

Hazel Phi: 9-type-aliases

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SYNTAX

BinOp	\oplus	$::=$	$\times \mid + \mid \rightarrow$
Kind	κ	$::=$	Type \mid KHole \mid $S_\kappa(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
Base Types	bse	$::=$	Int \mid Float \mid Bool
User Types	$\hat{\tau}$	$::=$	$t \mid$ bse \mid $\tau_1 \oplus \tau_2 \mid \langle \rangle^u \mid \langle \hat{\tau} \rangle^u \mid \lambda t::\mathbf{Type}.\hat{\tau} \mid \tau_1 \tau_2$
Internal Types	τ	$::=$	$t \mid$ bse \mid $\tau_1 \oplus \tau_2 \mid \langle \rangle^u \mid \langle \tau \rangle^u \mid \lambda t::\kappa.\tau \mid \tau_1 \tau_2$
Type Pattern			
User Expression			
Internal Expression			

DECLARATIVES

$\Delta; \Phi \vdash \tau ::> \kappa$ τ has principal (well formed) kind κ

$$\frac{}{\Delta; \Phi \vdash \mathbf{bse} ::> S_{\mathbf{Type}}(\mathbf{bse})} \quad (1)$$

$$\frac{t::\kappa \in \Phi}{\Delta; \Phi \vdash t ::> S_\kappa(t)} \quad (2)$$

$$\frac{\Delta; \Phi \vdash \tau_1 :: \mathbf{Type} \quad \Delta; \Phi \vdash \tau_2 :: \mathbf{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> S_{\mathbf{Type}}(\tau_1 \oplus \tau_2)} \quad (3)$$

$$\frac{u::\kappa \in \Delta}{\Delta; \Phi \vdash \langle \rangle^u ::> \kappa} \quad (4)$$

$$\frac{u::\kappa \in \Delta \quad \Delta; \Phi \vdash \tau :: \kappa_1}{\Delta; \Phi \vdash \langle \tau \rangle^u ::> \kappa} \quad (5)$$

$$\frac{u::\kappa \in \Delta \quad t \notin \text{dom}(\Phi)}{\Delta; \Phi \vdash \langle t \rangle^u ::> \kappa} \quad (6)$$

$$\frac{\Delta; \Phi, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau ::> S_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t::\kappa_1.\tau)} \quad (7)$$

$$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa \quad \Delta; \Phi \vdash \kappa \blacktriangleright \Pi_{t::\kappa_1}.\kappa_2 \quad \Delta; \Phi \vdash \tau_2 :: \kappa_1}{\Delta; \Phi \vdash \tau_1 \tau_2 ::> [\tau_2/t]\kappa_2} \quad (8)$$

$\Delta; \Phi \vdash \tau :: \kappa$ τ is well formed at kind κ

$$\frac{\Delta; \Phi \vdash \tau :: > \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \quad (9)$$

$$\frac{\Delta; \Phi \vdash \tau :: \mathbf{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau :: \kappa} \quad (10)$$

$$\frac{\Delta; \Phi \vdash \tau :: > \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \quad (11)$$

$$\frac{\Delta; \Phi \vdash \tau_2 :: \mathbf{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_2)} \quad (12)$$

$$\frac{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_3) \quad \Delta; \Phi \vdash \tau_3 :: \mathbf{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_2)} \quad (13)$$

$\Delta; \Phi \vdash \kappa \blacktriangleright \Pi_{t::\kappa_1} \cdot \kappa_2$ κ has matched Π -kind $\Pi_{t::\kappa_1} \cdot \kappa_2$

$$\frac{}{\Delta; \Phi \vdash \mathbf{KHole} \blacktriangleright \Pi_{t::\mathbf{KHole}} \cdot \mathbf{KHole}} \quad (14)$$

$$\frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1} \cdot \kappa_2}{\Delta; \Phi \vdash \kappa \blacktriangleright \Pi_{t::\kappa_1} \cdot \kappa_2} \quad (15)$$

