

July 30, 2021

SYNTAX

Kind $\kappa ::= \text{Type} \mid \text{KHole} \mid \text{S}_\kappa(\tau) \mid \Pi_{t::\kappa_1, \kappa_2}$
 User Types $\hat{\tau} ::= t \mid \text{bse} \mid \tau_1 \oplus \tau_2 \mid \emptyset^u \mid \langle \tau \rangle^u \mid \lambda t::\text{Type}.\hat{\tau} \mid \tau_1 \tau_2$
 Internal Types $\tau ::= t \mid \text{bse} \mid \tau_1 \oplus \tau_2 \mid \emptyset^u \mid \langle \tau \rangle^u \mid \langle \emptyset \rangle^u \mid \lambda t::\kappa.\tau \mid \tau_1 \tau_2$
 Base Types $\text{bse} ::= \text{Int} \mid \text{Float} \mid \text{Bool}$
 BinOp $\oplus ::= \times \mid + \mid \rightarrow$
 Type Pattern
 User Expression
 Internal Expression

DECLARATIVES

$\Delta; \Phi \vdash \tau ::> \kappa$ τ has principal (well formed) kind κ

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{bse} ::> \text{S}_{\text{Type}}(\text{bse})} \text{PK-Base}$ $\frac{\Delta; \Phi_1, t::\kappa_1, \Phi_2 \vdash \text{OK}}{\Delta; \Phi \vdash t ::> \text{S}_\kappa(t)} \text{PK-Var}$ $\frac{\Delta; \Phi \vdash \tau_1 :: \text{Type} \quad \Delta; \Phi \vdash \tau_2 :: \text{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> \text{S}_{\text{Type}}(\tau_1 \oplus \tau_2)} \text{PK-}\oplus$ $\frac{\Delta_1, u::\kappa_1, \Delta_2; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \emptyset^u ::> \text{S}_\kappa(\emptyset^u)} \text{PK-EHole}$ $\frac{\Delta_1, u::\kappa_1, \Delta_2; \Phi \vdash \text{OK} \quad \Delta; \Phi \vdash \tau::\kappa_1}{\Delta; \Phi \vdash \langle \tau \rangle^u ::> \text{S}_\kappa(\langle \tau \rangle^u)} \text{PK-NEHole}$ $\frac{\Delta_1, u::\kappa_1, \Delta_2; \Phi \vdash \text{OK} \quad t \notin \Phi}{\Delta; \Phi \vdash \langle \emptyset \rangle^u ::> \text{S}_\kappa(\langle \emptyset \rangle^u)} \text{PK-Unbound}$ $\frac{\Delta; \Phi_s, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau ::> \text{S}_{\Pi_{t::\kappa_1, \kappa_2}}(\lambda t::\kappa_1.\tau)} \text{PK-}\lambda$

$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa \quad \Delta; \Phi \vdash \kappa \overset{\text{!}}{\Pi} \Pi_{t::\kappa_1, \kappa_2} \quad \Delta; \Phi \vdash \tau_2 :: \kappa_1}{\Delta; \Phi \vdash \tau_1 \tau_2 ::> [\tau_2 / t] \kappa_2} \text{PK-Ap}$

$\Delta; \Phi \vdash \tau::\kappa$ τ is well formed at kind κ

$\frac{\Delta; \Phi \vdash \tau ::> \text{S}_\kappa(\tau)}{\Delta; \Phi \vdash \tau::\kappa} \text{WFaK-1}$

$\frac{\Delta; \Phi \vdash \tau ::> \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau::\kappa} \text{WFaK-Subeump}$

$\frac{\Delta; \Phi \vdash \tau ::> \kappa}{\Delta; \Phi \vdash \tau::\kappa} \text{WFaK-Reit}$

$\frac{\Delta; \Phi \vdash \tau::\kappa}{\Delta; \Phi \vdash \tau::\text{S}_\kappa(\tau)} \text{WFaK-Self}$

