

# Hazel Phi: 11-type-constructors

September 1, 2021

## NOTES

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This is (sadly) the last version for summer 2021.

See markdown for a brief TODO list.

## SYNTAX

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Kind	$\kappa$	$::=$	$\text{Type} \mid \text{KHole} \mid \text{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	$::=$	$t \mid \text{bse} \mid \tau_1 \oplus \tau_2 \mid \langle \rangle^u \mid \langle \hat{\tau} \rangle^u \mid \lambda t::\text{Type}.\hat{\tau} \mid \tau_1 \tau_2$
Internal Types	$\tau$	$::=$	$t \mid \text{bse} \mid \tau_1 \oplus \tau_2 \mid \langle \rangle^u \mid \langle \tau \rangle^u \mid \langle t \rangle^u \mid \lambda t::\kappa.\tau \mid \tau_1 \tau_2$
Base Types	$\text{bse}$	$::=$	$\text{Int} \mid \text{Float} \mid \text{Bool}$
BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Type Pattern			
User Expression			
Internal Expression			

## DECLARATIVES

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$\Delta; \Phi \vdash \tau ::> \kappa$   $\tau$  has principal (well formed) kind  $\kappa$

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{bse} ::> \text{S}_{\text{Type}}(\text{bse})} \text{PK-Base}$	$\frac{\Delta; \Phi_1, t::\kappa, \Phi_2 \vdash \text{OK}}{\Delta; \Phi \vdash t ::> \text{S}_{\kappa}(t)} \text{PK-Var}$
$\frac{\Delta; \Phi \vdash \tau_1::\text{Type} \quad \Delta; \Phi \vdash \tau_2::\text{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> \text{S}_{\text{Type}}(\tau_1 \oplus \tau_2)} \text{PK-}\oplus$	$\frac{\Delta_1, u::\kappa, \Delta_2; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \langle \rangle^u ::> \text{S}_{\kappa}(\langle \rangle^u)} \text{PK-EHole}$
$\frac{\Delta_1, u::\kappa, \Delta_2; \Phi \vdash \text{OK} \quad \Delta; \Phi \vdash \tau::\kappa_1}{\Delta; \Phi \vdash \langle \tau \rangle^u ::> \text{S}_{\kappa}(\langle \tau \rangle^u)} \text{PK-NEHole}$	$\frac{\Delta_1, u::\kappa, \Delta_2; \Phi \vdash \text{OK} \quad t \notin \Phi}{\Delta; \Phi \vdash \langle t \rangle^u ::> \text{S}_{\kappa}(\langle t \rangle^u)} \text{PK-Unbound}$
$\frac{\Delta; \Phi, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau ::> \text{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t::\kappa_1.\tau)} \text{PK-}\lambda$	
$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa \quad \Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2 \quad \Delta; \Phi \vdash \tau_2::\kappa_1}{\Delta; \Phi \vdash \tau_1 \tau_2 ::> [\tau_2/t]\kappa_2} \text{PK-Ap}$	

$\Delta; \Phi \vdash \tau :: \kappa$   $\tau$  is well formed at kind  $\kappa$

$$\frac{\Delta; \Phi \vdash \tau :: > \mathbf{S}_{\kappa}(\tau)}{\Delta; \Phi \vdash \tau :: \kappa} \text{WFaK-1} \quad \frac{\Delta; \Phi \vdash \tau :: > \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \text{WFaK-Subsump}$$

$$\frac{\Delta; \Phi \vdash \tau :: > \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \text{WFaK-Reit} \quad \frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau :: \mathbf{S}_{\kappa}(\tau)} \text{WFaK-Self}$$

$$\frac{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_3} \cdot \kappa_4 \quad \Delta; \Phi \vdash \Pi_{t::\kappa_3} \cdot \kappa_4 \lesssim \Pi_{t::\kappa_1} \cdot \kappa_2}{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_1} \cdot \kappa_2} \text{WFaK-PCSKTrans}$$

$$\frac{\Delta; \Phi \vdash \tau :: \mathbf{S}_{\kappa}(\tau_1) \quad \Delta; \Phi \vdash \tau_1 :: \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \text{WFaK-Flatten}$$

