Hazel Phi: 11-type-constructors

September 25, 2021

NOTES

Writing up the proof for unicity

SYNTAX

Kind	κ	$::=$	$\mathtt{Type} \mid \mathtt{KHole} \mid \mathsf{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	$::=$	$t \mid \mathtt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid \llparenthesis\rrparenthesis^{\mathtt{u}} \mid \llparenthesis\hat{\tau}\rrparenthesis^{\mathtt{u}} \mid \lambda t{::}\mathtt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \mathtt{bse} \mid \tau_1 \oplus \tau_2 \mid \llparenthesis\rrparenthesis^{\mathtt{u}} \mid \llparenthesis\tau\rrparenthesis^{\mathtt{u}} \mid \llparenthesis t\rrparenthesis^{\mathtt{u}} \mid \lambda t{::}\kappa.\tau \mid \tau_1\ \tau_2$
Base Types	$\mathtt{bse}$	$::=$	$\mathtt{Int} \mid \mathtt{Float} \mid \mathtt{Bool}$
BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Type Pattern			
User Expression			
Internal Expression			

DECLARATIVES

$\boxed{\Delta; \Phi \vdash \tau :\!:> \kappa}$ τ has principal (well formed) kind κ

$$\frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{bse} :\!:> \mathsf{S}_{\mathtt{Type}}(\mathtt{bse})}\ \texttt{PK-Base} \qquad \frac{\Delta; \Phi_1, t{::}\kappa, \Phi_2 \vdash \mathsf{OK}}{\Delta; \Phi \vdash t :\!:> \mathsf{S}_{\kappa}(t)}\ \texttt{PK-Var}$$

$$\frac{\Delta; \Phi \vdash \tau_1{::}\mathtt{Type} \qquad \Delta; \Phi \vdash \tau_2{::}\mathtt{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 :\!:> \mathsf{S}_{\mathtt{Type}}(\tau_1 \oplus \tau_2)}\ \texttt{PK-}\oplus \qquad \frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \llparenthesis\rrparenthesis^{\mathtt{u}} :\!:> \mathsf{S}_{\kappa}(\llparenthesis\rrparenthesis^{\mathtt{u}})}\ \texttt{PK-EHole}$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK} \qquad \Delta; \Phi \vdash \tau{::}\kappa_1}{\Delta; \Phi \vdash \llparenthesis\tau\rrparenthesis^{\mathtt{u}} :\!:> \mathsf{S}_{\kappa}(\llparenthesis\tau\rrparenthesis^{\mathtt{u}})}\ \texttt{PK-NEHole} \qquad \frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK} \qquad t \notin \Phi}{\Delta; \Phi \vdash \llparenthesis t\rrparenthesis^{\mathtt{u}} :\!:> \mathsf{S}_{\kappa}(\llparenthesis t\rrparenthesis^{\mathtt{u}})}\ \texttt{PK-Unbound}$$

$$\frac{\Delta; \underline{\Phi, t{::}\kappa_1} \vdash \tau :\!:> \kappa_2}{\Delta; \Phi \vdash \lambda t{::}\kappa_1.\tau :\!:> \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}\ \texttt{PK-}\lambda$$

$$\frac{\Delta; \Phi \vdash \tau_1 :\!:> \kappa \qquad \Delta; \Phi \vdash \kappa \blacktriangleright_{\Pi} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta; \Phi \vdash \tau_2{::}\kappa_1}{\Delta; \Phi \vdash \tau_1\ \tau_2 :\!:> [\tau_2/t]\kappa_2}\ \texttt{PK-Ap}$$

$\boxed{\Delta; \Phi \vdash \tau :: \kappa}$ τ is well formed at kind κ

$$\frac{\Delta; \Phi \vdash \tau ::> \mathsf{S}_{\kappa}(\tau)}{\Delta; \Phi \vdash \tau :: \kappa}\ \texttt{WFaK-1} \qquad \frac{\Delta; \Phi \vdash \tau ::> \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau :: \kappa}\ \texttt{WFaK-Subsump}$$

$$\frac{\Delta; \Phi \vdash \tau ::> \kappa}{\Delta; \Phi \vdash \tau :: \kappa}\ \texttt{WFaK-Reit} \qquad \frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau :: \mathsf{S}_{\kappa}(\tau)}\ \texttt{WFaK-Self}$$

$$\frac{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_3}.\kappa_4 \qquad \Delta; \Phi \vdash \Pi_{t::\kappa_3}.\kappa_4 \lesssim \Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_1}.\kappa_2}\ \texttt{WFaK-}\Pi\texttt{CSKTrans}$$

$$\frac{\Delta; \Phi \vdash \tau :: \mathsf{S}_{\kappa}(\tau_1) \qquad \Delta; \Phi \vdash \tau_1 :: \kappa}{\Delta; \Phi \vdash \tau :: \kappa}\ \texttt{WFaK-Flatten}$$

