# Hazel PHI: 10-modules

July 1, 2021

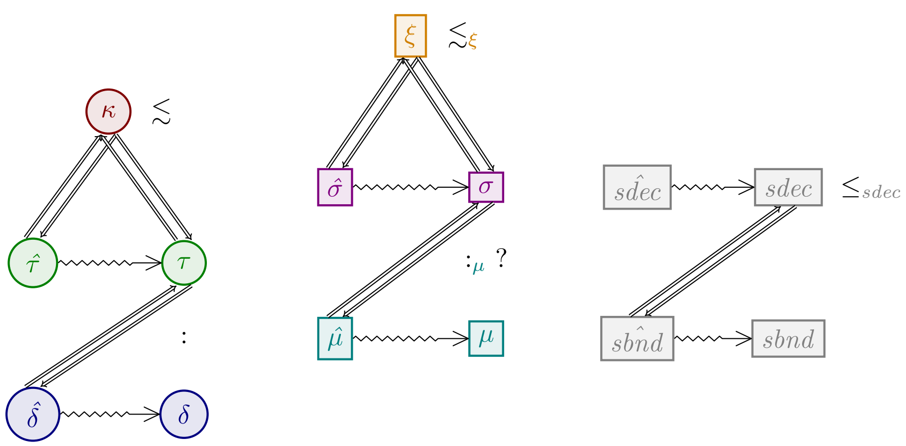
## prerequisites

- Hazel PHI: 9-type-aliases-redux
  - github
  - current commit: 4410cd565ce717707e580e44f64868d3175fe2a6
- (optional) Hazel PHI: 1-labeled-tuples
  - github
  - current commit: 0a7d0b53ee7286d03ea3be13a7ac91a86f1c90b1

## how to read

| | | | |
|---|---|---|---|
| 800000 | kinds | D08000 | signature kind |
| 008000 | types (constructors) | 800080 | signatures |
| 000080 | terms | 008080 | modules |

## notes




external typ/sig/mod syntax not written out yet (waiting for construction dust to settle); patterns not handled yet– will be left till end.

## syntax

| | | | | |
|---|---|---|---|---|
| kind | $\kappa$ | $::=$ | $\texttt{Type}$ | kind of types |
| | | $\mid$ | $\texttt{S}(\tau)$ | singleton kind |
| | | $\mid$ | $\texttt{KHole}$ | kind hole |
| | | $\mid$ | $\Pi_{t::\kappa_1}.\kappa_2$ | dependent function kind |

