

# Hazel PHI: 10-modules

June 28, 2021

## prerequisites

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- Hazel PHI: 9-type-aliases-redux
  - github
  - current commit: 4410cd565ce717707e580e44f64868d3175fe2a6
- (optional) Hazel PHI: 1-labeled-tuples
  - github
  - current commit: 0a7d0b53ee7286d03ea3be13a7ac91a86f1c90b1

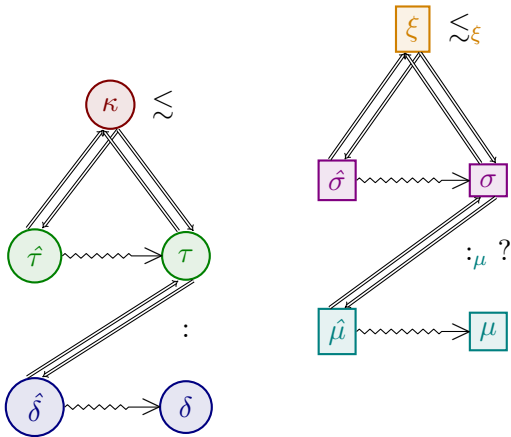
## how to read

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800000	kinds	D08000	signature kind
008000	types (constructors)	800080	signatures
000080	terms	008080	modules

## notes

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external typ/sig/mod syntax not written out yet (waiting for construction dust to settle); patterns not handled yet– will be left till end.

## syntax

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kind	$\kappa$	$::=$	<b>Type</b>	kind of types
			$S(\tau)$	singleton kind
			<b>KHole</b>	kind hole
			$\Pi_{t::\kappa_1}.\kappa_2$	dependent function kind

HTyp	$\tau$	$::=$ $t$ $bse$ $\tau_1 \oplus \tau_2$ $[\tau]$ $\lambda t :: \kappa. \tau$ $\tau_1 \tau_2$ $\{lab_1 \hookrightarrow \tau_1, \dots lab_n \hookrightarrow \tau_n\}$ $\mu.lab$ $()$ $(\tau)$	type variable base type type binop list type type function type application labelled product type (record) module type projection empty type hole nonempty type hole
base type	$bse$	$::=$ $\text{Int}$ $\text{Float}$ $\text{Bool}$	
HTyp BinOp	$\oplus$	$::=$ $\times$ $+$ $\rightarrow$	
external expression	$\hat{\delta}$	$::=$ $\dots$ $x$ $\text{signature } s = \hat{\sigma} \text{ in } \hat{\delta}$ $\text{module } m = \hat{\mu} \text{ in } \hat{\delta}$ $\text{module } m :_{\mu} s = \hat{\mu} \text{ in } \hat{\delta}$ $\text{functor something} = \text{something in } \hat{\delta}$ $\hat{\mu}.lab$	module term projection
internal expression	$\delta$	$::=$ $\dots$ $x$ $\text{signature } s = \sigma \text{ in } \delta$ $\text{module } m :_{\mu} s = \mu \text{ in } \delta$ $\text{functor something} = \text{something in } \delta$ $\mu.lab$	module term projection
signature kind	$\xi$	$::=$ $\text{SSigKind}(\sigma)$ $\text{SigKHole}$	
signature	$\sigma$	$::=$ $s$ $\{sdec s\}$ $\Pi_{m :_{\mu} \sigma_1} . \sigma_2$ $()$ $(s)$	signature variable structure signature functor signature empty signature hole nonempty signature hole
module	$\mu$	$::=$ $m$ $\{sbnds\}$ $\lambda m :_{\mu} \sigma . \mu$ $\mu_1 \mu_2$ $\mu.lab$ $()$ $(\mu)$	module variable structure functor functor application submodule projection empty module hole nonempty module hole
signature declarations	$sdec s$	$::=$ $\cdot$ $sdec, sdec s$	
signature declaration	$sdec$	$::=$ $\text{type } lab$ $\text{type } lab = \tau$ $\text{val } lab : \tau$ $\text{module } lab :_{\mu} \sigma$	

