

## SYNTAX

Kind  $\kappa ::= \text{Type} \mid \text{KHole} \mid \text{S}_\kappa(\tau) \mid \Pi_{t::\kappa_1, \kappa_2}$   
 User Types  $\hat{\tau} ::= t \mid \text{bse} \mid \tau_1 \oplus \tau_2 \mid \emptyset^u \mid \langle \hat{\tau} \rangle^u \mid \lambda t::\text{Type}.\hat{\tau} \mid \tau_1 \tau_2$   
 Internal Types  $\tau ::= t \mid \text{bse} \mid \tau_1 \oplus \tau_2 \mid \emptyset^u \mid \langle \hat{\tau} \rangle^u \mid \langle \emptyset \rangle^u \mid \lambda t::\kappa.\tau \mid \tau_1 \tau_2$   
 Base Types  $\text{bse} ::= \text{Int} \mid \text{Float} \mid \text{Bool}$   
 BinOp  $\oplus ::= \times \mid + \mid \rightarrow$   
 Type Pattern  
 User Expression  
 Internal Expression

## DECLARATIVES

$\Delta; \Phi \vdash \tau ::> \kappa$   $\tau$  has principal (well formed) kind  $\kappa$

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{bse} ::> \text{S}_{\text{Type}}(\text{bse})} \text{PK-Base}$   $\frac{\Delta; \Phi_1, t::\kappa_1, \Phi_2 \vdash \text{OK}}{\Delta; \Phi \vdash t ::> \text{S}_\kappa(t)} \text{PK-Var}$   $\frac{\Delta; \Phi \vdash \tau_1 :: \text{Type} \quad \Delta; \Phi \vdash \tau_2 :: \text{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> \text{S}_{\text{Type}}(\tau_1 \oplus \tau_2)} \text{PK-}\oplus$   $\frac{\Delta_1, u::\kappa_1, \Delta_2; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \emptyset^u ::> \text{S}_\kappa(\emptyset^u)} \text{PK-EHole}$   $\frac{\Delta_1, u::\kappa_1, \Delta_2; \Phi \vdash \text{OK} \quad \Delta; \Phi \vdash \tau::\kappa_1}{\Delta; \Phi \vdash \langle \hat{\tau} \rangle^u ::> \text{S}_\kappa(\langle \hat{\tau} \rangle^u)} \text{PK-NEHole}$   $\frac{\Delta_1, u::\kappa_1, \Delta_2; \Phi \vdash \text{OK} \quad t \notin \Phi}{\Delta; \Phi \vdash \langle \emptyset \rangle^u ::> \text{S}_\kappa(\langle \emptyset \rangle^u)} \text{PK-Unbound}$   $\frac{\Delta; \Phi_s, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau ::> \text{S}_{\Pi_{t::\kappa_1, \kappa_2}}(\lambda t::\kappa_1.\tau)} \text{PK-}\lambda$

$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa \quad \Delta; \Phi \vdash \kappa \overset{\text{!}}{\Pi} \Pi_{t::\kappa_1, \kappa_2} \quad \Delta; \Phi \vdash \tau_2 :: \kappa_1}{\Delta; \Phi \vdash \tau_1 \tau_2 ::> [\tau_2 / t] \kappa_2} \text{PK-Ap}$

$\Delta; \Phi \vdash \tau::\kappa$   $\tau$  is well formed at kind  $\kappa$

$\frac{\Delta; \Phi \vdash \tau ::> \text{S}_\kappa(\tau)}{\Delta; \Phi \vdash \tau::\kappa} \text{WFaK-1}$

$\frac{\Delta; \Phi \vdash \tau ::> \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau::\kappa} \text{WFaK-Subeump}$

$\frac{\Delta; \Phi \vdash \tau ::> \kappa}{\Delta; \Phi \vdash \tau::\kappa} \text{WFaK-Reit}$

$\frac{\Delta; \Phi \vdash \tau::\kappa}{\Delta; \Phi \vdash \tau::\text{S}_\kappa(\tau)} \text{WFaK-Self}$

$\frac{\Delta; \Phi \vdash \tau::\Pi_{t::\kappa_1, \kappa_2} \quad \Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_2} \lesssim \Pi_{t::\kappa_1, \kappa_2}}{\Delta; \Phi \vdash \tau::\Pi_{t::\kappa_1, \kappa_2}} \text{WFaK-}\Pi\text{CSKTrans}$