| | | | | |
|---|---|---|---|---|
| HTyp | $\tau$ | ::= | $t$ | type variable |
| | | \| | $bse$ | base type |
| | | \| | $\tau_1 \oplus \tau_2$ | type binop |
| | | \| | $[\tau]$ | list type |
| | | \| | $\lambda t{::}\kappa.\tau$ | type function |
| | | \| | $\tau_1\ \tau_2$ | type application |
| | | \| | $\{lab_1 \hookrightarrow \tau_1, \ldots\ lab_n \hookrightarrow \tau_n\}$ | labelled product type (record) |
| | | \| | $\mu.lab$ | module type projection |
| | | \| | $⦇⦈$ | empty type hole |
| | | \| | $⦇\tau⦈$ | nonempty type hole |
| base type | $bse$ | ::= | $\texttt{Int}$ | |
| | | \| | $\texttt{Float}$ | |
| | | \| | $\texttt{Bool}$ | |
| HTyp BinOp | $\oplus$ | ::= | $\times$ | |
| | | \| | $+$ | |
| | | \| | $\rightarrow$ | |
| external expression | $\hat{\delta}$ | ::= | ... | |
| | | \| | $x$ | |
| | | \| | $\texttt{signature}\ s = \hat{\sigma}\ \texttt{in}\ \hat{\delta}$ | |
| | | \| | $\texttt{module}\ m = \hat{\mu}\ \texttt{in}\ \hat{\delta}$ | |
| | | \| | $\texttt{module}\ m{:}_{\mu}s = \hat{\mu}\ \texttt{in}\ \hat{\delta}$ | |
| | | \| | $\texttt{functor something} = \texttt{something in}\ \hat{\delta}$ | |
| | | \| | $\hat{\mu}.lab$ | module term projection |
| internal expression | $\delta$ | ::= | ... | |
| | | \| | $x$ | |
| | | \| | $\texttt{signature}\ s = \sigma\ \texttt{in}\ \delta$ | |
| | | \| | $\texttt{module}\ m{:}_{\mu}s = \mu\ \texttt{in}\ \delta$ | |
| | | \| | $\texttt{functor something} = \texttt{something in}\ \delta$ | |
| | | \| | $\mu.lab$ | module term projection |
| signature kind | $\xi$ | ::= | $\texttt{SSigKind}(\sigma)$ | |
| | | \| | $\texttt{SigKHole}$ | |
| signature | $\sigma$ | ::= | $s$ | signature variable |
| | | \| | $\{sdecs\}$ | structure signature |
| | | \| | $\Pi_{m:_{\mu}\sigma_1}.\sigma_2$ | functor signature |
| | | \| | $⦇⦈$ | empty signature hole |
| | | \| | $⦇s⦈$ | nonempty signature hole |
| module | $\mu$ | ::= | $m$ | module variable |
| | | \| | $\{sbnds\}$ | structure |
| | | \| | $\lambda m{:}_{\mu}\sigma.\mu$ | functor |
| | | \| | $\mu_1\ \mu_2$ | functor application |
| | | \| | $\mu.lab$ | submodule projection |
| | | \| | $⦇⦈$ | empty module hole |
| | | \| | $⦇\mu⦈$ | nonempty module hole |
| signature declarations | $sdecs$ | ::= | $\cdot$ | |
| | | \| | $sdec, sdecs$ | |
| signature declaration | $sdec$ | ::= | $\texttt{type}\ lab$ | |
| | | \| | $\texttt{type}\ lab = \tau$ | |
| | | \| | $\texttt{val}\ lab{:}\tau$ | |
| | | \| | $\texttt{module}\ lab{:}_{\mu}\sigma$ | |
| | | \| | $\texttt{functor}\ lab{:}_{\mu}\sigma$ | |
| structure bindings | $sbnds$ | ::= | $\cdot$ | |
| | | \| | $sbnd, sbnds$ | |
| structure binding | $sbnd$ | ::= | $\texttt{type}\ t = \tau$ | |
| | | \| | $\texttt{let}\ x{:}\tau = \delta$ | |
| | | \| | $\texttt{module}\ m = \mu$ | |
| | | \| | $\texttt{module}\ m{:}_{\mu}s = \mu$ | |

$| \quad \texttt{functor}\ m{:}_\mu s = \mu$

## declarative statics

## contexts

$\Delta, ?; \Gamma, x{:}\tau; \Phi, t{::}\kappa; \Xi, m{:}_\mu\sigma; \Psi, s{::}_\sigma\xi$

## statics

scratch

$\boxed{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}$ $\kappa_1$ is a consistent subkind of $\kappa_2$

**KCSubsumption**

$$\frac{\textit{test}}{\textit{test}}$$

$\boxed{\Delta; \Phi; \Xi; \Psi \vdash \xi_1 \lesssim_\xi \xi_2}$ $\xi_1$ is a consistent sub signature kind of $\xi_2$

**nameMe**

$$\frac{\exists sdec_x \in sdecs_1 \ \ \textit{st}\ \ \Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(\{sdec_x\}) \lesssim_\xi \texttt{SSigKind}(\{sdec_2\}) \quad \Delta; \Phi, \mathsf{type}(\Delta; \Phi; \Xi; \Psi, sdec_2); \Xi, \mathsf{submodule}(sdec_2); \Psi \vdash \{sdecs_1\} \lesssim_\xi \{sdecs_2\}}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(\{sdec_{11}, sdec_{12}, sdecs_{13}\ \texttt{as}\ sdecs_1\}) \lesssim_\xi \texttt{SSigKind}(\{sdec_2, sdecs_2\})}$$