$\Delta; \Phi \vdash \kappa \Pi \Pi_{t::\kappa_1} \cdot \kappa_2$   $\kappa$  has matched  $\Pi$ -kind  $\Pi_{t::\kappa_1} \cdot \kappa_2$

$$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{KHole} \Pi \Pi_{t::\text{KHole}} \cdot \text{KHole}} \Pi\text{-KHole} \quad \frac{\Delta; \Phi \vdash \kappa \equiv \mathbf{S}_{\text{KHole}}(\tau)}{\Delta; \Phi \vdash \kappa \Pi \Pi_{t::\mathbf{S}_{\text{KHole}}(\tau)} \cdot \mathbf{S}_{\text{KHole}}(\tau \ t)} \Pi\text{-SKHole}$$

$$\frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1} \cdot \kappa_2}{\Delta; \Phi \vdash \kappa \Pi \Pi_{t::\kappa_1} \cdot \kappa_2} \Pi\text{-}\Pi$$

$\Delta; \Phi \vdash \kappa_1 \equiv > \kappa_2$   $\kappa_1$  singleton reduces to  $\kappa_2$

$$\frac{\Delta; \Phi \vdash \tau :: \mathbf{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \mathbf{S}_{\mathbf{S}_{\kappa}(\tau_1)}(\tau) \equiv > \mathbf{S}_{\kappa}(\tau_1)} \text{KEquiv-SKind}_{\text{SKind}} \quad \frac{\Delta; \Phi \vdash \kappa_1 \equiv > \kappa_2 \quad \Delta; \Phi \vdash \kappa_2 \equiv > \kappa_3}{\Delta; \Phi \vdash \kappa_1 \equiv > \kappa_3}$$

$\Delta; \Phi \vdash \kappa_1 \equiv^* \kappa_2$   $\kappa_1$  has singleton normal form  $\kappa_2$

$$\frac{\Delta; \Phi \vdash \kappa \equiv > \mathbf{S}_{\text{Type}}(\tau)}{\Delta; \Phi \vdash \kappa \equiv^* \mathbf{S}_{\text{Type}}(\tau)} \quad \frac{\Delta; \Phi \vdash \kappa \equiv > \mathbf{S}_{\text{KHole}}(\tau)}{\Delta; \Phi \vdash \kappa \equiv^* \mathbf{S}_{\text{KHole}}(\tau)} \quad \frac{\Delta; \Phi \vdash \kappa \equiv > \mathbf{S}_{\Pi_{t::\kappa_1} \cdot \kappa_2}(\tau)}{\Delta; \Phi \vdash \kappa \equiv^* \Pi_{t_1::\kappa_1} \cdot \mathbf{S}_{[t_1/t]\kappa_2}(\tau \ t_1)}$$

$\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$   $\kappa_1$  is equivalent to  $\kappa_2$

$$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \equiv \kappa} \text{KEquiv-Ref1} \qquad \frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \text{KEquiv-Symm}$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \text{KEquiv-Trans}$$

$$\frac{\Delta; \Phi \vdash \tau :: \mathbf{S}_\kappa(\tau_1)}{\Delta; \Phi \vdash \mathbf{S}_{\mathbf{S}_\kappa(\tau_1)}(\tau) \equiv \mathbf{S}_\kappa(\tau_1)} \text{KEquiv-SKind}_{\text{SKind}}$$

$$\frac{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_1} \cdot \kappa_2}{\Delta; \Phi \vdash \mathbf{S}_{\Pi_{t::\kappa_1} \cdot \kappa_2}(\tau) \equiv \Pi_{t_1::\kappa_1} \cdot \mathbf{S}_{[t_1/t]\kappa_2}(\tau \ t_1)} \text{KEquiv-SKind}_\Pi$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1} \cdot \kappa_2 \equiv \Pi_{t::\kappa_3} \cdot \kappa_4} \text{KEquiv-}\Pi$$

$$\frac{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2 \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \mathbf{S}_{\kappa_1}(\tau_1) \equiv \mathbf{S}_{\kappa_2}(\tau_2)} \text{KEquiv-SKind}$$