structure bindings	$sbnds$	$::=$		<code>functor</code> $lab:\mu\sigma$			
				<code>.</code>			
				$sbnd, sbnds$			
			structure binding	$sbnd$	$::=$		<code>type</code> $t = \tau$
							<code>let</code> $x:\tau = \delta$
	<code>module</code> $m = \mu$						
				<code>module</code> $m:\mu s = \mu$			
				<code>functor</code> $m:\mu s = \mu$			

## contexts

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$$\Delta, ?; \Gamma, x:\tau; \Phi, t::\kappa; \Xi, m:\mu\sigma; \Psi, s::\sigma\xi$$

## statics

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scratch

$\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$   $\kappa_1$  is a consistent subkind of  $\kappa_2$

KCSubsumption

$\frac{test}{test}$

$test$

$\Delta; \Phi; \Xi; \Psi \vdash \xi_1 \lesssim \xi_2$   $\xi_1$  is a consistent sub signature kind of  $\xi_2$

nameMe

$\exists sdec_x \in sdec_{s_1} \text{ st } \Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\{sdec_x\}) \lesssim \text{SSigKind}(\{sdec_2\})$   
 $\Delta; \Phi, \text{type}(sdec_2); \Xi, \text{submodule}(sdec_2); \Psi \vdash \{sdec_{s_1}\} \lesssim \{sdec_{s_2}\}$

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\{sdec_{11}, sdec_{12}, sdec_{13} \text{ as } sdec_{s_1}\}) \lesssim \text{SSigKind}(\{sdec_2, sdec_{s_2}\})$

singleType

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\{\text{type } lab = \tau\}) \lesssim \text{SSigKind}(\{\text{type } lab\})$

singleType2

$\Delta; \Phi; \Xi; \Psi \vdash \tau_1 \equiv \tau_2$

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\{\text{type } lab = \tau_1\}) \lesssim \text{SSigKind}(\{\text{type } lab = \tau_2\})$

singleType3

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\{\text{type } lab\}) \lesssim \text{SSigKind}(\{\text{type } lab\})$

singleVa

$\Delta; \Phi; \Xi; \Psi \vdash \tau_1 \equiv \tau_2$

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\{\text{val } lab:\tau_1\}) \lesssim \text{SSigKind}(\{\text{val } lab:\tau_2\})$

singleMod

$\Delta; \Phi; \Xi; \Psi \vdash \sigma_1 \Leftarrow \text{SSigKind}(\sigma_2)$

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\{\text{module } lab:\mu\sigma_1\}) \lesssim \text{SSigKind}(\{\text{module } lab:\mu\sigma_2\})$

nil

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\{sdec_{s_1}\}) \lesssim \text{SSigKind}(\{.\})$

varprop

$s::_{\sigma}\xi \in \Psi$

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(s) \lesssim \xi$

nameMe

$\Delta; \Phi; \Xi; \Psi \vdash \sigma_1 \Leftarrow \text{SSigKind}(\sigma_2)$

$\Delta; \Phi; \Xi; \Psi \vdash \text{SSigKind}(\sigma_1) \lesssim \text{SSigKind}(\sigma_2)$

CSubSigKindHoleL

$\Delta; \Phi; \Xi; \Psi \vdash \text{SigKHole} \lesssim \xi$

CSubSigKindHoleR

$\Delta; \Phi; \Xi; \Psi \vdash \xi \lesssim \text{SigKHole}$

$\Delta; \Phi; \Xi; \Psi \vdash \sigma \Rightarrow \xi$   $\sigma$  synthesizes signature kind  $\xi$