$\boxed{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \texttt{KHole} \overset{\blacktriangleright}{\Pi} \Pi_{t::\texttt{KHole}}.\texttt{KHole}}\ \overset{\blacktriangleright}{\Pi}\texttt{-KHole} \qquad \frac{\Delta; \Phi \vdash \kappa \equiv \mathsf{S}_{\texttt{KHole}}(\tau)}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\mathsf{S}_{\texttt{KHole}}(\tau)}.\mathsf{S}_{\texttt{KHole}}(\tau\ t)}\ \overset{\blacktriangleright}{\Pi}\texttt{-SKHole}$$

$$\frac{\Delta; \Phi \vdash \kappa \overset{*}{\equiv>} \Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}\ \overset{\blacktriangleright}{\Pi}\texttt{-}\Pi$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \equiv> \kappa_2}$ κ_1 singleton reduces to κ_2

$$\frac{\Delta; \Phi \vdash \mathsf{S}_{\mathsf{S}_{\kappa}(\tau_1)}(\tau)\ \mathsf{OK}}{\Delta; \Phi \vdash \mathsf{S}_{\mathsf{S}_{\kappa}(\tau_1)}(\tau) \equiv> \mathsf{S}_{\kappa}(\tau_1)}\ \equiv>\texttt{-1} \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv> \kappa_2 \qquad \Delta; \Phi \vdash \kappa_2 \equiv> \kappa_3}{\Delta; \Phi \vdash \kappa_1 \equiv> \kappa_3}\ \equiv>\texttt{-Trans}$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \overset{*}{\equiv>} \kappa_2}$ κ_1 has singleton normal form κ_2

$$\frac{\Delta; \Phi \vdash \kappa \equiv> \mathsf{S}_{\texttt{Type}}(\tau)}{\Delta; \Phi \vdash \kappa \overset{*}{\equiv>} \mathsf{S}_{\texttt{Type}}(\tau)}\ \overset{*}{\equiv>}\texttt{-Type} \qquad \frac{\Delta; \Phi \vdash \kappa \equiv> \mathsf{S}_{\texttt{KHole}}(\tau)}{\Delta; \Phi \vdash \kappa \overset{*}{\equiv>} \mathsf{S}_{\texttt{KHole}}(\tau)}\ \overset{*}{\equiv>}\texttt{-KHole}$$

$$\frac{\Delta; \Phi \vdash \kappa \equiv> \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau)}{\Delta; \Phi \vdash \kappa \overset{*}{\equiv>} \Pi_{t_1::\kappa_1}.\mathsf{S}_{[t_1/t]\kappa_2}(\tau\ t_1)}\ \overset{*}{\equiv>}\texttt{-}\Pi$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \equiv \kappa}\ \texttt{KEquiv-Refl} \qquad \frac{\Delta;\Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Symm}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Trans}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv\!> \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-SReduc} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \overset{*}{\equiv\!>} \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-SNorm}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\underline{\Phi, t{::}\kappa_1} \vdash \kappa_3 \equiv \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{KEquiv-}\Pi$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\kappa_1}(\tau_1) \equiv \mathsf{S}_{\kappa_2}(\tau_2)}\ \texttt{KEquiv-SKind}$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \texttt{KHole} \lesssim \kappa}\ \texttt{CSK-KHoleL} \qquad \frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \lesssim \texttt{KHole}}\ \texttt{CSK-KHoleR}$$

$$\frac{\Delta;\Phi \vdash \mathsf{S}_{\texttt{KHole}}(\tau)\ \mathsf{OK} \qquad \Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \mathsf{S}_{\texttt{KHole}}(\tau) \lesssim \kappa}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{L}$$

