# Hazel Phi: 9-type-aliases

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## SYNTAX

$$
\begin{array}{rlll}
\text{BinOp} & \oplus & ::= & \times \mid + \mid \to \\
\text{Kind} & \kappa & ::= & \mathtt{Type} \mid \mathtt{KHole} \mid \mathtt{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2 \\
\text{Base Types} & \mathtt{bse} & ::= & \mathtt{Int} \mid \mathtt{Float} \mid \mathtt{Bool} \\
\text{User Types} & \hat{\tau} & ::= & t \mid \mathtt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!|\,|\!)^{\mathtt{u}} \mid (\!|\hat{\tau}|\!)^{\mathtt{u}} \mid \lambda t{::}\mathtt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2} \\
\text{Internal Types} & \tau & ::= & t \mid \mathtt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!|\,|\!)^{\mathtt{u}} \mid (\!|\tau|\!)^{\mathtt{u}} \mid \lambda t{::}\kappa.\tau \mid \tau_1\ \tau_2 \\
\text{Type Pattern} & & & \\
\text{User Expression} & & & \\
\text{Internal Expression} & & &
\end{array}
$$

## Declaratives

$\boxed{\Delta;\Phi \vdash \tau :: > \kappa}$ $\tau$ has principal kind $\kappa$

$$\frac{}{\Delta;\Phi \vdash \mathtt{bse} ::> \mathtt{S}_{\mathtt{Type}}(\mathtt{bse})} \qquad \frac{t{::}\kappa \in \Phi}{\Delta;\Phi \vdash t ::> \mathtt{S}_{\kappa}(t)} \qquad \frac{\Delta;\Phi \vdash \tau_1{::}\mathtt{Type} \qquad \Delta;\Phi \vdash \tau_2{::}\mathtt{Type}}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 ::> \mathtt{S}_{\mathtt{Type}}(\tau_1 \oplus \tau_2)}$$

$$\frac{\mathrm{u}{::}\kappa \in \Delta}{\Delta;\Phi \vdash (\!|\,|\!)^{\mathtt{u}} ::> \kappa} \qquad \frac{\mathrm{u}{::}\kappa \in \Delta \qquad \Delta;\Phi \vdash \tau{::}\kappa_1}{\Delta;\Phi \vdash (\!|\tau|\!)^{\mathtt{u}} ::> \kappa} \qquad \frac{\mathrm{u}{::}\kappa \in \Delta \qquad t \notin \mathsf{dom}(\Phi)}{\Delta;\Phi \vdash (\!|t|\!)^{\mathtt{u}} ::> \kappa}$$

$$\frac{\Delta;\Phi, t{::}\kappa_1 \vdash \tau ::> \kappa_2}{\Delta;\Phi \vdash \lambda t{::}\kappa_1.\tau ::> \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}$$

$$\frac{\Delta;\Phi \vdash \tau_1 ::> \kappa \qquad \Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta;\Phi \vdash \tau_2{::}\kappa_1}{\Delta;\Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}$$

$\boxed{\Delta;\Phi\vdash\tau{::}\kappa}$ $\tau$ is well formed at kind $\kappa$

$$\frac{\Delta;\Phi\vdash\tau ::> \kappa}{\Delta;\Phi\vdash\tau{::}\kappa}\qquad\frac{\Delta;\Phi\vdash\tau{::}\mathtt{S}_\kappa(\tau_1)}{\Delta;\Phi\vdash\tau{::}\kappa}\qquad\frac{\Delta;\Phi\vdash\tau ::> \kappa_1\qquad\Delta;\Phi\vdash\kappa_1\lesssim\kappa}{\Delta;\Phi\vdash\tau{::}\kappa}$$

$$\frac{\Delta;\Phi\vdash\tau_2{::}\mathtt{S}_\kappa(\tau_1)}{\Delta;\Phi\vdash\tau_1{::}\mathtt{S}_\kappa(\tau_2)}\qquad\frac{\Delta;\Phi\vdash\tau_1{::}\mathtt{S}_\kappa(\tau_3)\qquad\Delta;\Phi\vdash\tau_3{::}\mathtt{S}_\kappa(\tau_2)}{\Delta;\Phi\vdash\tau_1{::}\mathtt{S}_\kappa(\tau_2)}$$

$\boxed{\Delta;\Phi\vdash\kappa\overset{\blacktriangleright}{_\Pi}\Pi_{t::\kappa_1}.\kappa_2}$ $\kappa$ has matched $\Pi$-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{}{\Delta;\Phi\vdash\mathtt{KHole}\overset{\blacktriangleright}{_\Pi}\Pi_{t::\mathtt{KHole}}.\mathtt{KHole}}\qquad\frac{\Delta;\Phi\vdash\kappa\equiv\Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi\vdash\kappa\overset{\blacktriangleright}{_\Pi}\Pi_{t::\kappa_1}.\kappa_2}$$