SynSigKndVar

$s::_{\sigma}\xi \in \Psi$

$\Delta; \Phi; \Xi; \Psi \vdash s \Rightarrow \text{SSigKind}(s)$

SynSigKndVarFail

$s \notin \text{dom}(\Psi)$

$\Delta; \Phi; \Xi; \Psi \vdash s \Rightarrow \text{SigKHole}$

$\{sdec_{s_1}\} \text{ wellformed?}$

$\vdash \{sdec_{s_1}\} \Rightarrow \text{SSigKind}(\{sdec_{s_1}\})$

SynSigKndSigHole

$u::_{\sigma}\xi \in \Delta$

$\Delta; \Phi; \Xi; \Psi \vdash \llbracket u \rrbracket \Rightarrow \xi$

SynSigKndSigHole

$u::_{\sigma}\xi \in \Delta$

$\Delta; \Phi; \Xi; \Psi \vdash s \Rightarrow \xi_1$

$\Delta; \Phi; \Xi; \Psi \vdash \llbracket s \rrbracket^u \Rightarrow \xi$

$\Delta; \Phi; \Xi; \Psi \vdash \sigma \Leftarrow \xi$   $\sigma$  analyzes against signature kind  $\xi$

$$\frac{\text{Sub} \quad \Delta; \Phi; \Xi; \Psi \vdash \sigma \Rightarrow \xi_1 \quad \Delta; \Phi; \Xi; \Psi \vdash \xi_1 \lesssim_{\xi} \xi}{\Delta; \Phi; \Xi; \Psi \vdash \sigma \Leftarrow \xi}$$

elab

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$\Gamma; \Phi; \Xi \vdash \hat{\delta} \Rightarrow \tau \rightsquigarrow \delta \dashv \Delta$   $\hat{\delta}$  synthesizes type  $\tau$  and elaborates to  $\delta$  with hole context  $\Delta$

$$\dots \quad \frac{\text{SynElabLetMod} \quad \Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \rightsquigarrow \mu \dashv \Delta_1 \quad \Gamma; \Phi; \Xi, m:_{\mu} \sigma \vdash \hat{\delta} \Rightarrow \tau \rightsquigarrow \delta \dashv \Delta_2}{\Gamma; \Phi; \Xi \vdash \text{module } m = \hat{\mu} \text{ in } \hat{\delta} \Rightarrow \tau \rightsquigarrow \text{module } m = \mu \text{ in } \delta \dashv \Delta_1 \cup \Delta_2}$$

$$\frac{\text{SynElabLetModAnn} \quad \Phi; \Xi \vdash \hat{\sigma} \Rightarrow \xi \rightsquigarrow \sigma \dashv \Delta_1 \quad \Gamma; \Phi; \Xi \vdash \hat{\mu} \Leftarrow \sigma \rightsquigarrow \mu \dashv \Delta_2 \quad \Gamma; \Phi; \Xi, m:_{\mu} \sigma \vdash \hat{\delta} \Rightarrow \tau \rightsquigarrow \delta \dashv \Delta_3}{\Gamma; \Phi; \Xi \vdash \text{module } m:_{\mu} \hat{\sigma} = \hat{\mu} \text{ in } \hat{\delta} \Rightarrow \tau \rightsquigarrow \text{module } m:_{\mu} \sigma = \mu \text{ in } \delta \dashv \Delta_1 \cup \Delta_2 \cup \Delta_3}$$

$$\frac{\text{SynElabModTermPrj} \quad \Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \rightsquigarrow \mu \dashv \Delta \quad \Phi; \Xi \vdash \sigma \Rightarrow \xi}{\Gamma; \Phi; \Xi \vdash \hat{\mu}.lab \Rightarrow \tau \rightsquigarrow \mu.lab \dashv \Delta}$$

