Hazel Phi: 9-type-aliases

July 20, 2021

SYNTAX

Kind	κ	$::=$	$\mathtt{Type} \mid \mathtt{KHole} \mid \mathsf{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	$::=$	$t \mid \mathtt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid \llparenthesis\rrparenthesis^{\mathtt{u}} \mid \llparenthesis\hat{\tau}\rrparenthesis^{\mathtt{u}} \mid \lambda t::\mathtt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \mathtt{bse} \mid \tau_1 \oplus \tau_2 \mid \llparenthesis\rrparenthesis^{\mathtt{u}} \mid \llparenthesis\tau\rrparenthesis^{\mathtt{u}} \mid \llparenthesis t\rrparenthesis^{\mathtt{u}} \mid \lambda t::\kappa.\tau \mid \tau_1\ \tau_2$
Base Types	$\mathtt{bse}$	$::=$	$\mathtt{Int} \mid \mathtt{Float} \mid \mathtt{Bool}$
BinOp	$\oplus$	$::=$	$\times \mid + \mid \to$
Type Pattern			
User Expression			
Internal Expression			

DECLARATIVES

$\boxed{\Delta;\Phi \vdash \tau ::> \kappa}$ τ has principal (well formed) kind κ

$$\frac{}{\Delta;\Phi \vdash \mathtt{bse} ::> \mathsf{S}_{\mathtt{Type}}(\mathtt{bse})}\ (1) \qquad \frac{t::\kappa \in \Phi}{\Delta;\Phi \vdash t ::> \mathsf{S}_{\kappa}(t)}\ (2)$$

$$\frac{\Delta;\Phi \vdash \tau_1::\mathtt{Type} \qquad \Delta;\Phi \vdash \tau_2::\mathtt{Type}}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 ::> \mathsf{S}_{\mathtt{Type}}(\tau_1 \oplus \tau_2)}\ (3) \qquad \frac{\mathtt{u}::\kappa \in \Delta}{\Delta;\Phi \vdash \llparenthesis\rrparenthesis^{\mathtt{u}} ::> \mathsf{S}_{\kappa}(\llparenthesis\rrparenthesis^{\mathtt{u}})}\ (4)$$

$$\frac{\mathtt{u}::\kappa \in \Delta \qquad \Delta;\Phi \vdash \tau::\kappa_1}{\Delta;\Phi \vdash \llparenthesis\tau\rrparenthesis^{\mathtt{u}} ::> \mathsf{S}_{\kappa}(\llparenthesis\tau\rrparenthesis^{\mathtt{u}})}\ (5) \qquad \frac{\mathtt{u}::\kappa \in \Delta \qquad t \notin \mathsf{dom}(\Phi)}{\Delta;\Phi \vdash \llparenthesis t\rrparenthesis^{\mathtt{u}} ::> \mathsf{S}_{\kappa}(\llparenthesis t\rrparenthesis^{\mathtt{u}})}\ (6)$$

$$\frac{\Delta;\Phi \vdash \kappa_1\ \mathsf{OK} \qquad \Delta;\Phi, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta;\Phi \vdash \lambda t::\kappa_1.\tau ::> \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t::\kappa_1.\tau)}\ (7)$$

$$\frac{\Delta;\Phi \vdash \tau_1 ::> \kappa \qquad \Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptscriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta;\Phi \vdash \tau_2::\kappa_1}{\Delta;\Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}\ (8)$$

$\boxed{\Delta; \Phi \vdash \tau :: \kappa}$ τ is well formed at kind κ

$$\frac{\Delta; \Phi \vdash \tau ::> \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \;(9) \qquad \frac{\Delta; \Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau :: \kappa} \;(10)$$

$$\frac{\Delta; \Phi \vdash \tau ::> \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \;(11)$$

$$\frac{\Delta; \Phi \vdash \tau_2 :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_2)} \;(12) \qquad \frac{\Delta; \Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_3) \qquad \Delta; \Phi \vdash \tau_3 :: \mathtt{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_2)} \;(13)$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau)} \;(14)$$

