

# Hazel Phi: 9-type-aliases

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## SYNTAX

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BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Kind	$\kappa$	$::=$	<b>Type</b> $\mid$ <b>KHole</b> $\mid$ $\mathbf{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
Base Types	<b>bse</b>	$::=$	<b>Int</b> $\mid$ <b>Float</b> $\mid$ <b>Bool</b>
User Types	$\hat{\tau}$	$::=$	$t \mid \mathbf{bse} \mid \tau_1 \oplus \tau_2 \mid \langle \rangle^u \mid \langle \hat{\tau} \rangle^u \mid \lambda t::\mathbf{Type}.\hat{\tau} \mid \tau_1 \tau_2$
Internal Types	$\tau$	$::=$	$t \mid \mathbf{bse} \mid \tau_1 \oplus \tau_2 \mid \langle \rangle^u \mid \langle \tau \rangle^u \mid \lambda t::\kappa.\tau \mid \tau_1 \tau_2$
Type Pattern			
User Expression			
Internal Expression			

## DECLARATIVES

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$\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$   $\kappa_1$  is a consistent subkind of  $\kappa_2$

$$\begin{array}{c}
 \frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \mathbf{KHole} \lesssim \kappa} \qquad \frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathbf{KHole}} \\
 \\
 \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \quad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \\
 \\
 \frac{\Delta; \Phi \vdash \tau::\kappa}{\Delta; \Phi \vdash \mathbf{S}_{\kappa}(\tau) \lesssim \kappa} \qquad \frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \quad \Delta; \Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4} \\
 \\
 \frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \quad \Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathbf{S}_{\kappa_1}(\tau_1) \lesssim \mathbf{S}_{\kappa_2}(\tau_2)}
 \end{array}$$

$\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$   $\kappa_1$  is equivalent to  $\kappa_2$

$$\begin{array}{c}
\frac{\Delta; \Phi \vdash \kappa \text{ OK}}{\Delta; \Phi \vdash \kappa \equiv \kappa} \quad \frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \quad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \\
\\
\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathbf{S}_{\kappa}(\tau_1) \equiv \mathbf{S}_{\kappa}(\tau_2)} \quad \frac{\Delta; \Phi \vdash \tau :: \mathbf{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \mathbf{S}_{\mathbf{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathbf{S}_{\kappa}(\tau_1)} \quad \frac{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_1} \cdot \kappa_2}{\Delta; \Phi \vdash \mathbf{S}_{\Pi_{t::\kappa_1} \cdot \kappa_2}(\tau) \equiv \Pi_{t::\kappa_1} \cdot \mathbf{S}_{\kappa_2}(\tau \ t)} \\
\\
\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1} \cdot \kappa_2 \equiv \Pi_{t::\kappa_3} \cdot \kappa_4}
\end{array}$$

$\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2$   $\tau_1$  is equivalent to  $\tau_2$  at kind  $\kappa$

$$\begin{array}{c}
\frac{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \\
\\
\begin{array}{ccc}
\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau \overset{\kappa}{\equiv} \tau} & \frac{\Delta; \Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} & \frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \quad \Delta; \Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \\
\Delta; \Phi \vdash \tau_1 \overset{\text{Type}}{\equiv} \tau_3 & \Delta; \Phi \vdash \tau_2 \overset{\text{Type}}{\equiv} \tau_4 & \Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2 \\
\hline
\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \overset{\text{Type}}{\equiv} \tau_3 \oplus \tau_4 & \Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1} \cdot \kappa_2}{\equiv} \tau_3 & \Delta; \Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4 \\
\hline
\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1} \cdot \kappa_2}{\equiv} \tau_2 & \Delta; \Phi \vdash \tau_1 \overset{[\tau_1/t] \kappa_2}{\equiv} \tau_3 \tau_4 & \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \overset{\kappa_2}{\equiv} \tau_2 \ t \\
\hline
\Delta; \Phi \vdash \tau_1 :: \Pi_{t::\kappa_1} \cdot \kappa_3 & \Delta; \Phi \vdash \tau_2 :: \Pi_{t::\kappa_1} \cdot \kappa_4 & \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \overset{\Pi_{t::\kappa_1} \cdot \kappa_2}{\equiv} \tau_2 \\
\hline
\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 & \Delta; \Phi \vdash \kappa_1 \equiv \kappa & \\
\hline
\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2
\end{array}
\end{array}$$

$\boxed{\Delta; \Phi \vdash \tau ::> \kappa}$   $\tau$  has principal kind  $\kappa$

$$\begin{array}{c}
\frac{}{\Delta; \Phi \vdash \mathbf{bse} ::> \mathbf{S}_{\mathbf{Type}}(\mathbf{bse})} \quad \frac{t::\kappa \in \Phi}{\Delta; \Phi \vdash t ::> \mathbf{S}_{\kappa}(t)} \quad \frac{\Delta; \Phi \vdash \tau_1 :: \mathbf{Type} \quad \Delta; \Phi \vdash \tau_2 :: \mathbf{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> \mathbf{S}_{\mathbf{Type}}(\tau_1 \oplus \tau_2)} \\
\\
\frac{u::\kappa \in \Delta}{\Delta; \Phi \vdash \langle \rangle^u ::> \kappa} \quad \frac{u::\kappa \in \Delta \quad \Delta; \Phi \vdash \tau :: \kappa_1}{\Delta; \Phi \vdash \langle \tau \rangle^u ::> \kappa} \quad \frac{u::\kappa \in \Delta \quad t \notin \text{dom}(\Phi)}{\Delta; \Phi \vdash \langle t \rangle^u ::> \kappa} \\
\\
\frac{\Delta; \Phi, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t::\kappa_1. \tau ::> \mathbf{S}_{\Pi_{t::\kappa_1}. \kappa_2}}(\lambda t::\kappa_1. \tau)} \\
\\
\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa \quad \Delta; \Phi \vdash \kappa \blacktriangleright \Pi_{t::\kappa_1}. \kappa_2 \quad \Delta; \Phi \vdash \tau_2 :: \kappa_1}{\Delta; \Phi \vdash \tau_1 \tau_2 ::> [\tau_2/t] \kappa_2}
\end{array}$$

$\boxed{\Delta; \Phi \vdash \kappa \blacktriangleright \Pi_{t::\kappa_1}. \kappa_2}$   $\kappa$  has matched  $\Pi$ -kind  $\Pi_{t::\kappa_1}. \kappa_2$

$$\frac{}{\Delta; \Phi \vdash \mathbf{KHole} \blacktriangleright \Pi_{t::\mathbf{KHole}}. \mathbf{KHole}} \quad \frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}. \kappa_2}{\Delta; \Phi \vdash \kappa \blacktriangleright \Pi_{t::\kappa_1}. \kappa_2}$$

$\boxed{\Delta; \Phi \vdash \tau :: \kappa}$   $\tau$  is well formed at kind  $\kappa$

$$\begin{array}{c}
\frac{\Delta; \Phi \vdash \tau ::> \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \quad \frac{\Delta; \Phi \vdash \tau :: \mathbf{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau :: \kappa} \quad \frac{\Delta; \Phi \vdash \tau ::> \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \\
\\
\frac{\Delta; \Phi \vdash \tau_2 :: \mathbf{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_2)} \quad \frac{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_3) \quad \Delta; \Phi \vdash \tau_3 :: \mathbf{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_2)}
\end{array}$$