$\frac{\Delta; \Phi \vdash \tau::\text{S}_\kappa(\tau_1) \quad \Delta; \Phi \vdash \tau_1::\kappa}{\Delta; \Phi \vdash \tau::\kappa} \text{WFaK-Flatten}$

$\Delta; \Phi \vdash \kappa \overset{\text{!}}{\Pi} \Pi_{t::\kappa_1, \kappa_2}$   $\kappa$  has matched  $\Pi$ -kind  $\Pi_{t::\kappa_1, \kappa_2}$

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{KHole} \overset{\text{!}}{\Pi} \Pi_{t::\text{KHole}, \text{KHole}} \text{KHole}} \text{-KHole}$

$\frac{\Delta; \Phi \vdash \kappa \equiv \text{S}_{\text{KHole}}(\tau)}{\Delta; \Phi \vdash \kappa \overset{\text{!}}{\Pi} \Pi_{t::\text{S}_{\text{KHole}}(\tau), \text{S}_{\text{KHole}}(\tau) t}} \text{-SKHole}$

$\frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1, \kappa_2}}{\Delta; \Phi \vdash \kappa \overset{\text{!}}{\Pi} \Pi_{t::\kappa_1, \kappa_2}} \text{-}\Pi$

$\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$   $\kappa_1$  is equivalent to  $\kappa_2$

$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \equiv \kappa} \text{KEquiv-Ref1}$

$\frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \text{KEquiv-Symm}$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \text{KEquiv-Trans}$

$\frac{\Delta; \Phi \vdash \tau::\text{S}_\kappa(\tau_1)}{\Delta; \Phi \vdash \text{S}_{\text{S}_\kappa(\tau_1)}(\tau) \equiv \text{S}_\kappa(\tau_1)} \text{KEquiv-SKind}_{\text{SKind}}$

$\frac{\Delta; \Phi \vdash \tau::\Pi_{t::\kappa_1, \kappa_2}}{\Delta; \Phi \vdash \text{S}_{\Pi_{t::\kappa_1, \kappa_2}}(\tau) \equiv \Pi_{t::\kappa_1, \kappa_2} \cdot \text{S}_{[t_1 / t] \kappa_2}(\tau t_1)} \text{KEquiv-SKind}_{\Pi}$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_2} \equiv \Pi_{t::\kappa_3, \kappa_4}} \text{KEquiv-}\Pi$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \text{S}_{\kappa_1}(\tau_1) \equiv \text{S}_{\kappa_2}(\tau_2)} \text{KEquiv-SKind}$

$\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$   $\kappa_1$  is a consistent subkind of  $\kappa_2$

$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \text{KHole} \lesssim \kappa} \text{CSK-KHoleL}$

$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \lesssim \text{KHole}} \text{CSK-KHoleR}$

$\frac{\Delta; \Phi \vdash \text{S}_{\text{KHole}}(\tau) \text{ OK} \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \text{S}_{\text{KHole}}(\tau) \lesssim \kappa} \text{CSK-SKind}_{\text{SoleL}}$

$\frac{\Delta; \Phi \vdash \kappa \text{ OK} \quad \Delta; \Phi \vdash \text{S}_{\text{KHole}}(\tau) \text{ OK}}{\Delta; \Phi \vdash \kappa \lesssim \text{S}_{\text{KHole}}(\tau)} \text{CSK-SKind}_{\text{SoleR}}$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \text{CSK-KEquiv}$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \quad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \text{CSK-Normal}$

$\frac{\Delta; \Phi \vdash \text{S}_\kappa(\tau) \text{ OK}}{\Delta; \Phi \vdash \text{S}_\kappa(\tau) \lesssim \kappa} \text{CSK-SKind}$

$\frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \quad \Delta; \Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_3} \lesssim \Pi_{t::\kappa_3, \kappa_4}} \text{CSK-}\Pi$

$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \quad \Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \text{S}_{\kappa_1}(\tau_1) \lesssim \text{S}_{\kappa_2}(\tau_2)} \text{CSK-?}$

$\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2$   $\tau_1$  is provably equivalent to  $\tau_2$  at kind  $\kappa$

$\frac{\Delta; \Phi \vdash \tau::\kappa}{\Delta; \Phi \vdash \tau \equiv \tau} \text{EquivAK-Ref1}$