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK} \qquad \Delta;\Phi \vdash \mathsf{S}_{\texttt{KHole}}(\tau)\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \lesssim \mathsf{S}_{\texttt{KHole}}(\tau)}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{R}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-KEquiv} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta;\Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-Normal}$$

$$\frac{\Delta;\Phi \vdash \mathsf{S}_{\kappa}(\tau)\ \mathsf{OK}}{\Delta;\Phi \vdash \mathsf{S}_{\kappa}(\tau) \lesssim \kappa}\ \texttt{CSK-SKind} \qquad \frac{\Delta;\Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta;\underline{\Phi, t{::}\kappa_3} \vdash \kappa_2 \lesssim \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{CSK-}\Pi$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta;\Phi \vdash \mathsf{S}_{\kappa_1}(\tau_1) \lesssim \mathsf{S}_{\kappa_2}(\tau_2)}\ \texttt{CSK-?}$$

$\boxed{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$ τ_1 is provably equivalent to τ_2 at kind κ

$$\frac{\Delta;\Phi \vdash \tau{::}\kappa}{\Delta;\Phi \vdash \tau \overset{\kappa}{\equiv} \tau}\ \texttt{EquivAK-Refl} \qquad \frac{\Delta;\Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Symm}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Trans}$$

$$\frac{\Delta;\Phi \vdash \tau_1 ::> \kappa_1 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \mathsf{S}_\kappa(\tau_2)}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-SKind}$$

$$\frac{\Delta;\Phi \vdash \tau_1{::}\Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta;\Phi \vdash \tau_2{::}\Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta;\underline{\Phi, t{::}\kappa_1} \vdash \tau_1\ t \overset{\kappa_2}{\equiv} \tau_2\ t}{\Delta;\Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}\ \texttt{EquivAK-}\Pi$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1\ \tau_2 \overset{[\tau_2/t]\kappa_2}{\equiv} \tau_3\ \tau_4}\ \texttt{EquivAK-Ap}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\mathsf{S}_\kappa(\tau)}{\equiv} \tau_2}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (1)$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\mathtt{Type}}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4}\ (2) \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\underline{\Phi, t{::}\kappa_1} \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \lambda t{::}\kappa_1.\tau_1 \overset{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t{::}\kappa_2.\tau_2}\ (3)$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (4)$$

$\boxed{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}$ κ is well formed

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \mathtt{Type}\ \mathsf{OK}}\ \texttt{KWF-Type} \qquad \frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \mathtt{KHole}\ \mathsf{OK}}\ \texttt{KWF-KHole} \qquad \frac{\Delta;\Phi \vdash \tau{::}\kappa}{\Delta;\Phi \vdash \mathsf{S}_\kappa(\tau)\ \mathsf{OK}}\ \texttt{KWF-SKind}$$

$$\frac{\Delta;\underline{\Phi, t{::}\kappa_1} \vdash \kappa_2\ \mathsf{OK}}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}}\ \texttt{KWF-}\Pi$$

$\boxed{\Delta;\Phi \vdash \mathsf{OK}}$ Context is well formed

$$\frac{}{\cdot;\cdot \vdash \mathsf{OK}}\ \texttt{CWF-Nil} \qquad \frac{t \notin \Phi \qquad \Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\underline{\Phi, t{::}\kappa} \vdash \mathsf{OK}}\ \texttt{CWF-TypVar} \qquad \frac{\mathrm{u} \notin \Delta \qquad \Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\underline{\Delta, \mathrm{u}{::}\kappa};\Phi \vdash \mathsf{OK}}\ \texttt{CWF-Hole}$$

Metatheory

subderivation preserving inferences:

- premiss
- COK (Context OK)
- PoS (premiss of subderivation)

Lemma 1 (COK). *If $\Delta;\Phi \vdash \mathcal{J}$, then $\Delta;\Phi \vdash \mathit{OK}$ in a subderivation (where $\Delta;\Phi \vdash \mathcal{J} \neq \Delta;\Phi \vdash \mathit{OK}$)*

Proof. By induction on derivations.
No interesting cases. □

Lemma 2 (Exchange).
If $\Delta;\Phi_1, t_{L1}::\kappa_{L1}, t_{L2}::\kappa_{L2}, \Phi_2 \vdash \mathcal{J}$ and $\Delta;\Phi_1, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1}, \Phi_2 \vdash \mathit{OK}$, then $\Delta;\Phi_1, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1}, \Phi_2 \vdash \mathcal{J}$

Proof. By induction on derivations.
No interesting cases.
(Only rules with Φ extended in the consequent are interesting, which is only CWF-TypVar, but when $\mathcal{J}$ is CWF, Exchange is identity) □