$\Phi; \Xi \vdash \hat{\tau} \Rightarrow \kappa \rightsquigarrow \tau \dashv \Delta$   $\hat{\tau}$  synthesizes kind  $\kappa$  and elaborates to  $\tau$  with hole context  $\Delta$

$$\dots \quad \frac{\text{SynElabModTypPrj} \quad \Phi; \Xi \vdash m \Rightarrow \sigma \rightsquigarrow m \dashv \Delta \quad \text{something} \sigma \kappa}{\Phi; \Xi \vdash m.lab \Rightarrow \kappa \rightsquigarrow m.lab \dashv \Delta}$$

$\Phi; \Xi \vdash \hat{\tau} \Leftarrow \kappa \rightsquigarrow \tau \dashv \Delta$   $\hat{\tau}$  analyzes against kind  $\kappa$  and elaborates to  $\tau$  with hole context  $\Delta$

$\Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \rightsquigarrow \mu \dashv \Delta$   $\hat{\mu}$  synthesizes signature  $\sigma$  and elaborates to  $\mu$  with hole context  $\Delta$

$$\frac{\text{SynElabModVar} \quad m:_{\mu} \sigma \in \Xi}{\Gamma; \Phi; \Xi \vdash m \Rightarrow \sigma \rightsquigarrow m \dashv \cdot}$$

$$\frac{\text{SynElabModVarFail} \quad m \notin \text{dom}(\Xi)}{\Gamma; \Phi; \Xi \vdash m \Rightarrow \emptyset \rightsquigarrow \langle m \rangle^u \dashv u:_{\mu} \emptyset}$$

SynElabConsStruct

$$\frac{\Gamma; \Phi; \Xi \vdash sbnd \Rightarrow sdec \rightsquigarrow sbnd \dashv \Delta_1 \quad \Gamma, \text{val}(sdec); \Phi, \text{type}(sdec); \Xi, \text{submodule}(sdec) \vdash \{sbnds\} \Rightarrow \{sdec\} \rightsquigarrow \{sbnds\} \dashv \Delta_2}{\Gamma; \Phi; \Xi \vdash \{sbnd, sbnds\} \Rightarrow \{sdec, sdec\} \rightsquigarrow \{sbnd, sbnds\} \dashv \Delta_1 \cup \Delta_2}$$

SynElabNilStruct

$$\frac{}{\Gamma; \Phi; \Xi \vdash \{\cdot\} \Rightarrow \{\cdot\} \rightsquigarrow \{\cdot\} \dashv \cdot}$$

SynElabEmptyModHole

$$\frac{}{\Gamma; \Phi; \Xi \vdash \emptyset^u \Rightarrow \emptyset \rightsquigarrow \emptyset^u \dashv u:_{\mu} \emptyset}$$

SynElabNonemptyModHole

$$\frac{}{\Gamma; \Phi; \Xi \vdash \langle m \rangle^u \Rightarrow \emptyset \rightsquigarrow \langle m \rangle^u \dashv u:_{\mu} \emptyset}$$

functor stuff

$\Gamma; \Phi; \Xi \vdash \hat{\mu} \Leftarrow \sigma \rightsquigarrow \mu \dashv \Delta$   $\hat{\mu}$  analyzes against signature  $\sigma$  and elaborates to  $\mu$  with hole context  $\Delta$

$$\frac{\text{AnaElabModSubsumption} \quad \Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \rightsquigarrow \mu \dashv \Delta}{\Gamma; \Phi; \Xi \vdash \hat{\mu} \Leftarrow \sigma \rightsquigarrow \mu \dashv \Delta}$$

$\boxed{\Gamma; \Phi; \Xi \vdash sbnd \Rightarrow sdec \rightsquigarrow sbnd \dashv \Delta}$   $sbnd$  synthesizes declaration  $sdec$  and elaborates to  $sbnd$  with hole context  $\Delta$

$$\frac{\text{SynElabTypeSbnd} \quad \Phi; \Xi \vdash \hat{\tau} \Rightarrow \kappa \rightsquigarrow \tau \dashv \Delta}{\Gamma; \Phi; \Xi \vdash \text{type } t = \hat{\tau} \Rightarrow \text{type } t = \tau \rightsquigarrow \text{type } t = \tau \dashv \Delta}$$