$\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$ κ_1 is equivalent to κ_2

$$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \equiv \kappa} \quad (16)$$

$$\frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \quad (17)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \quad (18)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathbf{S}_{\kappa}(\tau_1) \equiv \mathbf{S}_{\kappa}(\tau_2)} \quad (19)$$

$$\frac{\Delta; \Phi \vdash \tau :: \mathbf{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \mathbf{S}_{\mathbf{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathbf{S}_{\kappa}(\tau_1)} \quad (20)$$

$$\frac{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_1} \cdot \kappa_2}{\Delta; \Phi \vdash \mathbf{S}_{\Pi_{t::\kappa_1} \cdot \kappa_2}(\tau) \equiv \Pi_{t::\kappa_1} \cdot \mathbf{S}_{\kappa_2}(\tau \ t)} \quad (21)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1} \cdot \kappa_2 \equiv \Pi_{t::\kappa_3} \cdot \kappa_4} \quad (22)$$

$\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \mathbf{KHole} \lesssim \kappa} \quad (23)$$

$$\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathbf{KHole}} \quad (24)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \quad (25)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \quad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \quad (26)$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \mathbf{S}_{\kappa}(\tau) \lesssim \kappa} \quad (27)$$

$$\frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \quad \Delta; \Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1} \cdot \kappa_2 \lesssim \Pi_{t::\kappa_3} \cdot \kappa_4} \quad (28)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \quad \Delta; \Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathbf{S}_{\kappa_1}(\tau_1) \lesssim \mathbf{S}_{\kappa_2}(\tau_2)} \quad (29)$$

$\boxed{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2}$ τ_1 is equivalent to τ_2 at kind κ

$$\frac{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \quad (30)$$

$$\begin{array}{c} \frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau \equiv^{\kappa} \tau} \quad (31) \quad \frac{\Delta; \Phi \vdash \tau_2 \equiv^{\kappa} \tau_1}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \quad (32) \quad \frac{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_3 \quad \Delta; \Phi \vdash \tau_3 \equiv^{\kappa} \tau_1}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \quad (33) \\ \frac{\Delta; \Phi \vdash \tau_1 \equiv^{\text{Type}} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \equiv^{\text{Type}} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \equiv^{\text{Type}} \tau_3 \oplus \tau_4} \quad (34) \quad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t :: \kappa_1 \vdash \tau_1 \equiv^{\kappa} \tau_2}{\Delta; \Phi \vdash \lambda t :: \kappa_1. \tau_1 \equiv^{\Pi_{t :: \kappa_1}. \kappa} \lambda t :: \kappa_2. \tau_2} \quad (35) \\ \frac{\Delta; \Phi \vdash \tau_1 \equiv^{\Pi_{t :: \kappa_1}. \kappa_2} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \equiv^{\kappa_1} \tau_4}{\Delta; \Phi \vdash \tau_1 \tau_2 \equiv^{\tau_1/t, \kappa_2} \tau_3 \tau_4} \quad (36) \\ \frac{\Delta; \Phi \vdash \tau_1 :: \Pi_{t :: \kappa_1}. \kappa_3 \quad \Delta; \Phi \vdash \tau_2 :: \Pi_{t :: \kappa_1}. \kappa_4 \quad \Delta; \Phi, t :: \kappa_1 \vdash \tau_1 \equiv^{\kappa_2} \tau_2 \quad t}{\Delta; \Phi \vdash \tau_1 \equiv^{\Pi_{t :: \kappa_1}. \kappa_2} \tau_2} \quad (37) \\ \frac{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa_1} \tau_2 \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \quad (38) \end{array}$$

$\boxed{\Delta; \Phi \vdash \kappa \text{ OK}}$ κ is well formed

$$\overline{\Delta; \Phi \vdash \text{Type OK}} \quad (39)$$

$$\overline{\Delta; \Phi \vdash \text{KHole OK}} \quad (40)$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \mathbf{S}_{\kappa}(\tau) \text{ OK}} \quad (41)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \text{ OK} \quad \Delta; \Phi \vdash \kappa_2 \text{ OK}}{\Delta; \Phi \vdash \Pi_{t :: \kappa_1}. \kappa_2 \text{ OK}} \quad (42)$$