

$\text{BinOp } \oplus ::= \text{Product} \mid \text{Sum} \mid \text{Arrow}$
 $\text{Kind } \kappa ::= \text{Ty} \mid \text{KHole} \mid \text{S}(\tau)$
 $\text{ConstantTypes } c ::= \text{Int} \mid \text{Float} \mid \text{Bool}$
 $\text{UserHTyp } \hat{\tau} ::= c \mid \hat{\tau}_1 \oplus \hat{\tau}_2 \mid \text{list}(\hat{\tau}) \mid \langle \rangle^u \mid \langle \hat{\tau} \rangle^u$
 $\text{InternalHTyp } \tau ::= c \mid \tau_1 \oplus \tau_2 \mid \text{list}(\tau) \mid \langle \rangle^u \mid \langle \tau \rangle^u$
 $\text{TypeVars } t$
 $\text{UserTypePattern } \hat{\rho} ::= t \mid \langle \rangle^u \mid \langle t \rangle^u$
 $\text{UserExpression } \hat{e} ::= \text{type } \hat{\rho} = \hat{\tau} \text{ in } \hat{e} \mid \text{elided}$
 $\text{InternalExpression } \tau ::= \text{type bindings } \Phi \text{ in } e \mid \text{elided}$

$\boxed{\Delta; \Phi \vdash \kappa_1 <\sim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$\frac{\text{KHoleL}}{\Delta; \Phi \vdash \text{KHole} <\sim \kappa}$	$\frac{\text{KHoleR}}{\Delta; \Phi \vdash \kappa <\sim \text{KHole}}$	$\frac{\text{KRespectEquiv} \quad \Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 <\sim \kappa_2}$
$\frac{\text{KSubsumption} \quad \Delta; \Phi \vdash \tau : \text{Ty}}{\Delta; \Phi \vdash \text{S}(\tau) <\sim \text{Ty}}$		

$\boxed{t \text{ valid}}$ t is a valid type variable

t is valid if it is not a builtin-type or keyword, begins with an alpha char or underscore, and only contains alphanumeric characters, underscores, and primes.

$\boxed{\Delta; \Phi \vdash \kappa \text{ kind}}$ κ forms a kind

$\frac{\text{KFTy}}{\Delta; \Phi \vdash \text{Ty} \text{ kind}}$	$\frac{\text{KFHole}}{\Delta; \Phi \vdash \text{KHole} \text{ kind}}$	$\frac{\text{KFSing} \quad \Delta; \Phi \vdash \tau : \text{Ty}}{\Delta; \Phi \vdash \text{S}(\tau) \text{ kind}}$
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$\boxed{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\begin{array}{c}
\text{KESym} \\
\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_2 \equiv \kappa_1} \\
\\
\text{KESym} \quad \text{KETrans} \\
\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \quad \Delta; \Phi \vdash \kappa_2 \equiv \kappa_3}{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3} \\
\\
\text{KESingEquiv} \\
\frac{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \text{Ty}}{\Delta; \Phi \vdash S(\tau_1) \equiv S(\tau_2)}
\end{array}$$

$\boxed{\Delta; \Phi \vdash \tau : \kappa}$ τ is assigned kind κ

$$\begin{array}{c}
\text{KACnst} \quad \text{KAVar} \quad \text{KABinOp} \\
\frac{}{\Delta; \Phi \vdash c : \text{Ty}} \quad \frac{t : \kappa \in \Phi}{\Delta; \Phi \vdash t : \kappa} \quad \frac{\Delta; \Phi \vdash \tau_1 : \text{Ty} \quad \Delta; \Phi \vdash \tau_2 : \text{Ty}}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 : \text{Ty}} \\
\\
\text{KAList} \quad \text{KAEHole} \\
\frac{\Delta; \Phi \vdash \tau : \text{Ty}}{\Delta; \Phi \vdash \text{list}(\tau) : \text{Ty}} \quad \frac{u :: \kappa[\Phi'] \in \Delta \quad \Delta; \Phi \vdash \sigma : \Phi'}{\Delta; \Phi \vdash \bigoplus_{\sigma}^u : \kappa} \\
\\
\text{KANEHole} \quad \text{KASelfRecognition} \\
\frac{\Delta; \Phi \vdash \tau : \kappa' \quad u :: \kappa[\Phi'] \in \Delta \quad \Delta; \Phi \vdash \sigma : \Phi'}{\Delta; \Phi \vdash \bigoplus_{\sigma}^u : \kappa} \quad \frac{\Delta; \Phi \vdash \tau : \text{Ty}}{\Delta; \Phi \vdash \tau : S(\tau)} \\
\\
\text{KASubkind} \\
\frac{\Delta; \Phi \vdash \tau : \kappa_1 \quad \Delta; \Phi \vdash \kappa_1 < \sim \kappa_2}{\Delta; \Phi \vdash \tau : \kappa_2}
\end{array}$$

