Hazel Phi: 11-type-constructors

July 29, 2021

SYNTAX

Kind	κ	$::=$	$\mathtt{Type} \mid \mathtt{KHole} \mid \mathtt{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	$::=$	$t \mid \mathtt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid \llparenthesis\rrparenthesis^{\mathtt{u}} \mid \llparenthesis\hat{\tau}\rrparenthesis^{\mathtt{u}} \mid \lambda t{::}\mathtt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \mathtt{bse} \mid \tau_1 \oplus \tau_2 \mid \llparenthesis\rrparenthesis^{\mathtt{u}} \mid \llparenthesis\tau\rrparenthesis^{\mathtt{u}} \mid \llparenthesis t\rrparenthesis^{\mathtt{u}} \mid \lambda t{::}\kappa.\tau \mid \tau_1\ \tau_2$
Base Types	$\mathtt{bse}$	$::=$	$\mathtt{Int} \mid \mathtt{Float} \mid \mathtt{Bool}$
BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Type Pattern			
User Expression			
Internal Expression			

DECLARATIVES

$\boxed{\Delta;\Phi \vdash \tau ::> \kappa}$ τ has principal (well formed) kind κ

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \mathtt{bse} ::> \mathtt{S}_{\mathtt{Type}}(\mathtt{bse})}\ \texttt{PK-Base}$$

$$\frac{\Delta;\Phi_1, t{::}\kappa, \Phi_2 \vdash \mathsf{OK}}{\Delta;\Phi \vdash t ::> \mathtt{S}_{\kappa}(t)}\ \texttt{PK-Var}$$

$$\frac{\Delta;\Phi \vdash \tau_1{::}\mathtt{Type} \qquad \Delta;\Phi \vdash \tau_2{::}\mathtt{Type}}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 ::> \mathtt{S}_{\mathtt{Type}}(\tau_1 \oplus \tau_2)}\ \texttt{PK-}\oplus$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \llparenthesis\rrparenthesis^{\mathtt{u}} ::> \mathtt{S}_{\kappa}(\llparenthesis\rrparenthesis^{\mathtt{u}})}\ \texttt{PK-EHole}$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK} \qquad \Delta;\Phi \vdash \tau{::}\kappa_1}{\Delta;\Phi \vdash \llparenthesis\tau\rrparenthesis^{\mathtt{u}} ::> \mathtt{S}_{\kappa}(\llparenthesis\tau\rrparenthesis^{\mathtt{u}})}\ \texttt{PK-NEHole}$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK} \qquad t \notin \Phi}{\Delta;\Phi \vdash \llparenthesis t\rrparenthesis^{\mathtt{u}} ::> \mathtt{S}_{\kappa}(\llparenthesis t\rrparenthesis^{\mathtt{u}})}\ \texttt{PK-Unbound}$$

$$\frac{\Delta;\underline{\Phi, t{::}\kappa_1} \vdash \tau ::> \kappa_2}{\Delta;\Phi \vdash \lambda t{::}\kappa_1.\tau ::> \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}\ \texttt{PK-}\lambda$$

$$\frac{\Delta;\Phi \vdash \tau_1 ::> \kappa \qquad \Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{_{\Pi}} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta;\Phi \vdash \tau_2{::}\kappa_1}{\Delta;\Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}\ \texttt{PK-Ap}$$

$\boxed{\Delta;\Phi \vdash \tau{::}\kappa}$ τ is well formed at kind κ

$$\frac{\Delta;\Phi \vdash \tau \mathrel{::>} \mathsf{S}_{\kappa}(\tau)}{\Delta;\Phi \vdash \tau{::}\kappa}\ \texttt{WFaK-1} \qquad \frac{\Delta;\Phi \vdash \tau \mathrel{::>} \kappa_1 \qquad \Delta;\Phi \vdash \kappa_1 \lesssim \kappa}{\Delta;\Phi \vdash \tau{::}\kappa}\ \texttt{WFaK-Subsump}$$

