Hazel Phi: 11-type-constructors

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SYNTAX

Kind	κ	$::=$	$\mathtt{Type} \mid \mathtt{KHole} \mid \mathtt{S}_\kappa(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	$::=$	$t \mid \mathtt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!|\,|\!)^{\mathtt{u}} \mid (\!|\hat{\tau}|\!)^{\mathtt{u}} \mid \lambda t{::}\mathtt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \mathtt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!|\,|\!)^{\mathtt{u}} \mid (\!|\tau|\!)^{\mathtt{u}} \mid (\!|t|\!)^{\mathtt{u}} \mid \lambda t{::}\kappa.\tau \mid \tau_1\ \tau_2$
Base Types	$\mathtt{bse}$	$::=$	$\mathtt{Int} \mid \mathtt{Float} \mid \mathtt{Bool}$
BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Type Pattern			
User Expression			
Internal Expression			

DECLARATIVES

$\boxed{\Delta;\Phi \vdash \tau ::> \kappa}$ τ has principal (well formed) kind κ

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \mathtt{bse} ::> \mathtt{S}_{\mathtt{Type}}(\mathtt{bse})}\ \texttt{PK-Base}$$

$$\frac{\Delta;\Phi_1, t{::}\kappa, \Phi_2 \vdash \mathsf{OK}}{\Delta;\Phi \vdash t ::> \mathtt{S}_\kappa(t)}\ \texttt{PK-Var}$$

$$\frac{\Delta;\Phi \vdash \tau_1{::}\mathtt{Type} \qquad \Delta;\Phi \vdash \tau_2{::}\mathtt{Type}}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 ::> \mathtt{S}_{\mathtt{Type}}(\tau_1 \oplus \tau_2)}\ \texttt{PK-}\oplus$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash (\!|\,|\!)^{\mathtt{u}} ::> \mathtt{S}_\kappa((\!|\,|\!)^{\mathtt{u}})}\ \texttt{PK-EHole}$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK} \qquad \Delta;\Phi \vdash \tau{::}\kappa_1}{\Delta;\Phi \vdash (\!|\tau|\!)^{\mathtt{u}} ::> \mathtt{S}_\kappa((\!|\tau|\!)^{\mathtt{u}})}\ \texttt{PK-NEHole}$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK} \qquad t \notin \Phi}{\Delta;\Phi \vdash (\!|t|\!)^{\mathtt{u}} ::> \mathtt{S}_\kappa((\!|t|\!)^{\mathtt{u}})}\ \texttt{PK-Unbound}$$

$$\frac{\Delta;\underline{\Phi, t{::}\kappa_1} \vdash \tau ::> \kappa_2}{\Delta;\Phi \vdash \lambda t{::}\kappa_1.\tau ::> \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}\ \texttt{PK-}\lambda$$

$$\frac{\Delta;\Phi \vdash \tau_1 ::> \kappa \qquad \Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\underset{\Pi}{}} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta;\Phi \vdash \tau_2{::}\kappa_1}{\Delta;\Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}\ \texttt{PK-Ap}$$

$\boxed{\Delta;\Phi \vdash \tau::\kappa}$ τ is well formed at kind κ

$$\frac{\Delta;\Phi \vdash \tau ::> \mathsf{S}_{\kappa}(\tau)}{\Delta;\Phi \vdash \tau::\kappa}\ \texttt{WFaK-1} \qquad \frac{\Delta;\Phi \vdash \tau ::> \kappa_1 \qquad \Delta;\Phi \vdash \kappa_1 \lesssim \kappa}{\Delta;\Phi \vdash \tau::\kappa}\ \texttt{WFaK-Subsump}$$

$$\frac{\Delta;\Phi \vdash \tau ::> \kappa}{\Delta;\Phi \vdash \tau::\kappa}\ \texttt{WFaK-Reit} \qquad \frac{\Delta;\Phi \vdash \tau::\kappa}{\Delta;\Phi \vdash \tau::\mathsf{S}_{\kappa}(\tau)}\ \texttt{WFaK-Self}$$

$$\frac{\Delta;\Phi \vdash \tau::\Pi_{t::\kappa_3}.\kappa_4 \qquad \Delta;\Phi \vdash \Pi_{t::\kappa_3}.\kappa_4 \lesssim \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \tau::\Pi_{t::\kappa_1}.\kappa_2}\ \texttt{WFaK-}\Pi\texttt{CSKTrans}$$

$$\frac{\Delta;\Phi \vdash \tau::\mathsf{S}_{\kappa}(\tau_1) \qquad \Delta;\Phi \vdash \tau_1::\kappa}{\Delta;\Phi \vdash \tau::\kappa}\ \texttt{WFaK-Flatten}$$