$\boxed{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \kappa}$ τ_1 is equivalent to τ_2 and has kind κ_2

$$\begin{array}{c}
\text{KCERefl} \quad \text{KCESym} \\
\frac{}{\Delta; \Phi \vdash \tau \equiv \tau : \kappa} \quad \frac{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \kappa}{\Delta; \Phi \vdash \tau_2 \equiv \tau_1 : \kappa} \\
\\
\text{KCETrans} \quad \text{KCESingEquiv} \\
\frac{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \kappa \quad \Delta; \Phi \vdash \tau_2 \equiv \tau_3 : \kappa}{\Delta; \Phi \vdash \tau_1 \equiv \tau_3 : \kappa} \quad \frac{\Delta; \Phi \vdash \tau_1 : S(\tau_2)}{\Delta; \Phi \vdash \tau_1 \equiv \tau_2 : \text{Ty}}
\end{array}$$

$\boxed{\Phi \vdash \hat{\tau} \Rightarrow \kappa \rightsquigarrow \tau \dashv \Delta}$ $\hat{\tau}$ synthesizes kind κ and elaborates to τ

TElabSConst

$$\frac{}{\Phi \vdash c \Rightarrow \mathbf{Ty} \rightsquigarrow c \dashv \cdot}$$

TElabSBinOp

$$\frac{\Phi \vdash \hat{\tau}_1 \Leftarrow \mathbf{Ty} \rightsquigarrow \tau_1 : \mathbf{Ty} \dashv \Delta_1 \quad \Phi \vdash \hat{\tau}_2 \Leftarrow \mathbf{Ty} \rightsquigarrow \tau_2 : \mathbf{Ty} \dashv \Delta_2}{\Phi \vdash \hat{\tau}_1 \oplus \hat{\tau}_2 \Rightarrow \mathbf{Ty} \rightsquigarrow \tau_1 \oplus \tau_2 \dashv \Delta_1 \cup \Delta_2}$$

TElabSList

$$\frac{\Phi \vdash \hat{\tau} \Leftarrow \mathbf{Ty} \rightsquigarrow \tau : \mathbf{Ty} \dashv \Delta}{\Phi \vdash \mathbf{list}(\hat{\tau}) \Rightarrow \mathbf{Ty} \rightsquigarrow \mathbf{list}(\tau) \dashv \Delta}$$

TElabSVar

$$\frac{t : \kappa \in \Phi}{\Phi \vdash t \Rightarrow \kappa \rightsquigarrow t \dashv \cdot}$$

TElabSUVar

$$\frac{t \notin \Phi}{\Phi \vdash t \Rightarrow \mathbf{KHole} \rightsquigarrow (\textcolor{violet}{t})_{\text{id}(\Phi)}^u \dashv u :: (\textcolor{violet}{\llbracket \rrbracket})[\Phi]}$$

TElabSHole

$$\frac{}{\Phi \vdash (\textcolor{violet}{\llbracket \rrbracket})^u \Rightarrow \mathbf{KHole} \rightsquigarrow (\textcolor{violet}{\llbracket \rrbracket})_{\text{id}(\Phi)}^u \dashv u :: (\textcolor{violet}{\llbracket \rrbracket})[\Phi]}$$

TElabSNEHole

$$\frac{\Phi \vdash \hat{\tau} \Rightarrow \kappa \rightsquigarrow \tau \dashv \Delta}{\Phi \vdash (\textcolor{violet}{\hat{\tau}})^u \Rightarrow \mathbf{KHole} \rightsquigarrow (\textcolor{violet}{\tau})_{\text{id}(\Phi)}^u \dashv \Delta, u :: (\textcolor{violet}{\llbracket \rrbracket})[\Phi]}$$