$$\frac{\Delta;\Phi \vdash \tau \mathrel{::>} \kappa}{\Delta;\Phi \vdash \tau{::}\kappa}\ \texttt{WFaK-Reit} \qquad \frac{\Delta;\Phi \vdash \tau{::}\kappa}{\Delta;\Phi \vdash \tau{::}\mathsf{S}_{\kappa}(\tau)}\ \texttt{WFaK-Self}$$

$$\frac{\Delta;\Phi \vdash \tau{::}\Pi_{t::\kappa_3}.\kappa_4 \qquad \Delta;\Phi \vdash \Pi_{t::\kappa_3}.\kappa_4 \lesssim \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \tau{::}\Pi_{t::\kappa_1}.\kappa_2}\ \texttt{WFaK-}\Pi\texttt{CSKTrans}$$

$$\frac{\Delta;\Phi \vdash \tau{::}\mathsf{S}_{\kappa}(\tau_1) \qquad \Delta;\Phi \vdash \tau_1{::}\kappa}{\Delta;\Phi \vdash \tau{::}\kappa}\ \texttt{WFaK-Flatten}$$

$\boxed{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \texttt{KHole} \overset{\blacktriangleright}{\Pi} \Pi_{t::\texttt{KHole}}.\texttt{KHole}}\ \overset{\blacktriangleright}{\Pi}\texttt{-KHole} \qquad \frac{\Delta;\Phi \vdash \kappa \equiv \mathsf{S}_{\texttt{KHole}}(\tau)}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\mathsf{S}_{\texttt{KHole}}(\tau)}.\mathsf{S}_{\texttt{KHole}}(\tau\ t)}\ \overset{\blacktriangleright}{\Pi}\texttt{-SKHole}$$

$$\frac{\Delta;\Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}\ \overset{\blacktriangleright}{\Pi}\texttt{-}\Pi$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \equiv \kappa}\ \texttt{KEquiv-Refl} \qquad \frac{\Delta;\Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Symm}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Trans}$$

$$\frac{\Delta;\Phi \vdash \tau{::}\mathsf{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \mathsf{S}_{\mathsf{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathsf{S}_{\kappa}(\tau_1)}\ \texttt{KEquiv-SKind}_{\texttt{SKind}} \qquad \frac{\Delta;\Phi \vdash \tau{::}\Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t_1::\kappa_1}.\mathsf{S}_{[t_1/t]\kappa_2}(\tau\ t_1)}\ \texttt{KEquiv-SKind}_{\Pi}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\underline{\Phi, t{::}\kappa_1} \vdash \kappa_3 \equiv \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{KEquiv-}\Pi \qquad \frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\kappa_1}(\tau_1) \equiv \mathsf{S}_{\kappa_2}(\tau_2)}\ \texttt{KEquiv-SKind}$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{KHole} \lesssim \kappa}\ \texttt{CSK-KHoleL} \qquad \frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathtt{KHole}}\ \texttt{CSK-KHoleR}$$

$$\frac{\Delta; \Phi \vdash \mathtt{S}_{\mathtt{KHole}}(\tau) \ \mathsf{OK} \qquad \Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{S}_{\mathtt{KHole}}(\tau) \lesssim \kappa}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{L}$$

$$\frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK} \qquad \Delta; \Phi \vdash \mathtt{S}_{\mathtt{KHole}}(\tau) \ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathtt{S}_{\mathtt{KHole}}(\tau)}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{R}$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-KEquiv} \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-Normal}$$

$$\frac{\Delta; \Phi \vdash \mathtt{S}_\kappa(\tau) \ \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{S}_\kappa(\tau) \lesssim \kappa}\ \texttt{CSK-SKind} \qquad \frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta; \underline{\Phi, t{::}\kappa_3} \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{CSK-}\Pi$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta; \Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathtt{S}_{\kappa_1}(\tau_1) \lesssim \mathtt{S}_{\kappa_2}(\tau_2)}\ \texttt{CSK-?}$$

