Hazel Phi: 9-type-aliases

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Syntax

Kind	κ	$::=$	$\mathtt{Type} \mid \mathtt{KHole} \mid \mathtt{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	$::=$	$t \mid \mathtt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!\|\!)^{\mathtt{u}} \mid (\!\|\hat{\tau}\|\!)^{\mathtt{u}} \mid \lambda t{::}\mathtt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \mathtt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!\|\!)^{\mathtt{u}} \mid (\!\|\tau\|\!)^{\mathtt{u}} \mid (\!\|t\|\!)^{\mathtt{u}} \mid \lambda t{::}\kappa.\tau \mid \tau_1\ \tau_2$
Base Types	$\mathtt{bse}$	$::=$	$\mathtt{Int} \mid \mathtt{Float} \mid \mathtt{Bool}$
BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Type Pattern			
User Expression			
Internal Expression			

Declaratives

$\boxed{\Delta; \Phi \vdash \tau ::> \kappa}$ τ has principal (well formed) kind κ

$$\frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{bse} ::> \mathtt{S}_{\mathtt{Type}}(\mathtt{bse})}\ (1)
\qquad
\frac{\Delta; \Phi_1, t{::}\kappa, \Phi_2 \vdash \mathsf{OK}}{\Delta; \Phi \vdash t ::> \mathtt{S}_{\kappa}(t)}\ (2)$$

$$\frac{\Delta; \Phi \vdash \mathsf{OK} \qquad \Delta; \Phi \vdash \tau_1{::}\mathtt{Type} \qquad \Delta; \Phi \vdash \tau_2{::}\mathtt{Type}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 ::> \mathtt{S}_{\mathtt{Type}}(\tau_1 \oplus \tau_2)}\ (3)
\qquad
\frac{\Delta_1, \mathtt{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash (\!\|\!)^{\mathtt{u}} ::> \mathtt{S}_{\kappa}((\!\|\!)^{\mathtt{u}})}\ (4)$$

$$\frac{\Delta_1, \mathtt{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK} \qquad \Delta; \Phi \vdash \tau{::}\kappa_1}{\Delta; \Phi \vdash (\!\|\tau\|\!)^{\mathtt{u}} ::> \mathtt{S}_{\kappa}((\!\|\tau\|\!)^{\mathtt{u}})}\ (5)
\qquad
\frac{\Delta_1, \mathtt{u}{::}\kappa, \Delta_2; \Phi \vdash \mathsf{OK} \qquad t \notin \mathsf{dom}(\Phi)}{\Delta; \Phi \vdash (\!\|t\|\!)^{\mathtt{u}} ::> \mathtt{S}_{\kappa}((\!\|t\|\!)^{\mathtt{u}})}\ (6)$$

$$\frac{\Delta; \Phi \vdash \kappa_1\ \mathsf{OK} \qquad \Delta; \Phi, t{::}\kappa_1 \vdash \tau ::> \kappa_2}{\Delta; \Phi \vdash \lambda t{::}\kappa_1.\tau ::> \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}\ (7)$$

$$\frac{\Delta; \Phi \vdash \tau_1\ ::> \kappa \qquad \Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\underset{\Pi}{}} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta; \Phi \vdash \tau_2{::}\kappa_1}{\Delta; \Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}\ (8)$$

$\boxed{\Delta; \Phi \vdash \tau :: \kappa}$ τ is well formed at kind κ

$$\frac{\Delta; \Phi \vdash \tau ::> \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \text{ (9)} \qquad \frac{\Delta; \Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau_1) \qquad \Delta; \Phi \vdash \tau_1 :: \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \text{ (10)}$$

$$\frac{\Delta; \Phi \vdash \tau ::> \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \text{ (11)}$$

$$\frac{\Delta; \Phi \vdash \tau_2 :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_2)} \text{ (12)} \qquad \frac{\Delta; \Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_3) \qquad \Delta; \Phi \vdash \tau_3 :: \mathtt{S}_{\kappa}(\tau_2)}{\Delta; \Phi \vdash \tau_1 :: \mathtt{S}_{\kappa}(\tau_2)} \text{ (13)}$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau)} \text{ (14)}$$

$\boxed{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{\Delta; \Phi \vdash \kappa \equiv \mathtt{KHole}}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\mathtt{KHole}}.\mathtt{KHole}} \text{ (15)} \qquad \frac{\Delta; \Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2} \text{ (16)}$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \equiv \kappa} \text{ (17)} \qquad \frac{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \text{ (18)} \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta; \Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2} \text{ (19)}$$

$$\frac{\Delta; \Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \mathtt{S}_{\mathtt{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathtt{S}_{\kappa}(\tau_1)} \text{ (20)} \qquad \frac{\Delta; \Phi \vdash \tau :: \mathtt{KHole}}{\Delta; \Phi \vdash \mathtt{S}_{\mathtt{KHole}}(\tau) \equiv \mathtt{KHole}} \text{ (21)}$$

$$\frac{\Delta; \Phi \vdash \tau :: \Pi_{t::\kappa_1}.\kappa_2}{\Delta; \Phi \vdash \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t::\kappa_1}.\mathtt{S}_{\kappa_2}(\tau\ t)} \text{ (22)} \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4} \text{ (23)}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathtt{S}_{\kappa}(\tau_1) \equiv \mathtt{S}_{\kappa}(\tau_2)} \text{ (24)}$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{KHole} \lesssim \kappa} \ (25) \qquad \frac{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathtt{KHole}} \ (26)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \ (27) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2} \ (28)$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \mathtt{S}_\kappa(\tau) \lesssim \kappa} \ (29) \qquad \frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta; \Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4} \ (30)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathtt{S}_{\kappa_1}(\tau_1) \lesssim \mathtt{S}_{\kappa_2}(\tau_2)} \ (31)$$

