Hazel Phi: 11-type-constructors

July 29, 2021

SYNTAX

Kind	κ	$::=$	$\texttt{Type} \mid \texttt{KHole} \mid \texttt{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2$
User Types	$\hat{\tau}$	$::=$	$t \mid \texttt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!\|\!)^{\texttt{u}} \mid (\!\|\hat{\tau}\|\!)^{\texttt{u}} \mid \lambda t{::}\texttt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2}$
Internal Types	τ	$::=$	$t \mid \texttt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!\|\!)^{\texttt{u}} \mid (\!\|\tau\|\!)^{\texttt{u}} \mid (\!\|t\|\!)^{\texttt{u}} \mid \lambda t{::}\kappa.\tau \mid \tau_1\ \tau_2$
Base Types	$\texttt{bse}$	$::=$	$\texttt{Int} \mid \texttt{Float} \mid \texttt{Bool}$
BinOp	$\oplus$	$::=$	$\times \mid + \mid \rightarrow$
Type Pattern			
User Expression			
Internal Expression			

Declaratives

$\boxed{\Delta;\Phi \vdash \tau ::> \kappa}$ τ has principal (well formed) kind κ

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \texttt{bse} ::> \texttt{S}_{\texttt{Type}}(\texttt{bse})}\ \texttt{PK-Base}$$

$$\frac{\Delta;\Phi_1, t{::}\kappa, \Phi_2 \vdash \mathsf{OK}}{\Delta;\Phi \vdash t ::> \texttt{S}_{\kappa}(t)}\ \texttt{PK-Var}$$

$$\frac{\Delta;\Phi \vdash \tau_1{::}\texttt{Type} \qquad \Delta;\Phi \vdash \tau_2{::}\texttt{Type}}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 ::> \texttt{S}_{\texttt{Type}}(\tau_1 \oplus \tau_2)}\ \texttt{PK-}\oplus$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash (\!\|\!)^{\texttt{u}} ::> \texttt{S}_{\kappa}((\!\|\!)^{\texttt{u}})}\ \texttt{PK-EHole}$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK} \qquad \Delta;\Phi \vdash \tau{::}\kappa_1}{\Delta;\Phi \vdash (\!\|\tau\|\!)^{\texttt{u}} ::> \texttt{S}_{\kappa}((\!\|\tau\|\!)^{\texttt{u}})}\ \texttt{PK-NEHole}$$

$$\frac{\Delta_1, \mathrm{u}{::}\kappa, \Delta_2;\Phi \vdash \mathsf{OK} \qquad t \notin \Phi}{\Delta;\Phi \vdash (\!\|t\|\!)^{\texttt{u}} ::> \texttt{S}_{\kappa}((\!\|t\|\!)^{\texttt{u}})}\ \texttt{PK-Unbound}$$

$$\frac{\Delta;\underline{\Phi}, t{::}\kappa_1 \vdash \tau ::> \kappa_2}{\Delta;\Phi \vdash \lambda t{::}\kappa_1.\tau ::> \texttt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t{::}\kappa_1.\tau)}\ \texttt{PK-}\lambda$$

$$\frac{\Delta;\Phi \vdash \tau_1\ ::> \kappa \qquad \Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\underset{\Pi}{}} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta;\Phi \vdash \tau_2{::}\kappa_1}{\Delta;\Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}\ \texttt{PK-Ap}$$

$\boxed{\Delta;\Phi \vdash \tau{::}\kappa}$ τ is well formed at kind κ

$$\frac{\Delta;\Phi \vdash \tau ::> \mathsf{S}_{\kappa}(\tau)}{\Delta;\Phi \vdash \tau{::}\kappa}\ \texttt{WFaK-1} \qquad \frac{\Delta;\Phi \vdash \tau ::> \kappa_1 \qquad \Delta;\Phi \vdash \kappa_1 \lesssim \kappa}{\Delta;\Phi \vdash \tau{::}\kappa}\ \texttt{WFaK-Subsump}$$

