Algebraic Data Types for Hazel

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1 Syntax

$$
\begin{array}{llll}
\mathsf{HTyp} & \tau & ::= & \varnothing \mid \tau \to \tau \mid \alpha \mid \mu\pi.\tau \mid +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \mid +\{C(\tau);\ldots\} \mid \llparenthesis\rrparenthesis \mid \llparenthesis\alpha\rrparenthesis \\
\mathsf{HTypPat} & \pi & ::= & \alpha \mid \llparenthesis\rrparenthesis \\
\mathsf{HExp} & e & ::= & \varnothing \mid x \mid \lambda x{:}\tau.e \mid e(e) \mid e : \tau \mid \mathtt{inj}_C(e) \mid \mathtt{roll}(e) \mid \mathtt{unroll}(e) \\
 & & & \mid \llparenthesis\rrparenthesis^u \mid \llparenthesis e\rrparenthesis^u \mid \llparenthesis e\rrparenthesis^{u\blacktriangleright} \\
\mathsf{IHExp} & d & ::= & \varnothing \mid x \mid \lambda x{:}\tau.d \mid d(d) \mid \mathtt{inj}^{\tau}_C(d) \mid \mathtt{roll}^{\mu\alpha.\tau}(d) \mid \mathtt{unroll}(d) \\
 & & & \mid d\langle\tau \Rightarrow \tau\rangle \mid d\langle\tau \Rightarrow \llparenthesis\rrparenthesis \not\Rightarrow \tau\rangle \mid \llparenthesis\rrparenthesis^u_\sigma \mid \llparenthesis d\rrparenthesis^u_\sigma \mid \llparenthesis d\rrparenthesis^{u\blacktriangleright}_\sigma \\
\mathsf{HTag} & C & ::= & \mathbf{C} \mid \llparenthesis\rrparenthesis^u \mid \llparenthesis\mathbf{C}\rrparenthesis^u
\end{array}
$$

1.1 Context Extension

We write Θ, π to denote the extension of type variable context Θ with optional type variable name π.

$$
\Theta, \pi = \begin{cases} \Theta, \alpha & \pi = \alpha \\ \Theta & \pi = \llparenthesis\rrparenthesis \end{cases}
$$

2 Static Semantics

$\boxed{[\tau/\pi]\tau' = \tau''}$ τ'' is obtained by substituting τ for π in τ'

$$
\begin{array}{lcll}
[\tau/\llparenthesis\rrparenthesis]\tau' & = & \tau' & \\
[\tau/\alpha]\varnothing & = & \varnothing & \\
[\tau/\alpha](\tau_1 \to \tau_2) & = & [\tau/\alpha]\tau_1 \to [\tau/\alpha]\tau_1 & \\
[\tau/\alpha]\alpha & = & \tau & \\
[\tau/\alpha]\alpha_1 & = & \tau' & \text{when } \alpha \neq \alpha_1 \\
[\tau/\alpha]\mu\alpha_1.\tau_2 & = & \mu\alpha_1.[\tau/\alpha]\tau_2 & \text{when } \alpha \neq \alpha_1 \text{ and } \alpha_1 \notin \mathsf{FV}(\tau) \\
[\tau/\alpha]\mu\llparenthesis\rrparenthesis.\tau_2 & = & \mu\llparenthesis\rrparenthesis.[\tau/\alpha]\tau_2 & \\
[\tau/\alpha]+\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} & = & +\{C_i([\tau/\alpha]\tau_i)\}_{C_i \in \mathcal{C}} & \\
[\tau/\alpha]+\{C(\tau');\ldots\} & = & +\{C([\tau/\alpha]\tau');\ldots\} & \\
[\tau/\alpha]\llparenthesis\rrparenthesis & = & \llparenthesis\rrparenthesis & \\
[\alpha'/\alpha]\llparenthesis\alpha\rrparenthesis & = & \llparenthesis\alpha'\rrparenthesis & \\
[\alpha'/\alpha]\llparenthesis\alpha'\rrparenthesis & = & \llparenthesis\alpha'\rrparenthesis & \text{when } \alpha \neq \alpha'
\end{array}
$$

$\boxed{\Theta \vdash \tau\ \mathsf{valid}}$ τ is a valid type

$$\textsc{TVUnit}\quad \frac{}{\Theta \vdash \varnothing\ \mathsf{valid}}$$

$$\textsc{TVArr}\quad \frac{\Theta \vdash \tau_1\ \mathsf{valid} \qquad \Theta \vdash \tau_2\ \mathsf{valid}}{\Theta \vdash \tau_1 \to \tau_2\ \mathsf{valid}}$$

$$\textsc{TVVar}\quad \frac{\alpha \in \Theta}{\Theta \vdash \alpha\ \mathsf{valid}}$$

$$\textsc{TVRec}\quad \frac{\Theta, \pi \vdash \tau\ \mathsf{valid}}{\Theta \vdash \mu\pi.\tau\ \mathsf{valid}}$$

$$\textsc{TVSum1}\quad \frac{\{\Theta \vdash \tau_i\ \mathsf{valid}\}_{C_i \in \mathcal{C}}}{\Theta \vdash +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}}\ \mathsf{valid}}$$

$$\textsc{TVSum2}\quad \frac{\Theta \vdash \tau\ \mathsf{valid}}{\Theta \vdash +\{C(\tau); ...\}\ \mathsf{valid}}$$

$$\textsc{TVEHole}\quad \frac{}{\Theta \vdash \llparenthesis\rrparenthesis\ \mathsf{valid}}$$

$$\textsc{TVNEHole}\quad \frac{\alpha \notin \Theta}{\Theta \vdash \llparenthesis\alpha\rrparenthesis\ \mathsf{valid}}$$