$\frac{\Delta; \Phi \vdash \tau::\Pi_{t::\kappa_1, \kappa_2} \quad \Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_2} \lesssim \Pi_{t::\kappa_1, \kappa_2}}{\Delta; \Phi \vdash \tau::\Pi_{t::\kappa_1, \kappa_2}} \text{WFaK-}\Pi\text{CSKTrans}$

$\frac{\Delta; \Phi \vdash \tau::\text{S}_\kappa(\tau_1) \quad \Delta; \Phi \vdash \tau_1::\kappa}{\Delta; \Phi \vdash \tau::\kappa} \text{WFaK-Flatten}$

$\Delta; \Phi \vdash \kappa \overset{\text{!}}{\Pi} \Pi_{t::\kappa_1, \kappa_2}$ κ has matched Π -kind $\Pi_{t::\kappa_1, \kappa_2}$

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{KHole} \overset{\text{!}}{\Pi} \Pi_{t::\text{KHole}, \text{KHole}} \text{KHole}} \text{!-KHole}$

$\frac{\Delta; \Phi \vdash \kappa \equiv \text{S}_{\text{KHole}}(\tau)}{\Delta; \Phi \vdash \kappa \overset{\text{!}}{\Pi} \Pi_{t::\text{S}_{\text{KHole}}(\tau), \text{S}_{\text{KHole}}(\tau) t}} \text{!-SKHole}$

$\frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1, \kappa_2}}{\Delta; \Phi \vdash \kappa \overset{\text{!}}{\Pi} \Pi_{t::\kappa_1, \kappa_2}} \text{!-}\Pi$

$\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$ κ_1 is equivalent to κ_2

$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \equiv \kappa} \text{KEquiv-Ref1}$

$\frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \text{KEquiv-Symm}$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \text{KEquiv-Trans}$

$\frac{\Delta; \Phi \vdash \tau::\text{S}_\kappa(\tau_1)}{\Delta; \Phi \vdash \text{S}_{\text{S}_\kappa(\tau_1)}(\tau) \equiv \text{S}_\kappa(\tau_1)} \text{KEquiv-SKind}_{\text{SKind}}$

$\frac{\Delta; \Phi \vdash \tau::\Pi_{t::\kappa_1, \kappa_2}}{\Delta; \Phi \vdash \text{S}_{\Pi_{t::\kappa_1, \kappa_2}}(\tau) \equiv \Pi_{t::\kappa_1, \kappa_2} \cdot \text{S}_{[t_1 / t] \kappa_2}(\tau t_1)} \text{KEquiv-SKind}_{\Pi}$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_2} \equiv \Pi_{t::\kappa_3, \kappa_4}} \text{KEquiv-}\Pi$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \text{S}_{\kappa_1}(\tau_1) \equiv \text{S}_{\kappa_2}(\tau_2)} \text{KEquiv-SKind}$

$\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$ κ_1 is a consistent subkind of κ_2

$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \text{KHole} \lesssim \kappa} \text{CSK-KHoleL}$

$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \lesssim \text{KHole}} \text{CSK-KHoleR}$

$\frac{\Delta; \Phi \vdash \text{S}_{\text{KHole}}(\tau) \text{ OK} \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \text{S}_{\text{KHole}}(\tau) \lesssim \kappa} \text{CSK-SKind}_{\text{SoleL}}$

$\frac{\Delta; \Phi \vdash \kappa \text{ OK} \quad \Delta; \Phi \vdash \text{S}_{\text{KHole}}(\tau) \text{ OK}}{\Delta; \Phi \vdash \kappa \lesssim \text{S}_{\text{KHole}}(\tau)} \text{CSK-SKind}_{\text{SoleR}}$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \text{CSK-KEquiv}$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \quad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \text{CSK-Normal}$

