Hazel Phi: 11-type-constructors

July 31, 2021

SYNTAX

Kind	κ	$::=$	$\texttt{Type} \mid \texttt{KHole} \mid \texttt{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	$::=$	$t \mid \texttt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!\mid\!)^{\texttt{u}} \mid (\!\mid\hat{\tau}\mid\!)^{\texttt{u}} \mid \lambda t{::}\texttt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \texttt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!\mid\!)^{\texttt{u}} \mid (\!\mid\tau\mid\!)^{\texttt{u}} \mid (\!\mid t\mid\!)^{\texttt{u}} \mid \lambda t{::}\kappa.\tau \mid \tau_1\ \tau_2$
Base Types	$\texttt{bse}$	$::=$	$\texttt{Int} \mid \texttt{Float} \mid \texttt{Bool}$
BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Type Pattern			
User Expression			
Internal Expression			

DECLARATIVES

$\boxed{\Delta; \Phi \vdash \tau ::> \kappa}$ τ has principal (well formed) kind κ

$$\frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \texttt{bse} ::> \texttt{S}_{\texttt{Type}}(\texttt{bse})}\ \texttt{PK-Base} \qquad \frac{\Delta; \Phi_1, t{::}\kappa, \Phi_2 \vdash \mathsf{OK}}{\Delta; \Phi \vdash t ::> \texttt{S}_{\kappa}(t)}\ \texttt{PK-Var}$$

$$\frac{\Delta; \Phi \vdash \tau_1{::}\texttt{Type} \qquad \Delta; \Phi \vdash \tau_2{::}\texttt{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> \texttt{S}_{\texttt{Type}}(\tau_1 \oplus \tau_2)}\ \texttt{PK-}\oplus \qquad \frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash (\!\mid\!)^{\texttt{u}} ::> \texttt{S}_{\kappa}((\!\mid\!)^{\texttt{u}})}\ \texttt{PK-EHole}$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK} \qquad \Delta; \Phi \vdash \tau{::}\kappa_1}{\Delta; \Phi \vdash (\!\mid\tau\mid\!)^{\texttt{u}} ::> \texttt{S}_{\kappa}((\!\mid\tau\mid\!)^{\texttt{u}})}\ \texttt{PK-NEHole} \qquad \frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK} \qquad t \notin \Phi}{\Delta; \Phi \vdash (\!\mid t\mid\!)^{\texttt{u}} ::> \texttt{S}_{\kappa}((\!\mid t\mid\!)^{\texttt{u}})}\ \texttt{PK-Unbound}$$

$$\frac{\Delta; \underline{\Phi, t{::}\kappa_1} \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t{::}\kappa_1.\tau ::> \texttt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}\ \texttt{PK-}\lambda$$

$$\frac{\Delta; \Phi \vdash \tau_1 ::> \kappa \qquad \Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta; \Phi \vdash \tau_2{::}\kappa_1}{\Delta; \Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}\ \texttt{PK-Ap}$$

$\boxed{\Delta;\Phi \vdash \tau :: \kappa}$ $\quad \tau$ is well formed at kind κ

$$\frac{\Delta;\Phi \vdash \tau ::> \mathsf{S}_{\kappa}(\tau)}{\Delta;\Phi \vdash \tau :: \kappa}\ \texttt{WFaK-1} \qquad \frac{\Delta;\Phi \vdash \tau ::> \kappa_1 \qquad \Delta;\Phi \vdash \kappa_1 \lesssim \kappa}{\Delta;\Phi \vdash \tau :: \kappa}\ \texttt{WFaK-Subsump}$$

$$\frac{\Delta;\Phi \vdash \tau ::> \kappa}{\Delta;\Phi \vdash \tau :: \kappa}\ \texttt{WFaK-Reit} \qquad \frac{\Delta;\Phi \vdash \tau :: \kappa}{\Delta;\Phi \vdash \tau :: \mathsf{S}_{\kappa}(\tau)}\ \texttt{WFaK-Self}$$

$$\frac{\Delta;\Phi \vdash \tau :: \Pi_{t::\kappa_3}.\kappa_4 \qquad \Delta;\Phi \vdash \Pi_{t::\kappa_3}.\kappa_4 \lesssim \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \tau :: \Pi_{t::\kappa_1}.\kappa_2}\ \texttt{WFaK-}\Pi\texttt{CSKTrans}$$

$$\frac{\Delta;\Phi \vdash \tau :: \mathsf{S}_{\kappa}(\tau_1) \qquad \Delta;\Phi \vdash \tau_1 :: \kappa}{\Delta;\Phi \vdash \tau :: \kappa}\ \texttt{WFaK-Flatten}$$