$\boxed{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \texttt{KHole} \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\texttt{KHole}}.\texttt{KHole}}\ \overset{\blacktriangleright}{\scriptstyle\Pi}\texttt{-KHole} \qquad \frac{\Delta;\Phi \vdash \kappa \equiv \mathsf{S}_{\texttt{KHole}}(\tau)}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\mathsf{S}_{\texttt{KHole}}(\tau)}.\mathsf{S}_{\texttt{KHole}}(\tau\ t)}\ \overset{\blacktriangleright}{\scriptstyle\Pi}\texttt{-SKHole}$$

$$\frac{\Delta;\Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2}\ \overset{\blacktriangleright}{\scriptstyle\Pi}\texttt{-}\Pi$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \equiv \kappa}\ \texttt{KEquiv-Refl} \qquad \frac{\Delta;\Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Symm}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Trans}$$

$$\frac{\Delta;\Phi \vdash \tau::\mathsf{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \mathsf{S}_{\mathsf{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathsf{S}_{\kappa}(\tau_1)}\ \texttt{KEquiv-SKind}_{\texttt{SKind}} \qquad \frac{\Delta;\Phi \vdash \tau::\Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t_1::\kappa_1}.\mathsf{S}_{[t_1/t]\kappa_2}(\tau\ t_1)}\ \texttt{KEquiv-SKind}_{\Pi}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\underline{\Phi, t::\kappa_1} \vdash \kappa_3 \equiv \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{KEquiv-}\Pi \qquad \frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\kappa_1}(\tau_1) \equiv \mathsf{S}_{\kappa_2}(\tau_2)}\ \texttt{KEquiv-SKind}$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \texttt{KHole} \lesssim \kappa}\ \texttt{CSK-KHoleL} \qquad \frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \lesssim \texttt{KHole}}\ \texttt{CSK-KHoleR}$$

$$\frac{\Delta; \Phi \vdash \texttt{S}_{\texttt{KHole}}(\tau) \ \mathsf{OK} \qquad \Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \texttt{S}_{\texttt{KHole}}(\tau) \lesssim \kappa}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{L}$$

$$\frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK} \qquad \Delta; \Phi \vdash \texttt{S}_{\texttt{KHole}}(\tau) \ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \lesssim \texttt{S}_{\texttt{KHole}}(\tau)}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{R}$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-KEquiv} \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-Normal}$$

$$\frac{\Delta; \Phi \vdash \texttt{S}_{\kappa}(\tau) \ \mathsf{OK}}{\Delta; \Phi \vdash \texttt{S}_{\kappa}(\tau) \lesssim \kappa}\ \texttt{CSK-SKind} \qquad \frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta; \underline{\Phi, t{::}\kappa_3} \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{CSK-}\Pi$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \texttt{S}_{\kappa_1}(\tau_1) \lesssim \texttt{S}_{\kappa_2}(\tau_2)}\ \texttt{CSK-?}$$

$\boxed{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$ τ_1 is provably equivalent to τ_2 at kind κ

$$\frac{\Delta; \Phi \vdash \tau{::}\kappa}{\Delta; \Phi \vdash \tau \overset{\kappa}{\equiv} \tau}\ \texttt{EquivAK-Refl} \qquad \frac{\Delta; \Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Symm}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Trans}$$

$$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \equiv \texttt{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-SKind}$$

$$\frac{\Delta; \Phi \vdash \tau_1{::}\Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta; \Phi \vdash \tau_2{::}\Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta; \underline{\Phi, t{::}\kappa_1} \vdash \tau_1\ t \overset{\kappa_2}{\equiv} \tau_2\ t}{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}\ \texttt{EquivAK-}\Pi$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1\ \tau_2 \overset{[\tau_2/t]\kappa_2}{\equiv} \tau_3\ \tau_4}\ \texttt{EquivAK-Ap}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\texttt{S}_{\kappa}(\tau)}{\equiv} \tau_2}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (1)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\texttt{Type}}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \overset{\texttt{Type}}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \overset{\texttt{Type}}{\equiv} \tau_3 \oplus \tau_4}\ (2) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \underline{\Phi, t{::}\kappa_1} \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \lambda t{::}\kappa_1.\tau_1 \overset{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t{::}\kappa_2.\tau_2}\ (3)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (4)$$