$\frac{\Delta; \Phi \vdash \text{S}_\kappa(\tau) \text{ OK}}{\Delta; \Phi \vdash \text{S}_\kappa(\tau) \lesssim \kappa} \text{CSK-SKind}$

$\frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \quad \Delta; \Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_3} \lesssim \Pi_{t::\kappa_3, \kappa_4}} \text{CSK-}\Pi$

$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \quad \Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \text{S}_{\kappa_1}(\tau_1) \lesssim \text{S}_{\kappa_2}(\tau_2)} \text{CSK-?}$

$\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2$ τ_1 is provably equivalent to τ_2 at kind κ

$\frac{\Delta; \Phi \vdash \tau::\kappa}{\Delta; \Phi \vdash \tau \equiv \tau} \text{EquivAK-Ref1}$

$\frac{\Delta; \Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \text{EquivAK-Symm}$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \quad \Delta; \Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \text{EquivAK-Trans}$

$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 \equiv \text{S}_\kappa(\tau_2)}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \text{EquivAK-SKind}$

$\frac{\Delta; \Phi \vdash \tau_1::\Pi_{t::\kappa_1, \kappa_3} \quad \Delta; \Phi \vdash \tau_2::\Pi_{t::\kappa_1, \kappa_4} \quad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 t \overset{\kappa_3}{\equiv} \tau_2 t}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \text{EquivAK-}\Pi$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1, \kappa_3}}{\equiv} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \tau_2 \overset{[\tau_3 / t] \kappa_3}{\equiv} \tau_3 \tau_4} \text{EquivAK-Ap}$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\text{S}_\kappa(\tau)}{\equiv} \tau_2}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2} (1)$

$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau_1 \equiv \lambda t::\kappa_2.\tau_2} (3)$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2} (4)$

$\frac{\Delta; \Phi \vdash \tau_1 \overset{\text{Type}}{\equiv} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \overset{\text{Type}}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \overset{\text{Type}}{\equiv} \tau_3 \oplus \tau_4} (2)$

$\Delta; \Phi \vdash \kappa \text{ OK}$ κ is well formed

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{Type OK}} \text{KWF-Type}$

$\frac{\Delta; \Phi \vdash \text{OK}}{\Delta; \Phi \vdash \text{KHole OK}} \text{KWF-KHole}$

$\frac{\Delta; \Phi \vdash \tau::\kappa}{\Delta; \Phi \vdash \text{S}_\kappa(\tau) \text{ OK}} \text{KWF-SKind}$

$\frac{\Delta; \Phi, t::\kappa_1 \vdash \kappa_2 \text{ OK}}{\Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_2} \text{ OK}} \text{KWF-}\Pi$

$\Delta; \Phi \vdash \text{OK}$ Context is well formed

$\frac{}{\cdot, \cdot \vdash \text{OK}} \text{CWF-Nil}$

$\frac{t \notin \Delta \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi, t::\kappa \vdash \text{OK}} \text{CWF-TypVar}$

$\frac{u \notin \Delta \quad \Delta; \Phi \vdash \kappa \text{ OK}}{\Delta, u::\kappa; \Phi \vdash \text{OK}} \text{CWF-Hole}$

METATHEORY

Lemma 1 (COK). If $\Delta; \Phi \vdash \mathcal{J}$, then $\Delta; \Phi \vdash \text{OK}$ in a subderivation (where $\Delta; \Phi \vdash \mathcal{J} \neq \Delta; \Phi \vdash \text{OK}$)

Proof. By simultaneous induction on derivations.
 No interesting cases. □

Lemma 2 (Exchange).

If $\Delta; \Phi_1, t_{L1}::\kappa_{L1}, t_{L2}::\kappa_{L2}, \Phi_2 \vdash \mathcal{J}$ and $\Delta; \Phi_1, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1}, \Phi_2 \vdash \text{OK}$, then $\Delta; \Phi_1, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1}, \Phi_2 \vdash \mathcal{J}$

Proof. By induction on derivations.
 No interesting cases.