$$\frac{\text{SynElabValSbnd} \quad \Phi; \Xi \vdash \hat{\tau} \Rightarrow \kappa \rightsquigarrow \tau \dashv \Delta_1 \quad \Gamma; \Phi; \Xi \vdash \hat{\delta} \Leftarrow \tau \rightsquigarrow \delta \dashv \Delta_2}{\Gamma; \Phi; \Xi \vdash \text{let } x:\hat{\tau} = \hat{\delta} \Rightarrow \text{val } x:\tau \rightsquigarrow \text{let } x:\tau = \delta \dashv \Delta_1 \cup \Delta_2}$$

$$\frac{\text{SynElabModSbnd} \quad \Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \rightsquigarrow \mu \dashv \Delta}{\Gamma; \Phi; \Xi \vdash \text{module } m = \hat{\mu} \Rightarrow \text{module } m:_{\mu}\sigma \rightsquigarrow \text{module } m:_{\mu}\sigma = \mu \dashv \Delta}$$

$$\frac{\text{SynElabModAnnSbnd} \quad \Phi; \Xi \vdash \hat{\sigma} \Rightarrow \xi \rightsquigarrow \sigma_1 \dashv \Delta_1 \quad \Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma_2 \rightsquigarrow \mu \dashv \Delta_2 \quad \Phi; \Xi; \Psi \vdash \sigma_2 \Leftarrow \xi}{\Gamma; \Phi; \Xi \vdash \text{module } m:_{\mu}\hat{\sigma} = \hat{\mu} \Rightarrow \text{module } m:_{\mu}\sigma_1 \rightsquigarrow \text{module } m:_{\mu}\sigma_1 = \mu \dashv \Delta_1 \cup \Delta_2}$$

$\boxed{\Gamma; \Phi; \Xi \vdash sbnd \Leftarrow sdec \rightsquigarrow sbnd \dashv \Delta}$   $sbnd$  analyzes against declaration  $sdec$  and elaborates to  $sbnd$  with hole context  $\Delta$

$\boxed{\vdash s\hat{dec} \Rightarrow sdecknd \rightsquigarrow sdec \dashv \Delta}$

$\boxed{\Phi; \Xi; \Psi \vdash \hat{\sigma} \Rightarrow \xi \rightsquigarrow \sigma \dashv \Delta}$   $\hat{\sigma}$  synthesizes signature kind  $\xi$  and elaborates to  $\sigma$  with hole context  $\Delta$

$$\frac{\text{SynSigEmptyHole} \quad \Phi; \Xi; \Psi \vdash \emptyset^u \Rightarrow \text{SigKHole} \rightsquigarrow \emptyset^u \dashv u::_{\sigma}\text{SigKHole}}{\text{SynSigNonEmptyHole}}$$

$\boxed{\Phi; \Xi \vdash \hat{\sigma} \Leftarrow \xi \rightsquigarrow \sigma \dashv \Delta}$   $\hat{\sigma}$  analyzes against signature kind  $\xi$  and elaborates to  $\sigma$  with hole context  $\Delta$

misc

$$\begin{aligned} \text{val}(sdec) &= \begin{cases} lab:\tau & sdec \equiv \text{val } lab:\tau \\ \cdot & \text{otherwise} \end{cases} \\ \text{type}(sdec) &= \begin{cases} lab::\text{Type} & sdec \equiv \text{type } lab \\ lab::\kappa & sdec \equiv \text{type } lab = \tau \\ & \text{where } \vdash \tau \Rightarrow \kappa \\ \cdot & \text{otherwise} \end{cases} \\ \text{submodule}(sdec) &= \begin{cases} lab:_{\mu}\sigma & sdec \equiv \text{module } lab:_{\mu}\sigma \\ \cdot & \text{otherwise} \end{cases} \end{aligned}$$