$\boxed{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$ τ_1 is provably equivalent to τ_2 at kind κ

$$\frac{\Delta; \Phi \vdash \tau{::}\kappa}{\Delta; \Phi \vdash \tau \stackrel{\kappa}{\equiv} \tau}\ \texttt{EquivAK-Refl} \qquad \frac{\Delta; \Phi \vdash \tau_2 \stackrel{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Symm}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_3 \stackrel{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Trans}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \ {::>}\ \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \equiv \mathtt{S}_\kappa(\tau_2)}{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-SKind}$$

$$\frac{\Delta; \Phi \vdash \tau_1{::}\Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta; \Phi \vdash \tau_2{::}\Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta; \underline{\Phi, t{::}\kappa_1} \vdash \tau_1 \ t \stackrel{\kappa_2}{\equiv} \tau_2 \ t}{\Delta; \Phi \vdash \tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}\ \texttt{EquivAK-}\Pi$$

$$\frac{\Delta; \Phi \vdash \tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \stackrel{\kappa_1}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \ \tau_2 \stackrel{[\tau_2/t]\kappa_2}{\equiv} \tau_3 \ \tau_4}\ \texttt{EquivAK-Ap}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \stackrel{\mathtt{S}_\kappa(\tau)}{\equiv} \tau_2}{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ (1)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \stackrel{\mathtt{Type}}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \stackrel{\mathtt{Type}}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \stackrel{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4}\ (2) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \underline{\Phi, t{::}\kappa_1} \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \lambda t{::}\kappa_1.\tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t{::}\kappa_2.\tau_2}\ (3)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2 \qquad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ (4)$$

$\boxed{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}$ κ is well formed

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \mathtt{Type}\ \mathsf{OK}}\ \texttt{KWF-Type} \qquad \frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \mathtt{KHole}\ \mathsf{OK}}\ \texttt{KWF-KHole} \qquad \frac{\Delta;\Phi \vdash \tau{::}\kappa}{\Delta;\Phi \vdash \mathsf{S}_\kappa(\tau)\ \mathsf{OK}}\ \texttt{KWF-SKind}$$

$$\frac{\Delta;\underline{\Phi, t{::}\kappa_1} \vdash \kappa_2\ \mathsf{OK}}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}}\ \texttt{KWF-}\Pi$$

$\boxed{\Delta;\Phi \vdash \mathsf{OK}}$ Context is well formed

$$\frac{}{\cdot;\cdot \vdash \mathsf{OK}}\ \texttt{CWF-Nil} \qquad \frac{t \notin \Phi \qquad \Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\underline{\Phi, t{::}\kappa} \vdash \mathsf{OK}}\ \texttt{CWF-TypVar} \qquad \frac{\mathrm{u} \notin \Delta \qquad \Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\underline{\Delta, \mathrm{u}{::}\kappa};\Phi \vdash \mathsf{OK}}\ \texttt{CWF-Hole}$$

Metatheory

Lemma 1 (COK). *If $\Delta;\Phi \vdash \mathcal{J}$, then $\Delta;\Phi \vdash \mathsf{OK}$*

Proof. By simultaneous induction on derivations.
No interesting cases. □

Lemma 2 (Exchange).
If $\Delta;\Phi_1, t_{L1}{::}\kappa_{L1}, t_{L2}{::}\kappa_{L2}, \Phi_2 \vdash \mathcal{J}$ and $\Delta;\Phi_1, t_{L2}{::}\kappa_{L2}, t_{L1}{::}\kappa_{L1}, \Phi_2 \vdash \mathsf{OK}$, then $\Delta;\Phi_1, t_{L2}{::}\kappa_{L2}, t_{L1}{::}\kappa_{L1}, \Phi_2 \vdash \mathcal{J}$

Proof. By induction on derivations.
No interesting cases.
(Only rules with Φ extended in the consequent are interesting, which is only CWF-TypVar, but when $\mathcal{J}$ is CWF, Exchange is identity) □

Corollary 3 (Marked-Exchange).
If $\Delta;\underline{\underline{\Phi, t_{L1}{::}\kappa_{L1}}, t_{L2}{::}\kappa_{L2}} \vdash \mathcal{J}$ and $\Delta;\underline{\underline{\Phi, t_{L2}{::}\kappa_{L2}}, t_{L1}{::}\kappa_{L1}} \vdash \mathsf{OK}$, then $\Delta;\underline{\underline{\Phi, t_{L2}{::}\kappa_{L2}}, t_{L1}{::}\kappa_{L1}} \vdash \mathcal{J}$