$$\frac{\Delta;\Phi \vdash \tau ::> \kappa}{\Delta;\Phi \vdash \tau{::}\kappa}\ \texttt{WFaK-Reit} \qquad \frac{\Delta;\Phi \vdash \tau{::}\kappa}{\Delta;\Phi \vdash \tau{::}\mathsf{S}_{\kappa}(\tau)}\ \texttt{WFaK-Self}$$

$$\frac{\Delta;\Phi \vdash \tau{::}\Pi_{t::\kappa_3}.\kappa_4 \qquad \Delta;\Phi \vdash \Pi_{t::\kappa_3}.\kappa_4 \lesssim \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \tau{::}\Pi_{t::\kappa_1}.\kappa_2}\ \texttt{WFaK-}\Pi\texttt{CSKTrans}$$

$$\frac{\Delta;\Phi \vdash \tau{::}\mathsf{S}_{\kappa}(\tau_1) \qquad \Delta;\Phi \vdash \tau_1{::}\kappa}{\Delta;\Phi \vdash \tau{::}\kappa}\ \texttt{WFaK-Flatten}$$

$\boxed{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{\Delta;\Phi \vdash \mathsf{OK}}{\Delta;\Phi \vdash \texttt{KHole} \overset{\blacktriangleright}{\Pi} \Pi_{t::\texttt{KHole}}.\texttt{KHole}}\ \overset{\blacktriangleright}{\Pi}\texttt{-KHole} \qquad \frac{\Delta;\Phi \vdash \kappa \equiv \mathsf{S}_{\texttt{KHole}}(\tau)}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\mathsf{S}_{\texttt{KHole}}(\tau)}.\mathsf{S}_{\texttt{KHole}}(\tau\ t)}\ \overset{\blacktriangleright}{\Pi}\texttt{-SKHole}$$

$$\frac{\Delta;\Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \kappa \overset{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}\ \overset{\blacktriangleright}{\Pi}\texttt{-}\Pi$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{\Delta;\Phi \vdash \kappa\ \mathsf{OK}}{\Delta;\Phi \vdash \kappa \equiv \kappa}\ \texttt{KEquiv-Refl} \qquad \frac{\Delta;\Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Symm}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}\ \texttt{KEquiv-Trans}$$

$$\frac{\Delta;\Phi \vdash \tau{::}\mathsf{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \mathsf{S}_{\mathsf{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathsf{S}_{\kappa}(\tau_1)}\ \texttt{KEquiv-SKind}_{\texttt{SKind}} \qquad \frac{\Delta;\Phi \vdash \tau{::}\Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t_1::\kappa_1}.\mathsf{S}_{[t_1/t]\kappa_2}(\tau\ t_1)}\ \texttt{KEquiv-SKind}_{\Pi}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\underline{\Phi}, t{::}\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{KEquiv-}\Pi \qquad \frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \mathsf{S}_{\kappa_1}(\tau_1) \equiv \mathsf{S}_{\kappa_2}(\tau_2)}\ \texttt{KEquiv-SKind}$$

$\boxed{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{\Delta; \Phi \vdash \kappa\ \mathsf{OK}}{\Delta; \Phi \vdash \texttt{KHole} \lesssim \kappa}\ \texttt{CSK-KHoleL} \qquad \frac{\Delta; \Phi \vdash \kappa\ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \lesssim \texttt{KHole}}\ \texttt{CSK-KHoleR}$$

$$\frac{\Delta; \Phi \vdash \mathsf{S}_{\texttt{KHole}}(\tau)\ \mathsf{OK} \qquad \Delta; \Phi \vdash \kappa\ \mathsf{OK}}{\Delta; \Phi \vdash \mathsf{S}_{\texttt{KHole}}(\tau) \lesssim \kappa}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{L}$$

$$\frac{\Delta; \Phi \vdash \kappa\ \mathsf{OK} \qquad \Delta; \Phi \vdash \mathsf{S}_{\texttt{KHole}}(\tau)\ \mathsf{OK}}{\Delta; \Phi \vdash \kappa \lesssim \mathsf{S}_{\texttt{KHole}}(\tau)}\ \texttt{CSK-SKind}_{\texttt{KHole}}\texttt{R}$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-KEquiv} \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta; \Phi \vdash \kappa_3 \lesssim \kappa_4 \qquad \Delta; \Phi \vdash \kappa_4 \equiv \kappa_2}{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2}\ \texttt{CSK-Normal}$$