**single**

$$\frac{\Delta; \Phi; \Xi; \Psi \vdash sdec_1 \leq_{sdec} sdec_2}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(\{sdec_1\}) \lesssim_\xi \texttt{SSigKind}(\{sdec_2\})}$$

**nil**

$$\frac{}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(\{sdecs\}) \lesssim_\xi \texttt{SSigKind}(\{\cdot\})}$$

**varprop**

$$\frac{s{::}_\sigma\xi \in \Psi}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(s) \lesssim_\xi \xi}$$

**nameMe?delete?**

$$\frac{\sigma_1 \neq s \qquad \Delta; \Phi; \Xi; \Psi \vdash \sigma_1 \ \Leftarrow\ \texttt{SSigKind}(\sigma_2)}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(\sigma_1) \lesssim_\xi \texttt{SSigKind}(\sigma_2)}$$

**funct**

$$\frac{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(\sigma_{21}) \lesssim_\xi \texttt{SSigKind}(\sigma_{11}) \qquad \Delta; \Phi; \Xi, m{:}_\mu\sigma_{11}; \Psi \vdash \texttt{SSigKind}(\sigma_{12}) \lesssim_\xi \texttt{SSigKind}(\sigma_{22})}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}(\Pi_{m:_\mu\sigma_{11}}.\sigma_{12}) \lesssim_\xi \texttt{SSigKind}(\Pi_{m:_\mu\sigma_{21}}.\sigma_{22})}$$

**holes**

$$\frac{\Delta; \Phi; \Xi; \Psi \vdash (\!|\!)^{\mathsf{u}} \ \Leftarrow\ \xi}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}((\!|\!)^{\mathsf{u}}) \lesssim_\xi \xi}$$

**neholes**

$$\frac{\Delta; \Phi; \Xi; \Psi \vdash (\!|\mathsf{s}|\!)^{\mathsf{u}} \ \Leftarrow\ \xi}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SSigKind}((\!|\mathsf{s}|\!)^{\mathsf{u}}) \lesssim_\xi \xi}$$

**CSubSigKindHoleL**

$$\frac{}{\Delta; \Phi; \Xi; \Psi \vdash \texttt{SigKHole} \lesssim_\xi \xi}$$

**CSubSigKindHoleR**

$$\frac{}{\Delta; \Phi; \Xi; \Psi \vdash \xi \lesssim_\xi \texttt{SigKHole}}$$

$\boxed{\Delta; \Phi; \Xi; \Psi \vdash \sigma \ \Rightarrow\ \xi}$ $\sigma$ synthesizes signature kind $\xi$

**SynSigKndVar**

$$\frac{s{::}_\sigma\xi \in \Psi}{\Delta; \Phi; \Xi; \Psi \vdash s \ \Rightarrow\ \texttt{SSigKind}(s)}$$

**SynSigKndVarFail**

$$\frac{s \notin \mathsf{dom}(\Psi)}{\Delta; \Phi; \Xi; \Psi \vdash s \ \Rightarrow\ \texttt{SigKHole}}$$

$$\frac{\{sdecs\} \textit{wellformed?}}{\vdash \{sdecs\} \ \Rightarrow\ \texttt{SSigKind}(\{sdecs\})}$$

$$\frac{\Delta; \Phi; \Xi, m{:}_\mu\sigma_1; \Psi \vdash \sigma_2 \ \Rightarrow\ \xi}{\Delta; \Phi; \Xi; \Psi \vdash \Pi_{m:_\mu\sigma_1}.\sigma_2 \ \Rightarrow\ \texttt{SSigKind}(\Pi_{m:_\mu\sigma_1}.\sigma_2)}$$

**SynSigKndSigHole**

$$\frac{u{::}_\sigma\xi \in \Delta}{\Delta; \Phi; \Xi; \Psi \vdash (\!|\!)^{\mathsf{u}} \ \Rightarrow\ \xi}$$