Lemma 4 (Weakening).
If $\Delta;\Phi \vdash \mathcal{J}$ and $\Delta;\underline{\Phi, t_L{::}\kappa_L} \vdash \mathsf{OK}$, then $\Delta;\underline{\Phi, t_L{::}\kappa_L} \vdash \mathcal{J}$

Lemma 5 (OK-PK). *If $\Delta;\Phi \vdash \tau\ {::}{>}\ \kappa$, then $\Delta;\Phi \vdash \kappa\ \mathsf{OK}$*

Lemma 6 (OK-WFaK). *If $\Delta;\Phi \vdash \tau{::}\kappa$, then $\Delta;\Phi \vdash \kappa\ \mathsf{OK}$*

Lemma 7 (OK-MatchPi). *If $\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2$, then $\Delta;\Phi \vdash \kappa\ \mathsf{OK}$ and $\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}$*

Lemma 8 (OK-KEquiv). *If $\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2$, then $\Delta;\Phi \vdash \kappa_1\ \mathsf{OK}$ and $\Delta;\Phi \vdash \kappa_2\ \mathsf{OK}$*

Lemma 9 (OK-CSK). *If $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$, then $\Delta;\Phi \vdash \kappa_1\ \mathsf{OK}$ and $\Delta;\Phi \vdash \kappa_2\ \mathsf{OK}$*

Lemma 10 (OK-EquivAK). *If $\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2$, then $\Delta;\Phi \vdash \tau_1{::}\kappa$ and $\Delta;\Phi \vdash \tau_2{::}\kappa$ and $\Delta;\Phi \vdash \kappa\ \mathsf{OK}$*

Lemma 11 (OK-Substitution).
If $\Delta;\Phi \vdash \tau_L{::}\kappa_{L1}$ and $\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \kappa_{L2}\ \mathsf{OK}$, then $\Delta;\Phi \vdash [\tau_L/t_L]\kappa_{L2}\ \mathsf{OK}$
(induction on $\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \kappa_{L2}\ \mathsf{OK}$)

Lemma 12 (K-Substitution).
If $\Delta;\Phi \vdash \tau_{L1}{::}\kappa_{L1}$ and $\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \tau_{L2}{::}\kappa_{L2}$, then $\Delta;\Phi \vdash [\tau_{L1}/t_L]\tau_{L2}{::}[\tau_{L1}/t_L]\kappa_{L2}$
(induction on $\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \tau_{L2}{::}\kappa_{L2}$)

Proof. By simultaneous induction on derivations.
The interesting cases per lemma:

Weakening

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{d}{\Delta;\underline{\Phi, t::\kappa_1} \vdash \tau ::> \kappa_2}\;\texttt{premiss}
      \qquad
      \dfrac{
        \dfrac{a}{\Delta;\Phi \vdash \kappa_2\ \mathsf{OK}}\;\texttt{Weakening}
        \qquad
        \dfrac{b}{c}
      }{\Delta; \underline{\underline{\underline{\Phi}, t::\kappa_1}, t_L::\kappa_L} \vdash \mathsf{OK}}\;\texttt{CWF-TypVar}
    }{\Delta; \underline{\underline{\underline{\Phi}, t::\kappa_1}, t_L::\kappa_L} \vdash \tau ::> \kappa_2}\;\texttt{Weakening}
  }{\Delta; \underline{\underline{\underline{\Phi}, t_L::\kappa_L}, t::\kappa_1} \vdash \mathsf{OK}}\;\texttt{CWF-TypVar}
}{
  \dfrac{\Delta; \underline{\underline{\underline{\Phi}, t_L::\kappa_L}, t::\kappa_1} \vdash \tau ::> \kappa_2}{\Delta; \underline{\Phi, t_L::\kappa_L} \vdash \lambda t::\kappa_1.\tau ::> \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t::\kappa_1.\tau)}\;\texttt{PK-}\lambda
}\;\texttt{Marked-Exchange}
$$