$\boxed{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ $\quad \kappa$ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \texttt{KHole} \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\texttt{KHole}}.\texttt{KHole}}\ \overset{\blacktriangleright}{\scriptstyle\Pi}\texttt{-KHole} \qquad \frac{\Delta;\Phi \vdash \kappa \equiv \mathsf{S}_{\texttt{KHole}}(\tau)}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\mathsf{S}_{\texttt{KHole}}(\tau)}.\mathsf{S}_{\texttt{KHole}}(\tau\ t)}\ \overset{\blacktriangleright}{\scriptstyle\Pi}\texttt{-SKHole}$$

$$\frac{\Delta;\Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2}\ \overset{\blacktriangleright}{\scriptstyle\Pi}\texttt{-}\Pi$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$ $\quad \kappa_1$ is equivalent to κ_2

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \equiv \kappa}\ \texttt{KEquiv-Refl} \qquad \frac{\Delta;\Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Symm}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Trans}$$

$$\frac{\Delta;\Phi \vdash \tau :: \mathsf{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \mathsf{S}_{\mathsf{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathsf{S}_{\kappa}(\tau_1)}\ \texttt{KEquiv-SKind}_{\texttt{SKind}}$$

$$\frac{\Delta;\Phi \vdash \tau :: \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t_1::\kappa_1}.\mathsf{S}_{[t_1/t]\kappa_2}(\tau\ t_1)}\ \texttt{KEquiv-SKind}_{\Pi}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\underline{\Phi, t::\kappa_1} \vdash \kappa_3 \equiv \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{KEquiv-}\Pi$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\kappa_1}(\tau_1) \equiv \mathsf{S}_{\kappa_2}(\tau_2)}\ \texttt{KEquiv-SKind}$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta;\Phi \vdash \kappa \ \mathsf{OK}}{\Delta;\Phi \vdash \mathtt{KHole} \lesssim \kappa}\ \texttt{CSK-KHoleL} \qquad \frac{\Delta;\Phi \vdash \kappa \ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \lesssim \mathtt{KHole}}\ \texttt{CSK-KHoleR}$$

$$\frac{\Delta;\Phi \vdash \mathsf{S}_{\mathtt{KHole}}(\tau)\ \mathsf{OK} \qquad \Delta;\Phi \vdash \kappa \ \mathsf{OK}}{\Delta;\Phi \vdash \mathsf{S}_{\mathtt{KHole}}(\tau) \lesssim \kappa}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{L}$$

$$\frac{\Delta;\Phi \vdash \kappa \ \mathsf{OK} \qquad \Delta;\Phi \vdash \mathsf{S}_{\mathtt{KHole}}(\tau)\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \lesssim \mathsf{S}_{\mathtt{KHole}}(\tau)}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{R}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-KEquiv} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta;\Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-Normal}$$

$$\frac{\Delta;\Phi \vdash \mathsf{S}_{\kappa}(\tau)\ \mathsf{OK}}{\Delta;\Phi \vdash \mathsf{S}_{\kappa}(\tau) \lesssim \kappa}\ \texttt{CSK-SKind} \qquad \frac{\Delta;\Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta;\underline{\Phi, t{::}\kappa_3} \vdash \kappa_2 \lesssim \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{CSK-}\Pi$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta;\Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2}{\Delta;\Phi \vdash \mathsf{S}_{\kappa_1}(\tau_1) \lesssim \mathsf{S}_{\kappa_2}(\tau_2)}\ \texttt{CSK-?}$$