Corollary 3 (Marked-Exchange).
If $\Delta;\underline{\underline{\Phi, t_{L1}::\kappa_{L1}}, t_{L2}::\kappa_{L2}} \vdash \mathcal{J}$ and $\Delta;\underline{\underline{\Phi, t_{L2}::\kappa_{L2}}, t_{L1}::\kappa_{L1}} \vdash \mathit{OK}$, then $\Delta;\underline{\underline{\Phi, t_{L2}::\kappa_{L2}}, t_{L1}::\kappa_{L1}} \vdash \mathcal{J}$

Proof. Exchange when $\Phi_2 = \cdot$ □

Lemma 4 (Weakening).
If $\Delta;\Phi \vdash \mathcal{J}$ and $\Delta;\underline{\Phi, t_L::\kappa_L} \vdash \mathit{OK}$ and $t_L \notin \mathcal{J}$ and $\forall t \in \kappa_L, t \notin \mathcal{J}$, then $\Delta;\underline{\Phi, t_L::\kappa_L} \vdash \mathcal{J}$

Proof. see addendum □

Lemma 5 (K-Substitution).
*If $\Delta;\Phi \vdash \tau_{L1}::\kappa_{L1}$ and $\Delta;\underline{\Phi, t_L::\kappa_{L1}} \vdash \tau_{L2}::\kappa_{L2}$, then $\Delta;\Phi \vdash [\tau_{L1}/t_L]\tau_{L2}::[\tau_{L1}/t_L]\kappa_{L2}$
(induction on $\Delta;\underline{\Phi, t_L::\kappa_{L1}} \vdash \tau_{L2}::\kappa_{L2}$)*

Lemma 6 (PK-Substitution). *If $\Delta;\Phi \vdash \tau_{L1}::\kappa_{L1}$ and $\Delta;\underline{\Phi, t_L::\kappa_{L1}} \vdash \tau_{L2} ::> \kappa_{L2}$ and $\Delta;\Phi \vdash [\tau_{L1}/t_L]\tau_{L2} ::> \kappa_{L3}$, then $\Delta;\Phi \vdash [\tau_{L2}/t_L]\kappa_{L2} \equiv \kappa_{L3}$*

Lemma 7 (OK-Substitution).
*If $\Delta;\Phi \vdash \tau_L::\kappa_{L1}$ and $\Delta;\underline{\Phi, t_L::\kappa_{L1}} \vdash \kappa_{L2}$ OK, then $\Delta;\Phi \vdash [\tau_L/t_L]\kappa_{L2}$ OK
(induction on $\Delta;\underline{\Phi, t_L::\kappa_{L1}} \vdash \kappa_{L2}$ OK)*

Theorem 8 (OK-PK). *If $\Delta;\Phi \vdash \tau ::> \kappa$, then $\Delta;\Phi \vdash \kappa$ OK*

Theorem 9 (OK-WFaK). *If $\Delta;\Phi \vdash \tau::\kappa$, then $\Delta;\Phi \vdash \kappa$ OK*

Theorem 10 (OK-MatchPi). *If $\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2$, then $\Delta;\Phi \vdash \kappa$ OK and $\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2$ OK*

Theorem 11 (OK-KEquiv). *If $\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2$, then $\Delta;\Phi \vdash \kappa_1$ OK and $\Delta;\Phi \vdash \kappa_2$ OK*

Theorem 12 (OK-CSK). *If $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$, then $\Delta;\Phi \vdash \kappa_1$ OK and $\Delta;\Phi \vdash \kappa_2$ OK*

Theorem 13 (OK-EquivAK). *If $\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2$, then $\Delta;\Phi \vdash \tau_1::\kappa$ and $\Delta;\Phi \vdash \tau_2::\kappa$ and $\Delta;\Phi \vdash \kappa$ OK*

Proof. see addendum □

Proof.

Weakening

By induction on derivations.

Note: When applying Weakening in the induction, check that the left premiss is always a subderivation, and check variable exclusion conditions are satisfied (usually checked elsewhere in the derivation).

