$$
\begin{array}{rlll}
\textsf{BinOp} & \oplus & ::= & \texttt{Product} \mid \texttt{Sum} \mid \texttt{Arrow} \\
\textsf{Kind} & \kappa & ::= & \texttt{Ty} \mid \texttt{KHole} \mid \texttt{S}(\tau) \\
\textsf{ConstantTypes} & c & ::= & \texttt{Int} \mid \texttt{Float} \mid \texttt{Bool} \\
\textsf{UserHTyp} & \hat{\tau} & ::= & c \mid \hat{\tau}_1 \oplus \hat{\tau}_2 \mid \texttt{list}(\hat{\tau}) \mid (\!|\,|\!)^u \mid (\!|\hat{\tau}|\!)^u \\
\textsf{InternalHTyp} & \tau & ::= & c \mid \tau_1 \oplus \tau_2 \mid \texttt{list}(\tau) \mid (\!|\,|\!)^u \mid (\!|\tau|\!)^u \\
\textsf{TypeVars} & t & & \\
\textsf{UserTypePattern} & \hat{\rho} & ::= & t \mid (\!|\,|\!)^u \mid (\!|t|\!)^u \\
\textsf{UserExpression} & \hat{e} & ::= & \texttt{type}\ \hat{\rho}\ \texttt{=}\ \hat{\tau}\ \texttt{in}\ \hat{e} \mid \textit{elided} \\
\textsf{InternalExpression} & \tau & ::= & \texttt{type bindings}\ \Phi\ \texttt{in}\ e \mid \textit{elided}
\end{array}
$$

$\boxed{\Delta; \Phi \vdash \kappa_1 <\sim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$
\texttt{KCHoleL}\ \frac{}{\Delta; \Phi \vdash \texttt{KHole} <\sim \kappa}
\qquad
\texttt{KCHoleR}\ \frac{}{\Delta; \Phi \vdash \kappa <\sim \texttt{KHole}}
\qquad
\texttt{KCRespectEquiv}\ \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 <\sim \kappa_2}
$$

$$
\texttt{KCSubsumption}\ \frac{\Delta; \Phi \vdash \tau : \texttt{Ty}}{\Delta; \Phi \vdash \texttt{S}(\tau) <\sim \texttt{Ty}}
$$

$\boxed{t\ \texttt{valid}}$ t is a valid type variable

t is valid if it is not a builtin-type or keyword, begins with an alpha char or underscore, and only contains alphanumeric characters, underscores, and primes.

$\boxed{\Delta; \Phi \vdash \kappa\ \texttt{kind}}$ κ forms a kind

$$
\texttt{KFSing}\ \frac{\Delta; \Phi \vdash \tau : \texttt{Ty}}{\Delta; \Phi \vdash \texttt{S}(\tau)\ \texttt{kind}}
$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\texttt{KERefl}\quad \frac{}{\Delta; \Phi \vdash \kappa \equiv \kappa} \qquad \texttt{KESymm}\quad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1} \qquad \texttt{KETrans}\quad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \Phi \vdash \kappa_2 \equiv \kappa_3}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3}$$

$$\texttt{KESingEquiv}\quad \frac{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \texttt{Ty}}{\Delta; \Phi \vdash \texttt{S}(\tau_1) \equiv \texttt{S}(\tau_2)}$$

$\boxed{\Delta; \Phi \vdash \tau : \kappa}$ τ is assigned kind κ

$$\texttt{KAConst}\quad \frac{}{\Delta; \Phi \vdash c : \texttt{Ty}} \qquad \texttt{KAVar}\quad \frac{t : \kappa \in \Phi}{\Delta; \Phi \vdash t : \kappa} \qquad \texttt{KABinOp}\quad \frac{\Delta; \Phi \vdash \tau_1 : \texttt{Ty} \Delta; \Phi \vdash \tau_2 : \texttt{Ty}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 : \texttt{Ty}}$$

$$\texttt{KAList}\quad \frac{\Delta; \Phi \vdash \tau : \texttt{Ty}}{\Delta; \Phi \vdash \texttt{list}(\tau) : \texttt{Ty}} \qquad \texttt{KAEHole}\quad \frac{u :: \kappa[\Phi'] \in \Delta \qquad \Delta; \Phi \vdash \sigma : \Phi'}{\Delta; \Phi \vdash (\!|\,|\!)^u_\sigma : \kappa}$$