$$\frac{\Delta; \Phi \vdash \mathsf{S}_\kappa(\tau)\ \mathsf{OK}}{\Delta; \Phi \vdash \mathsf{S}_\kappa(\tau) \lesssim \kappa}\ \texttt{CSK-SKind} \qquad \frac{\Delta; \Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta; \underline{\Phi}, t{::}\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta; \Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4}\ \texttt{CSK-}\Pi$$

$$\frac{\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \qquad \Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2}{\Delta; \Phi \vdash \mathsf{S}_{\kappa_1}(\tau_1) \lesssim \mathsf{S}_{\kappa_2}(\tau_2)}\ \texttt{CSK-?}$$

$\boxed{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}$ τ_1 is provably equivalent to τ_2 at kind κ

$$\frac{\Delta; \Phi \vdash \tau{::}\kappa}{\Delta; \Phi \vdash \tau \overset{\kappa}{\equiv} \tau}\ \texttt{EquivAK-Refl} \qquad \frac{\Delta; \Phi \vdash \tau_2 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Symm}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_3 \overset{\kappa}{\equiv} \tau_1}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-Trans}$$

$$\frac{\Delta; \Phi \vdash \tau_1 :: > \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \equiv \mathsf{S}_\kappa(\tau_2)}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ \texttt{EquivAK-SKind}$$

$$\frac{\Delta; \Phi \vdash \tau_1{::}\Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta; \Phi \vdash \tau_2{::}\Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta; \underline{\Phi};, t{::}\kappa_1 \vdash \tau_1\ t \overset{\kappa_2}{\equiv} \tau_2\ t}{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}\ \texttt{EquivAK-}\Pi$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \overset{\kappa_1}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1\ \tau_2 \overset{[\tau_2/t]\kappa_2}{\equiv} \tau_3\ \tau_4}\ \texttt{EquivAK-Ap}$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\mathsf{S}_\kappa(\tau)}{\equiv} \tau_2}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (1)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\mathsf{Type}}{\equiv} \tau_3 \qquad \Delta; \Phi \vdash \tau_2 \overset{\mathsf{Type}}{\equiv} \tau_4}{\Delta; \Phi \vdash \tau_1 \oplus \tau_2 \overset{\mathsf{Type}}{\equiv} \tau_3 \oplus \tau_4}\ (2) \qquad \frac{\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta; \underline{\Phi}, t{::}\kappa_1 \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta; \Phi \vdash \lambda t{::}\kappa_1.\tau_1 \overset{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t{::}\kappa_2.\tau_2}\ (3)$$

$$\frac{\Delta; \Phi \vdash \tau_1 \overset{\kappa_1}{\equiv} \tau_2 \qquad \Delta; \Phi \vdash \kappa_1 \equiv \kappa}{\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}\ (4)$$

$\boxed{\Delta; \Phi \vdash \kappa \ \mathsf{OK}}$ κ is well formed

$$\frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{Type}\ \mathsf{OK}}\ \texttt{KWF-Type} \qquad \frac{\Delta; \Phi \vdash \mathsf{OK}}{\Delta; \Phi \vdash \mathtt{KHole}\ \mathsf{OK}}\ \texttt{KWF-KHole} \qquad \frac{\Delta; \Phi \vdash \tau :: \kappa}{\Delta; \Phi \vdash \mathsf{S}_{\kappa}(\tau)\ \mathsf{OK}}\ \texttt{KWF-SKind}$$

$$\frac{\Delta; \underline{\Phi}, t :: \kappa_1 \vdash \kappa_2\ \mathsf{OK}}{\Delta; \Phi \vdash \Pi_{t :: \kappa_1}.\kappa_2\ \mathsf{OK}}\ \texttt{KWF-}\Pi$$

