModAnnSbnd

$$\Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma_2 \rightsquigarrow \mu \dashv \Delta_2 \qquad \Phi; \Xi$$

 $\frac{\dot{\Phi};\Xi\vdash\hat{\sigma}\ \Rightarrow\ \xi\leadsto\sigma_1\dashv\Delta_1\qquad \Gamma;\Phi;\Xi\vdash\hat{\mu}\ \Rightarrow\ \sigma_2\leadsto\mu\dashv\Delta_2\qquad \Phi;\Xi;\Psi\vdash\sigma_2\ \Leftarrow\ \xi}{\Gamma;\Phi;\Xi\vdash\mathrm{module}\ m:_{\mu}\hat{\sigma}=\hat{\mu}\ \Rightarrow\ \mathrm{module}\ m:_{\mu}\sigma_1\leadsto\mathrm{module}\ m:_{\mu}\sigma_1=\mu\dashv\Delta_1\cup\Delta_2}$ 

 $\Gamma; \Phi; \Xi \vdash s\hat{bnd} \Leftarrow sdec \leadsto sbnd \dashv \Delta$ sbnd analyzes against declaration sdec and elaborates to sbnd with hole context  $\Delta$ 

$$\frac{\Gamma; \Phi; \Xi; l\Psi \vdash s\hat{bnd} \Rightarrow sdec_1 \leadsto sbnd \dashv \Delta \qquad \Delta; \Phi; \Xi; \Psi \vdash sdec_1 \leq_{sdec} sdec}{\Gamma; \Phi; \Xi; \Psi \vdash s\hat{bnd} \Leftarrow sdec \leadsto sbnd \dashv \Delta}$$

$$\Gamma; \Phi; \Xi; \Psi \vdash s\hat{bnd} \Leftarrow sdec \leadsto sbnd \dashv \Delta$$

 $\Gamma; \Phi; \Xi; \Psi \vdash s \hat{dec} \leadsto s dec \dashv \Delta$  |  $s \hat{dec}$  elaborates to s dec with hole context  $\Delta$ 

 $\overline{\Gamma;\Phi;\Xi;\Psi\vdash \mathsf{type}\;\mathit{lab}\leadsto \mathsf{type}\;\mathit{lab}\dashv\cdot}$ 

$$\Gamma; \Phi; \Xi; \Psi \vdash \hat{\tau} \implies \kappa \leadsto \tau \dashv \Delta$$

 $\frac{\Gamma; \Phi; \Xi; \Psi \vdash \hat{\tau} \implies \kappa \leadsto \tau \dashv \Delta}{\Gamma; \Phi; \Xi; \Psi \vdash \mathsf{type} \ lab = \hat{\tau} \leadsto \mathsf{type} \ lab = \tau \dashv \Delta}$ 

$$\begin{array}{c} \operatorname{val} & \operatorname{mod} \\ \Gamma; \Phi; \Xi; \Psi \vdash \hat{\tau} \ \Rightarrow \ \kappa \leadsto \tau \dashv \Delta \\ \hline \Gamma; \Phi; \Xi; \Psi \vdash \operatorname{val} \ \mathit{lab} : \hat{\tau} \leadsto \operatorname{val} \ \mathit{lab} : \tau \dashv \Delta \\ \end{array} \qquad \begin{array}{c} \operatorname{mod} \\ \Gamma; \Phi; \Xi; \Psi \vdash \hat{\sigma} \ \Rightarrow \ \xi \leadsto \sigma \dashv \Delta \\ \hline \Gamma; \Phi; \Xi; \Psi \vdash \operatorname{module} \ \mathit{lab} : \mu \hat{\sigma} \dashv \Delta \\ \end{array}$$

$$\Gamma; \Phi; \Xi; \Psi \vdash \hat{\sigma} \Rightarrow \xi \leadsto \sigma \dashv \Delta$$

we're going to need HOFunctors so we don't need to preclude users from typing a functor into a module and vice versa

 $\Phi; \Xi; \Psi \vdash \hat{\sigma} \Rightarrow \xi \leadsto \sigma \dashv \Delta$  |  $\hat{\sigma}$  synthesizes signature kind  $\xi$  and elaborates to  $\sigma$  with hole context  $\Delta$ 

SynSigEmptyHole

SynSigNonEmptyHole

 $\overline{\Phi;\Xi;\Psi\vdash (\!\!)^{\mathbf{u}} \Rightarrow \mathsf{SigKHole} \leadsto (\!\!)^{\mathbf{u}}\dashv u::_{\sigma}\mathsf{SigKHole}}$ 

 $\Phi;\Xi\vdash\hat{\sigma} \leftarrow \xi \leadsto \sigma\dashv\Delta$   $\hat{\sigma}$  analyzes against signature kind  $\xi$  and elaborates to  $\sigma$  with hole context  $\Delta$ 

misc

$$\mathsf{val}(sdec) = \begin{cases} lab{:}\tau & sdec = \mathtt{val}\ lab{:}\tau \\ \cdot & \text{otherwise} \end{cases}$$
 
$$\mathsf{type}(cntxts,sdec) = \begin{cases} lab{:}\mathsf{Type} & sdec = \mathsf{type}\ lab \\ lab{:}\mathsf{:}\kappa & sdec = \mathsf{type}\ lab = \tau \\ & \text{where}\ cntxts \vdash \tau \ \Rightarrow \ \kappa \\ \cdot & \text{otherwise} \end{cases}$$
 
$$\mathsf{submodule}(sdec) = \begin{cases} lab{:}_{\mu}\sigma & sdec = \mathsf{module}\ lab{:}_{\mu}\sigma \\ \cdot & \text{otherwise} \end{cases}$$