$\frac{\Delta; \Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \text{EquivAK-Symm}$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \quad \Delta; \Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \text{EquivAK-Trans}$

$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 \equiv \text{S}_\kappa(\tau_2)}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \text{EquivAK-SKind}$

$\frac{\Delta; \Phi \vdash \tau_1::\Pi_{t::\kappa_1, \kappa_3} \quad \Delta; \Phi \vdash \tau_2::\Pi_{t::\kappa_1, \kappa_4} \quad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 t \overset{\kappa_3}{\equiv} \tau_2 t}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \text{EquivAK-}\Pi$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1, \kappa_3}}{\equiv} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \tau_2 \overset{[\tau_2 / t] \kappa_2}{\equiv} \tau_3 \tau_4} \text{EquivAK-Ap}$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\text{S}_\kappa(\tau)}{\equiv} \tau_2}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2} (1)$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau_1 \equiv \lambda t::\kappa_2.\tau_2} (3)$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2} (4)$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\text{Type}}{\equiv} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \overset{\text{Type}}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \overset{\text{Type}}{\equiv} \tau_3 \oplus \tau_4} (2)$

$\Delta; \Phi \vdash \kappa \text{ OK}$   $\kappa$  is well formed

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{Type OK}} \text{KWF-Type}$

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{KHole OK}} \text{KWF-KHole}$

$\frac{\Delta; \Phi \vdash \tau::\kappa}{\Delta; \Phi \vdash \text{S}_\kappa(\tau) \text{ OK}} \text{KWF-SKind}$

$\frac{\Delta; \Phi, t::\kappa_1 \vdash \kappa_2 \text{ OK}}{\Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_2} \text{ OK}} \text{KWF-}\Pi$

$\Delta; \Phi \vdash \text{OK}$  Context is well formed

$\frac{}{\cdot, \cdot \vdash \text{OK}} \text{CWF-Nil}$

$\frac{t \notin \Delta \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi, t::\kappa \vdash \text{OK}} \text{CWF-TypVar}$

$\frac{u \notin \Delta \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta, u::\kappa; \Phi \vdash \text{OK}} \text{CWF-Hole}$

## METATHEORY

**Lemma 1** (COK). If  $\Delta; \Phi \vdash \mathcal{J}$ , then  $\Delta; \Phi \vdash \text{OK}$

*Proof.* By simultaneous induction on derivations.  
 No interesting cases. □

**Lemma 2** (Exchange).

If  $\Delta; \Phi_1, t_{L1}::\kappa_{L1}, t_{L2}::\kappa_{L2}, \Phi_2 \vdash \mathcal{J}$  and  $\Delta; \Phi_1, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1}, \Phi_2 \vdash \text{OK}$ , then  $\Delta; \Phi_1, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1}, \Phi_2 \vdash \mathcal{J}$

*Proof.* By induction on derivations.  
 No interesting cases.

(Only rules with  $\Phi$  extended in the consequent are interesting, which is only CWF-TypVar, but when  $\mathcal{J}$  is CWF, Exchange is identity) □

**Corollary 3** (Marked-Exchange).

If  $\Delta; \Phi, t_{L1}::\kappa_{L1}, t_{L2}::\kappa_{L2} \vdash \mathcal{J}$  and  $\Delta; \Phi, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1} \vdash \text{OK}$ , then  $\Delta; \Phi, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1} \vdash \mathcal{J}$

**Lemma 4** (Weakening).

If  $\Delta; \Phi \vdash \mathcal{J}$  and  $\Delta; \Phi, t_L::\kappa_L \vdash \text{OK}$  and  $t_L \notin \mathcal{J}$  and  $\forall t \in \kappa_L, t \notin \mathcal{J}$ , then  $\Delta; \Phi, t_L::\kappa_L \vdash \mathcal{J}$

*Proof.* By induction on derivations.

When applying Weakening in the induction, check that the left premiss is always a subderivation.