$\boxed{\tau \sim \tau'}$ τ and τ' are consistent

$$\textsc{TCRefl}\quad \frac{}{\tau \sim \tau}$$

$$\textsc{TCEHole1}\quad \frac{}{\llparenthesis\rrparenthesis \sim \tau}$$

$$\textsc{TCEHole2}\quad \frac{}{\tau \sim \llparenthesis\rrparenthesis}$$

$$\textsc{TCNEHole1}\quad \frac{}{\llparenthesis\alpha\rrparenthesis \sim \tau}$$

$$\textsc{TCNEHole2}\quad \frac{}{\tau \sim \llparenthesis\alpha\rrparenthesis}$$

$$\textsc{TCArr}\quad \frac{\tau_1 \sim \tau_1' \qquad \tau_2 \sim \tau_2'}{\tau_1 \to \tau_2 \sim \tau_1' \to \tau_2'}$$

$$\textsc{TCRec}\quad \frac{\tau \sim \tau'}{\mu\pi.\tau \sim \mu\pi.\tau'}$$

$$\textsc{TCRecHole1}\quad \frac{\alpha \notin \mathsf{FV}(\tau) \qquad \tau \sim \tau'}{\mu\llparenthesis\rrparenthesis.\tau \sim \mu\alpha.\tau'}$$

$$\textsc{TCRecHole2}\quad \frac{\alpha \notin \mathsf{FV}(\tau) \qquad \tau \sim \tau'}{\mu\alpha.\tau \sim \mu\llparenthesis\rrparenthesis.\tau'}$$

$$\textsc{TCSum1}\quad \frac{\{\tau_i \sim \tau_i'\}_{C_i \in \mathcal{C}}}{+\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \sim +\{C_i(\tau_i')\}_{C_i \in \mathcal{C}}}$$

$$\textsc{TCSum2}\quad \frac{\tau \sim \tau'}{+\{C(\tau); ...\} \sim +\{C(\tau'); ...\}}$$

$$\textsc{TCSum2C}\quad \frac{C \neq C'}{+\{C(\tau); ...\} \sim +\{C'(\tau'); ...\}}$$

$$\textsc{TCSum12}\quad \frac{C_j \in \mathcal{C} \qquad \tau \sim \tau_j}{+\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \sim +\{C_j(\tau); ...\}}$$

$$\textsc{TCSum21}\quad \frac{C_j \in \mathcal{C} \qquad \tau \sim \tau_j}{+\{C_j(\tau); ...\} \sim +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}}}$$

2.1 Bidirectional Typing

We call $[\mu\pi.\tau/\pi]\tau$ the *unrolling* of recursive type $\mu\pi.\tau$.

Theorem 1 (Synthetic Type Validity). *If* $\Gamma \vdash e \Rightarrow \tau$ *then* $\emptyset \vdash \tau\ \mathsf{valid}$.

Theorem 2 (Consistency Preserves Validity). *If* $\Theta \vdash \tau\ \mathsf{valid}$ *and* $\tau \sim \tau'$ *then* $\Theta \vdash \tau'\ \mathsf{valid}$.

$\boxed{\tau \blacktriangleright_{\to} \tau_1 \to \tau_2}$ τ has matched arrow type $\tau_1 \to \tau_2$

$$\textsc{MAHole}\quad \frac{}{\llparenthesis\rrparenthesis \blacktriangleright_{\to} \llparenthesis\rrparenthesis \to \llparenthesis\rrparenthesis}$$

$$\textsc{MAArr}\quad \frac{}{\tau_1 \to \tau_2 \blacktriangleright_{\to} \tau_1 \to \tau_2}$$

$\boxed{\tau \blacktriangleright_{\mu} \mu\pi.\tau'}$ τ has matched recursive type $\mu\pi.\tau'$

$$\textsc{MRRec}\quad \frac{}{\mu\pi.\tau \blacktriangleright_{\mu} \mu\pi.\tau}$$

$$\textsc{MRHole}\quad \frac{}{\llparenthesis\rrparenthesis \blacktriangleright_{\mu} \mu\llparenthesis\rrparenthesis.\llparenthesis\rrparenthesis}$$

$\boxed{\tau \blacktriangleright_{+} +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}}}$ τ has matched sum type $+\{C_i(\tau_i)\}_{C_i \in \mathcal{C}}$

$$\textsc{MSFinite}\quad \frac{}{+\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \blacktriangleright_{+} +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}}}$$

$$\textsc{MSElided}\quad \frac{}{+\{C(\tau); ...\} \blacktriangleright_{+} +\{C(\tau)\}}$$

$\boxed{\Gamma \vdash e \Rightarrow \tau}$ e synthesizes type τ

$$\textsc{SUnit}\ \frac{}{\Gamma \vdash \varnothing \Rightarrow \varnothing} \qquad \textsc{SVar}\ \frac{x : \tau \in \Gamma}{\Gamma \vdash x \Rightarrow \tau} \qquad \textsc{SVarFree}\ \frac{x \notin \mathsf{dom}(\Gamma)}{\Gamma \vdash \llparenthesis x \rrparenthesis^u \Rightarrow \llparenthesis\rrparenthesis} \qquad \textsc{SLam}\ \frac{\emptyset \vdash \tau\ \mathsf{valid} \qquad \Gamma, x : \tau \vdash e \Rightarrow \tau'}{\Gamma \vdash \lambda x{:}\tau.e \Rightarrow \tau \to \tau'}$$

$$\textsc{SApp}\ \frac{\Gamma \vdash e_1 \Rightarrow \tau_1 \qquad \tau_1 \blacktriangleright_{\to} \tau_2 \to \tau \qquad \Gamma \vdash e_2 \Leftarrow \tau_2}{\Gamma \vdash e_1(e_2) \Rightarrow \tau} \qquad \textsc{SAppNotArr}\ \frac{\Gamma \vdash e_1 \Rightarrow \tau_1 \qquad \tau_1 \nsim \llparenthesis\rrparenthesis \to \llparenthesis\rrparenthesis \qquad \Gamma \vdash e_2 \Leftarrow \llparenthesis\rrparenthesis}{\Gamma \vdash \llparenthesis e_1 \rrparenthesis^{u\blacktriangleright}(e_2) \Rightarrow \llparenthesis\rrparenthesis}$$

$$\textsc{SAsc}\ \frac{\emptyset \vdash \tau\ \mathsf{valid} \qquad \Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash e : \tau \Rightarrow \tau} \qquad \textsc{SRollErr}\ \frac{\Gamma \vdash e \Leftarrow \llparenthesis\rrparenthesis}{\Gamma \vdash \llparenthesis \mathtt{roll}(e) \rrparenthesis^u \Rightarrow \mu\llparenthesis\rrparenthesis.\llparenthesis\rrparenthesis} \qquad \textsc{SUnroll}\ \frac{\Gamma \vdash e \Rightarrow \tau \qquad \tau \blacktriangleright_{\mu} \mu\pi.\tau'}{\Gamma \vdash \mathtt{unroll}(e) \Rightarrow [\mu\pi.\tau'/\pi]\tau'}$$