$\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$   $\kappa_1$  is a consistent subkind of  $\kappa_2$

$$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \mathbf{KHole} \lesssim \kappa} \text{CSK-KHoleL} \qquad \frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathbf{KHole}} \text{CSK-KHoleR}$$

$$\frac{\Delta; \Phi \vdash \mathbf{S}_{\mathbf{KHole}}(\tau) \text{ OK} \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \mathbf{S}_{\mathbf{KHole}}(\tau) \lesssim \kappa} \text{CSK-SKind}_{\mathbf{KHoleL}}$$

$$\frac{\Delta; \Phi \vdash \kappa \text{ OK} \quad \Delta; \Phi \vdash \mathbf{S}_{\mathbf{KHole}}(\tau) \text{ OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathbf{S}_{\mathbf{KHole}}(\tau)} \text{CSK-SKind}_{\mathbf{KHoleR}}$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \text{CSK-KEquiv} \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \quad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \text{CSK-Normal}$$

$$\frac{\Delta; \Phi \vdash \mathbf{S}_\kappa(\tau) \text{ OK}}{\Delta; \Phi \vdash \mathbf{S}_\kappa(\tau) \lesssim \kappa} \text{CSK-SKind} \qquad \frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \quad \Delta; \Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1} \cdot \kappa_2 \lesssim \Pi_{t::\kappa_3} \cdot \kappa_4} \text{CSK-}\Pi$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \quad \Delta; \Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathbf{S}_{\kappa_1}(\tau_1) \lesssim \mathbf{S}_{\kappa_2}(\tau_2)} \text{CSK-?}$$

$\boxed{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2}$   $\tau_1$  is provably equivalent to  $\tau_2$  at kind  $\kappa$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau \equiv^{\kappa} \tau} \text{EquivAK-Ref1} \qquad \frac{\Delta; \Phi \vdash \tau_2 \equiv^{\kappa} \tau_1}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \text{EquivAK-Symm}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_3 \quad \Delta; \Phi \vdash \tau_3 \equiv^{\kappa} \tau_1}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \text{EquivAK-Trans}$$

$$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 \equiv \mathbf{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \text{EquivAK-SKind}$$

$$\frac{\Delta; \Phi \vdash \tau_1 :: \Pi_{t::\kappa_1} \kappa_3 \quad \Delta; \Phi \vdash \tau_2 :: \Pi_{t::\kappa_1} \kappa_4 \quad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \equiv^{\kappa_2} \tau_2 \ t}{\Delta; \Phi \vdash \tau_1 \equiv^{\Pi_{t::\kappa_1} \kappa_2} \tau_2} \text{EquivAK-}\Pi$$

$$\frac{\Delta; \Phi \vdash \tau_1 \equiv^{\Pi_{t::\kappa_1} \kappa_2} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \equiv^{\kappa_1} \tau_4}{\Delta; \Phi \vdash \tau_1 \tau_2 \equiv^{[\tau_2/t]\kappa_2} \tau_3 \tau_4} \text{EquivAK-Ap}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \equiv^{\mathbf{S}_{\kappa}(\tau)} \tau_2}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} (1)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \equiv^{\text{Type}} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \equiv^{\text{Type}} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \equiv^{\text{Type}} \tau_3 \oplus \tau_4} (2)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \equiv^{\kappa} \tau_2}{\Delta; \Phi \vdash \lambda t::\kappa_1. \tau_1 \equiv^{\Pi_{t::\kappa_1} \kappa} \lambda t::\kappa_2. \tau_2} (3)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa_1} \tau_2 \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} (4)$$