(PoS = premiss of subderivation)

## Weakening

$\frac{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \text{OK} \quad \text{IH}}{\Delta; \Phi \vdash \kappa_L \text{ OK}} \text{PoS} \quad \frac{\Delta; \Phi, \underline{t}::\kappa_I \vdash \tau ::= \kappa_S \quad \text{premiss}}{\Delta; \Phi, \underline{t}::\kappa_I \vdash \text{OK}} \text{COK} \quad \frac{}{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \kappa_L \text{ OK}} \text{Weakening}$	$\frac{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \text{OK} \quad \text{IH} \quad \frac{\underline{t_L} \notin \mathcal{J} \quad \text{IH}}{\underline{t_L} \neq \underline{t}} \quad \frac{\underline{t_L} \notin \mathcal{J} \quad \text{IH}}{\underline{t_L} \notin \kappa_I}}{\underline{t_L} \notin \Phi} \text{PoS} \quad \frac{}{\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \text{OK}} \text{CWF-TypVar}$	$\frac{\Delta; \Phi, \underline{t}::\kappa_I \vdash \tau ::= \kappa_S \quad \text{premiss}}{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \tau ::= \kappa_S} \text{PoS} \quad \frac{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \text{OK} \quad \text{COK}}{\underline{t} \notin \Phi} \text{PoS} \quad \frac{\underline{t_L} \notin \mathcal{J} \quad \text{IH} \quad \underline{t} \in \mathcal{J}}{\underline{t} \neq \underline{t_L}} \quad \frac{\forall \hat{t} \in \kappa_L, \hat{t} \notin \mathcal{J} \quad \text{IH} \quad \underline{t} \in \mathcal{J}}{\underline{t} \notin \kappa_L} \text{PoS}$
$\frac{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \tau ::= \kappa_S \quad \text{premiss}}{\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \tau ::= \kappa_S} \text{Weakening}$	$\frac{\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \text{OK} \quad \text{Weakening}}{\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \text{OK}} \text{CWF-TypVar}$	$\frac{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \text{OK} \quad \text{premiss} \quad \frac{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \text{OK} \quad \text{premiss}}{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \kappa_I \text{ OK}} \text{PoS} \quad \frac{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \kappa_I \text{ OK}}{\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \text{OK}} \text{Weakening} \quad \frac{}{\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \text{OK}} \text{CWF-TypVar}$
	$\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \tau ::= \kappa_S \quad \text{Marked-Exchange} \quad \text{PK-}\lambda$	
	$\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \tau ::= \kappa_S \quad \Delta; \Phi, \underline{t_L}::\kappa_L \vdash \lambda \underline{t_L}::\kappa_I, \tau ::= \mathbf{S}_{\Pi_{t_L}, \kappa_I, \kappa_S}(\lambda \underline{t_L}::\kappa_I, \tau)$	
$\frac{\Delta; \Phi \vdash \kappa_I \equiv \kappa_S \quad \text{premiss}}{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \kappa_I \equiv \kappa_S} \text{Weakening} \quad \frac{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \text{OK} \quad \text{IH}}{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \kappa_I \equiv \kappa_S} \text{Marked-Exchange}$	$\frac{\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_L \vdash \kappa_S \equiv \kappa_I \quad \Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_I \vdash \text{OK}}{\Delta; \Phi, \underline{t_L}::\kappa_L, \underline{t_L}::\kappa_I \vdash \kappa_S \equiv \kappa_I} \text{Marked-Exchange}$	$\frac{}{\Delta; \Phi, \underline{t_L}::\kappa_L \vdash \Pi_{t_L, \kappa_I, \kappa_S} \equiv \Pi_{t_L, \kappa_I, \kappa_S}} \text{KEquiv-II}$

□

**Lemma 5** (OK-PK). *If  $\Delta; \Phi \vdash \tau ::> \kappa$ , then  $\Delta; \Phi \vdash \kappa$  OK*

**Lemma 6** (OK-WFaK). *If  $\Delta; \Phi \vdash \tau :: \kappa$ , then  $\Delta; \Phi \vdash \kappa$  OK*

**Lemma 7** (OK-MatchPi). *If  $\Delta; \Phi \vdash \kappa \overset{\bullet}{\Pi} \Pi_{t::\kappa_1, \kappa_2}$ , then  $\Delta; \Phi \vdash \kappa$  OK and  $\Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_2}$  OK*

**Lemma 8** (OK-KEquiv). *If  $\Delta; \Phi \vdash \kappa_I \equiv \kappa_2$ , then  $\Delta; \Phi \vdash \kappa_I$  OK and  $\Delta; \Phi \vdash \kappa_2$  OK*