$$\textsc{SUnrollNotRec}\ \frac{\Gamma \vdash e \Rightarrow \tau \qquad \tau \nsim \mu\llparenthesis\rrparenthesis.\llparenthesis\rrparenthesis}{\Gamma \vdash \mathtt{unroll}\big(\llparenthesis e \rrparenthesis^{u\blacktriangleright}\big) \Rightarrow \llparenthesis\rrparenthesis} \qquad \textsc{SInj}\ \frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash \mathtt{inj}_C(e) \Rightarrow +\{C(\tau); \ldots\}} \qquad \textsc{SEHole}\ \frac{}{\Gamma \vdash \llparenthesis\rrparenthesis^u \Rightarrow \llparenthesis\rrparenthesis} \qquad \textsc{SNEHole}\ \frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash \llparenthesis e \rrparenthesis^u \Rightarrow \llparenthesis\rrparenthesis}$$

$\boxed{\Gamma \vdash e \Leftarrow \tau}$ e analyzes against type τ

$$\textsc{ARoll}\ \frac{\tau \blacktriangleright_{\mu} \mu\pi.\tau' \qquad \Gamma \vdash e \Leftarrow [\mu\pi.\tau'/\pi]\tau'}{\Gamma \vdash \mathtt{roll}(e) \Leftarrow \tau} \qquad \textsc{ARollNotRec}\ \frac{\tau \nsim \mu\llparenthesis\rrparenthesis.\llparenthesis\rrparenthesis \qquad \Gamma \vdash e \Leftarrow \llparenthesis\rrparenthesis}{\Gamma \vdash \llparenthesis \mathtt{roll}(e) \rrparenthesis^u \Leftarrow \tau} \qquad \textsc{AInjHole}\ \frac{\Gamma \vdash e \Leftarrow \llparenthesis\rrparenthesis}{\Gamma \vdash \mathtt{inj}_C(e) \Leftarrow \llparenthesis\rrparenthesis}$$

$$\textsc{AInj}\ \frac{\tau \blacktriangleright_{+} +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C_j \in \mathcal{C} \qquad \Gamma \vdash e \Leftarrow \tau_j}{\Gamma \vdash \mathtt{inj}_{C_j}(e) \Leftarrow \tau} \qquad \textsc{AInjTagErr}\ \frac{\tau \blacktriangleright_{+} +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C \notin \mathcal{C} \qquad \Gamma \vdash e \Leftarrow \llparenthesis\rrparenthesis}{\Gamma \vdash \llparenthesis \mathtt{inj}_C(e) \rrparenthesis^u \Leftarrow \tau}$$

$$\textsc{AInjUnexpectedArg}\ \frac{\tau \blacktriangleright_{+} +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C_j \in \mathcal{C} \qquad \tau_j = \varnothing \qquad e \neq \varnothing \qquad \Gamma \vdash e \Leftarrow \llparenthesis\rrparenthesis}{\Gamma \vdash \llparenthesis \mathtt{inj}_{C_j}(e) \rrparenthesis^u \Leftarrow \tau}$$

$$\textsc{AInjExpectedArg}\ \frac{\tau \blacktriangleright_{+} +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C_j \in \mathcal{C} \qquad \tau_j \neq \varnothing}{\Gamma \vdash \llparenthesis \mathtt{inj}_{C_j}(\varnothing) \rrparenthesis^u \Leftarrow \tau} \qquad \textsc{ASubsume}\ \frac{\Gamma \vdash e \Rightarrow \tau' \qquad \tau' \sim \tau}{\Gamma \vdash e \Leftarrow \tau}$$

2.2 Typed Elaboration

Theorem 3 (Synthetic Typed Elaboration Validity). *If* $\Gamma \vdash e \Rightarrow \tau \leadsto d \dashv \Delta$ *then* $\emptyset \vdash \tau\ \mathsf{valid}$.

$\boxed{\Gamma \vdash e \Rightarrow \tau \leadsto d \dashv \Delta}$ e synthesizes type τ and elaborates to d

$$\textsc{ESUnit}\ \frac{}{\Gamma \vdash \varnothing \Rightarrow \varnothing \leadsto \varnothing \dashv \emptyset} \qquad \textsc{ESVar}\ \frac{x : \tau \in \Gamma}{\Gamma \vdash x \Rightarrow \tau \leadsto x \dashv \emptyset} \qquad \textsc{ESVarFree}\ \frac{x \notin \mathsf{dom}(\Gamma)}{\Gamma \vdash \llparenthesis x \rrparenthesis^u \Rightarrow \llparenthesis\rrparenthesis \leadsto \llparenthesis x \rrparenthesis^u_{\mathsf{id}(\Gamma)} \dashv u :: \llparenthesis\rrparenthesis[\Gamma]}$$

$$\textsc{ESLam}\ \frac{\emptyset \vdash \tau\ \mathsf{valid} \qquad \Gamma, x : \tau \vdash e \Rightarrow \tau' \leadsto d \dashv \Delta}{\Gamma \vdash \lambda x{:}\tau.e \Rightarrow \tau \to \tau' \leadsto \lambda x{:}\tau.d \dashv \Delta}$$

$$\textsc{ESApp}\ \frac{\Gamma \vdash e_1 \Rightarrow \tau_1 \qquad \tau_1 \blacktriangleright_{\to} \tau_2 \to \tau \qquad \Gamma \vdash e_1 \Leftarrow \tau_2 \to \tau \leadsto d_1 : \tau_1' \dashv \Delta_1 \qquad \Gamma \vdash e_2 \Leftarrow \tau_2 \leadsto d_2 : \tau_2' \dashv \Delta_2}{\Gamma \vdash e_1(e_2) \Rightarrow \tau \leadsto (d_1\langle \tau_1' \Rightarrow \tau_1 \rangle)(d_2\langle \tau_2' \Rightarrow \tau_2 \rangle) \dashv \Delta_1 \cup \Delta_2}$$