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\dfrac{}{\Delta;\Phi,t::\kappa_1\vdash\tau::>\kappa_2}\,\texttt{premiss}
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\dfrac{\dfrac{}{\Delta;\Phi,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{IH}}{t_L\notin\Phi}\,\texttt{PoS}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\texttt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t_L\neq t}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\texttt{IH}}{t_L\notin\kappa_1}
}{t_L\notin\Phi,t::\kappa_1}
\quad
\dfrac{
\dfrac{\dfrac{}{\Delta;\Phi,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{IH}}{\Delta;\Phi\vdash\kappa_L\ \mathsf{OK}}\,\texttt{PoS}
\quad
\dfrac{\dfrac{}{\Delta;\Phi,t::\kappa_1\vdash\tau::>\kappa_2}\,\texttt{premiss}}{\Delta;\Phi,t::\kappa_1\vdash\mathsf{OK}}\,\texttt{COK}
}{\Delta;\Phi,t::\kappa_1\vdash\kappa_L\ \mathsf{OK}}\,\texttt{Weakening}
}{\Delta;\Phi,t::\kappa_1,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{CWF-TypVar}
}{\Delta;\Phi,t::\kappa_1,t_L::\kappa_L\vdash\tau::>\kappa_2}\,\texttt{Weakening}
\qquad
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\dfrac{\dfrac{\dfrac{}{\Delta;\Phi,t::\kappa_1\vdash\tau::>\kappa_2}\,\texttt{premiss}}{\Delta;\Phi,t::\kappa_1\vdash\mathsf{OK}}\,\texttt{COK}}{t\notin\Phi}\,\texttt{PoS}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\texttt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t\neq t_L}
\quad
\dfrac{\dfrac{}{\forall l\in\kappa_L,\,l\notin\mathcal{J}}\,\texttt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t\notin\kappa_L}
}{t\notin\Phi,t_L::\kappa_L}
\quad
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\dfrac{\dfrac{\dfrac{}{\Delta;\Phi,t::\kappa_1\vdash\tau::>\kappa_2}\,\texttt{premiss}}{\Delta;\Phi,t::\kappa_1\vdash\mathsf{OK}}\,\texttt{COK}}{\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}}\,\texttt{PoS}
\quad
\dfrac{}{\Delta;\Phi,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{IH}
}{\Delta;\Phi,t_L::\kappa_L\vdash\kappa_1\ \mathsf{OK}}\,\texttt{Weakening}
}{\Delta;\Phi,t_L::\kappa_L,t::\kappa_1\vdash\mathsf{OK}}\,\texttt{CWF-TypVar}
}{
\dfrac{\Delta;\Phi,t_L::\kappa_L,t::\kappa_1\vdash\tau::>\kappa_2}{\Delta;\Phi,t_L::\kappa_L\vdash\lambda t::\kappa_1.\tau::>\mathrm{S}_{\Pi t::\kappa_1.\kappa_2}(\lambda t::\kappa_1.\tau)}\,\texttt{PK-}\lambda
}\,\texttt{Marked-Exchange}
$$