$\boxed{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2}$ $\kappa_1$ is equivalent to $\kappa_2$

$$\frac{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta;\Phi\vdash\kappa\equiv\kappa}\qquad\frac{\Delta;\Phi\vdash\kappa_2\equiv\kappa_1}{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2}\qquad\frac{\Delta;\Phi\vdash\kappa_1\equiv\kappa_3\qquad\Delta;\Phi\vdash\kappa_3\equiv\kappa_2}{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2}$$

$$\frac{\Delta;\Phi\vdash\tau_1\overset{\kappa}{\equiv}\tau_2}{\Delta;\Phi\vdash\mathtt{S}_\kappa(\tau_1)\equiv\mathtt{S}_\kappa(\tau_2)}\qquad\frac{\Delta;\Phi\vdash\tau{::}\mathtt{S}_\kappa(\tau_1)}{\Delta;\Phi\vdash\mathtt{S}_{\mathtt{S}_\kappa(\tau_1)}(\tau)\equiv\mathtt{S}_\kappa(\tau_1)}\qquad\frac{\Delta;\Phi\vdash\tau{::}\Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi\vdash\mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau)\equiv\Pi_{t::\kappa_1}.\mathtt{S}_{\kappa_2}(\tau\ t)}$$

$$\frac{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2\qquad\Delta;\Phi,t{::}\kappa_1\vdash\kappa_3\equiv\kappa_4}{\Delta;\Phi\vdash\Pi_{t::\kappa_1}.\kappa_2\equiv\Pi_{t::\kappa_3}.\kappa_4}$$

$\boxed{\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2}$ $\kappa_1$ is a consistent subkind of $\kappa_2$

$$\frac{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta;\Phi\vdash\mathtt{KHole}\lesssim\kappa}\qquad\frac{\Delta;\Phi\vdash\kappa\ \mathsf{OK}}{\Delta;\Phi\vdash\kappa\lesssim\mathtt{KHole}}$$

$$\frac{\Delta;\Phi\vdash\kappa_1\equiv\kappa_2}{\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2}\qquad\frac{\Delta;\Phi\vdash\kappa_1\equiv\kappa_3\qquad\Delta;\Phi\vdash\kappa_3\lesssim\kappa_4\qquad\Delta;\Phi\vdash\kappa_4\equiv\kappa_2}{\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2}$$

$$\frac{\Delta;\Phi\vdash\tau{::}\kappa}{\Delta;\Phi\vdash\mathtt{S}_\kappa(\tau)\lesssim\kappa}\qquad\frac{\Delta;\Phi\vdash\kappa_3\lesssim\kappa_1\qquad\Delta;\Phi,t{::}\kappa_3\vdash\kappa_2\lesssim\kappa_4}{\Delta;\Phi\vdash\Pi_{t::\kappa_1}.\kappa_2\lesssim\Pi_{t::\kappa_3}.\kappa_4}$$

$$\frac{\Delta;\Phi\vdash\kappa_1\lesssim\kappa_2\qquad\Delta;\Phi\vdash\tau_1\overset{\kappa_1}{\equiv}\tau_2}{\Delta;\Phi\vdash\mathtt{S}_{\kappa_1}(\tau_1)\lesssim\mathtt{S}_{\kappa_2}(\tau_2)}$$

$\boxed{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$ $\tau_1$ is equivalent to $\tau_2$ at kind $\kappa$

$$\frac{\Delta;\Phi \vdash \tau_1 :: \mathtt{S}_\kappa(\tau_2)}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau :: \kappa}{\Delta;\Phi \vdash \tau \overset{\kappa}{\equiv} \tau} \qquad \frac{\Delta;\Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \qquad \frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \quad \Delta;\Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\mathtt{Type}}{\equiv} \tau_3 \quad \Delta;\Phi \vdash \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta;\Phi, t::\kappa_1 \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \lambda t::\kappa_1.\tau_1 \overset{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t::\kappa_2.\tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \quad \Delta;\Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1\ \tau_2 \overset{[\tau_1/t]\kappa_2}{\equiv} \tau_3\ \tau_4}$$

$$\frac{\Delta;\Phi \vdash \tau_1 :: \Pi_{t::\kappa_1}.\kappa_3 \quad \Delta;\Phi \vdash \tau_2 :: \Pi_{t::\kappa_1}.\kappa_4 \quad \Delta;\Phi, t::\kappa_1 \vdash \tau_1\ t \overset{\kappa_2}{\equiv} \tau_2\ t}{\Delta;\Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \quad \Delta;\Phi \vdash \kappa_1 \equiv \kappa}{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$$