$\boxed{\Phi \vdash \hat{\tau} \Leftarrow \kappa_1 \rightsquigarrow \tau : \kappa_2 \dashv \Delta}$ $\hat{\tau}$ analyzes against kind κ_1 and elaborates to τ of consistent kind κ_2

TElabASubsume

$$\frac{\hat{\tau} \neq t \text{ where } t \notin \Phi \quad \hat{\tau} \neq (\hat{\tau}')^u \quad \Phi \vdash \hat{\tau} \Rightarrow \kappa' \rightsquigarrow \tau \dashv \Delta \quad \kappa \sim \kappa'}{\Phi \vdash \hat{\tau} \Leftarrow \kappa \rightsquigarrow \tau : \kappa' \dashv \Delta}$$

TElabAUVar

$$\frac{t \notin \Phi}{\Phi \vdash t \Leftarrow \text{KHole} \rightsquigarrow (\hat{t})_{\text{id}(\Phi)}^u : \text{KHole} \dashv u :: \llbracket \Phi \rrbracket}$$

TElabAEHole

$$\frac{}{\Phi \vdash \llbracket \cdot \rrbracket^u \Leftarrow \kappa \rightsquigarrow \llbracket \cdot \rrbracket_{\text{id}(\Phi)}^u : \kappa \dashv u :: \kappa[\Phi]}$$

TElabANEHole

$$\frac{\Phi \vdash \hat{\tau} \Rightarrow \kappa' \rightsquigarrow \tau \dashv \Delta}{\Phi \vdash (\hat{\tau})^u \Leftarrow \kappa \rightsquigarrow (\tau)_{\text{id}(\Phi)}^u : \kappa \dashv \Delta, u :: \kappa[\Phi]}$$

$\boxed{\Delta_1; \Phi_1 \vdash \hat{\rho} \Leftarrow \tau \dashv \Phi_2; \Delta_2}$ $\hat{\rho}$ analyzes against τ yielding new tyvar and hole bindings

RESVar

$$\frac{t \text{ valid} \quad \Delta; \Phi \vdash \tau : \kappa}{\Delta; \Phi \vdash t \Leftarrow \tau \dashv t :: \kappa; \cdot}$$

RESEHole

$$\frac{}{\Delta; \Phi \vdash \llbracket \cdot \rrbracket^u \Leftarrow \tau \dashv \cdot; u :: \llbracket \Phi \rrbracket}$$

RESVarHole

$$\frac{t \neg \text{valid}}{\Delta; \Phi \vdash (\hat{t})^u \Leftarrow \tau \dashv \cdot; u :: \llbracket \Phi \rrbracket}$$

$\boxed{\Gamma; \Phi \vdash \hat{e} \Rightarrow \hat{\tau} \rightsquigarrow e \dashv \Delta}$ \hat{e} synthesizes type τ and elaborates to e

ESDefine

$$\frac{\Phi_1 \vdash \hat{\tau} \Rightarrow \kappa \rightsquigarrow \tau \dashv \Delta_1 \quad \Delta_1; \Phi_1 \vdash \hat{\rho} \Leftarrow \tau \dashv \Phi_2; \Delta_2 \quad \Gamma; \Phi_1 \cup \Phi_2 \vdash \hat{e} \Rightarrow \tau_1 \rightsquigarrow e \dashv \Delta_3}{\Gamma; \Phi_1 \vdash \text{type } \hat{\rho} = \hat{\tau} \text{ in } \hat{e} \Rightarrow \tau_1 \rightsquigarrow \text{type bindings } \Phi_2 \text{ in } e \dashv \Delta_1 \cup \Delta_2 \cup \Delta_3}$$

$\boxed{\Gamma; \Phi \vdash e : \tau}$ e is assigned type τ

DEDefine

$$\frac{\Gamma; \Phi_1 \cup \Phi_2 \vdash e : \tau}{\Gamma; \Phi_1 \vdash \text{type bindings } \Phi_2 \text{ in } e : \tau}$$