ESAppNotArr

$$\frac{\Gamma \vdash e_1 \Rightarrow \tau_1 \leadsto d_1 \dashv \Delta_1 \qquad \tau_1 \nsim (\!|\,|\!) \to (\!|\,|\!) \qquad \Gamma \vdash e_2 \Leftarrow (\!|\,|\!) \leadsto d_2 : \tau_2' \dashv \Delta_2}{\Gamma \vdash (\!|e_1|\!)^{u\blacktriangleright}(e_2) \Rightarrow (\!|\,|\!) \leadsto (\!|d_1|\!)^{u\blacktriangleright}_{\mathsf{id}(\Gamma)}(d_2\langle \tau_2' \Rightarrow (\!|\,|\!)\rangle) \dashv \Delta_1 \cup \Delta_2, u :: (\!|\,|\!) \to (\!|\,|\!)[\Gamma]}$$

ESAsc

$$\frac{\emptyset \vdash \tau\ \mathsf{valid} \qquad \Gamma \vdash e \Leftarrow \tau \leadsto d : \tau' \dashv \Delta}{\Gamma \vdash e : \tau \Rightarrow \tau \leadsto d\langle \tau' \Rightarrow \tau \rangle \dashv \Delta}$$

ESRollErr

$$\frac{\Gamma \vdash e \Leftarrow (\!|\,|\!) \leadsto d : \tau \dashv \Delta}{\Gamma \vdash (\!|\mathtt{roll}(e)|\!)^u \Rightarrow \mu(\!|\,|\!).(\!|\,|\!) \leadsto (\!|\mathtt{roll}^{\mu(\!|\,|\!).(\!|\,|\!)}(d\langle \tau \Rightarrow (\!|\,|\!)\rangle)|\!)^u_{\mathsf{id}(\Gamma)} \dashv \Delta, u :: \mu(\!|\,|\!).(\!|\,|\!)[\Gamma]}$$

ESUnroll

$$\frac{\Gamma \vdash e \Rightarrow \tau \leadsto d \dashv \Delta \qquad \tau \blacktriangleright_\mu \mu\pi.\tau'}{\Gamma \vdash \mathtt{unroll}(e) \Rightarrow [\mu\pi.\tau'/\pi]\tau' \leadsto \mathtt{unroll}(d\langle \tau \Rightarrow \mu\pi.\tau'\rangle) \dashv \Delta}$$

ESUnrollNotRec

$$\frac{\Gamma \vdash e \Rightarrow \tau \leadsto d \dashv \Delta \qquad \tau \nsim \mu(\!|\,|\!).(\!|\,|\!)}{\Gamma \vdash \mathtt{unroll}\big((\!|e|\!)^{u\blacktriangleright}\big) \Rightarrow (\!|\,|\!) \leadsto \mathtt{unroll}\Big((\!|d|\!)^{u\blacktriangleright}_{\mathsf{id}(\Gamma)}\Big) \dashv \Delta, u :: \mu(\!|\,|\!).(\!|\,|\!)[\Gamma]}$$

ESInj

$$\frac{\Gamma \vdash e \Rightarrow \tau \leadsto d \dashv \Delta \qquad \tau' = +\{C(\tau); ...\}}{\Gamma \vdash \mathtt{inj}_C(e) \Rightarrow \tau' \leadsto \mathtt{inj}^{\tau'}_C(d) \dashv \Delta}$$

ESEHole

$$\frac{}{\Gamma \vdash (\!|\,|\!)^u \Rightarrow (\!|\,|\!) \leadsto (\!|\,|\!)^u_{\mathsf{id}(\Gamma)} \dashv u :: (\!|\,|\!)[\Gamma]}$$

ESNEHole

$$\frac{\Gamma \vdash e \Rightarrow \tau \leadsto d \dashv \Delta}{\Gamma \vdash (\!|e|\!)^u \Rightarrow (\!|\,|\!) \leadsto (\!|d|\!)^u_{\mathsf{id}(\Gamma)} \dashv \Delta, u :: (\!|\,|\!)[\Gamma]}$$

$\boxed{\Gamma \vdash e \Leftarrow \tau_1 \leadsto d : \tau_2 \dashv \Delta}$ e analyzes against type τ_1 and elaborates to d of consistent type τ_2

EARoll

$$\frac{\tau \blacktriangleright_\mu \mu\pi.\tau' \qquad \Gamma \vdash e \Leftarrow [\mu\pi.\tau'/\pi]\tau' \leadsto d : \tau'' \dashv \Delta}{\Gamma \vdash \mathtt{roll}(e) \Leftarrow \tau \leadsto \mathtt{roll}^{\mu\pi.\tau'}(d\langle \tau'' \Rightarrow [\mu\pi.\tau'/\pi]\tau'\rangle) : \mu\pi.\tau' \dashv \Delta}$$

EARollNotRec

$$\frac{\tau \nsim \mu(\!|\,|\!).(\!|\,|\!) \qquad \Gamma \vdash e \Leftarrow (\!|\,|\!) \leadsto d : \tau' \dashv \Delta}{\Gamma \vdash (\!|\mathtt{roll}(e)|\!)^u \Leftarrow \tau \leadsto (\!|\mathtt{roll}^{\mu(\!|\,|\!).(\!|\,|\!)}(d)|\!)^u_{\mathsf{id}(\Gamma)} : \mu(\!|\,|\!).(\!|\,|\!) \dashv \Delta, u :: \mu(\!|\,|\!).(\!|\,|\!)[\Gamma]}$$

EAInjHole

$$\frac{\Gamma \vdash e \Leftarrow (\!|\,|\!) \leadsto d : \tau \dashv \Delta \qquad \tau' = +\{C(\tau); ...\}}{\Gamma \vdash \mathtt{inj}_C(e) \Leftarrow (\!|\,|\!) \leadsto \mathtt{inj}^{\tau'}_C(d) : \tau' \dashv \Delta}$$