(Only rules with Φ extended in the consequent are interesting, which is only CWF-TypVar, but when \mathcal{J} is CWF, Exchange is identity) □

Corollary 3 (Marked-Exchange).

If $\Delta; \Phi, t_{L1}::\kappa_{L1}, t_{L2}::\kappa_{L2} \vdash \mathcal{J}$ and $\Delta; \Phi, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1} \vdash \text{OK}$, then $\Delta; \Phi, t_{L2}::\kappa_{L2}, t_{L1}::\kappa_{L1} \vdash \mathcal{J}$

Lemma 4 (Weakening).

If $\Delta; \Phi \vdash \mathcal{J}$ and $\Delta; \Phi, t_L::\kappa_L \vdash \text{OK}$ and $t_L \notin \mathcal{J}$ and $\forall t \in \kappa_L, t \notin \mathcal{J}$, then $\Delta; \Phi, t_L::\kappa_L \vdash \mathcal{J}$

Proof. By induction on derivations.

When applying Weakening in the induction, check that the left premiss is always a subderivation, and check variable exclusion conditions are satisfied.

(PoS = premiss of subderivation)

Weakening

$\frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \text{OK}}{\Delta; \Phi \vdash \kappa_L \text{ OK}} \text{IH}}{\Delta; \Phi, t :: \kappa_I \vdash \tau ::> \kappa_g} \text{PoS} \quad \frac{\frac{\frac{\Delta; \Phi, t :: \kappa_I \vdash \tau ::> \kappa_g}{\Delta; \Phi, t :: \kappa_I \vdash \text{OK}} \text{premiss}}{\Delta; \Phi, t :: \kappa_I \vdash \kappa_L \text{ OK}} \text{COK}$	$\frac{\frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \text{OK}}{t_L \notin \Phi} \text{IH}}{t_L \notin \Phi, t :: \kappa_I} \text{PoS} \quad \frac{\frac{t_L \notin \mathcal{J}}{t_L \neq t} \text{IH} \quad \frac{t \in \mathcal{J}}{t_L \notin \mathcal{J}} \text{IH}}{t_L \notin \Phi, t :: \kappa_I} \text{CWF-TypVar}$	$\frac{\frac{\frac{\frac{\Delta; \Phi, t :: \kappa_I \vdash \tau ::> \kappa_g}{\Delta; \Phi, t :: \kappa_I \vdash \text{OK}} \text{premiss}}{t \notin \Phi} \text{COK} \quad \frac{\frac{\frac{t_L \notin \mathcal{J}}{t \neq t_L} \text{IH} \quad \frac{t \in \mathcal{J}}{t \notin \kappa_L} \text{IH}}{t \notin \Phi, t_L :: \kappa_L} \text{PoS}}{t \notin \Phi, t_L :: \kappa_L} \text{CWF-TypVar}$	$\frac{\frac{\frac{\frac{\Delta; \Phi, t :: \kappa_I \vdash \text{OK}}{\Delta; \Phi \vdash \kappa_I \text{ OK}} \text{premiss}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \text{ OK}} \text{PoS} \quad \frac{\frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \text{OK}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \text{ OK}} \text{premiss}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \text{ OK}} \text{Weakening} \quad \frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \text{ OK}}{\Delta; \Phi, t_L :: \kappa_L, t :: \kappa_I \vdash \text{OK}} \text{CWF-TypVar}$	$\frac{\frac{\Delta; \Phi, t_L :: \kappa_L, t_L :: \kappa_L \vdash \tau ::> \kappa_g}{\Delta; \Phi, t_L :: \kappa_L, t_L :: \kappa_L \vdash \tau ::> \kappa_g} \text{Weakening} \quad \frac{\Delta; \Phi, t_L :: \kappa_L, t :: \kappa_I \vdash \tau ::> \kappa_g}{\Delta; \Phi, t_L :: \kappa_L, t :: \kappa_I \vdash \lambda t :: \kappa_I. \tau ::> \mathbf{S}_{\Pi_{t :: \kappa_I}. \kappa_g}(\lambda t :: \kappa_I. \tau)} \text{Marked-Exchange} \quad \frac{\Delta; \Phi, t_L :: \kappa_L, t :: \kappa_I \vdash \text{OK}}{\Delta; \Phi, t_L :: \kappa_L, t_L :: \kappa_L \vdash \text{OK}} \text{Marked-Exchange} \quad \frac{\Delta; \Phi, t_L :: \kappa_L, t :: \kappa_I \vdash \text{OK}}{\Delta; \Phi, t_L :: \kappa_L, t_L :: \kappa_L \vdash \text{OK}} \text{PK-}\lambda$
