Hazel Phi: 9-type-aliases

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SYNTAX

BinOp	$\oplus$	$::=$	$\times \mid + \mid \to$
Kind	κ	$::=$	$\texttt{Type} \mid \texttt{KHole} \mid \texttt{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
Base Types	$\texttt{bse}$	$::=$	$\texttt{Int} \mid \texttt{Float} \mid \texttt{Bool}$
User Types	$\hat{\tau}$	$::=$	$t \mid \texttt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!\|\,\|\!)^{\mathtt{u}} \mid (\!\|\hat{\tau}\|\!)^{\mathtt{u}} \mid \lambda t::\texttt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \texttt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!\|\,\|\!)^{\mathtt{u}} \mid (\!\|\tau\|\!)^{\mathtt{u}} \mid \lambda t::\kappa.\tau \mid \tau_1\ \tau_2$
Type Pattern			
User Expression			
Internal Expression			

DECLARATIVES

$\boxed{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \texttt{KHole} \lesssim \kappa} \qquad \frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \lesssim \texttt{KHole}}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta;\Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}$$

$$\frac{\Delta;\Phi \vdash \tau::\kappa}{\Delta;\Phi \vdash \texttt{S}_{\kappa}(\tau) \lesssim \kappa} \qquad \frac{\Delta;\Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta;\Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta;\Phi \vdash \texttt{S}_{\kappa_1}(\tau_1) \lesssim \texttt{S}_{\kappa_2}(\tau_2)}$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \equiv \kappa} \qquad \frac{\Delta;\Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \mathtt{S}_{\kappa}(\tau_1) \equiv \mathtt{S}_{\kappa}(\tau_2)} \qquad \frac{\Delta;\Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \mathtt{S}_{\mathtt{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathtt{S}_{\kappa}(\tau_1)} \qquad \frac{\Delta;\Phi \vdash \tau :: \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t::\kappa_1}.\mathtt{S}_{\kappa_2}(\tau\ t)}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4}$$

$\boxed{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$ τ_1 is equivalent to τ_2 at kind κ

$$\frac{\Delta;\Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_2)}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau :: \kappa}{\Delta;\Phi \vdash \tau \stackrel{\kappa}{\equiv} \tau} \qquad \frac{\Delta;\Phi \vdash \tau_2 \stackrel{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2} \qquad \frac{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_3 \stackrel{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\mathtt{Type}}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \stackrel{\mathtt{Type}}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 \stackrel{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\Phi, t::\kappa_1 \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \lambda t::\kappa_1.\tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t::\kappa_2.\tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \stackrel{\kappa_1}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1\ \tau_2 \stackrel{[\tau_1/t]\kappa_2}{\equiv} \tau_3\ \tau_4}$$

$$\frac{\Delta;\Phi \vdash \tau_1 :: \Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta;\Phi \vdash \tau_2 :: \Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta;\Phi, t::\kappa_1 \vdash \tau_1\ t \stackrel{\kappa_2}{\equiv} \tau_2\ t}{\Delta;\Phi \vdash \tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$$

$\boxed{\Delta; \Phi \vdash \tau ::> \kappa}$ τ has principal kind κ

$$\frac{}{\Delta; \Phi \vdash \texttt{bse} ::> \texttt{S}_{\texttt{Type}}(\texttt{bse})} \qquad \frac{t::\kappa \in \Phi}{\Delta; \Phi \vdash t ::> \texttt{S}_{\kappa}(t)} \qquad \frac{\Delta; \Phi \vdash \tau_1::\texttt{Type} \qquad \Delta; \Phi \vdash \tau_2::\texttt{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> \texttt{S}_{\texttt{Type}}(\tau_1 \oplus \tau_2)}$$

$$\frac{\mathtt{u}::\kappa \in \Delta}{\Delta; \Phi \vdash (\!|\,|\!)^{\mathtt{u}} ::> \kappa} \qquad \frac{\mathtt{u}::\kappa \in \Delta \qquad \Delta; \Phi \vdash \tau::\kappa_1}{\Delta; \Phi \vdash (\!|\tau|\!)^{\mathtt{u}} ::> \kappa} \qquad \frac{\mathtt{u}::\kappa \in \Delta \qquad t \notin \mathsf{dom}(\Phi)}{\Delta; \Phi \vdash (\!|t|\!)^{\mathtt{u}} ::> \kappa}$$

$$\frac{\Delta; \Phi, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau ::> \texttt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t::\kappa_1.\tau)}$$

$$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa \qquad \Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta; \Phi \vdash \tau_2::\kappa_1}{\Delta; \Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}$$

$\boxed{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{}{\Delta; \Phi \vdash \texttt{KHole} \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\texttt{KHole}}.\texttt{KHole}} \qquad \frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2}$$

$\boxed{\Delta; \Phi \vdash \tau::\kappa}$ τ is well formed at kind κ

$$\frac{\Delta; \Phi \vdash \tau ::> \kappa}{\Delta; \Phi \vdash \tau::\kappa} \qquad \frac{\Delta; \Phi \vdash \tau::\texttt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau::\kappa} \qquad \frac{\Delta; \Phi \vdash \tau ::> \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau::\kappa}$$

$$\frac{\Delta; \Phi \vdash \tau_2::\texttt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau_1::\texttt{S}_{\kappa}(\tau_2)} \qquad \frac{\Delta; \Phi \vdash \tau_1::\texttt{S}_{\kappa}(\tau_3) \qquad \Delta; \Phi \vdash \tau_3::\texttt{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1::\texttt{S}_{\kappa}(\tau_2)}$$