$\boxed{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$ τ_1 is provably equivalent to τ_2 at kind κ

$$\frac{\Delta;\Phi \vdash \tau{::}\kappa}{\Delta;\Phi \vdash \tau \stackrel{\kappa}{\equiv} \tau}\ \texttt{EquivAK-Refl} \qquad \frac{\Delta;\Phi \vdash \tau_2 \stackrel{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Symm}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_3 \stackrel{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Trans}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \ {::>}\ \kappa_1 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \mathsf{S}_{\kappa}(\tau_2)}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-SKind}$$

$$\frac{\Delta;\Phi \vdash \tau_1{::}\Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta;\Phi \vdash \tau_2{::}\Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta;\underline{\Phi, t{::}\kappa_1} \vdash \tau_1\ t \stackrel{\kappa_2}{\equiv} \tau_2\ t}{\Delta;\Phi \vdash \tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}\ \texttt{EquivAK-}\Pi$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \stackrel{\kappa_1}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1\ \tau_2 \stackrel{[\tau_2/t]\kappa_2}{\equiv} \tau_3\ \tau_4}\ \texttt{EquivAK-Ap}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\mathsf{S}_{\kappa}(\tau)}{\equiv} \tau_2}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ (1)$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\mathtt{Type}}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \stackrel{\mathtt{Type}}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 \stackrel{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4}\ (2) \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\underline{\Phi, t{::}\kappa_1} \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \lambda t{::}\kappa_1.\tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t{::}\kappa_2.\tau_2}\ (3)$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}\ (4)$$

$\boxed{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}$ κ is well formed

$$\frac{\Delta;\Phi\vdash\mathsf{OK}}{\Delta;\Phi\vdash\mathtt{Type}\ \mathsf{OK}}\ \texttt{KWF-Type}\qquad\frac{\Delta;\Phi\vdash\mathsf{OK}}{\Delta;\Phi\vdash\mathtt{KHole}\ \mathsf{OK}}\ \texttt{KWF-KHole}\qquad\frac{\Delta;\Phi\vdash\tau::\kappa}{\Delta;\Phi\vdash\mathsf{S}_{\kappa}(\tau)\ \mathsf{OK}}\ \texttt{KWF-SKind}$$

$$\frac{\Delta;\underline{\Phi,t::\kappa_1}\vdash\kappa_2\ \mathsf{OK}}{\Delta;\Phi\vdash\Pi_{t::\kappa_1}.\kappa_2\ \mathsf{OK}}\ \texttt{KWF-}\Pi$$

$\boxed{\Delta;\Phi\vdash\mathsf{OK}}$ Context is well formed

$$\frac{}{\cdot;\cdot\vdash\mathsf{OK}}\ \texttt{CWF-Nil}\qquad\frac{t\notin\Phi\qquad\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta;\underline{\Phi,t::\kappa}\vdash\mathsf{OK}}\ \texttt{CWF-TypVar}\qquad\frac{\mathrm{u}\notin\Delta\qquad\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\underline{\Delta,\mathrm{u}::\kappa};\Phi\vdash\mathsf{OK}}\ \texttt{CWF-Hole}$$

Metatheory

subderivation preserving inferences:

- premiss
- COK (Context OK)
- PoS (premiss of subderivation)

Lemma 1 (COK). *If $\Delta;\Phi\vdash\mathcal{J}$, then $\Delta;\Phi\vdash\mathsf{OK}$ in a subderivation (where $\Delta;\Phi\vdash\mathcal{J}\neq\Delta;\Phi\vdash\mathsf{OK}$)*

Proof. By induction on derivations.
No interesting cases. □

Lemma 2 (Exchange).
If $\Delta;\Phi_1,t_{L1}::\kappa_{L1},t_{L2}::\kappa_{L2},\Phi_2\vdash\mathcal{J}$ and $\Delta;\Phi_1,t_{L2}::\kappa_{L2},t_{L1}::\kappa_{L1},\Phi_2\vdash\mathsf{OK}$, then $\Delta;\Phi_1,t_{L2}::\kappa_{L2},t_{L1}::\kappa_{L1},\Phi_2\vdash\mathcal{J}$