$\frac{\frac{\Delta; \Phi \vdash \kappa_I \equiv \kappa_g}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \equiv \kappa_g} \text{premiss} \quad \frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \text{OK}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \equiv \kappa_g} \text{IH}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \equiv \kappa_g} \text{Weakening}$	$\frac{\frac{\frac{\Delta; \Phi, t :: \kappa_I \vdash \kappa_g \equiv \kappa_4}{\Delta; \Phi, t :: \kappa_I \vdash \text{OK}} \text{premiss}}{\Delta; \Phi, t :: \kappa_I \vdash \text{OK}} \text{COK} \quad \frac{\frac{\frac{\Delta; \Phi, t :: \kappa_I \vdash \text{OK}}{\Delta; \Phi \vdash \kappa_I \text{ OK}} \text{PoS} \quad \frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \text{OK}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \text{ OK}} \text{IH}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \text{ OK}} \text{Weakening} \quad \frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \text{ OK}}{\Delta; \Phi, t_L :: \kappa_L, t :: \kappa_I \vdash \text{OK}} \text{CWF-TypVar}}{\Delta; \Phi, t_L :: \kappa_L, t :: \kappa_I \vdash \kappa_g \equiv \kappa_4} \text{Marked-Exchange}$	$\frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \equiv \kappa_g}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \equiv \kappa_g} \text{premiss} \quad \frac{\frac{\Delta; \Phi, t_L :: \kappa_L \vdash \text{OK}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \equiv \kappa_g} \text{IH}}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \equiv \kappa_g} \text{Weakening} \quad \frac{\frac{\Delta; \Phi, t :: \kappa_I, t_L :: \kappa_L \vdash \kappa_g \equiv \kappa_4}{\Delta; \Phi, t_L :: \kappa_L \vdash \kappa_I \equiv \kappa_g} \text{CWF-TypVar} \quad \frac{\Delta; \Phi, t_L :: \kappa_L, t :: \kappa_I \vdash \kappa_g \equiv \kappa_4}{\Delta; \Phi, t_L :: \kappa_L \vdash \Pi_{t :: \kappa_I}. \kappa_g \equiv \Pi_{t :: \kappa_g}. \kappa_4} \text{KEquiv-II}$		

Lemma 5 (OK-PK). *If $\Delta; \Phi \vdash \tau ::> \kappa$, then $\Delta; \Phi \vdash \kappa$ OK*

Lemma 6 (OK-WFaK). *If $\Delta; \Phi \vdash \tau :: \kappa$, then $\Delta; \Phi \vdash \kappa$ OK*

Lemma 7 (OK-MatchPi). *If $\Delta; \Phi \vdash \kappa \Pi \dot{\vdash} \Pi_{t::\kappa_1, \kappa_2}$, then $\Delta; \Phi \vdash \kappa$ OK and $\Delta; \Phi \vdash \Pi_{t::\kappa_1, \kappa_2}$ OK*

Lemma 8 (OK-KEquiv). *If $\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$, then $\Delta; \Phi \vdash \kappa_1$ OK and $\Delta; \Phi \vdash \kappa_2$ OK*

