$$
\begin{array}{rcll}
\textsf{BinOp} & \oplus & ::= & \texttt{Product} \mid \texttt{Sum} \mid \texttt{Arrow} \\
\textsf{Kind} & \kappa & ::= & \texttt{Ty} \mid \texttt{KHole} \\
\textsf{ConstantTypes} & c & ::= & \texttt{Int} \mid \texttt{Float} \mid \texttt{Bool} \\
\textsf{UserHTyp} & \hat{\tau} & ::= & c \mid \hat{\tau}_1 \oplus \hat{\tau}_2 \mid \texttt{list}(\hat{\tau}) \mid (\!|\,|\!)^u \mid (\!|\hat{\tau}|\!)^u \\
\textsf{InternalHTyp} & \tau & ::= & c \mid \tau_1 \oplus \tau_2 \mid \texttt{list}(\tau) \mid (\!|\,|\!)^u \mid (\!|\tau|\!)^u
\end{array}
$$

$\boxed{\kappa_1 \sim \kappa_2}$ κ_1 is consistent with κ_2

$$
\texttt{KCHole}\ \frac{}{\texttt{KHole} \sim \texttt{Ty}}
\qquad
\texttt{KCSymm}\ \frac{\kappa_1 \sim \kappa_2}{\kappa_2 \sim \kappa_1}
\qquad
\texttt{KCRefl}\ \frac{}{\kappa \sim \kappa}
$$

$\boxed{\Phi \vdash \hat{\tau} \Rightarrow \kappa \rightsquigarrow \tau \dashv \Delta}$ $\hat{\tau}$ synthesizes kind κ and elaborates to τ

$$
\texttt{TElabSConst}\ \frac{}{\Phi \vdash c \Rightarrow \texttt{Ty} \rightsquigarrow c \dashv \cdot}
$$

$$
\texttt{TElabSBinOp}\ \frac{\Phi \vdash \hat{\tau}_1 \Leftarrow \texttt{Ty} \rightsquigarrow \tau_1 : \texttt{Ty} \dashv \Delta_1 \qquad \Phi \vdash \hat{\tau}_2 \Leftarrow \texttt{Ty} \rightsquigarrow \tau_2 : \texttt{Ty} \dashv \Delta_2}{\Phi \vdash \hat{\tau}_1 \oplus \hat{\tau}_2 \Rightarrow \texttt{Ty} \rightsquigarrow \tau_1 \oplus \tau_2 \dashv \Delta_1 \cup \Delta_2}
$$

$$
\texttt{TElabSList}\ \frac{\Phi \vdash \hat{\tau} \Leftarrow \texttt{Ty} \rightsquigarrow \tau : \texttt{Ty} \dashv \Delta}{\Phi \vdash \texttt{list}(\hat{\tau}) \Rightarrow \texttt{Ty} \rightsquigarrow \texttt{list}(\tau) \dashv \Delta}
\qquad
\texttt{TElabSVar}\ \frac{t : \kappa \in \Phi}{\Phi \vdash t \Rightarrow \kappa \rightsquigarrow t \dashv \cdot}
$$

$$
\texttt{TElabSUVar}\ \frac{t \notin \Phi}{\Phi \vdash t \Rightarrow \texttt{KHole} \rightsquigarrow (\!|t|\!)^u_{\mathsf{id}(\Phi)} \dashv u :: (\!|\,|\!)[\Phi]}
$$

$$
\texttt{TElabSHole}\ \frac{}{\Phi \vdash (\!|\,|\!)^u \Rightarrow \texttt{KHole} \rightsquigarrow (\!|\,|\!)^u_{\mathsf{id}(\Phi)} \dashv u :: (\!|\,|\!)[\Phi]}
$$

$$
\texttt{TElabSNEHole}\ \frac{\Phi \vdash \hat{\tau} \Rightarrow \kappa \rightsquigarrow \tau \dashv \Delta}{\Phi \vdash (\!|\hat{\tau}|\!)^u \Rightarrow \texttt{KHole} \rightsquigarrow (\!|\tau|\!)^u_{\mathsf{id}(\Phi)} \dashv \Delta, u :: (\!|\,|\!)[\Phi]}
$$

$\boxed{\Phi \vdash \hat{\tau} \Leftarrow \kappa_1 \leadsto \tau : \kappa_2 \dashv \Delta}$ $\hat{\tau}$ analyzes against kind κ_1 and elaborates to τ of consistent kind κ_2

TElabASubsume

$$\frac{\hat{\tau} \neq (\!|\,|\!)^u \qquad \hat{\tau} \neq (\!|\hat{\tau}'|\!)^u \qquad \begin{array}{c}\hat{\tau} \neq t \text{ where } t \notin \Phi \\ \Phi \vdash \hat{\tau} \Rightarrow \kappa' \leadsto \tau \dashv \Delta\end{array} \qquad \kappa \sim \kappa'}{\Phi \vdash \hat{\tau} \Leftarrow \kappa \leadsto \tau : \kappa' \dashv \Delta}$$

TElabAUVar

$$\frac{t \notin \Phi}{\Phi \vdash t \Leftarrow \texttt{KHole} \leadsto (\!|t|\!)^u_{\mathsf{id}(\Phi)} : \texttt{KHole} \dashv u :: (\!|\,|\!)[\Phi]}$$

TElabAEHole

$$\frac{}{\Phi \vdash (\!|\,|\!)^u \Leftarrow \kappa \leadsto (\!|\,|\!)^u_{\mathsf{id}(\Phi)} : \kappa \dashv u :: \kappa[\Phi]}$$

TElabANEHole

$$\frac{\Phi \vdash \hat{\tau} \Rightarrow \kappa' \leadsto \tau \dashv \Delta}{\Phi \vdash (\!|\hat{\tau}|\!)^u \Leftarrow \kappa \leadsto (\!|\tau|\!)^u_{\mathsf{id}(\Phi)} : \kappa \dashv \Delta, u :: \kappa[\Phi]}$$

$\boxed{\Delta; \Phi \vdash \tau : \kappa}$ $\hat{\tau}$ is assigned kind κ

KAConst

$$\frac{}{\Delta; \Phi \vdash c : \texttt{Ty}}$$

KAVar

$$\frac{t : \kappa \in \Phi}{\Delta; \Phi \vdash t : \kappa}$$

KABinOp

$$\frac{\Delta; \Phi \vdash \tau_1 : \texttt{Ty}\Delta; \Phi \vdash \tau_2 : \texttt{Ty}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 : \texttt{Ty}}$$

KAList

$$\frac{\Delta; \Phi \vdash \tau : \texttt{Ty}}{\Delta; \Phi \vdash \texttt{list}(\tau) : \texttt{Ty}}$$

KAEHole

$$\frac{u :: \kappa[\Phi'] \in \Delta \qquad \Delta; \Phi \vdash \sigma : \Phi'}{\Delta; \Phi \vdash (\!|\,|\!)^u_\sigma : \kappa}$$

KANEHole

$$\frac{\Delta; \Phi \vdash \tau : \kappa' \qquad u :: \kappa[\Phi'] \in \Delta \qquad \Delta; \Phi \vdash \sigma : \Phi'}{\Delta; \Phi \vdash (\!|\tau|\!)^u_\sigma : \kappa}$$