$\boxed{\Delta; \Phi \vdash \kappa \text{ OK}}$   $\kappa$  is well formed

$$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{Type OK}} \text{KWF-Type}$$

$$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{KHole OK}} \text{KWF-KHole}$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \mathbf{S}_{\kappa}(\tau) \text{ OK}} \text{KWF-SKind}$$

$$\frac{\Delta; \Phi, t::\kappa_1 \vdash \kappa_2 \text{ OK}}{\Delta; \Phi \vdash \Pi_{t::\kappa_1} \kappa_2 \text{ OK}} \text{KWF-}\Pi$$

$\boxed{\Delta; \Phi \vdash \text{OK}}$  Context is well formed

$$\frac{}{.; \cdot \text{ OK}} \text{CWF-Nil}$$

$$\frac{t \notin \Phi \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi, t::\kappa \vdash \text{OK}} \text{CWF-TypVar}$$

$$\frac{u \notin \Delta \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta, u::\kappa; \Phi \vdash \text{OK}} \text{CWF-Hole}$$

## METATHEORY

subderivation preserving inferences:

- premiss
- COK (Context OK)
- PoS (premiss of subderivation)

**Lemma 1** (COK). *If  $\Delta; \Phi \vdash \mathcal{J}$ , then  $\Delta; \Phi \vdash OK$  in a subderivation (where  $\Delta; \Phi \vdash \mathcal{J} \neq \Delta; \Phi \vdash OK$ )*

*Proof.* By induction on derivations.

No interesting cases. □

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**Lemma 2** (Exchange).

*If  $\Delta; \Phi_1, t_{L1}::\kappa_{L1}, t_{L2}::\kappa_{L2}, \Phi_2 \vdash \mathcal{J}$  and  $\Delta; \Phi_1, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1}, \Phi_2 \vdash OK$ , then  $\Delta; \Phi_1, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1}, \Phi_2 \vdash \mathcal{J}$*

*Proof.* By induction on derivations.

No interesting cases.

(Only rules with  $\Phi$  extended in the consequent are interesting, which is only CWF-TypVar, but when  $\mathcal{J}$  is CWF, Exchange is identity) □

**Corollary 3** (Marked-Exchange).

*If  $\Delta; \Phi, \underline{t_{L1}::\kappa_{L1}}, \underline{t_{L2}::\kappa_{L2}} \vdash \mathcal{J}$  and  $\Delta; \Phi, \underline{t_{L2}::\kappa_{L2}}, \underline{t_{L1}::\kappa_{L1}} \vdash OK$ , then  $\Delta; \Phi, \underline{t_{L2}::\kappa_{L2}}, \underline{t_{L1}::\kappa_{L1}} \vdash \mathcal{J}$*

*Proof.* Exchange when  $\Phi_2 = \cdot$  □

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**Lemma 4** (Weakening).

*If  $\Delta; \Phi \vdash \mathcal{J}$  and  $\Delta; \Phi, \underline{t_L::\kappa_L} \vdash OK$  and  $t_L \notin \mathcal{J}$  and  $\forall t \in \kappa_L, t \notin \mathcal{J}$ , then  $\Delta; \Phi, \underline{t_L::\kappa_L} \vdash \mathcal{J}$*

*Proof.* see addendum □

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**Lemma 5** (K-Substitution).

*If  $\Delta; \Phi \vdash \tau_{L1}::\kappa_{L1}$  and  $\Delta; \Phi, \underline{t_L::\kappa_{L1}} \vdash \tau_{L2}::\kappa_{L2}$ , then  $\Delta; \Phi \vdash [\tau_{L1}/t_L]\tau_{L2}::[\tau_{L1}/t_L]\kappa_{L2}$   
(induction on  $\Delta; \Phi, \underline{t_L::\kappa_{L1}} \vdash \tau_{L2}::\kappa_{L2}$ )*

**Lemma 6** (PK-Substitution). *If  $\Delta; \Phi \vdash \tau_{L1}::\kappa_{L1}$  and  $\Delta; \Phi, \underline{t_L::\kappa_{L1}} \vdash \tau_{L2}::\kappa_{L2}$  and  $\Delta; \Phi \vdash [\tau_{L1}/t_L]\tau_{L2}::\kappa_{L3}$ , then  $\Delta; \Phi \vdash [\tau_{L2}/t_L]\kappa_{L2} \equiv \kappa_{L3}$*