**SynSigKndSigHole**

$$\frac{u{::}_\sigma\xi \in \Delta \qquad \Delta; \Phi; \Xi; \Psi \vdash s \ \Rightarrow\ \xi_1}{\Delta; \Phi; \Xi; \Psi \vdash (\!|s|\!)^{\mathsf{u}} \ \Rightarrow\ \xi}$$

$\boxed{\Delta;\Phi;\Xi;\Psi \vdash \sigma \ \Leftarrow\ \xi}$ $\sigma$ analyzes against signature kind $\xi$

$$\frac{\Delta;\Phi;\Xi;\Psi \vdash \sigma \ \Rightarrow\ \xi_1 \qquad \Delta;\Phi;\Xi;\Psi \vdash \xi_1 \lesssim_\xi \xi}{\Delta;\Phi;\Xi;\Psi \vdash \sigma \ \Leftarrow\ \xi}\ \texttt{Sub}$$

$\boxed{\Delta;\Phi;\Xi;\Psi \vdash sdec_1 \leq_{sdec} sdec_2}$ $sdec_1$ is a subsdec of $sdec_2$

$$\frac{}{\Delta;\Phi;\Xi;\Psi \vdash \texttt{type}\ lab = \tau \leq_{sdec} \texttt{type}\ lab}\ \texttt{singleType}$$

$$\frac{\Delta;\Phi;\Xi;\Psi \vdash \tau_1 \equiv \tau_2}{\Delta;\Phi;\Xi;\Psi \vdash \texttt{type}\ lab = \tau_1 \leq_{sdec} \texttt{type}\ lab = \tau_2}\ \texttt{singleType2}$$

$$\frac{}{\Delta;\Phi;\Xi;\Psi \vdash \texttt{type}\ lab \leq_{sdec} \texttt{type}\ lab}\ \texttt{singleType3}$$

$$\frac{\Delta;\Phi;\Xi;\Psi \vdash \tau_1 \equiv \tau_2}{\Delta;\Phi;\Xi;\Psi \vdash \texttt{val}\ lab{:}\tau_1 \leq_{sdec} \texttt{val}\ lab{:}\tau_2}\ \texttt{singleVa}$$

$$\frac{\Delta;\Phi;\Xi;\Psi \vdash \sigma_1 \ \Leftarrow\ \texttt{SSigKind}(\sigma_2)}{\Delta;\Phi;\Xi;\Psi \vdash \texttt{module}\ lab{:}_\mu\sigma_1 \leq_{sdec} \texttt{module}\ lab{:}_\mu\sigma_2}\ \texttt{singleMod}$$

# elab

$\boxed{\Gamma;\Phi;\Xi \vdash \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \delta \dashv \Delta}$ $\hat{\delta}$ synthesizes type $\tau$ and elaborates to $\delta$ with hole context $\Delta$

$\ldots$

$$\frac{\Gamma;\Phi;\Xi \vdash \hat{\mu} \ \Rightarrow\ \sigma \rightsquigarrow \mu \dashv \Delta_1 \qquad \Gamma;\Phi;\Xi, m{:}_\mu\sigma \vdash \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \delta \dashv \Delta_2}{\Gamma;\Phi;\Xi \vdash \texttt{module}\ m = \hat{\mu}\ \texttt{in}\ \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \texttt{module}\ m = \mu\ \texttt{in}\ \delta \dashv \Delta_1 \cup \Delta_2}\ \texttt{SynElabLetMod}$$

$$\frac{\Phi;\Xi \vdash \hat{\sigma} \ \Rightarrow\ \xi \rightsquigarrow \sigma \dashv \Delta_1 \qquad \Gamma;\Phi;\Xi \vdash \hat{\mu} \ \Leftarrow\ \sigma \rightsquigarrow \mu \dashv \Delta_2 \qquad \Gamma;\Phi;\Xi, m{:}_\mu\sigma \vdash \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \delta \dashv \Delta_3}{\Gamma;\Phi;\Xi \vdash \texttt{module}\ m{:}_\mu\hat{\sigma} = \hat{\mu}\ \texttt{in}\ \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \texttt{module}\ m{:}_\mu\sigma = \mu\ \texttt{in}\ \delta \dashv \Delta_1 \cup \Delta_2 \cup \Delta_3}\ \texttt{SynElabLetModAnn}$$

$$\frac{\Gamma;\Phi;\Xi \vdash \hat{\mu} \ \Rightarrow\ \sigma \rightsquigarrow \mu \dashv \Delta \qquad \Phi;\Xi \vdash \sigma \ \Rightarrow\ \xi}{\Gamma;\Phi;\Xi \vdash \hat{\mu}.lab \ \Rightarrow\ \tau \rightsquigarrow \mu.lab \dashv \Delta}\ \texttt{SynElabModTermPrj}$$

