Hazel Phi: 9-type-aliases

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Syntax

BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Kind	κ	$::=$	$\texttt{Type} \mid \texttt{KHole} \mid \texttt{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
Base Types	$\texttt{bse}$	$::=$	$\texttt{Int} \mid \texttt{Float} \mid \texttt{Bool}$
User Types	$\hat{\tau}$	$::=$	$t \mid \texttt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!\|\!)^{\texttt{u}} \mid (\!\|\hat{\tau}\|\!)^{\texttt{u}} \mid \lambda t{::}\texttt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \texttt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!\|\!)^{\texttt{u}} \mid (\!\|\tau\|\!)^{\texttt{u}} \mid \lambda t{::}\kappa.\tau \mid \tau_1\ \tau_2$
Type Pattern			
User Expression			
Internal Expression			

Declaratives

$\boxed{\Delta;\Phi \vdash \tau ::> \kappa}$ τ has principal (well formed) kind κ

$$\frac{}{\Delta;\Phi \vdash \texttt{bse} ::> \texttt{S}_{\texttt{Type}}(\texttt{bse})}\ (1) \qquad \frac{t{::}\kappa \in \Phi}{\Delta;\Phi \vdash t ::> \texttt{S}_{\kappa}(t)}\ (2)$$

$$\frac{\Delta;\Phi \vdash \tau_1{::}\texttt{Type} \qquad \Delta;\Phi \vdash \tau_2{::}\texttt{Type}}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 ::> \texttt{S}_{\texttt{Type}}(\tau_1 \oplus \tau_2)}\ (3) \qquad \frac{\texttt{u}{::}\kappa \in \Delta}{\Delta;\Phi \vdash (\!\|\!)^{\texttt{u}} ::> \kappa}\ (4)$$

$$\frac{\texttt{u}{::}\kappa \in \Delta \qquad \Delta;\Phi \vdash \tau{::}\kappa_1}{\Delta;\Phi \vdash (\!\|\tau\|\!)^{\texttt{u}} ::> \kappa}\ (5) \qquad \frac{\texttt{u}{::}\kappa \in \Delta \qquad t \notin \mathsf{dom}(\Phi)}{\Delta;\Phi \vdash (\!\|t\|\!)^{\texttt{u}} ::> \kappa}\ (6)$$

$$\frac{\Delta;\Phi, t{::}\kappa_1 \vdash \tau ::> \kappa_2}{\Delta;\Phi \vdash \lambda t{::}\kappa_1.\tau ::> \texttt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}\ (7)$$

$$\frac{\Delta;\Phi \vdash \tau_1 ::> \kappa \qquad \Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\underset{\Pi}{}} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta;\Phi \vdash \tau_2{::}\kappa_1}{\Delta;\Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}\ (8)$$

$\boxed{\Delta;\Phi \vdash \tau :: \kappa}$ τ is well formed at kind κ

$$\frac{\Delta;\Phi \vdash \tau ::> \kappa}{\Delta;\Phi \vdash \tau :: \kappa} \; (9) \qquad \frac{\Delta;\Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \tau :: \kappa} \; (10)$$

$$\frac{\Delta;\Phi \vdash \tau ::> \kappa_1 \qquad \Delta;\Phi \vdash \kappa_1 \lesssim \kappa}{\Delta;\Phi \vdash \tau :: \kappa} \; (11)$$

$$\frac{\Delta;\Phi \vdash \tau_2 :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_2)} \; (12) \qquad \frac{\Delta;\Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_3) \qquad \Delta;\Phi \vdash \tau_3 :: \mathtt{S}_{\kappa}(\tau_2)}{\Delta;\Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_2)} \; (13)$$

$\boxed{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{}{\Delta;\Phi \vdash \mathtt{KHole} \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\mathtt{KHole}}.\mathtt{KHole}} \; (14) \qquad \frac{\Delta;\Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2} \; (15)$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta;\Phi \vdash \kappa \ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \equiv \kappa} \; (16) \qquad \frac{\Delta;\Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2} \; (17) \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2} \; (18)$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \mathtt{S}_{\kappa}(\tau_1) \equiv \mathtt{S}_{\kappa}(\tau_2)} \; (19) \qquad \frac{\Delta;\Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \mathtt{S}_{\mathtt{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathtt{S}_{\kappa}(\tau_1)} \; (20)$$

$$\frac{\Delta;\Phi \vdash \tau :: \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t::\kappa_1}.\mathtt{S}_{\kappa_2}(\tau\ t)} \; (21) \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4} \; (22)$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta;\Phi \vdash \kappa \ \mathsf{OK}}{\Delta;\Phi \vdash \mathtt{KHole} \lesssim \kappa} \; (23) \qquad \frac{\Delta;\Phi \vdash \kappa \ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \lesssim \mathtt{KHole}} \; (24)$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2} \; (25) \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta;\Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2} \; (26)$$

$$\frac{\Delta;\Phi \vdash \tau :: \kappa}{\Delta;\Phi \vdash \mathtt{S}_{\kappa}(\tau) \lesssim \kappa} \; (27) \qquad \frac{\Delta;\Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta;\Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4} \; (28)$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta;\Phi \vdash \mathtt{S}_{\kappa_1}(\tau_1) \lesssim \mathtt{S}_{\kappa_2}(\tau_2)} \; (29)$$

$\boxed{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$ τ_1 is equivalent to τ_2 at kind κ

$$\frac{\Delta;\Phi \vdash \tau_1 :: \mathtt{S}_\kappa(\tau_2)}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (30)$$

$$\frac{\Delta;\Phi \vdash \tau :: \kappa}{\Delta;\Phi \vdash \tau \overset{\kappa}{\equiv} \tau}\ (31) \qquad \frac{\Delta;\Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (32) \qquad \frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (33)$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\mathtt{Type}}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4}\ (34) \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\Phi, t::\kappa_1 \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \lambda t::\kappa_1.\tau_1 \overset{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t::\kappa_2.\tau_2}\ (35)$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1\ \tau_2 \overset{[\tau_1/t]\kappa_2}{\equiv} \tau_3\ \tau_4}\ (36)$$

$$\frac{\Delta;\Phi \vdash \tau_1 :: \Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta;\Phi \vdash \tau_2 :: \Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta;\Phi, t::\kappa_1 \vdash \tau_1\ t \overset{\kappa_2}{\equiv} \tau_2\ t}{\Delta;\Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}\ (37)$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (38)$$

$\boxed{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}$ κ is well formed

$$\frac{}{\Delta;\Phi \vdash \mathtt{Type}\ \mathsf{OK}}\ (39) \qquad \frac{}{\Delta;\Phi \vdash \mathtt{KHole}\ \mathsf{OK}}\ (40) \qquad \frac{\Delta;\Phi \vdash \tau :: \kappa}{\Delta;\Phi \vdash \mathtt{S}_\kappa(\tau)\ \mathsf{OK}}\ (41)$$

$$\frac{\Delta;\Phi \vdash \kappa_1\ \mathsf{OK} \qquad \Delta;\Phi \vdash \kappa_2\ \mathsf{OK}}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}}\ (42)$$