Hazel Phi: 9-type-aliases

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SYNTAX

Kind	κ	::=	$\mathtt{Type} \mid \mathtt{KHole} \mid \mathsf{S}_\kappa(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	::=	$t \mid \mathtt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!\|\!)^{\mathtt{u}} \mid (\!\|\hat{\tau}\|\!)^{\mathtt{u}} \mid \lambda t::\mathtt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	::=	$t \mid \mathtt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!\|\!)^{\mathtt{u}} \mid (\!\|\tau\|\!)^{\mathtt{u}} \mid (\!\|t\|\!)^{\mathtt{u}} \mid \lambda t::\kappa.\tau \mid \tau_1\ \tau_2$
Base Types	$\mathtt{bse}$	::=	$\mathtt{Int} \mid \mathtt{Float} \mid \mathtt{Bool}$
BinOp	$\oplus$	::=	$\times \mid + \mid \rightarrow$
Type Pattern			
User Expression			
Internal Expression			

DECLARATIVES

$\boxed{\Delta; \Phi \vdash \tau ::> \kappa}$ τ has principal (well formed) kind κ

$$\frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{bse} ::> \mathsf{S}_{\mathtt{Type}}(\mathtt{bse})} \quad (1)$$

$$\frac{\Delta; \Phi_1, t::\kappa, \Phi_2 \vdash \mathsf{OK}}{\Delta; \Phi \vdash t ::> \mathsf{S}_\kappa(t)} \quad (2)$$

$$\frac{\Delta; \Phi \vdash \mathsf{OK} \qquad \Delta; \Phi \vdash \tau_1 ::\mathtt{Type} \qquad \Delta; \Phi \vdash \tau_2 ::\mathtt{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> \mathsf{S}_{\mathtt{Type}}(\tau_1 \oplus \tau_2)} \quad (3)$$

$$\frac{\Delta_1, \mathtt{u}::\kappa, \Delta_2; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash (\!\|\!)^{\mathtt{u}} ::> \mathsf{S}_\kappa((\!\|\!)^{\mathtt{u}})} \quad (4)$$

$$\frac{\Delta_1, \mathtt{u}::\kappa, \Delta_2; \Phi \vdash \mathsf{OK} \qquad \Delta; \Phi \vdash \tau::\kappa_1}{\Delta; \Phi \vdash (\!\|\tau\|\!)^{\mathtt{u}} ::> \mathsf{S}_\kappa((\!\|\tau\|\!)^{\mathtt{u}})} \quad (5)$$

$$\frac{\Delta_1, \mathtt{u}::\kappa, \Delta_2; \Phi \vdash \mathsf{OK} \qquad t \notin \mathsf{dom}(\Phi)}{\Delta; \Phi \vdash (\!\|t\|\!)^{\mathtt{u}} ::> \mathsf{S}_\kappa((\!\|t\|\!)^{\mathtt{u}})} \quad (6)$$

$$\frac{\Delta; \Phi \vdash \kappa_1\ \mathsf{OK} \qquad \Delta; \Phi, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau ::> \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t::\kappa_1.\tau)} \quad (7)$$

$$\frac{\Delta; \Phi \vdash \tau_1\ ::> \kappa \qquad \Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta; \Phi \vdash \tau_2::\kappa_1}{\Delta; \Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2} \quad (8)$$

$\boxed{\Delta; \Phi \vdash \tau{::}\kappa}$ τ is well formed at kind κ

$$\frac{\Delta; \Phi \vdash \tau ::> \kappa}{\Delta; \Phi \vdash \tau{::}\kappa}\ (9) \qquad \frac{\Delta; \Phi \vdash \tau{::}\mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau{::}\kappa}\ (10)$$

$$\frac{\Delta; \Phi \vdash \tau ::> \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau{::}\kappa}\ (11)$$

$$\frac{\Delta; \Phi \vdash \tau_2{::}\mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau_1{::}\mathtt{S}_{\kappa}(\tau_2)}\ (12) \qquad \frac{\Delta; \Phi \vdash \tau_1{::}\mathtt{S}_{\kappa}(\tau_3) \qquad \Delta; \Phi \vdash \tau_3{::}\mathtt{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1{::}\mathtt{S}_{\kappa}(\tau_2)}\ (13)$$

$$\frac{\Delta; \Phi \vdash \tau{::}\kappa}{\Delta; \Phi \vdash \tau{::}\mathtt{S}_{\kappa}(\tau)}\ (14)$$