$\boxed{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{\Delta; \Phi \vdash \kappa \equiv \mathtt{KHole}}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\mathtt{KHole}}.\mathtt{KHole}} \;(15) \qquad \frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2} \;(16)$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta; \Phi \vdash \kappa \;\mathsf{OK}}{\Delta; \Phi \vdash \kappa \equiv \kappa} \;(17) \qquad \frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \;(18) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \;(19)$$

$$\frac{\Delta; \Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \mathtt{S}_{\mathtt{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathtt{S}_{\kappa}(\tau_1)} \;(20) \qquad \frac{\Delta; \Phi \vdash \tau :: \mathtt{KHole}}{\Delta; \Phi \vdash \mathtt{S}_{\mathtt{KHole}}(\tau) \equiv \mathtt{KHole}} \;(21)$$

$$\frac{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t::\kappa_1}.\mathtt{S}_{\kappa_2}(\tau\ t)} \;(22) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4} \;(23)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathtt{S}_{\kappa}(\tau_1) \equiv \mathtt{S}_{\kappa}(\tau_2)} \;(24)$$

$\boxed{\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta;\Phi\vdash\mathtt{KHole}\lesssim\kappa}\ (25)\qquad\frac{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta;\Phi\vdash\kappa\lesssim\mathtt{KHole}}\ (26)$$

$$\frac{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2}{\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2}\ (27)\qquad\frac{\Delta;\Phi\vdash\kappa_1\equiv\kappa_3\quad\Delta;\Phi\vdash\kappa_3\lesssim\kappa_4\quad\Delta;\Phi\vdash\kappa_4\equiv\kappa_2}{\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2}\ (28)$$

$$\frac{\Delta;\Phi\vdash\tau::\kappa}{\Delta;\Phi\vdash\mathtt{S}_\kappa(\tau)\lesssim\kappa}\ (29)\qquad\frac{\Delta;\Phi\vdash\kappa_3\lesssim\kappa_1\quad\Delta;\Phi,t::\kappa_3\vdash\kappa_2\lesssim\kappa_4}{\Delta;\Phi\vdash\Pi_{t::\kappa_1}.\kappa_2\lesssim\Pi_{t::\kappa_3}.\kappa_4}\ (30)$$

$$\frac{\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2\quad\Delta;\Phi\vdash\tau_1\overset{\kappa_1}{\equiv}\tau_2}{\Delta;\Phi\vdash\mathtt{S}_{\kappa_1}(\tau_1)\lesssim\mathtt{S}_{\kappa_2}(\tau_2)}\ (31)$$

$\boxed{\Delta;\Phi\vdash\tau_1\overset{\kappa}{\equiv}\tau_2}$ τ_1 is equivalent to τ_2 at kind κ

$$\frac{\Delta;\Phi\vdash\tau_1::\mathtt{S}_\kappa(\tau_2)}{\Delta;\Phi\vdash\tau_1\overset{\kappa}{\equiv}\tau_2}\ (32)$$

$$\frac{\Delta;\Phi\vdash\tau::\kappa}{\Delta;\Phi\vdash\tau\overset{\kappa}{\equiv}\tau}\ (33)\qquad\frac{\Delta;\Phi\vdash\tau_2\overset{\kappa}{\equiv}\tau_1}{\Delta;\Phi\vdash\tau_1\overset{\kappa}{\equiv}\tau_2}\ (34)\qquad\frac{\Delta;\Phi\vdash\tau_1\overset{\kappa}{\equiv}\tau_3\quad\Delta;\Phi\vdash\tau_3\overset{\kappa}{\equiv}\tau_1}{\Delta;\Phi\vdash\tau_1\overset{\kappa}{\equiv}\tau_2}\ (35)$$