$\boxed{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}$ κ is well formed

$$\frac{\Delta;\Phi\vdash\mathsf{OK}}{\Delta;\Phi\vdash\mathtt{Type}\ \mathsf{OK}}\ \texttt{KWF-Type}\qquad\frac{\Delta;\Phi\vdash\mathsf{OK}}{\Delta;\Phi\vdash\mathtt{KHole}\ \mathsf{OK}}\ \texttt{KWF-KHole}\qquad\frac{\Delta;\Phi\vdash\tau{::}\kappa}{\Delta;\Phi\vdash\mathtt{S}_{\kappa}(\tau)\ \mathsf{OK}}\ \texttt{KWF-SKind}$$

$$\frac{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\kappa_2\ \mathsf{OK}}{\Delta;\Phi\vdash\Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}}\ \texttt{KWF-}\Pi$$

$\boxed{\Delta;\Phi \vdash \mathsf{OK}}$ Context is well formed

$$\frac{}{\cdot;\cdot\vdash\mathsf{OK}}\ \texttt{CWF-Nil}\qquad\frac{t\notin\Phi\qquad\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta;\underline{\Phi, t{::}\kappa}\vdash\mathsf{OK}}\ \texttt{CWF-TypVar}\qquad\frac{\mathrm{u}\notin\Delta\qquad\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\underline{\Delta,\mathrm{u}{::}\kappa};\Phi\vdash\mathsf{OK}}\ \texttt{CWF-Hole}$$

Metatheory

Lemma 1 (COK)**.** *If* $\Delta;\Phi\vdash\mathcal{J}$, *then* $\Delta;\Phi\vdash \mathsf{OK}$

Proof. By simultaneous induction on derivations.
No interesting cases. □

Lemma 2 (Exchange)**.**
If $\Delta;\Phi_1, t_{L1}{::}\kappa_{L1}, t_{L2}{::}\kappa_{L2}, \Phi_2\vdash\mathcal{J}$ *and* $\Delta;\Phi_1, t_{L2}{::}\kappa_{L2}, t_{L1}{::}\kappa_{L1}, \Phi_2\vdash \mathsf{OK}$, *then* $\Delta;\Phi_1, t_{L2}{::}\kappa_{L2}, t_{L1}{::}\kappa_{L1}, \Phi_2\vdash\mathcal{J}$

Proof. By simultaneous induction on derivations.
No interesting cases.
(Only rules with Φ extended in the consequent are interesting, which is only CWF-TypVar, but when $\mathcal{J}$ is CWF, Exchange is identity) □

Lemma 3 (Weakening)**.**
If $\Delta;\Phi\vdash\mathcal{J}$ *and* $\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash \mathsf{OK}$, *then* $\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\mathcal{J}$

Lemma 4 (OK-PK)**.** *If* $\Delta;\Phi\vdash\tau ::> \kappa$, *then* $\Delta;\Phi\vdash\kappa\ \mathsf{OK}$

Lemma 5 (OK-WFaK)**.** *If* $\Delta;\Phi\vdash\tau{::}\kappa$, *then* $\Delta;\Phi\vdash\kappa\ \mathsf{OK}$

Lemma 6 (OK-MatchPi)**.** *If* $\Delta;\Phi\vdash\kappa \mathrel{\overset{\blacktriangleright}{\scriptstyle\Pi}} \Pi_{t::\kappa_1}.\kappa_2$, *then* $\Delta;\Phi\vdash\kappa\ \mathsf{OK}$ *and* $\Delta;\Phi\vdash\Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}$

Lemma 7 (OK-KEquiv)**.** *If* $\Delta;\Phi\vdash\kappa_1\equiv\kappa_2$, *then* $\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa_2\ \mathsf{OK}$

Lemma 8 (OK-CSK)**.** *If* $\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2$, *then* $\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa_2\ \mathsf{OK}$

Lemma 9 (OK-EquivAK)**.** *If* $\Delta;\Phi\vdash\tau_1\overset{\kappa}{\equiv}\tau_2$, *then* $\Delta;\Phi\vdash\tau_1{::}\kappa$ *and* $\Delta;\Phi\vdash\tau_2{::}\kappa$ *and* $\Delta;\Phi\vdash\kappa\ \mathsf{OK}$

Lemma 10 (OK-Substitution)**.**
If $\Delta;\Phi\vdash\tau_L{::}\kappa_{L1}$ *and* $\Delta;\Phi, t_L{::}\kappa_{L1}\vdash\kappa_{L2}\ \mathsf{OK}$, *then* $\Delta;\Phi\vdash[\tau_L/t_L]\kappa_{L2}\ \mathsf{OK}$
(induction on $\Delta;\Phi, t_L{::}\kappa_{L1}\vdash\kappa_{L2}\ \mathsf{OK}$*)*