Weakening.		$\Delta;\Phi\vdash\kappa_1$ OK	by subderivation premiss
		$\Delta;\underline{\Phi,t_L{::}\kappa_L}\vdash$ OK	by IH
		$\Delta;\underline{\Phi,t_L{::}\kappa_L}\vdash\kappa_1$ OK	by Weakening on subderivation premiss
		$\Delta;\underline{\underline{\Phi,t_L{::}\kappa_L},t{::}\kappa_1}\vdash$ OK	by CWF-TypVar
		$\Delta;\underline{\underline{\Phi,t{::}\kappa_1},t_L{::}\kappa_L}\vdash$ OK	by ?
		$\Delta;\underline{\underline{\Phi,t{::}\kappa_1},t_L{::}\kappa_L}\vdash\tau ::> \kappa_2$	by Weakening on premiss
		$\Delta;\underline{\underline{\Phi,t_L{::}\kappa_L},t{::}\kappa_1}\vdash\tau ::> \kappa_2$	by Marked-Exchange
		$\Delta;\underline{\Phi,t_L{::}\kappa_L}\vdash\lambda t{::}\kappa_1.\tau ::> \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)$	by PK-λ
OK-PK.	PK-Base	$\Delta;\Phi\vdash\mathtt{bse}{::}\mathtt{S}_{\mathtt{Type}}(\mathtt{bse})$	by (9)
		$\Delta;\Phi\vdash\mathtt{bse}{::}\mathtt{Type}$	by (10)
	*	$\Delta;\Phi\vdash\mathtt{S}_{\mathtt{Type}}(\mathtt{bse})$ OK	by (43)
	*	$\Delta;\Phi\vdash$ OK	by premiss
	PK-Ap		bad
OK-WFaK.	(12)	$\Delta;\Phi\vdash\tau_2{::}\kappa$	by (10)
	*	$\Delta;\Phi\vdash\mathtt{S}_{\kappa}(\tau_2)$ OK	by (43)
OK-KEquiv.	(22)	$\Delta;\Phi\vdash\tau\ t ::> \kappa$	
OK-Substitution.	(41)	$\Delta;\Phi,t_L{::}\kappa_{L1}\vdash$ OK	premiss (41)
		$\Delta;\Phi\vdash\kappa_{L1}$ OK	by subderivation premiss (46)
	*	$\Delta;\Phi\vdash$ OK	by OK-KWF
	*	$\Delta;\Phi\vdash[\tau_L/t_L]\mathtt{Type}$ OK	by (41) and degenerate subst
	(43)	$\Delta;\Phi,t_L{::}\kappa_{L1}\vdash\tau{::}\kappa$	premiss (43)
		$\Delta;\Phi,t_L{::}\kappa_{L1}\vdash$ OK	by OK-WFaK
		$\Delta;\Phi\vdash\kappa_{L1}$ OK	by subderivation premiss (46)
	*	$\Delta;\Phi\vdash$ OK	by OK-KWF
		$\Delta;\Phi\vdash[\tau_{L1}/t_L]\tau{::}[\tau_{L1}/t_L]\kappa$	by K-Substitution on premiss
	*	$\Delta;\Phi\vdash[\tau_L/t_L]\mathtt{S}_{\kappa}(\tau)$ OK	by (43)

$\square$

Lemma 13 (PK-Unicity)**.** *If* $\Delta;\Phi\vdash\tau_L ::> \kappa_{L1}$ *and* $\Delta;\Phi\vdash\tau_L ::> \kappa_{L2}$ *then* κ_{L1} *is* κ_{L2}

Lemma 14. *If* $\Delta;\Phi\vdash\tau ::> \kappa_1$ *and* $\Delta;\Phi\vdash\tau{::}\kappa_2$*, then* $\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2$

Lemma 15. *If* $\Delta;\Phi\vdash\kappa_1\lesssim\mathtt{S}_{\kappa_2}(\tau)$*, then* $\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2$