$\boxed{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$ τ_1 is equivalent to τ_2 at kind κ

$$\frac{\Delta; \Phi \vdash \tau_1 :: \mathtt{S}_\kappa(\tau_2)}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \ (32)$$

$$\frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \tau \overset{\kappa}{\equiv} \tau} \ (33) \qquad \frac{\Delta; \Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \ (34) \qquad \frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \ (35)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\mathtt{Type}}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \overset{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4} \ (36) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \lambda t::\kappa_1.\tau_1 \overset{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t::\kappa_2.\tau_2} \ (37)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \ \tau_2 \overset{[\tau_1/t]\kappa_2}{\equiv} \tau_3 \ \tau_4} \ (38)$$

$$\frac{\Delta; \Phi \vdash \tau_1 :: \Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta; \Phi \vdash \tau_2 :: \Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta; \Phi, t::\kappa_1 \vdash \tau_1 \ t \overset{\kappa_2}{\equiv} \tau_2 \ t}{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2} \ (39)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2} \ (40)$$

$\boxed{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}$ κ is well formed

$$\frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{Type} \ \mathsf{OK}} \ (41) \qquad \frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{KHole} \ \mathsf{OK}} \ (42) \qquad \frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \mathtt{S}_\kappa(\tau) \ \mathsf{OK}} \ (43)$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \ \mathsf{OK} \qquad \Delta; \Phi, t::\kappa_1 \vdash \kappa_2 \ \mathsf{OK}}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \ \mathsf{OK}} \ (44)$$

$\boxed{\Delta; \Phi \vdash \mathsf{OK}}$ Context is well formed

$$\frac{}{; \vdash \mathsf{OK}}\ (45) \qquad \frac{\Delta; \Phi \vdash \kappa\ \mathsf{OK}}{\Delta; \Phi, t{::}\kappa \vdash \mathsf{OK}}\ (46) \qquad \frac{\Delta; \Phi \vdash \kappa\ \mathsf{OK}}{\Delta, \mathtt{u}{::}\kappa; \Phi \vdash \mathsf{OK}}\ (47)$$

Variables implicitly assumed to be fresh as necessary

Metatheory

Lemma 1 (substitution)**.** *If* $\Delta; \Phi \vdash \tau_1{::}\kappa_1$ *and* $\Delta; \Phi, t{::}\kappa_1 \vdash \tau_2{::}\kappa_2$*, then* $\Delta; \Phi \vdash [\tau_1/t]\tau_2{::}\kappa_2$

Lemma 2. *If* $\Delta; \Phi \vdash \tau ::> \kappa$*, then* $\Delta; \Phi \vdash \mathit{OK}$ *and* $\Delta; \Phi \vdash \kappa\ \mathit{OK}$

Lemma 3. *If* $\Delta; \Phi \vdash \tau{::}\kappa$*, then* $\Delta; \Phi \vdash \mathit{OK}$ *and* $\Delta; \Phi \vdash \kappa\ \mathit{OK}$

Lemma 4. *If* $\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t::\kappa_1}.\kappa_2$*, then* $\Delta; \Phi \vdash \mathit{OK}$ *and* $\Delta; \Phi \vdash \kappa\ \mathit{OK}$ *and* $\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2\ \mathit{OK}$

Lemma 5. *If* $\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$*, then* $\Delta; \Phi \vdash \mathit{OK}$ *and* $\Delta; \Phi \vdash \kappa_1\ \mathit{OK}$ *and* $\Delta; \Phi \vdash \kappa_2\ \mathit{OK}$

Lemma 6. *If* $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$*, then* $\Delta; \Phi \vdash \mathit{OK}$ *and* $\Delta; \Phi \vdash \kappa_1\ \mathit{OK}$ *and* $\Delta; \Phi \vdash \kappa_2\ \mathit{OK}$

Lemma 7. *If* $\Delta; \Phi \vdash \kappa\ \mathit{OK}$*, then* $\Delta; \Phi \vdash \mathit{OK}$

Proof. By simultaneous rule induction/length of proof.
The interesting cases per lemma:

L1.	(1)	$\Delta; \Phi \vdash \mathtt{bse}{::}\mathtt{S}_{\mathtt{Type}}(\mathtt{bse})$	by (9)
		$\Delta; \Phi \vdash \mathtt{bse}{::}\mathtt{Type}$	by (10)
	*	$\Delta; \Phi \vdash \mathtt{S}_{\mathtt{Type}}(\mathtt{bse})\ \mathsf{OK}$	by (43)
	*	$\Delta; \Phi \vdash \mathsf{OK}$	by premiss
	(8)		bad
L2.	(12)	$\Delta; \Phi \vdash \tau_2{::}\kappa$	by (10)
	*	$\Delta; \Phi \vdash \mathtt{S}_{\kappa}(\tau_2)\ \mathsf{OK}$	by (43)
L4.	(22)	$\Delta; \Phi \vdash \tau\ t ::> \kappa$	

□

Lemma 8. *If* $\Delta; \Phi \vdash \tau ::> \kappa_1$ *and* $\Delta; \Phi \vdash \tau{::}\kappa_2$*, then* $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$