Lemma 11 (K-Substitution)**.**
If $\Delta;\Phi\vdash\tau_{L1}{::}\kappa_{L1}$ *and* $\Delta;\Phi, t_L{::}\kappa_{L1}\vdash\tau_{L2}{::}\kappa_{L2}$, *then* $\Delta;\Phi\vdash[\tau_{L1}/t_L]\tau_{L2}{::}[\tau_{L1}/t_L]\kappa_{L2}$
(induction on $\Delta;\Phi, t_L{::}\kappa_{L1}\vdash\tau_{L2}{::}\kappa_{L2}$*)*

Proof. By simultaneous induction on derivations.
The interesting cases per lemma:

Weakening.	PK-λ	$\Delta;\Phi, t_L{::}\kappa_2 \vdash \mathsf{OK}$	by IH
		$\Delta;\Phi, t{::}\kappa_1 \vdash \mathsf{OK}$	by COK on premiss
		$\Delta;\Phi \vdash \kappa_1\ \mathsf{OK}$	by premiss
		$\Delta;\Phi, t_L{::}\kappa_L \vdash \kappa_1\ \mathsf{OK}$	by Weakening on premiss
		$\Delta;\Phi, t_L{::}\kappa_L, t{::}\kappa_1 \vdash \mathsf{OK}$	
		$\Delta;\Phi, t{::}\kappa_1, t_L{::}\kappa_L \vdash \tau ::> \kappa_2$	
		$\Delta;\Phi, t_L{::}\kappa_L, t{::}\kappa_1 \vdash \tau ::> \kappa_2$	
OK-PK.	PK-Base	$\Delta;\Phi \vdash \mathtt{bse}{::}\mathtt{S}_{\mathtt{Type}}(\mathtt{bse})$	by (9)
		$\Delta;\Phi \vdash \mathtt{bse}{::}\mathtt{Type}$	by (10)
	*	$\Delta;\Phi \vdash \mathtt{S}_{\mathtt{Type}}(\mathtt{bse})\ \mathsf{OK}$	by (43)
	*	$\Delta;\Phi \vdash \mathsf{OK}$	by premiss
	PK-Ap		bad
OK-WFaK.	(12)	$\Delta;\Phi \vdash \tau_2{::}\kappa$	by (10)
	*	$\Delta;\Phi \vdash \mathtt{S}_\kappa(\tau_2)\ \mathsf{OK}$	by (43)
OK-KEquiv.	(22)	$\Delta;\Phi \vdash \tau\ t ::> \kappa$	
OK-Substitution.	(41)	$\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \mathsf{OK}$	premiss (41)
		$\Delta;\Phi \vdash \kappa_{L1}\ \mathsf{OK}$	by subderivation premiss (46)
	*	$\Delta;\Phi \vdash \mathsf{OK}$	by OK-KWF
	*	$\Delta;\Phi \vdash [\tau_L/t_L]\mathtt{Type}\ \mathsf{OK}$	by (41) and degenerate subst
	(43)	$\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \tau{::}\kappa$	premiss (43)
		$\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \mathsf{OK}$	by OK-WFaK
		$\Delta;\Phi \vdash \kappa_{L1}\ \mathsf{OK}$	by subderivation premiss (46)
	*	$\Delta;\Phi \vdash \mathsf{OK}$	by OK-KWF
		$\Delta;\Phi \vdash [\tau_{L1}/t_L]\tau{::}[\tau_{L1}/t_L]\kappa$	by K-Substitution on premiss
	*	$\Delta;\Phi \vdash [\tau_L/t_L]\mathtt{S}_\kappa(\tau)\ \mathsf{OK}$	by (43)

□

Lemma 12 (PK-Unicity)**.** *If* $\Delta;\Phi \vdash \tau_L ::> \kappa_{L1}$ *and* $\Delta;\Phi \vdash \tau_L ::> \kappa_{L2}$ *then* κ_{L1} *is* κ_{L2}

Lemma 13. *If* $\Delta;\Phi \vdash \tau ::> \kappa_1$ *and* $\Delta;\Phi \vdash \tau{::}\kappa_2$*, then* $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$

Lemma 14. *If* $\Delta;\Phi \vdash \kappa_1 \lesssim \mathtt{S}_{\kappa_2}(\tau)$*, then* $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$