Proof. By induction on derivations.
No interesting cases.
(Only rules with Φ extended in the consequent are interesting, which is only CWF-TypVar, but when $\mathcal{J}$ is CWF, Exchange is identity) □

Corollary 3 (Marked-Exchange).
If $\Delta;\underline{\underline{\Phi,t_{L1}::\kappa_{L1}},t_{L2}::\kappa_{L2}}\vdash\mathcal{J}$ and $\Delta;\underline{\underline{\Phi,t_{L2}::\kappa_{L2}},t_{L1}::\kappa_{L1}}\vdash\mathsf{OK}$, then $\Delta;\underline{\underline{\Phi,t_{L2}::\kappa_{L2}},t_{L1}::\kappa_{L1}}\vdash\mathcal{J}$

Proof. Exchange when $\Phi_2=\cdot$ □

Lemma 4 (Weakening)**.**
If $\Delta;\Phi\vdash\mathcal{J}$ *and* $\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash \mathit{OK}$ *and* $t_L\notin\mathcal{J}$ *and* $\forall t\in\kappa_L, t\notin\mathcal{J}$*, then* $\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\mathcal{J}$

Proof. By induction on derivations.
When applying Weakening in the induction, check that the left premiss is always a subderivation, and check variable exclusion conditions are satisfied (usually checked elsewhere in the derivation).