EAInj

$$\frac{\tau \blacktriangleright_+ \tau' \qquad \tau' = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C_j \in \mathcal{C} \qquad \Gamma \vdash e \Leftarrow \tau_j \leadsto d : \tau_j' \dashv \Delta}{\Gamma \vdash \mathtt{inj}_{C_j}(e) \Leftarrow \tau \leadsto \mathtt{inj}^{\tau'}_{C_j}\big(d\langle \tau_j' \Rightarrow \tau_j\rangle\big) : \tau \dashv \Delta}$$

EAInjTagErr

$$\frac{\tau \blacktriangleright_+ +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C \notin \mathcal{C} \qquad \Gamma \vdash e \Leftarrow (\!|\,|\!) \leadsto d : \tau' \dashv \Delta \qquad \tau'' = +\{C(\tau'); ...\}}{\Gamma \vdash (\!|\mathtt{inj}_C(e)|\!)^u \Leftarrow \tau \leadsto (\!|\mathtt{inj}^{\tau''}_C(d)\langle \tau' \Rightarrow (\!|\,|\!)\rangle|\!)^u_{\mathsf{id}(\Gamma)} : (\!|\,|\!) \dashv \Delta, u :: \tau[\Gamma]}$$

$$\textsc{EAInjUnexpectedArg1}\quad \frac{\begin{array}{c}\tau = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C_j \in \mathcal{C} \qquad \tau_j = \varnothing \qquad e \neq \varnothing \\ \Gamma \vdash e \Leftarrow \llparenthesis\rrparenthesis \rightsquigarrow d : \tau'_j \dashv \Delta \qquad \tau' = +\left\{\{C_i(\tau_i)\}_{C_i \in \mathcal{C} \setminus C_j} \cup \{C_j(\tau'_j)\}\right\}\end{array}}{\Gamma \vdash \llparenthesis \mathtt{inj}_{C_j}(e) \rrparenthesis^u \Leftarrow \tau \rightsquigarrow \llparenthesis \mathtt{inj}^{\tau'}_{C_j}\big(d\langle \tau'_j \Rightarrow \varnothing\rangle\big)\rrparenthesis^u_{\mathsf{id}(\Gamma)} : \tau \dashv \Delta, u :: \tau[\Gamma]}$$

$$\textsc{EAInjUnexpectedArg2}\quad \frac{\tau = +\{C(\varnothing); ...\} \qquad e \neq \varnothing \qquad \Gamma \vdash e \Leftarrow \llparenthesis\rrparenthesis \rightsquigarrow d : \tau' \dashv \Delta \qquad \tau'' = +\{C(\tau'); ...\}}{\Gamma \vdash \llparenthesis \mathtt{inj}_{C}(e) \rrparenthesis^u \Leftarrow \tau \rightsquigarrow \llparenthesis \mathtt{inj}^{\tau''}_{C}(d\langle \tau' \Rightarrow \varnothing\rangle)\rrparenthesis^u_{\mathsf{id}(\Gamma)} : \tau \dashv \Delta, u :: \tau[\Gamma]}$$

$$\textsc{EAInjExpectedArg1}\quad \frac{\tau = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C_j \in \mathcal{C} \qquad \tau_j \neq \varnothing \qquad \tau' = +\left\{\{C_i(\tau_i)\}_{C_i \in \mathcal{C} \setminus C_j} \cup \{C_j(\varnothing)\}\right\}}{\Gamma \vdash \llparenthesis \mathtt{inj}_{C_j}(\varnothing) \rrparenthesis^u \Leftarrow \tau \rightsquigarrow \llparenthesis \mathtt{inj}^{\tau'}_{C_j}(\varnothing\langle \varnothing \Rightarrow \llparenthesis\rrparenthesis\rangle)\rrparenthesis^u_{\mathsf{id}(\Gamma)} : \tau \dashv \Delta, u :: \tau[\Gamma]}$$

$$\textsc{EAInjExpectedArg2}\quad \frac{\tau = +\{C(\tau'); ...\} \qquad \tau' \neq \varnothing \qquad \tau'' = +\{C(\varnothing); ...\}}{\Gamma \vdash \llparenthesis \mathtt{inj}_{C}(\varnothing) \rrparenthesis^u \Leftarrow \tau \rightsquigarrow \llparenthesis \mathtt{inj}^{\tau''}_{C}(\varnothing\langle \varnothing \Rightarrow \llparenthesis\rrparenthesis\rangle)\rrparenthesis^u_{\mathsf{id}(\Gamma)} : \tau \dashv \Delta, u :: \tau[\Gamma]}$$

$$\textsc{EASubsume}\quad \frac{e \neq \llparenthesis\rrparenthesis^u \qquad e \neq \llparenthesis e' \rrparenthesis^u \qquad \Gamma \vdash e \Rightarrow \tau' \rightsquigarrow d \dashv \Delta \qquad \tau \sim \tau'}{\Gamma \vdash e \Leftarrow \tau \rightsquigarrow d : \tau' \dashv \Delta}$$

$$\textsc{EAEHole}\quad \frac{}{\Gamma \vdash \llparenthesis\rrparenthesis^u \Leftarrow \tau \rightsquigarrow \llparenthesis\rrparenthesis^u_{\mathsf{id}(\Gamma)} : \tau \dashv u :: \tau[\Gamma]}$$

$$\textsc{EANEHole}\quad \frac{\Gamma \vdash e \Rightarrow \tau' \rightsquigarrow d \dashv \Delta}{\Gamma \vdash \llparenthesis e \rrparenthesis^u \Leftarrow \tau \rightsquigarrow \llparenthesis d \rrparenthesis^u_{\mathsf{id}(\Gamma)} : \tau \dashv u :: \tau[\Gamma]}$$