$\boxed{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{\Delta; \Phi \vdash \kappa \equiv \mathtt{KHole}}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\mathtt{KHole}}.\mathtt{KHole}}\ (15) \qquad \frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}\ (16)$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta; \Phi \vdash \kappa\ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \equiv \kappa}\ (17) \qquad \frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}\ (18) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}\ (19)$$

$$\frac{\Delta; \Phi \vdash \tau{::}\mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \mathtt{S}_{\mathtt{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathtt{S}_{\kappa}(\tau_1)}\ (20) \qquad \frac{\Delta; \Phi \vdash \tau{::}\mathtt{KHole}}{\Delta; \Phi \vdash \mathtt{S}_{\mathtt{KHole}}(\tau) \equiv \mathtt{KHole}}\ (21)$$

$$\frac{\Delta; \Phi \vdash \tau{::}\Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t::\kappa_1}.\mathtt{S}_{\kappa_2}(\tau\ t)}\ (22) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \Phi, t{::}\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4}\ (23)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathtt{S}_{\kappa}(\tau_1) \equiv \mathtt{S}_{\kappa}(\tau_2)}\ (24)$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{KHole} \lesssim \kappa} \ (25) \qquad \frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathtt{KHole}} \ (26)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \ (27) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \ (28)$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \mathtt{S}_{\kappa}(\tau) \lesssim \kappa} \ (29) \qquad \frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta; \Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4} \ (30)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathtt{S}_{\kappa_1}(\tau_1) \lesssim \mathtt{S}_{\kappa_2}(\tau_2)} \ (31)$$

$\boxed{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$ τ_1 is equivalent to τ_2 at kind κ

$$\frac{\Delta; \Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \ (32)$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau \overset{\kappa}{\equiv} \tau} \ (33) \qquad \frac{\Delta; \Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \ (34) \qquad \frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \ (35)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\mathtt{Type}}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4} \ (36) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau_1 \overset{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t::\kappa_2.\tau_2} \ (37)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \ \tau_2 \overset{[\tau_1/t]\kappa_2}{\equiv} \tau_3 \ \tau_4} \ (38)$$

$$\frac{\Delta; \Phi \vdash \tau_1 :: \Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta; \Phi \vdash \tau_2 :: \Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \ t \overset{\kappa_2}{\equiv} \tau_2 \ t}{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2} \ (39)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \ (40)$$

$\boxed{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}$ κ is well formed

$$\frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{Type} \ \mathsf{OK}} \ (41) \qquad \frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{KHole} \ \mathsf{OK}} \ (42) \qquad \frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \mathtt{S}_{\kappa}(\tau) \ \mathsf{OK}} \ (43)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \ \mathsf{OK} \qquad \Delta; \Phi \vdash \kappa_2 \ \mathsf{OK}}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \ \mathsf{OK}} \ (44)$$

$\boxed{\Delta;\Phi\vdash\mathsf{OK}}$ Context is well formed

$$\frac{}{;\vdash\mathsf{OK}}\ (45)\qquad\frac{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta;\Phi,t::\kappa\vdash\mathsf{OK}}\ (46)\qquad\frac{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta,\mathrm{u}::\kappa;\Phi\vdash\mathsf{OK}}\ (47)$$

Variables implicitly assumed to be fresh as necessary

Metatheory

Lemma 1. *If* $\Delta;\Phi\vdash\tau ::> \kappa$*, then* $\Delta;\Phi\vdash\mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa\ \mathsf{OK}$

Lemma 2. *If* $\Delta;\Phi\vdash\tau::\kappa$*, then* $\Delta;\Phi\vdash\mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa\ \mathsf{OK}$

Lemma 3. *If* $\Delta;\Phi\vdash\kappa\overset{\blacktriangleright}{\scriptscriptstyle\Pi}\Pi_{t::\kappa_1}.\kappa_2$*, then* $\Delta;\Phi\vdash\mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa\ \mathsf{OK}$ *and* $\Delta;\Phi\vdash\Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}$

Lemma 4. *If* $\Delta;\Phi\vdash\kappa_1\equiv\kappa_2$*, then* $\Delta;\Phi\vdash\mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa_2\ \mathsf{OK}$

Lemma 5. *If* $\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2$*, then* $\Delta;\Phi\vdash\mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}$ *and* $\Delta;\Phi\vdash\kappa_2\ \mathsf{OK}$

Lemma 6. *If* $\Delta;\Phi\vdash\kappa\ \mathsf{OK}$*, then* $\Delta;\Phi\vdash\mathsf{OK}$

Proof. By simultaneous rule induction/length of proof.
The interesting cases per lemma:

L1.

(1)	$\Delta;\Phi\vdash\mathtt{bse}::\mathrm{S}_{\mathtt{Type}}(\mathtt{bse})$	by (9)
	$\Delta;\Phi\vdash\mathtt{bse}::\mathtt{Type}$	by (10)
	$\Delta;\Phi\vdash\mathrm{S}_{\mathtt{Type}}(\mathtt{bse})\ \mathsf{OK}$	by (43)
(8)		text

L2.

L3.

L4.

L5. (25): L6 + (42)

L6. (43): L2

□

Lemma 7. *If* $\Delta;\Phi\vdash\tau ::> \kappa_1$ *and* $\Delta;\Phi\vdash\tau::\kappa_2$*, then* $\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2$