Weakening

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\dfrac{}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\tau ::> \kappa_2}\,\mathtt{premiss}
\quad
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\dfrac{\dfrac{}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\mathsf{OK}}\,\mathtt{IH}}{t_L\notin\Phi}\,\mathtt{PoS}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\mathtt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t_L\neq t}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\mathtt{IH}}{t_L\notin\kappa_1}
}{t_L\notin\underline{\Phi, t{::}\kappa_1}}
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\dfrac{\dfrac{}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\mathsf{OK}}\,\mathtt{IH}}{\Delta;\Phi\vdash\kappa_L\ \mathsf{OK}}\,\mathtt{PoS}
\quad
\dfrac{\dfrac{}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\tau ::> \kappa_2}\,\mathtt{premiss}}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\mathsf{OK}}\,\mathtt{COK}
}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\kappa_L\ \mathsf{OK}}\,\mathtt{Weakening}
}{\Delta;\underline{\underline{\Phi, t{::}\kappa_1}, t_L{::}\kappa_L}\vdash\mathsf{OK}}\,\mathtt{CWF\text{-}TypVar}
}{\Delta;\underline{\underline{\Phi, t{::}\kappa_1}, t_L{::}\kappa_L}\vdash\tau ::> \kappa_2}\,\mathtt{Weakening}
\quad
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\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\mathtt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t\neq t_L}
\quad
\dfrac{\dfrac{}{\forall\dot{t}\in\kappa_L, \dot{t}\notin\mathcal{J}}\,\mathtt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t\notin\kappa_L}
}{t\notin\underline{\Phi, t_L{::}\kappa_L}}
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\dfrac{\dfrac{\dfrac{}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\tau ::> \kappa_2}\,\mathtt{premiss}}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\mathsf{OK}}\,\mathtt{COK}}{\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}}\,\mathtt{PoS}
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}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\kappa_1\ \mathsf{OK}}\,\mathtt{Weakening}
}{\Delta;\underline{\underline{\Phi, t_L{::}\kappa_L}, t{::}\kappa_1}\vdash\mathsf{OK}}\,\mathtt{CWF\text{-}TypVar}
}{
\dfrac{\Delta;\underline{\underline{\Phi, t_L{::}\kappa_L}, t{::}\kappa_1}\vdash\tau ::> \kappa_2}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\lambda t{::}\kappa_1.\tau ::> \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}\,\mathtt{PK\text{-}\lambda}
}\,\mathtt{Marked\text{-}Exchange}
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\dfrac{}{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2}\,\mathtt{premiss}
\quad
\dfrac{}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\mathsf{OK}}\,\mathtt{IH}
}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\kappa_1\equiv\kappa_2}\,\mathtt{Weakening}
\quad
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\dfrac{}{\Delta;\Phi, t{::}\kappa_1\vdash\tau ::> \kappa_2}\,\mathtt{premiss}
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\dfrac{\dfrac{}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\mathsf{OK}}\,\mathtt{IH}}{t_L\notin\Phi}\,\mathtt{PoS}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\mathtt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t_L\neq t}
\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\mathtt{IH}}{t_L\notin\kappa_1}
}{t_L\notin\underline{\Phi, t{::}\kappa_1}}
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\dfrac{\dfrac{}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\mathsf{OK}}\,\mathtt{IH}}{\Delta;\Phi\vdash\kappa_L\ \mathsf{OK}}\,\mathtt{PoS}
\quad
\dfrac{\dfrac{}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\kappa_3\equiv\kappa_4}\,\mathtt{premiss}}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\mathsf{OK}}\,\mathtt{COK}
}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\kappa_L\ \mathsf{OK}}\,\mathtt{Weakening}
}{\Delta;\underline{\underline{\Phi, t{::}\kappa_1}, t_L{::}\kappa_L}\vdash\mathsf{OK}}\,\mathtt{CWF\text{-}TypVar}
}{\Delta;\underline{\underline{\Phi, t{::}\kappa_1}, t_L{::}\kappa_L}\vdash\tau ::> \kappa_2}\,\mathtt{Weakening}
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\quad
\dfrac{\dfrac{}{t_L\notin\mathcal{J}}\,\mathtt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t\neq t_L}
\quad
\dfrac{\dfrac{}{\forall\dot{t}\in\kappa_L, \dot{t}\notin\mathcal{J}}\,\mathtt{IH}\quad\dfrac{}{t\in\mathcal{J}}}{t\notin\kappa_L}
}{t\notin\underline{\Phi, t_L{::}\kappa_L}}
\quad
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\dfrac{\dfrac{\dfrac{}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\kappa_3\equiv\kappa_4}\,\mathtt{premiss}}{\Delta;\underline{\Phi, t{::}\kappa_1}\vdash\mathsf{OK}}\,\mathtt{COK}}{\Delta;\Phi\vdash\kappa_1\ \mathsf{OK}}\,\mathtt{PoS}
\quad
\dfrac{}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\mathsf{OK}}\,\mathtt{IH}
}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\kappa_1\ \mathsf{OK}}\,\mathtt{Weakening}
}{\Delta;\underline{\underline{\Phi, t_L{::}\kappa_L}, t{::}\kappa_1}\vdash\mathsf{OK}}\,\mathtt{CWF\text{-}TypVar}
}{\Delta;\underline{\underline{\Phi, t_L{::}\kappa_L}, t{::}\kappa_1}\vdash\kappa_3\equiv\kappa_4}\,\mathtt{Marked\text{-}Exchange}
}{\Delta;\underline{\Phi, t_L{::}\kappa_L}\vdash\Pi_{t::\kappa_1}.\kappa_2\equiv\Pi_{t::\kappa_3}.\kappa_4}\,\mathtt{KEquiv\text{-}\Pi}
\qquad
\dfrac{}{sim}\,\mathtt{CSK\text{-}\Pi}
\qquad
\dfrac{}{sim}\,\mathtt{EquivAK\text{-}\Pi}
\qquad
\dfrac{}{sim}\,\mathtt{KWF\text{-}\Pi}
$$

$\square$

Lemma 5 (OK-PK)**.** *If* $\Delta;\Phi \vdash \tau ::> \kappa$*, then* $\Delta;\Phi \vdash \kappa$ *OK*

Lemma 6 (OK-WFaK)**.** *If* $\Delta;\Phi \vdash \tau{::}\kappa$*, then* $\Delta;\Phi \vdash \kappa$ *OK*

Lemma 7 (OK-MatchPi)**.** *If* $\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2$*, then* $\Delta;\Phi \vdash \kappa$ *OK and* $\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2$ *OK*

Lemma 8 (OK-KEquiv)**.** *If* $\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2$*, then* $\Delta;\Phi \vdash \kappa_1$ *OK and* $\Delta;\Phi \vdash \kappa_2$ *OK*