2.3 Type Assignment

$\boxed{\Delta; \Gamma \vdash d : \tau}$ d is assigned type τ

$$\textsc{TAUnit}\quad \frac{}{\Delta; \Gamma \vdash \varnothing : \varnothing} \qquad \textsc{TAVar}\quad \frac{x : \tau \in \Gamma}{\Delta; \Gamma \vdash x : \tau} \qquad \textsc{TALam}\quad \frac{\emptyset \vdash \tau\ \mathsf{valid} \qquad \Delta; \Gamma, x : \tau \vdash d : \tau'}{\Delta; \Gamma \vdash \lambda x{:}\tau.d : \tau \to \tau'}$$

$$\textsc{TAApp}\quad \frac{\Delta; \Gamma \vdash d_1 : \tau_2 \to \tau \qquad \Delta; \Gamma \vdash d_2 : \tau_2}{\Delta; \Gamma \vdash d_1(d_2) : \tau}$$

$$\textsc{TARoll}\quad \frac{\emptyset \vdash \mu\pi.\tau\ \mathsf{valid} \qquad \Delta; \Gamma \vdash d : [\mu\pi.\tau/\pi]\tau}{\Delta; \Gamma \vdash \mathtt{roll}^{\mu\pi.\tau}(d) : \mu\pi.\tau} \qquad \textsc{TAUnroll}\quad \frac{\Delta; \Gamma \vdash d : \mu\pi.\tau}{\Delta; \Gamma \vdash \mathtt{unroll}(d) : [\mu\pi.\tau/\pi]\tau}$$

$$\textsc{TAInj}\quad \frac{\tau \blacktriangleright_{+} +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad C_j \in \mathcal{C} \qquad \Delta; \Gamma \vdash d : \tau_j}{\Delta; \Gamma \vdash \mathtt{inj}^{\tau}_{C_j}(d) : \tau} \qquad \textsc{TAEHole}\quad \frac{u :: \tau[\Gamma'] \in \Delta \qquad \Delta; \Gamma \vdash \sigma : \Gamma'}{\Delta; \Gamma \vdash \llparenthesis\rrparenthesis^u_{\sigma} : \tau}$$

$$\textsc{TANEHole}\quad \frac{\Delta; \Gamma \vdash d : \tau' \qquad u :: \tau[\Gamma'] \in \Delta \qquad \Delta; \Gamma \vdash \sigma : \Gamma'}{\Delta; \Gamma \vdash \llparenthesis d \rrparenthesis^u_{\sigma} : \tau} \qquad \textsc{TAMHole}\quad \frac{\Delta; \Gamma \vdash d : \tau' \qquad u :: \tau[\Gamma'] \in \Delta \qquad \Delta; \Gamma \vdash \sigma : \Gamma'}{\Delta; \Gamma \vdash \llparenthesis d \rrparenthesis^{u\blacktriangleright}_{\sigma} : \tau}$$

$$\textsc{TACast}\quad \frac{\Delta; \Gamma \vdash d : \tau_1 \qquad \tau_1 \sim \tau_2}{\Delta; \Gamma \vdash d\langle \tau_1 \Rightarrow \tau_2 \rangle : \tau_2} \qquad \textsc{TAFailedCast}\quad \frac{\Delta; \Gamma \vdash d : \tau_1 \qquad \tau_1\ \text{ground} \qquad \tau_2\ \text{ground} \qquad \tau_1 \neq \tau_2}{\Delta; \Gamma \vdash d\langle \tau_1 \Rightarrow \llparenthesis\rrparenthesis \not\Rightarrow \tau_2 \rangle : \tau_2}$$

3 Dynamic Semantics

$\boxed{\tau \text{ ground}}$ τ is a ground type

GArr
$$\frac{}{\llparenthesis\rrparenthesis \to \llparenthesis\rrparenthesis \text{ ground}}$$

GRec
$$\frac{}{\mu\llparenthesis\rrparenthesis.\llparenthesis\rrparenthesis \text{ ground}}$$

GSum
$$\frac{\tau \blacktriangleright_{+} +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \qquad \{\tau_i = \varnothing \lor \tau_i = \llparenthesis\rrparenthesis\}_{C_i \in \mathcal{C}}}{\tau \text{ ground}}$$

$\boxed{\tau \blacktriangleright_{\mathsf{ground}} \tau'}$ τ has matched ground type τ'

MGArr
$$\frac{\tau_1 \to \tau_2 \neq \llparenthesis\rrparenthesis \to \llparenthesis\rrparenthesis}{\tau_1 \to \tau_2 \blacktriangleright_{\mathsf{ground}} \llparenthesis\rrparenthesis \to \llparenthesis\rrparenthesis}$$

MGRec
$$\frac{\tau \neq \llparenthesis\rrparenthesis}{\mu\pi.\tau \blacktriangleright_{\mathsf{ground}} \mu\llparenthesis\rrparenthesis.\llparenthesis\rrparenthesis}$$

MGSum1
$$\frac{\begin{array}{c}\{(\tau_i = \varnothing \implies \tau_i' = \varnothing) \land (\tau_i \neq \varnothing \implies \tau_i' = \llparenthesis\rrparenthesis)\}_{C_i \in \mathcal{C}} \\ +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \neq +\{C_i(\tau_i')\}_{C_i \in \mathcal{C}}\end{array}}{+\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \blacktriangleright_{\mathsf{ground}} +\{C_i(\tau_i')\}_{C_i \in \mathcal{C}}}$$

MGSum2
$$\frac{\tau \neq \varnothing \qquad \tau \neq \llparenthesis\rrparenthesis}{+\{C(\tau);\ldots\} \blacktriangleright_{\mathsf{ground}} +\{C(\llparenthesis\rrparenthesis);\ldots\}}$$

$\boxed{d \text{ final}}$ d is final

FBoxedVal
$$\frac{d \text{ boxedval}}{d \text{ final}}$$

FIndet
$$\frac{d \text{ indet}}{d \text{ final}}$$

$\boxed{d \text{ val}}$ d is a value

VUnit
$$\frac{}{\varnothing \text{ val}}$$

VLam
$$\frac{}{\lambda x{:}\tau.d \text{ val}}$$

VRoll
$$\frac{d \text{ val}}{\mathtt{roll}^{\mu\pi.\tau}(d) \text{ val}}$$

VInj
$$\frac{d \text{ val}}{\mathtt{inj}_{\mathbf{C}}^{\tau}(d) \text{ val}}$$

$\boxed{d \text{ boxedval}}$ d is a boxed value

BVVal
$$\frac{d \text{ val}}{d \text{ boxedval}}$$

BVRoll
$$\frac{d \text{ boxedval}}{\mathtt{roll}^{\mu\pi.\tau}(d) \text{ boxedval}}$$

BVInj
$$\frac{d \text{ boxedval}}{\mathtt{inj}_{\mathbf{C}}^{\tau}(d) \text{ boxedval}}$$