$\boxed{\Phi;\Xi \vdash \hat{\tau} \ \Rightarrow\ \kappa \rightsquigarrow \tau \dashv \Delta}$ $\hat{\tau}$ synthesizes kind $\kappa$ and elaborates to $\tau$ with hole context $\Delta$

$\ldots$

$$\frac{\Phi;\Xi \vdash m \ \Rightarrow\ \sigma \rightsquigarrow m \dashv \Delta \qquad something\sigma\kappa}{\Phi;\Xi \vdash m.lab \ \Rightarrow\ \kappa \rightsquigarrow m.lab \dashv \Delta}\ \texttt{SynElabModTypPrj}$$

$\boxed{\Phi;\Xi \vdash \hat{\tau} \ \Leftarrow\ \kappa \rightsquigarrow \tau \dashv \Delta}$ $\hat{\tau}$ analyzes against kind $\kappa$ and elaborates to $\tau$ with hole context $\Delta$

$\boxed{\Gamma;\Phi;\Xi \vdash \hat{\mu} \ \Rightarrow\ \sigma \rightsquigarrow \mu \dashv \Delta}$ $\hat{\mu}$ synthesizes signature $\sigma$ and elaborates to $\mu$ with hole context $\Delta$

$$\frac{m{:}_\mu\sigma \in \Xi}{\Gamma;\Phi;\Xi \vdash m \ \Rightarrow\ \sigma \rightsquigarrow m \dashv \cdot}\ \texttt{SynElabModVar}$$

$$\frac{m \notin \mathsf{dom}(\Xi)}{\Gamma;\Phi;\Xi \vdash m \ \Rightarrow\ (\!|\,|\!) \rightsquigarrow (\!|m|\!)^{\mathtt{u}} \dashv u{:}_\mu(\!|\,|\!)}\ \texttt{SynElabModVarFail}$$

$$\frac{\Gamma;\Phi;\Xi \vdash \hat{sbnd} \ \Rightarrow\ sdec \rightsquigarrow sbnd \dashv \Delta_1 \qquad \Gamma, \mathsf{val}(sdec); \Phi, \mathsf{type}(\Delta_1;\Phi;\Xi;\Psi, sdec); \Xi, \mathsf{submodule}(sdec) \vdash \{\hat{sbnds}\} \ \Rightarrow\ \{sdecs\} \rightsquigarrow \{sbnds\} \dashv \Delta_2}{\Gamma;\Phi;\Xi \vdash \{\hat{sbnd}, \hat{sbnds}\} \ \Rightarrow\ \{sdec, sdecs\} \rightsquigarrow \{sbnd, sbnds\} \dashv \Delta_1 \cup \Delta_2}\ \texttt{SynElabConsStruct}$$

$$\frac{}{\Gamma;\Phi;\Xi \vdash \{\cdot\} \ \Rightarrow\ \{\cdot\} \rightsquigarrow \{\cdot\} \dashv \cdot}\ \texttt{SynElabNilStruct}$$

$$\frac{}{\Gamma;\Phi;\Xi \vdash (\!|\,|\!)^{\mathtt{u}} \ \Rightarrow\ (\!|\,|\!) \rightsquigarrow (\!|\,|\!)^{\mathtt{u}} \dashv u{:}_\mu(\!|\,|\!)}\ \texttt{SynElabEmptyModHole}$$

$$\frac{}{\Gamma;\Phi;\Xi \vdash (\!|m|\!)^{\mathtt{u}} \ \Rightarrow\ (\!|\,|\!) \rightsquigarrow (\!|m|\!)^{\mathtt{u}} \dashv u{:}_\mu(\!|\,|\!)}\ \texttt{SynElabNonemptyModHole}$$