$\boxed{\Delta; \Phi \vdash \mathsf{OK}}$ Context is well formed

$$\frac{}{\cdot; \cdot \vdash \mathsf{OK}}\ \texttt{CWF-Nil} \qquad \frac{t \notin \Phi \qquad \Delta; \Phi \vdash \kappa\ \mathsf{OK}}{\Delta; \underline{\Phi}, t :: \kappa \vdash \mathsf{OK}}\ \texttt{CWF-TypVar} \qquad \frac{\mathrm{u} \notin \Delta \qquad \Delta; \Phi \vdash \kappa\ \mathsf{OK}}{\underline{\Delta}, \mathrm{u} :: \kappa; \Phi \vdash \mathsf{OK}}\ \texttt{CWF-Hole}$$

Metatheory

Lemma 1 (COK). *If $\Delta; \Phi \vdash \mathcal{J}$, then $\Delta; \Phi \vdash \mathsf{OK}$*

Proof. By simultaneous induction on derivations.
No interesting cases. □

Lemma 2 (Exchange).
If $\Delta; \Phi_1, t_{L1} :: \kappa_{L1}, t_{L2} :: \kappa_{L2}, \Phi_2 \vdash \mathcal{J}$ and $\Delta; \Phi_1, t_{L2} :: \kappa_{L2}, t_{L1} :: \kappa_{L1}, \Phi_2 \vdash \mathsf{OK}$, then $\Delta; \Phi_1, t_{L2} :: \kappa_{L2}, t_{L1} :: \kappa_{L1}, \Phi_2 \vdash \mathcal{J}$

Proof. By simultaneous induction on derivations.
No interesting cases.
(Only rules with Φ extended in the consequent are interesting, which is only CWF-TypVar, but when $\mathcal{J}$ is CWF, Exchange is identity) □

Lemma 3 (Weakening).
If $\Delta; \Phi \vdash \mathcal{J}$ and $\Delta; \underline{\Phi}, t_L :: \kappa_L \vdash \mathsf{OK}$, then $\Delta; \underline{\Phi}, t_L :: \kappa_L \vdash \mathcal{J}$

Lemma 4 (OK-PK). *If $\Delta; \Phi \vdash \tau :: > \kappa$, then $\Delta; \Phi \vdash \kappa$ OK*

Lemma 5 (OK-WFaK). *If $\Delta; \Phi \vdash \tau :: \kappa$, then $\Delta; \Phi \vdash \kappa$ OK*

Lemma 6 (OK-MatchPi). *If $\Delta; \Phi \vdash \kappa \overset{\blacktriangleright}{\scriptstyle\Pi} \Pi_{t :: \kappa_1}.\kappa_2$, then $\Delta; \Phi \vdash \kappa$ OK and $\Delta; \Phi \vdash \Pi_{t :: \kappa_1}.\kappa_2$ OK*

Lemma 7 (OK-KEquiv). *If $\Delta; \Phi \vdash \kappa_1 \equiv \kappa_2$, then $\Delta; \Phi \vdash \kappa_1$ OK and $\Delta; \Phi \vdash \kappa_2$ OK*

Lemma 8 (OK-CSK). *If $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$, then $\Delta; \Phi \vdash \kappa_1$ OK and $\Delta; \Phi \vdash \kappa_2$ OK*

Lemma 9 (OK-EquivAK). *If $\Delta; \Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2$, then $\Delta; \Phi \vdash \tau_1 :: \kappa$ and $\Delta; \Phi \vdash \tau_2 :: \kappa$ and $\Delta; \Phi \vdash \kappa$ OK*

Lemma 10 (OK-Substitution).
If $\Delta; \Phi \vdash \tau_L :: \kappa_{L1}$ and $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \kappa_{L2}$ OK, then $\Delta; \Phi \vdash [\tau_L / t_L]\kappa_{L2}$ OK
(induction on $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \kappa_{L2}$ OK)

Lemma 11 (K-Substitution).
If $\Delta; \Phi \vdash \tau_{L1} :: \kappa_{L1}$ and $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \tau_{L2} :: \kappa_{L2}$, then $\Delta; \Phi \vdash [\tau_{L1} / t_L]\tau_{L2} :: [\tau_{L1} / t_L]\kappa_{L2}$
(induction on $\Delta; \Phi, t_L :: \kappa_{L1} \vdash \tau_{L2} :: \kappa_{L2}$)