BVArrCast
$$\frac{\tau_1 \to \tau_2 \neq \tau_3 \to \tau_4 \qquad d \text{ boxedval}}{d\langle \tau_1 \to \tau_2 \Rightarrow \tau_3 \to \tau_4 \rangle \text{ boxedval}}$$

BVRecCast
$$\frac{\mu\pi.\tau \neq \mu\pi'.\tau' \qquad d \text{ boxedval}}{d\langle \mu\pi.\tau \Rightarrow \mu\pi'.\tau' \rangle \text{ boxedval}}$$

BVSumCast1
$$\frac{\begin{array}{c}\tau = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \\ \tau' = +\{C_i(\tau_i')\}_{C_i \in \mathcal{C}'} \\ \tau \neq \tau' \qquad d \text{ boxedval}\end{array}}{d\langle \tau \Rightarrow \tau' \rangle \text{ boxedval}}$$

BVSumCast2
$$\frac{\begin{array}{c}\tau = +\{C(\tau_1);\ldots\} \\ \tau' = +\{C(\tau_1');\ldots\} \\ \tau_1 \neq \tau_1' \qquad d \text{ boxedval}\end{array}}{d\langle \tau \Rightarrow \tau' \rangle \text{ boxedval}}$$

BVSumCast12
$$\frac{\begin{array}{c}\tau = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \\ \tau' = +\{C_j(\tau_j);\ldots\} \\ C_j \in \mathcal{C} \qquad \tau_j \neq \tau_i \qquad d \text{ boxedval}\end{array}}{d\langle \tau \Rightarrow \tau' \rangle \text{ boxedval}}$$

BVSumCast21
$$\frac{\begin{array}{c}\tau = +\{C_j(\tau_j);\ldots\} \\ \tau' = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \\ C_j \in \mathcal{C} \qquad \tau_i \neq \tau_j \qquad d \text{ boxedval}\end{array}}{d\langle \tau \Rightarrow \tau' \rangle \text{ boxedval}}$$

BVHoleCast
$$\frac{d \text{ boxedval} \qquad \tau \text{ ground}}{d\langle \tau \Rightarrow \llparenthesis\rrparenthesis \rangle \text{ boxedval}}$$

$\boxed{d \text{ indet}}$ d is indeterminate

IEHole
$$\frac{}{⦇⦈^u_\sigma \text{ indet}}$$

INEHole
$$\frac{d \text{ final}}{⦇d⦈^u_\sigma \text{ indet}}$$

IApp
$$\frac{d_1 \neq d_1'\langle \tau_1 \to \tau_2 \Rightarrow \tau_3 \to \tau_4 \rangle \qquad d_1 \text{ indet} \qquad d_2 \text{ final}}{d_1(d_2) \text{ indet}}$$

IRoll
$$\frac{d \text{ indet}}{\texttt{roll}^{\mu\pi.\tau}(d) \text{ indet}}$$

IUnroll
$$\frac{d \text{ indet}}{\texttt{unroll}(d) \text{ indet}}$$

IInj
$$\frac{d \text{ indet}}{\texttt{inj}^\tau_{\mathbf{C}}(d) \text{ indet}}$$

IInjHole
$$\frac{C \neq \mathbf{C} \qquad d \text{ final}}{\texttt{inj}^\tau_C(d) \text{ indet}}$$

ICastGroundHole
$$\frac{d \text{ indet} \qquad \tau \text{ ground}}{d\langle \tau \Rightarrow ⦇⦈ \rangle \text{ indet}}$$

ICastHoleGround
$$\frac{d \neq d'\langle \tau' \Rightarrow ⦇⦈ \rangle \qquad d \text{ indet} \qquad \tau \text{ ground}}{d\langle ⦇⦈ \Rightarrow \tau \rangle \text{ indet}}$$

ICastArr
$$\frac{\tau_1 \to \tau_2 \neq \tau_3 \to \tau_4 \qquad d \text{ indet}}{d\langle \tau_1 \to \tau_2 \Rightarrow \tau_3 \to \tau_4 \rangle \text{ indet}}$$

ICastRec
$$\frac{\mu\pi.\tau \neq \mu\pi'.\tau' \qquad d \text{ indet}}{d\langle \mu\pi.\tau \Rightarrow \mu\pi'.\tau' \rangle \text{ indet}}$$

ICastSum1
$$\frac{\begin{array}{c}\tau = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \\ \tau' = +\{C_i(\tau_i')\}_{C_i \in \mathcal{C}} \\ \tau \neq \tau' \qquad d \text{ indet}\end{array}}{d\langle \tau \Rightarrow \tau' \rangle \text{ indet}}$$

ICastSum2
$$\frac{\begin{array}{c}\tau = +\{C(\tau_1); \ldots\} \\ \tau' = +\{C(\tau_1'); \ldots\} \\ \tau_1 \neq \tau_1' \qquad d \text{ indet}\end{array}}{d\langle \tau \Rightarrow \tau' \rangle \text{ indet}}$$

ICastSum12
$$\frac{\begin{array}{c}\tau = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \\ \tau' = +\{C_j(\tau_j); \ldots\} \\ C_j \in \mathcal{C} \qquad \tau_j \neq \tau_i \qquad d \text{ indet}\end{array}}{d\langle \tau \Rightarrow \tau' \rangle \text{ indet}}$$

ICastSum21
$$\frac{\begin{array}{c}\tau = +\{C_j(\tau_j); \ldots\} \\ \tau' = +\{C_i(\tau_i)\}_{C_i \in \mathcal{C}} \\ C_j \in \mathcal{C} \qquad \tau_i \neq \tau_j \qquad d \text{ indet}\end{array}}{d\langle \tau \Rightarrow \tau' \rangle \text{ indet}}$$

IFailedCast
$$\frac{d \text{ final} \qquad \tau_1 \text{ ground} \qquad \tau_2 \text{ ground} \qquad \tau_1 \nsim \tau_2}{d\langle \tau_1 \Rightarrow ⦇⦈ \not\Rightarrow \tau_2 \rangle \text{ indet}}$$

$\boxed{d \longrightarrow d'}$ d takes an instruction transition to d'