$$\texttt{KANEHole}\quad \frac{\Delta; \Phi \vdash \tau : \kappa' \qquad u :: \kappa[\Phi'] \in \Delta \qquad \Delta; \Phi \vdash \sigma : \Phi'}{\Delta; \Phi \vdash (\!|\tau|\!)^u_\sigma : \kappa} \qquad \texttt{KASelfRecognition}\quad \frac{\Delta; \Phi \vdash \tau : \texttt{Ty}}{\Delta; \Phi \vdash \tau : \texttt{S}(\tau)}$$

$$\texttt{KASubkind}\quad \frac{\Delta; \Phi \vdash \tau : \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 <\sim \kappa_2}{\Delta; \Phi \vdash \tau : \kappa_2}$$

$\boxed{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \kappa}$ τ_1 is equivalent to τ_2 and has kind κ_2

$$\texttt{KCERefl}\quad \frac{}{\Delta; \Phi \vdash \tau \equiv \tau : \kappa} \qquad \texttt{KCESymm}\quad \frac{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \kappa}{\Delta; \Phi \vdash \tau_2 \equiv \tau_1 : \kappa}$$

$$\texttt{KCETrans}\quad \frac{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \kappa \qquad \Delta; \Phi \vdash \tau_2 \equiv \tau_3 : \kappa}{\Delta; \Phi \vdash \tau_1 \equiv \tau_3 : \kappa} \qquad \texttt{KCESingEquiv}\quad \frac{\Delta; \Phi \vdash \tau_1 : \texttt{S}(\tau_2)}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \texttt{Ty}}$$

$\boxed{\Phi \vdash \hat{\tau} \Rightarrow \kappa \leadsto \tau \dashv \Delta}$ $\hat{\tau}$ synthesizes kind κ and elaborates to τ

$$\texttt{TElabSConst}\quad \frac{}{\Phi \vdash c \Rightarrow \texttt{Ty} \leadsto c \dashv \cdot}$$

$$\texttt{TElabSBinOp}\quad \frac{\Phi \vdash \hat{\tau}_1 \Leftarrow \texttt{Ty} \leadsto \tau_1 : \texttt{Ty} \dashv \Delta_1 \qquad \Phi \vdash \hat{\tau}_2 \Leftarrow \texttt{Ty} \leadsto \tau_2 : \texttt{Ty} \dashv \Delta_2}{\Phi \vdash \hat{\tau}_1 \oplus \hat{\tau}_2 \Rightarrow \texttt{Ty} \leadsto \tau_1 \oplus \tau_2 \dashv \Delta_1 \cup \Delta_2}$$

$$\texttt{TElabSList}\quad \frac{\Phi \vdash \hat{\tau} \Leftarrow \texttt{Ty} \leadsto \tau : \texttt{Ty} \dashv \Delta}{\Phi \vdash \texttt{list(}\hat{\tau}\texttt{)} \Rightarrow \texttt{Ty} \leadsto \texttt{list(}\tau\texttt{)} \dashv \Delta} \qquad \texttt{TElabSVar}\quad \frac{t : \kappa \in \Phi}{\Phi \vdash t \Rightarrow \kappa \leadsto t \dashv \cdot}$$

$$\texttt{TElabSUVar}\quad \frac{t \notin \Phi}{\Phi \vdash t \Rightarrow \texttt{KHole} \leadsto (\!| t |\!)^u_{\mathsf{id}(\Phi)} \dashv u :: (\!|\,|\!)[\Phi]}$$

$$\texttt{TElabSHole}\quad \frac{}{\Phi \vdash (\!|\,|\!)^u \Rightarrow \texttt{KHole} \leadsto (\!|\,|\!)^u_{\mathsf{id}(\Phi)} \dashv u :: (\!|\,|\!)[\Phi]}$$

$$\texttt{TElabSNEHole}\quad \frac{\Phi \vdash \hat{\tau} \Rightarrow \kappa \leadsto \tau \dashv \Delta}{\Phi \vdash (\!| \hat{\tau} |\!)^u \Rightarrow \texttt{KHole} \leadsto (\!| \tau |\!)^u_{\mathsf{id}(\Phi)} \dashv \Delta, u :: (\!|\,|\!)[\Phi]}$$

