

# Hazel Phi: 9-type-aliases

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## SYNTAX

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BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Kind	$\kappa$	$::=$	$\mathbf{Type} \mid \mathbf{KHole} \mid \mathbf{S}_\kappa(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
Base Types	$\mathbf{bse}$	$::=$	$\mathbf{Int} \mid \mathbf{Float} \mid \mathbf{Bool}$
User Types	$\hat{\tau}$	$::=$	$t \mid \mathbf{bse} \mid \tau_1 \oplus \tau_2 \mid \langle \rangle^u \mid \langle \hat{\tau} \rangle^u \mid \lambda t::\mathbf{Type}.\hat{\tau} \mid \tau_1 \tau_2$
Internal Types	$\hat{\tau}$	$::=$	$t \mid \mathbf{bse} \mid \tau_1 \oplus \tau_2 \mid \langle \rangle^u \mid \langle \tau \rangle^u \mid \lambda t::\kappa.\tau \mid \tau_1 \tau_2$
Type Pattern			
User Expression			
Internal Expression			

## DECLARATIVES

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$\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$   $\kappa_1$  is a consistent subkind of  $\kappa_2$

$$\frac{}{\Delta; \Phi \vdash \mathbf{KHole} \lesssim \kappa} \quad \frac{}{\Delta; \Phi \vdash \kappa \lesssim \mathbf{KHole}} \quad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \quad \frac{\Delta; \Phi \vdash \tau::\kappa}{\Delta; \Phi \vdash \mathbf{S}_\kappa(\tau) \lesssim \kappa}$$

$\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$   $\kappa_1$  is equivalent to  $\kappa_2$

$$\frac{}{\Delta; \Phi \vdash \kappa \equiv \kappa} \quad \frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \quad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \quad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathbf{S}_\kappa(\tau_1) \equiv \mathbf{S}_\kappa(\tau_2)} \quad \frac{\Delta; \Phi \vdash \tau::\mathbf{S}_\kappa(\tau_1)}{\Delta; \Phi \vdash \mathbf{S}_{\mathbf{S}_\kappa(\tau_1)}(\tau) \equiv \mathbf{S}_\kappa(\tau_1)} \quad \frac{\Delta; \Phi \vdash \tau::\Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \mathbf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t::\kappa_1}.\mathbf{S}_{\kappa_2}(\tau \ t)}$$

$\boxed{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2}$   $\tau_1$  is equivalent to  $\tau_2$  at kind  $\kappa$

$$\begin{array}{c}
\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau \equiv^{\kappa} \tau} \quad \frac{\Delta; \Phi \vdash \tau_2 \equiv^{\kappa} \tau_1}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \quad \frac{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_3 \quad \Delta; \Phi \vdash \tau_3 \equiv^{\kappa} \tau_1}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2} \\
\\
\frac{\Delta; \Phi \vdash \tau_1 \stackrel{\text{Type}}{\equiv} \tau_3 \quad \Delta; \Phi \vdash \tau_2 \stackrel{\text{Type}}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \stackrel{\text{Type}}{\equiv} \tau_3 \oplus \tau_4} \quad \frac{\Delta; \Phi \vdash \tau_1 :: \mathbf{S}_{\kappa}(\tau_2) \quad \Delta; \Phi \vdash \tau_2 :: \kappa}{\Delta; \Phi \vdash \tau_1 \stackrel{\mathbf{S}_{\kappa}(\tau_2)}{\equiv} \tau_2} \\
\\
\frac{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa_1} \tau_2 \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2}
\end{array}$$

$\boxed{\Delta; \Phi \vdash \tau_1 \equiv \tau_2}$   $\tau_1$  is equivalent to  $\tau_2$  at “top” kind

$$\frac{\Delta; \Phi \vdash \tau_1 \equiv^{\kappa} \tau_2}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2} \quad \frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau \equiv \tau} \quad \frac{\Delta; \Phi \vdash \tau_2 \equiv \tau_1}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2} \quad \frac{\Delta; \Phi \vdash \tau_1 \equiv \tau_3 \quad \Delta; \Phi \vdash \tau_3 \equiv \tau_2}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2}$$

$\boxed{\Delta; \Phi \vdash \tau :: > \kappa}$   $\tau$  has principal kind  $\kappa$

$$\overline{\Delta; \Phi \vdash \mathbf{bse} :: > \mathbf{S}_{\text{Type}}(\mathbf{bse})}$$