ITApp
$$\frac{[d_2 \text{ final}]}{(\lambda x{:}\tau.d_1)(d_2) \longrightarrow [d_2/x]d_1}$$

ITUnroll
$$\frac{[d \text{ final}]}{\texttt{unroll}(\texttt{roll}^{\mu\pi.\tau}(d)) \longrightarrow d}$$

ITAppCast
$$\frac{[d_1 \text{ final}] \qquad [d_2 \text{ final}] \qquad \tau_1 \to \tau_2 \neq \tau_1' \to \tau_2'}{d_1\langle \tau_1 \to \tau_2 \Rightarrow \tau_1' \to \tau_2' \rangle(d_2) \longrightarrow (d_1(d_2\langle \tau_1' \Rightarrow \tau_1 \rangle))\langle \tau_2 \Rightarrow \tau_2' \rangle}$$

ITUnrollCast
$$\frac{[d \text{ final}] \qquad \mu\pi.\tau \neq \mu\pi'.\tau'}{\texttt{unroll}(d\langle \mu\pi.\tau \Rightarrow \mu\pi'.\tau' \rangle) \longrightarrow \texttt{unroll}(d)\langle [\mu\pi.\tau/\pi]\tau \Rightarrow [\mu\pi'.\tau'/\pi']\tau' \rangle}$$

ITCastId
$$\frac{[d \text{ final}]}{d\langle \tau \Rightarrow \tau \rangle \longrightarrow d}$$

ITCastSucceed
$$\frac{[d \text{ final}] \qquad \tau \text{ ground}}{d\langle \tau \Rightarrow ⦇⦈ \Rightarrow \tau \rangle \longrightarrow d}$$

ITCastFail
$$\frac{[d \text{ final}] \qquad \tau_1 \neq \tau_2 \qquad \tau_1 \text{ ground} \qquad \tau_2 \text{ ground}}{d\langle \tau_1 \Rightarrow ⦇⦈ \Rightarrow \tau_2 \rangle \longrightarrow d\langle \tau_1 \Rightarrow ⦇⦈ \not\Rightarrow \tau_2 \rangle}$$

ITGround
$$\frac{[d \text{ final}] \qquad \tau \blacktriangleright_{\mathsf{ground}} \tau'}{d\langle \tau \Rightarrow ⦇⦈ \rangle \longrightarrow d\langle \tau \Rightarrow \tau' \Rightarrow ⦇⦈ \rangle}$$

ITExpand
$$\frac{[d \text{ final}] \qquad \tau \blacktriangleright_{\mathsf{ground}} \tau'}{d\langle ⦇⦈ \Rightarrow \tau \rangle \longrightarrow d\langle ⦇⦈ \Rightarrow \tau' \Rightarrow \tau \rangle}$$

$$\begin{array}{lcl}\mathsf{EvalCtx} \ \mathcal{E} & ::= & \circ \mid \mathcal{E}(d) \mid d(\mathcal{E}) \mid \texttt{roll}^{\mu\pi.\tau}(\mathcal{E}) \mid \texttt{unroll}(\mathcal{E}) \mid \texttt{inj}^\tau_C(\mathcal{E}) \mid ⦇\mathcal{E}⦈^u_\sigma \mid ⦇\mathcal{E}⦈^{u\blacktriangleright}_\sigma \\ & & \mid \mathcal{E}\langle \tau \Rightarrow \tau \rangle \mid \mathcal{E}\langle \tau \Rightarrow ⦇⦈ \not\Rightarrow \tau \rangle\end{array}$$

$\boxed{d = \mathcal{E}\{d'\}}$ d is obtained by placing d' at the mark in $\mathcal{E}$

$$\textsc{FHOuter}\ \frac{}{d = \circ\{d\}} \qquad \textsc{FHApp1}\ \frac{d_1 = \mathcal{E}\{d_1'\}}{d_1(d_2) = \mathcal{E}(d_2)\{d_1'\}} \qquad \textsc{FHApp2}\ \frac{[d_1 \text{ final}] \qquad d_2 = \mathcal{E}\{d_2'\}}{d_1(d_2) = d_1(\mathcal{E})\{d_2'\}}$$

$$\textsc{FHRoll}\ \frac{d = \mathcal{E}\{d'\}}{\texttt{roll}^{\mu\pi.\tau}(d) = \texttt{roll}^{\mu\pi.\tau}(\mathcal{E})\{d'\}}$$

$$\textsc{FHUnroll}\ \frac{d = \mathcal{E}\{d'\}}{\texttt{unroll}(d) = \texttt{unroll}(\mathcal{E})\{d'\}} \qquad \textsc{FHInj}\ \frac{d = \mathcal{E}\{d'\}}{\texttt{inj}_C^{\tau}(d) = \texttt{inj}_C^{\tau}(\mathcal{E})\{d'\}}$$

$$\textsc{FNEHoleInside}\ \frac{d = \mathcal{E}\{d'\}}{(\!|d|\!)^u_\sigma = (\!|\mathcal{E}|\!)^u_\sigma\{d'\}} \qquad \textsc{FMHoleInside}\ \frac{d = \mathcal{E}\{d'\}}{(\!|d|\!)^{u\blacktriangleright}_\sigma = (\!|\mathcal{E}|\!)^{u\blacktriangleright}_\sigma\{d'\}}$$

$$\textsc{FHCastInside}\ \frac{d = \mathcal{E}\{d'\}}{d\langle\tau_1 \Rightarrow \tau_2\rangle = \mathcal{E}\langle\tau_1 \Rightarrow \tau_2\rangle\{d'\}} \qquad \textsc{FHFailedCast}\ \frac{d = \mathcal{E}\{d'\}}{d\langle\tau_1 \Rightarrow (\!|\,|\!) \not\Rightarrow \tau_2\rangle = \mathcal{E}\langle\tau_1 \Rightarrow (\!|\,|\!) \not\Rightarrow \tau_2\rangle\{d'\}}$$

$\boxed{d \mapsto d'}$ d steps to d'

$$\textsc{Step}\ \frac{d = \mathcal{E}\{d_0\} \qquad d_0 \longrightarrow d_0' \qquad d' = \mathcal{E}\{d_0'\}}{d \mapsto d'}$$