$\boxed{\Phi \vdash \hat{\tau} \Leftarrow \kappa_1 \leadsto \tau : \kappa_2 \dashv \Delta}$ $\hat{\tau}$ analyzes against kind κ_1 and elaborates to τ of consistent kind κ_2

TElabASubsume

$$\frac{\hat{\tau} \neq t \text{ where } t \notin \Phi \\ \hat{\tau} \neq (\!|\,|\!)^u \qquad \hat{\tau} \neq (\!|\hat{\tau}'|\!)^u \qquad \Phi \vdash \hat{\tau} \Rightarrow \kappa' \leadsto \tau \dashv \Delta \qquad \kappa \sim \kappa'}{\Phi \vdash \hat{\tau} \Leftarrow \kappa \leadsto \tau : \kappa' \dashv \Delta}$$

TElabAUVar

$$\frac{t \notin \Phi}{\Phi \vdash t \Leftarrow \texttt{KHole} \leadsto (\!|t|\!)^u_{\mathsf{id}(\Phi)} : \texttt{KHole} \dashv u :: (\!|\,|\!)[\Phi]}$$

TElabAEHole

$$\frac{}{\Phi \vdash (\!|\,|\!)^u \Leftarrow \kappa \leadsto (\!|\,|\!)^u_{\mathsf{id}(\Phi)} : \kappa \dashv u :: \kappa[\Phi]}$$

TElabANEHole

$$\frac{\Phi \vdash \hat{\tau} \Rightarrow \kappa' \leadsto \tau \dashv \Delta}{\Phi \vdash (\!|\hat{\tau}|\!)^u \Leftarrow \kappa \leadsto (\!|\tau|\!)^u_{\mathsf{id}(\Phi)} : \kappa \dashv \Delta, u :: \kappa[\Phi]}$$

$\boxed{\Delta_1; \Phi_1 \vdash \hat{\rho} \leftarrow \tau \dashv \Phi_2; \Delta_2}$ $\hat{\rho}$ analyzes against τ yielding new tyvar and hole bindings

RESVar

$$\frac{t \ \texttt{valid} \qquad \Delta; \Phi \vdash \tau : \kappa}{\Delta; \Phi \vdash t \leftarrow \tau \dashv t :: \kappa; \cdot}$$

RESEHole

$$\frac{}{\Delta; \Phi \vdash (\!|\,|\!)^u \leftarrow \tau \dashv \cdot; u :: (\!|\,|\!)[\Phi]}$$

RESVarHole

$$\frac{t \ \neg\texttt{valid}}{\Delta; \Phi \vdash (\!|t|\!)^u \leftarrow \tau \dashv \cdot; u :: (\!|\,|\!)[\Phi]}$$

$\boxed{\Gamma; \Phi \vdash \hat{e} \Rightarrow \hat{\tau} \leadsto e \dashv \Delta}$ $\hat{e}$ synthesizes type τ and elaborates to e

ESDefine

$$\frac{\Phi_1 \vdash \hat{\tau} \Rightarrow \kappa \leadsto \tau \dashv \Delta_1 \\ \Delta_1; \Phi_1 \vdash \hat{\rho} \leftarrow \tau \dashv \Phi_2; \Delta_2 \qquad \Gamma; \Phi_1 \cup \Phi_2 \vdash \hat{e} \Rightarrow \tau_1 \leadsto e \dashv \Delta_3}{\Gamma; \Phi_1 \vdash \texttt{type}\ \hat{\rho}\ \texttt{=}\ \hat{\tau}\ \texttt{in}\ \hat{e} \Rightarrow \tau_1 \leadsto \texttt{type bindings}\ \Phi_2\ \texttt{in}\ e \dashv \Delta_1 \cup \Delta_2 \cup \Delta_3}$$

$\boxed{\Gamma; \Phi \vdash e : \tau}$ e is assigned type τ

DEDefine

$$\frac{\Gamma; \Phi_1 \cup \Phi_2 \vdash e : \tau}{\Gamma; \Phi_1 \vdash \texttt{type bindings } \Phi_2 \texttt{ in } e : \tau}$$