$$\frac{\Delta;\Phi\vdash\tau_1\overset{\mathtt{Type}}{\equiv}\tau_3\quad\Delta;\Phi\vdash\tau_2\overset{\mathtt{Type}}{\equiv}\tau_4}{\Delta;\Phi\vdash\tau_1\oplus\tau_2\overset{\mathtt{Type}}{\equiv}\tau_3\oplus\tau_4}\ (36)\qquad\frac{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2\quad\Delta;\Phi,t::\kappa_1\vdash\tau_1\overset{\kappa}{\equiv}\tau_2}{\Delta;\Phi\vdash\lambda t::\kappa_1.\tau_1\overset{\Pi_{t::\kappa_1}.\kappa}{\equiv}\lambda t::\kappa_2.\tau_2}\ (37)$$

$$\frac{\Delta;\Phi\vdash\tau_1\overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv}\tau_3\quad\Delta;\Phi\vdash\tau_2\overset{\kappa_1}{\equiv}\tau_4}{\Delta;\Phi\vdash\tau_1\ \tau_2\overset{[\tau_1/t]\kappa_2}{\equiv}\tau_3\ \tau_4}\ (38)$$

$$\frac{\Delta;\Phi\vdash\tau_1::\Pi_{t::\kappa_1}.\kappa_3\quad\Delta;\Phi\vdash\tau_2::\Pi_{t::\kappa_1}.\kappa_4\quad\Delta;\Phi,t::\kappa_1\vdash\tau_1\ t\overset{\kappa_2}{\equiv}\tau_2\ t}{\Delta;\Phi\vdash\tau_1\overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv}\tau_2}\ (39)$$

$$\frac{\Delta;\Phi\vdash\tau_1\overset{\kappa_1}{\equiv}\tau_2\quad\Delta;\Phi\vdash\kappa_1\equiv\kappa}{\Delta;\Phi\vdash\tau_1\overset{\kappa}{\equiv}\tau_2}\ (40)$$

$\boxed{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}$ κ is well formed

$$\frac{}{\Delta;\Phi\vdash\mathtt{Type}\ \mathsf{OK}}\ (41)\qquad\frac{}{\Delta;\Phi\vdash\mathtt{KHole}\ \mathsf{OK}}\ (42)\qquad\frac{\Delta;\Phi\vdash\tau::\kappa}{\Delta;\Phi\vdash\mathtt{S}_\kappa(\tau)\ \mathsf{OK}}\ (43)$$

$$\frac{\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}\quad\Delta;\Phi\vdash\kappa_2\ \mathsf{OK}}{\Delta;\Phi\vdash\Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}}\ (44)$$

Metatheory

Contexts implicitly assumed to be wellformed in the usual way.

Definition 1 (size of kinds a la Stone)**.**

$$
\begin{array}{lcl}
size(\texttt{Type}) & = & 1 \\
size(\texttt{KHole}) & = & 1 \\
size(\mathtt{S}_{\kappa}(\tau)) & = & size(\kappa) + 2 \\
size(\Pi_{t::\kappa_1}.\kappa_2) & = & size(\kappa_1) + size(\kappa_2) + 2
\end{array}
$$

Lemma 1. *If* $\Delta; \Phi \vdash \tau ::> \kappa$*, then* $\Delta; \Phi \vdash \kappa$ *OK*

Lemma 2. *If* $\Delta; \Phi \vdash \tau :: \kappa$*, then* $\Delta; \Phi \vdash \kappa$ *OK*

Lemma 3. *If* $\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$*, then* $\Delta; \Phi \vdash \kappa_1$ *OK and* $\Delta; \Phi \vdash \kappa_2$ *OK*

Lemma 4. *If* $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$*, then* $\Delta; \Phi \vdash \kappa_1$ *OK and* $\Delta; \Phi \vdash \kappa_2$ *OK*

Lemma 5. *If* $\Delta; \Phi \vdash \tau ::> \kappa_1$ *and* $\Delta; \Phi \vdash \tau :: \kappa_2$*, then* $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$