`functor stuff`

$\boxed{\Gamma;\Phi;\Xi \vdash \hat{\mu} \;\Leftarrow\; \sigma \rightsquigarrow \mu \dashv \Delta}$ $\hat{\mu}$ analyzes against signature $\sigma$ and elaborates to $\mu$ with hole context $\Delta$

AnaElabModSubsumption
$$\frac{\Gamma;\Phi;\Xi \vdash \hat{\mu} \;\Rightarrow\; \sigma \rightsquigarrow \mu \dashv \Delta}{\Gamma;\Phi;\Xi \vdash \hat{\mu} \;\Leftarrow\; \sigma \rightsquigarrow \mu \dashv \Delta}$$

$\boxed{\Gamma;\Phi;\Xi \vdash \hat{sbnd} \;\Rightarrow\; sdec \rightsquigarrow sbnd \dashv \Delta}$ $\hat{sbnd}$ synthesizes declaration $sdec$ and elaborates to $sbnd$ with hole context $\Delta$

SynElabTypeSbnd
$$\frac{\Phi;\Xi \vdash \hat{\tau} \;\Rightarrow\; \kappa \rightsquigarrow \tau \dashv \Delta}{\Gamma;\Phi;\Xi \vdash \texttt{type}\ t = \hat{\tau} \;\Rightarrow\; \texttt{type}\ t = \tau \rightsquigarrow \texttt{type}\ t = \tau \dashv \Delta}$$

SynElabValSbnd
$$\frac{\Phi;\Xi \vdash \hat{\tau} \;\Rightarrow\; \kappa \rightsquigarrow \tau \dashv \Delta_1 \qquad \Gamma;\Phi;\Xi \vdash \hat{\delta} \;\Leftarrow\; \tau \rightsquigarrow \delta \dashv \Delta_2}{\Gamma;\Phi;\Xi \vdash \texttt{let}\ x{:}\hat{\tau} = \hat{\delta} \;\Rightarrow\; \texttt{val}\ x{:}\tau \rightsquigarrow \texttt{let}\ x{:}\tau = \delta \dashv \Delta_1 \cup \Delta_2}$$

SynElabModSbnd
$$\frac{\Gamma;\Phi;\Xi \vdash \hat{\mu} \;\Rightarrow\; \sigma \rightsquigarrow \mu \dashv \Delta}{\Gamma;\Phi;\Xi \vdash \texttt{module}\ m = \hat{\mu} \;\Rightarrow\; \texttt{module}\ m{:}_{\mu}\sigma \rightsquigarrow \texttt{module}\ m{:}_{\mu}\sigma = \mu \dashv \Delta}$$

SynElabModAnnSbnd
$$\frac{\Phi;\Xi \vdash \hat{\sigma} \;\Rightarrow\; \xi \rightsquigarrow \sigma_1 \dashv \Delta_1 \qquad \Gamma;\Phi;\Xi \vdash \hat{\mu} \;\Rightarrow\; \sigma_2 \rightsquigarrow \mu \dashv \Delta_2 \qquad \Phi;\Xi;\Psi \vdash \sigma_2 \;\Leftarrow\; \xi}{\Gamma;\Phi;\Xi \vdash \texttt{module}\ m{:}_{\mu}\hat{\sigma} = \hat{\mu} \;\Rightarrow\; \texttt{module}\ m{:}_{\mu}\sigma_1 \rightsquigarrow \texttt{module}\ m{:}_{\mu}\sigma_1 = \mu \dashv \Delta_1 \cup \Delta_2}$$