Lemma 9 (OK-CSK). *If $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$, then $\Delta; \Phi \vdash \kappa_1$ OK and $\Delta; \Phi \vdash \kappa_2$ OK*

Lemma 10 (OK-EquivAK). *If $\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{=} \tau_2$, then $\Delta; \Phi \vdash \tau_1 :: \kappa$ and $\Delta; \Phi \vdash \tau_2 :: \kappa$ and $\Delta; \Phi \vdash \kappa$ OK*

Lemma 11 (OK-Substitution).
*If $\Delta; \Phi \vdash \tau_L :: \kappa_{L1}$ and $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \kappa_{L2}$ OK, then $\Delta; \Phi \vdash [\tau_L/t_L] \kappa_{L2}$ OK
(induction on $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \kappa_{L2}$ OK)*

Lemma 12 (K-Substitution).
*If $\Delta; \Phi \vdash \tau_{L1} :: \kappa_{L1}$ and $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \tau_{L2} :: \kappa_{L2}$, then $\Delta; \Phi \vdash [\tau_{L1}/t_L]\tau_{L2} :: [\tau_{L1}/t_L]\kappa_{L2}$
(induction on $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \tau_{L2} :: \kappa_{L2}$)*

Proof. By simultaneous induction on derivations.

The interesting cases per lemma.

	OK-FAK.	PK-DAS.	$\Delta, \Phi \vdash \text{bse}::\text{Type}$ OK
	*	*	$\Delta, \Phi \vdash \text{S}_{\text{Type}}(\text{bse})$ OK
		*	$\Delta, \Phi \vdash \text{OK}$
	OK-WFaK.	PK-Ap	$\Delta, \Phi \vdash \tau_{\mathcal{G}}::\kappa$
	*	(12)	$\Delta, \Phi \vdash \text{S}_{\kappa}(\tau_{\mathcal{G}})$ OK
	OK-KEquiv.	(22)	$\Delta, \Phi \vdash \tau \text{ t.t.} > \kappa$ OK
(PoS = premiss of subderivation)	OK-Substitution.	(41)	$\Delta, \Phi, t_L::\kappa_{L,I} \vdash \text{OK}$
		*	$\Delta, \Phi \vdash \kappa_{L,I}$ OK
		*	$\Delta, \Phi \vdash \text{OK}$
		*	$\Delta, \Phi \vdash [\tau_L/t_L]\text{Type}$ OK
		(43)	$\Delta, \Phi, t_L::\kappa_{L,I} \vdash \tau::\kappa$
			$\Delta, \Phi, t_L::\kappa_{L,I} \vdash \text{OK}$
			$\Delta, \Phi \vdash \kappa_{L,I}$ OK
		*	$\Delta, \Phi \vdash \text{OK}$
		*	$\Delta, \Phi \vdash [\tau_{L,I}/t_L]::[\tau_{L,I}/t_L]\kappa$
		*	$\Delta, \Phi \vdash [\tau_L/t_L]\text{S}_{\kappa}(\tau)$ OK

by (9)
by (10)
by (43)
by premiss
bad
by (10)
by (43)

premiss (41)
by subderivation premiss (46)
by OK-KWF
by (41) and degenerate subst
premiss (43)
by OK-WFaK
by subderivation premiss (46)
by OK-KWF
by K-Substitution on premiss
by (43)

Lemma 13 (PK-Unicity). *If $\Delta; \Phi \vdash \tau_L ::> \kappa_{L1}$ and $\Delta; \Phi \vdash \tau_L ::> \kappa_{L2}$ then κ_{L1} is κ_{L2}*

Lemma 14. *If $\Delta; \Phi \vdash \tau :: > \kappa_1$ and $\Delta; \Phi \vdash \tau :: \kappa_2$, then $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$*

Lemma 15. *If $\Delta; \Phi \vdash \kappa_1 \lesssim \mathbf{S}_{\kappa_2}(\tau)$, then $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$*