Proof. By simultaneous induction on derivations.
The interesting cases per lemma:

Weakening.	PK-λ	$\Delta; \Phi, t_L{::}\kappa_2 \vdash \mathsf{OK}$	by IH
		$\Delta; \Phi, t{::}\kappa_1 \vdash \mathsf{OK}$	by COK on premiss
		$\Delta; \Phi \vdash \kappa_1\ \mathsf{OK}$	by premiss
		$\Delta; \Phi, t_L{::}\kappa_L \vdash \kappa_1\ \mathsf{OK}$	by Weakening on premiss
		$\Delta; \Phi, t_L{::}\kappa_L, t{::}\kappa_1 \vdash \mathsf{OK}$	
		$\Delta; \Phi, t{::}\kappa_1, t_L{::}\kappa_L \vdash \tau :{:}{>}\ \kappa_2$	
		$\Delta; \Phi, t_L{::}\kappa_L, t{::}\kappa_1 \vdash \tau :{:}{>}\ \kappa_2$	
OK-PK.	PK-Base	$\Delta; \Phi \vdash \mathtt{bse}{::}\mathtt{S}_{\mathtt{Type}}(\mathtt{bse})$	by (9)
		$\Delta; \Phi \vdash \mathtt{bse}{::}\mathtt{Type}$	by (10)
	*	$\Delta; \Phi \vdash \mathtt{S}_{\mathtt{Type}}(\mathtt{bse})\ \mathsf{OK}$	by (43)
	*	$\Delta; \Phi \vdash \mathsf{OK}$	by premiss
	PK-Ap		bad
OK-WFaK.	(12)	$\Delta; \Phi \vdash \tau_2{::}\kappa$	by (10)
	*	$\Delta; \Phi \vdash \mathtt{S}_{\kappa}(\tau_2)\ \mathsf{OK}$	by (43)
OK-KEquiv.	(22)	$\Delta; \Phi \vdash \tau\ t :{:}{>}\ \kappa$	
OK-Substitution.	(41)	$\Delta; \Phi, t_L{::}\kappa_{L1} \vdash \mathsf{OK}$	premiss (41)
		$\Delta; \Phi \vdash \kappa_{L1}\ \mathsf{OK}$	by subderivation premiss (46)
	*	$\Delta; \Phi \vdash \mathsf{OK}$	by OK-KWF
	*	$\Delta; \Phi \vdash [\tau_L/t_L]\mathtt{Type}\ \mathsf{OK}$	by (41) and degenerate subst
	(43)	$\Delta; \Phi, t_L{::}\kappa_{L1} \vdash \tau{::}\kappa$	premiss (43)
		$\Delta; \Phi, t_L{::}\kappa_{L1} \vdash \mathsf{OK}$	by OK-WFaK
		$\Delta; \Phi \vdash \kappa_{L1}\ \mathsf{OK}$	by subderivation premiss (46)
	*	$\Delta; \Phi \vdash \mathsf{OK}$	by OK-KWF
		$\Delta; \Phi \vdash [\tau_{L1}/t_L]\tau{::}[\tau_{L1}/t_L]\kappa$	by K-Substitution on premiss
	*	$\Delta; \Phi \vdash [\tau_L/t_L]\mathtt{S}_{\kappa}(\tau)\ \mathsf{OK}$	by (43)

$\square$

Lemma 12 (PK-Unicity)**.** *If* $\Delta; \Phi \vdash \tau_L :{:}{>}\ \kappa_{L1}$ *and* $\Delta; \Phi \vdash \tau_L :{:}{>}\ \kappa_{L2}$ *then* κ_{L1} *is* κ_{L2}

Lemma 13. *If* $\Delta; \Phi \vdash \tau :{:}{>}\ \kappa_1$ *and* $\Delta; \Phi \vdash \tau{::}\kappa_2$*, then* $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$

Lemma 14. *If* $\Delta; \Phi \vdash \kappa_1 \lesssim \mathtt{S}_{\kappa_2}(\tau)$*, then* $\Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2$