$\boxed{\Gamma;\Phi;\Xi \vdash \hat{sbnd} \;\Leftarrow\; sdec \rightsquigarrow sbnd \dashv \Delta}$ $\hat{sbnd}$ analyzes against declaration $sdec$ and elaborates to $sbnd$ with hole context $\Delta$

subsump
$$\frac{\Gamma;\Phi;\Xi;l\Psi \vdash \hat{sbnd} \;\Rightarrow\; sdec_1 \rightsquigarrow sbnd \dashv \Delta \qquad \Delta;\Phi;\Xi;\Psi \vdash sdec_1 \leq_{sdec} sdec}{\Gamma;\Phi;\Xi;\Psi \vdash \hat{sbnd} \;\Leftarrow\; sdec \rightsquigarrow sbnd \dashv \Delta}$$

$\boxed{\Gamma;\Phi;\Xi;\Psi \vdash \hat{sdec} \rightsquigarrow sdec \dashv \Delta}$ $\hat{sdec}$ elaborates to $sdec$ with hole context $\Delta$

opq
$$\frac{}{\Gamma;\Phi;\Xi;\Psi \vdash \texttt{type}\ lab \rightsquigarrow \texttt{type}\ lab \dashv \cdot}$$

trn
$$\frac{\Gamma;\Phi;\Xi;\Psi \vdash \hat{\tau} \;\Rightarrow\; \kappa \rightsquigarrow \tau \dashv \Delta}{\Gamma;\Phi;\Xi;\Psi \vdash \texttt{type}\ lab = \hat{\tau} \rightsquigarrow \texttt{type}\ lab = \tau \dashv \Delta}$$

val
$$\frac{\Gamma;\Phi;\Xi;\Psi \vdash \hat{\tau} \;\Rightarrow\; \kappa \rightsquigarrow \tau \dashv \Delta}{\Gamma;\Phi;\Xi;\Psi \vdash \texttt{val}\ lab{:}\hat{\tau} \rightsquigarrow \texttt{val}\ lab{:}\tau \dashv \Delta}$$

mod
$$\frac{\Gamma;\Phi;\Xi;\Psi \vdash \hat{\sigma} \;\Rightarrow\; \xi \rightsquigarrow \sigma \dashv \Delta}{\Gamma;\Phi;\Xi;\Psi \vdash \texttt{module}\ lab{:}_{\mu}\hat{\sigma} \rightsquigarrow \texttt{module}\ lab{:}_{\mu}\sigma \dashv \Delta}$$

`we're going to need HOFunctors so we don't need to preclude users from typing a functor into a module and vice versa`

$\boxed{\Phi;\Xi;\Psi \vdash \hat{\sigma} \;\Rightarrow\; \xi \rightsquigarrow \sigma \dashv \Delta}$ $\hat{\sigma}$ synthesizes signature kind $\xi$ and elaborates to $\sigma$ with hole context $\Delta$

SynSigEmptyHole
$$\frac{}{\Phi;\Xi;\Psi \vdash (\!|\!|)^{u} \;\Rightarrow\; \texttt{SigKHole} \rightsquigarrow (\!|\!|)^{u} \dashv u{::}_{\sigma}\texttt{SigKHole}}$$

SynSigNonEmptyHole

$\boxed{\Phi;\Xi \vdash \hat{\sigma} \;\Leftarrow\; \xi \rightsquigarrow \sigma \dashv \Delta}$ $\hat{\sigma}$ analyzes against signature kind $\xi$ and elaborates to $\sigma$ with hole context $\Delta$

`misc`

$$\mathsf{val}(sdec) = \begin{cases} lab{:}\tau & sdec = \texttt{val}\ lab{:}\tau \\ \cdot & \text{otherwise} \end{cases}$$

$$\mathsf{type}(cntxts, sdec) = \begin{cases} lab{::}\texttt{Type} & sdec = \texttt{type}\ lab \\ lab{::}\kappa & sdec = \texttt{type}\ lab = \tau \\ & \text{where } cntxts \vdash \tau \ \Rightarrow \ \kappa \\ \cdot & \text{otherwise} \end{cases}$$

$$\mathsf{submodule}(sdec) = \begin{cases} lab{:}_{\mu}\sigma & sdec = \texttt{module}\ lab{:}_{\mu}\sigma \\ \cdot & \text{otherwise} \end{cases}$$