**Lemma 9** (OK-CSK). *If  $\Delta; \Phi \vdash \kappa_I \lesssim \kappa_2$ , then  $\Delta; \Phi \vdash \kappa_I$  OK and  $\Delta; \Phi \vdash \kappa_2$  OK*

**Lemma 10** (OK-EquivAK). *If  $\Delta; \Phi \vdash \tau_I \overset{\kappa}{=} \tau_2$ , then  $\Delta; \Phi \vdash \tau_I :: \kappa$  and  $\Delta; \Phi \vdash \tau_2 :: \kappa$  and  $\Delta; \Phi \vdash \kappa$  OK*

**Lemma 11** (OK-Substitution).

*If  $\Delta; \Phi \vdash \tau_L :: \kappa_{L1}$  and  $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \kappa_{L2}$  OK, then  $\Delta; \Phi \vdash [\tau_L/t_L] \kappa_{L2}$  OK*

*(induction on  $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \kappa_{L2}$  OK)*

**Lemma 12** (K-Substitution).

*If  $\Delta; \Phi \vdash \tau_{L1} :: \kappa_{L1}$  and  $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \tau_{L2} :: \kappa_{L2}$ , then  $\Delta; \Phi \vdash [\tau_{L1}/t_L] \tau_{L2} :: [\tau_{L1}/t_L] \kappa_{L2}$*

*(induction on  $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \tau_{L2} :: \kappa_{L2}$ )*

*Proof.* By simultaneous induction on derivations.

The interesting cases per lemma:

(PoS = premiss of subderivation)	OK-PK.	PK-Base	$\Delta; \Phi \vdash \mathbf{bse} :: \mathbf{S_{Type}}(\mathbf{bse})$ $\Delta; \Phi \vdash \mathbf{bse} :: \mathbf{Type}$ $\Delta; \Phi \vdash \mathbf{S_{Type}}(\mathbf{bse})$ OK $\Delta; \Phi \vdash \text{OK}$	by (9) by (10) by (43) by premiss
		*		bad
		*		by (10)
	OK-WFaK.	PK-Ap (12)	$\Delta; \Phi \vdash \tau_2 :: \kappa$ $\Delta; \Phi \vdash \mathbf{S}_{\kappa}(\tau_2)$ OK	by (43)
		*	$\Delta; \Phi \vdash \tau \ t ::> \kappa$ $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \text{OK}$ $\Delta; \Phi \vdash \kappa_{L1}$ OK $\Delta; \Phi \vdash \text{OK}$	premiss (41) by subderivation premiss (46) by OK-KWF
	OK-KEquiv.	(22)	$\Delta; \Phi, t_L :: \kappa_{L1} \vdash \text{OK}$ $\Delta; \Phi \vdash \kappa_{L1}$ OK	by (41) and degenerate subst
	OK-Substitution.	(41)	$\Delta; \Phi, t_L :: \kappa_{L1} \vdash \tau :: \kappa$ $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \text{OK}$ $\Delta; \Phi \vdash \kappa_{L1}$ OK $\Delta; \Phi \vdash \text{OK}$	premiss (43) by OK-WFaK by subderivation premiss (46) by OK-KWF
		*	$\Delta; \Phi \vdash [\tau_{L1}/t_L] \tau :: [\tau_{L1}/t_L] \kappa$	by K-Substitution on premiss
		*	$\Delta; \Phi \vdash [\tau_L/t_L] \mathbf{S}_{\kappa}(\tau)$ OK	by (43)
		(43)		
		*		
		*		

□

**Lemma 13** (PK-Unicity). *If  $\Delta; \Phi \vdash \tau_L ::> \kappa_{L1}$  and  $\Delta; \Phi \vdash \tau_L ::> \kappa_{L2}$  then  $\kappa_{L1}$  is  $\kappa_{L2}$*

**Lemma 14.** *If  $\Delta; \Phi \vdash \tau ::> \kappa_I$  and  $\Delta; \Phi \vdash \tau :: \kappa_2$ , then  $\Delta; \Phi \vdash \kappa_I \lesssim \kappa_2$*

**Lemma 15.** *If  $\Delta; \Phi \vdash \kappa_I \lesssim \mathbf{S}_{\kappa_2}(\tau)$ , then  $\Delta; \Phi \vdash \kappa_I \lesssim \kappa_2$*