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\dfrac{
\dfrac{\dfrac{}{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2}\,\texttt{premiss}\quad\dfrac{}{\Delta;\Phi,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{IH}}{\Delta;\Phi,t_L::\kappa_L\vdash\kappa_1\equiv\kappa_2}\,\texttt{Weakening}
\qquad
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\dfrac{}{\Delta;\Phi,t::\kappa_1\vdash\tau::>\kappa_2}\,\texttt{premiss}
\qquad
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\dfrac{\dfrac{}{\Delta;\Phi,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{IH}}{t_L\notin\Phi}\,\texttt{PoS}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\texttt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t_L\neq t}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\texttt{IH}}{t_L\notin\kappa_1}
}{t_L\notin\Phi,t::\kappa_1}
\quad
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\dfrac{\dfrac{}{\Delta;\Phi,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{IH}}{\Delta;\Phi\vdash\kappa_L\ \mathsf{OK}}\,\texttt{PoS}
\quad
\dfrac{\dfrac{}{\Delta;\Phi,t::\kappa_1\vdash\kappa_3\equiv\kappa_4}\,\texttt{premiss}}{\Delta;\Phi,t::\kappa_1\vdash\mathsf{OK}}\,\texttt{COK}
}{\Delta;\Phi,t::\kappa_1\vdash\kappa_L\ \mathsf{OK}}\,\texttt{Weakening}
}{\Delta;\Phi,t::\kappa_1,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{CWF-TypVar}
}{\Delta;\Phi,t::\kappa_1,t_L::\kappa_L\vdash\tau::>\kappa_2}\,\texttt{Weakening}
\qquad
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\dfrac{\dfrac{\dfrac{}{\Delta;\Phi,t::\kappa_1\vdash\kappa_3\equiv\kappa_4}\,\texttt{premiss}}{\Delta;\Phi,t::\kappa_1\vdash\mathsf{OK}}\,\texttt{COK}}{t\notin\Phi}\,\texttt{PoS}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\texttt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t\neq t_L}
\quad
\dfrac{\dfrac{}{\forall l\in\kappa_L,\,l\notin\mathcal{J}}\,\texttt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t\notin\kappa_L}
}{t\notin\Phi,t_L::\kappa_L}
\quad
\dfrac{
\dfrac{\dfrac{\dfrac{}{\Delta;\Phi,t::\kappa_1\vdash\kappa_3\equiv\kappa_4}\,\texttt{premiss}}{\Delta;\Phi,t::\kappa_1\vdash\mathsf{OK}}\,\texttt{COK}}{\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}}\,\texttt{PoS}
\quad
\dfrac{}{\Delta;\Phi,t_L::\kappa_L\vdash\mathsf{OK}}\,\texttt{IH}
}{\Delta;\Phi,t_L::\kappa_L\vdash\kappa_1\ \mathsf{OK}}\,\texttt{Weakening}
}{\Delta;\Phi,t_L::\kappa_L,t::\kappa_1\vdash\mathsf{OK}}\,\texttt{CWF-TypVar}
}{\Delta;\Phi,t_L::\kappa_L,t::\kappa_1\vdash\kappa_3\equiv\kappa_4}\,\texttt{Marked-Exchange}
}{\Delta;\Phi,t_L::\kappa_L\vdash\Pi_{t::\kappa_1}.\kappa_3\equiv\Pi_{t::\kappa_2}.\kappa_4}\,\texttt{KEquiv-}\Pi
\qquad
\dfrac{}{sim}\,\texttt{CEK-}\Pi
\qquad
\dfrac{}{sim}\,\texttt{EquivAK-}\Pi
\qquad
\dfrac{}{sim}\,\texttt{KWF-}\Pi
$$

O?K-.*

By simultaneous induction on derivations.

The interesting cases per lemma:

K-Substitution by type size??

OK-Substitution

OK-PK

$$
\dfrac{\dfrac{\dfrac{}{\Delta;\Phi\vdash\mathtt{bse}::>\mathrm{S}_{\mathtt{Type}}(\mathtt{bse})}\,\texttt{premiss}}{\Delta;\Phi\vdash\mathtt{bse}::\mathtt{Type}}\,\texttt{WFaK-1}}{\Delta;\Phi\vdash\mathrm{S}_{\mathtt{Type}}(\mathtt{bse})\ \mathsf{OK}}\,\texttt{KWF-SKind}
\qquad\qquad
\dfrac{}{\Delta;\Phi\vdash[\tau_2/t]\kappa_2\ \mathsf{OK}}\,\texttt{OK-Substitution}
$$

OK-WFaK $\square$

Theorem 14 (PK-Unicity). *If* $\Delta;\Phi \vdash \tau_L ::> \kappa_{L1}$ *and* $\Delta;\Phi \vdash \tau_L ::> \kappa_{L2}$ *then* $\kappa_{L1} = \kappa_{L2}$

Proof. There's actually a slight problem here. In PK-Ap, $\overset{\blacktriangleright}{\Pi}$ doesn't have unicity since $\overset{\blacktriangleright}{\Pi}$ -Π uses $\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2$, where "the same left side can have mutliple right sides" (transitivity), causing unicity to fail for PK.
Only thought about this quickly but we might need a seperate HO singleton normalization scheme that is essentially the KEquiv-SKind$_{\text{SKind}}$, KEquiv-SKind$_{\Pi}$ fragment of kind equality, and have the HO singleton normalization judgment have unicity.
Kind of gross but...
(and add other HO kinds, namely Σ, when they get here)
And as cleanup, we can remove the normalization fragment from kind equality and add a single rule (yay more sim judgments!)
Also we need some unicity lemmas for contexts like for PK-EHole and co. □

Theorem 15 (PK-Principality). *If* $\Delta;\Phi \vdash \tau ::> \kappa_1$ *and* $\Delta;\Phi \vdash \tau :: \kappa_2$*, then* $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$

Proof. From definition of $\Delta;\Phi \vdash \tau :: \kappa$ and CSK-SKind □

Theorem 16 (why is this here?). *If* $\Delta;\Phi \vdash \kappa_1 \lesssim \mathtt{S}_{\kappa_2}(\tau)$*, then* $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$

Elaboration

TODO