**Lemma 7** (OK-Substitution).

*If  $\Delta; \Phi \vdash \tau_L::\kappa_{L1}$  and  $\Delta; \Phi, \underline{t_L::\kappa_{L1}} \vdash \kappa_{L2} OK$ , then  $\Delta; \Phi \vdash [\tau_L/t_L]\kappa_{L2} OK$   
(induction on  $\Delta; \Phi, \underline{t_L::\kappa_{L1}} \vdash \kappa_{L2} OK$ )*

**Theorem 8** (OK-PK). *If  $\Delta; \Phi \vdash \tau::\kappa$ , then  $\Delta; \Phi \vdash \kappa OK$*

**Theorem 9** (OK-WFaK). *If  $\Delta; \Phi \vdash \tau::\kappa$ , then  $\Delta; \Phi \vdash \kappa OK$*

**Theorem 10** (OK-MatchPi). *If  $\Delta; \Phi \vdash \kappa \blacktriangleright \Pi_{t::\kappa_1}.\kappa_2$ , then  $\Delta; \Phi \vdash \kappa OK$  and  $\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 OK$*

**Theorem 11** (OK-KEquiv). *If  $\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$ , then  $\Delta; \Phi \vdash \kappa_1 OK$  and  $\Delta; \Phi \vdash \kappa_2 OK$*

**Theorem 12** (OK-CSK). *If  $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$ , then  $\Delta; \Phi \vdash \kappa_1 OK$  and  $\Delta; \Phi \vdash \kappa_2 OK$*

**Theorem 13** (OK-EquivAK). *If  $\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2$ , then  $\Delta; \Phi \vdash \tau_1::\kappa$  and  $\Delta; \Phi \vdash \tau_2::\kappa$  and  $\Delta; \Phi \vdash \kappa OK$*

*Proof.* see addendum □

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By induction on derivations.

By simultaneous induction on derivations.

**K-Substitution** by type size??

OK-PK



$$\overline{\Delta; \Phi \vdash [\tau_2 / t] \kappa_2 \text{ OK}} \text{ OK-Substitution}$$

**Theorem 14** (PK-Unicity). *If  $\Delta; \Phi \vdash \tau_L ::> \kappa_{L1}$  and  $\Delta; \Phi \vdash \tau_L ::> \kappa_{L2}$  then  $\kappa_{L1} = \kappa_{L2}$*

*Proof.* There's actually a slight problem here. In PK-Ap,  $\blacktriangleright_{\Pi}$  doesn't have unicity since  $\blacktriangleright_{\Pi}$ - $\Pi$  uses  $\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$ , where “the same left side can have mutiple right sides” (transitivity), causing unicity to fail for PK.

Only thought about this quickly but we might need a separate HO singleton normalization scheme that is essentially the  $\text{KEquiv-SKind}_{\text{SKind}}$ ,  $\text{KEquiv-SKind}_{\Pi}$  fragment of kind equality, and have the HO singleton normalization judgment have unicity.

Kind of gross but...

(and add other HO kinds, namely  $\Sigma$ , when they get here)

And as cleanup, we can remove the normalization fragment from kind equality and add a single rule (yay more sim judgments!)

Also we need some unicity lemmas for contexts like for PK-EHole and co.

□

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**Theorem 15** (PK-Principality). *If  $\Delta; \Phi \vdash \tau ::> \kappa_1$  and  $\Delta; \Phi \vdash \tau ::\kappa_2$ , then  $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$*

*Proof.* From definition of  $\Delta; \Phi \vdash \tau ::\kappa$  and CSK-SKind

□

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**Theorem 16** (why is this here?). *If  $\Delta; \Phi \vdash \kappa_1 \lesssim \mathbf{S}_{\kappa_2}(\tau)$ , then  $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$*

ELABORATION

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TODO