Lemma 9 (OK-CSK)**.** *If* $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$*, then* $\Delta;\Phi \vdash \kappa_1$ *OK and* $\Delta;\Phi \vdash \kappa_2$ *OK*

Lemma 10 (OK-EquivAK)**.** *If* $\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2$*, then* $\Delta;\Phi \vdash \tau_1{::}\kappa$ *and* $\Delta;\Phi \vdash \tau_2{::}\kappa$ *and* $\Delta;\Phi \vdash \kappa$ *OK*

Lemma 11 (OK-Substitution)**.**
If $\Delta;\Phi \vdash \tau_L{::}\kappa_{L1}$ *and* $\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \kappa_{L2}$ *OK, then* $\Delta;\Phi \vdash [\tau_L/t_L]\kappa_{L2}$ *OK*
(induction on $\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \kappa_{L2}$ *OK)*

Lemma 12 (K-Substitution)**.**
If $\Delta;\Phi \vdash \tau_{L1}{::}\kappa_{L1}$ *and* $\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \tau_{L2}{::}\kappa_{L2}$*, then* $\Delta;\Phi \vdash [\tau_{L1}/t_L]\tau_{L2}{::}[\tau_{L1}/t_L]\kappa_{L2}$
(induction on $\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \tau_{L2}{::}\kappa_{L2}$*)*

Proof. By simultaneous induction on derivations.
The interesting cases per lemma:

OK-PK.	PK-Base	$\Delta;\Phi \vdash \mathtt{bse}{::}\mathtt{S}_{\mathtt{Type}}(\mathtt{bse})$	by (9)
		$\Delta;\Phi \vdash \mathtt{bse}{::}\mathtt{Type}$	by (10)
	*	$\Delta;\Phi \vdash \mathtt{S}_{\mathtt{Type}}(\mathtt{bse})$ OK	by (43)
	*	$\Delta;\Phi \vdash$ OK	by premiss
	PK-Ap		bad
OK-WFaK.	(12)	$\Delta;\Phi \vdash \tau_2{::}\kappa$	by (10)
	*	$\Delta;\Phi \vdash \mathtt{S}_\kappa(\tau_2)$ OK	by (43)
OK-KEquiv.	(22)	$\Delta;\Phi \vdash \tau\ t ::> \kappa$	
OK-Substitution.	(41)	$\Delta;\Phi, t_L{::}\kappa_{L1} \vdash$ OK	premiss (41)
		$\Delta;\Phi \vdash \kappa_{L1}$ OK	by subderivation premiss (46)
	*	$\Delta;\Phi \vdash$ OK	by OK-KWF
	*	$\Delta;\Phi \vdash [\tau_L/t_L]\mathtt{Type}$ OK	by (41) and degenerate subst
	(43)	$\Delta;\Phi, t_L{::}\kappa_{L1} \vdash \tau{::}\kappa$	premiss (43)
		$\Delta;\Phi, t_L{::}\kappa_{L1} \vdash$ OK	by OK-WFaK
		$\Delta;\Phi \vdash \kappa_{L1}$ OK	by subderivation premiss (46)
	*	$\Delta;\Phi \vdash$ OK	by OK-KWF
		$\Delta;\Phi \vdash [\tau_{L1}/t_L]\tau{::}[\tau_{L1}/t_L]\kappa$	by K-Substitution on premiss
	*	$\Delta;\Phi \vdash [\tau_L/t_L]\mathtt{S}_\kappa(\tau)$ OK	by (43)

□

Lemma 13 (PK-Unicity)**.** *If* $\Delta;\Phi \vdash \tau_L ::> \kappa_{L1}$ *and* $\Delta;\Phi \vdash \tau_L ::> \kappa_{L2}$ *then* κ_{L1} *is* κ_{L2}

Lemma 14. *If* $\Delta;\Phi \vdash \tau ::> \kappa_1$ *and* $\Delta;\Phi \vdash \tau{::}\kappa_2$*, then* $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$

Lemma 15. *If* $\Delta;\Phi \vdash \kappa_1 \lesssim \mathtt{S}_{\kappa_2}(\tau)$*, then* $\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2$