Hazel Phi: 9-type-aliases

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Syntax

$$
\begin{array}{rcll}
\text{BinOp} & \oplus & ::= & \times \mid + \mid \rightarrow \\
\text{Kind} & \kappa & ::= & \mathtt{Type} \mid \mathtt{KHole} \mid \mathtt{S}_{\kappa}(\tau) \mid \Pi_{t::\kappa_1}.\kappa_2 \\
\text{Base Types} & \mathtt{bse} & ::= & \mathtt{Int} \mid \mathtt{Float} \mid \mathtt{Bool} \\
\text{User Types} & \hat{\tau} & ::= & t \mid \mathtt{bse} \mid \hat{\tau_1} \oplus \hat{\tau_2} \mid (\!|\,|\!)^{\mathtt{u}} \mid (\!|\hat{\tau}|\!)^{\mathtt{u}} \mid \lambda t::\mathtt{Type}.\hat{\tau} \mid \hat{\tau_1}\ \hat{\tau_2} \\
\text{Internal Types} & \tau & ::= & t \mid \mathtt{bse} \mid \tau_1 \oplus \tau_2 \mid (\!|\,|\!)^{\mathtt{u}} \mid (\!|\tau|\!)^{\mathtt{u}} \mid \lambda t::\kappa.\tau \mid \tau_1\ \tau_2 \\
\text{Type Pattern} & & & \\
\text{User Expression} & & & \\
\text{Internal Expression} & & &
\end{array}
$$

Declaratives

$\boxed{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

$$\frac{}{\Delta;\Phi \vdash \mathtt{KHole} \lesssim \kappa} \qquad \frac{}{\Delta;\Phi \vdash \kappa \lesssim \mathtt{KHole}}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \lesssim \kappa_2} \qquad \frac{\Delta;\Phi \vdash \tau::\kappa}{\Delta;\Phi \vdash \mathtt{S}_{\kappa}(\tau) \lesssim \kappa} \qquad \frac{\Delta;\Phi \vdash \kappa_3 \lesssim \kappa_1 \qquad \Delta;\Phi, t::\kappa_3 \vdash \kappa_2 \lesssim \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \lesssim \Pi_{t::\kappa_3}.\kappa_4}$$

$\boxed{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$ κ_1 is equivalent to κ_2

$$\frac{}{\Delta;\Phi \vdash \kappa \equiv \kappa} \qquad \frac{\Delta;\Phi \vdash \kappa_2 \equiv \kappa_1}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_3 \qquad \Delta;\Phi \vdash \kappa_3 \equiv \kappa_2}{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \overset{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \mathtt{S}_{\kappa}(\tau_1) \equiv \mathtt{S}_{\kappa}(\tau_2)} \qquad \frac{\Delta;\Phi \vdash \tau::\mathtt{S}_{\kappa}(\tau_1)}{\Delta;\Phi \vdash \mathtt{S}_{\mathtt{S}_{\kappa}(\tau_1)}(\tau) \equiv \mathtt{S}_{\kappa}(\tau_1)} \qquad \frac{\Delta;\Phi \vdash \tau::\Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \mathtt{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\tau) \equiv \Pi_{t::\kappa_1}.\mathtt{S}_{\kappa_2}(\tau\ t)}$$

$$\frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\Phi, t::\kappa_1 \vdash \kappa_3 \equiv \kappa_4}{\Delta;\Phi \vdash \Pi_{t::\kappa_1}.\kappa_2 \equiv \Pi_{t::\kappa_3}.\kappa_4}$$

$\boxed{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$ τ_1 is equivalent to τ_2 at kind κ

$$\frac{\Delta;\Phi \vdash \tau_1 :: \mathsf{S}_{\kappa}(\tau_2) \qquad \Delta;\Phi \vdash \tau_2 :: \kappa}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau :: \kappa}{\Delta;\Phi \vdash \tau \stackrel{\kappa}{\equiv} \tau} \qquad \frac{\Delta;\Phi \vdash \tau_2 \stackrel{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2} \qquad \frac{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_3 \stackrel{\kappa}{\equiv} \tau_1}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\mathtt{Type}}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \stackrel{\mathtt{Type}}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 \stackrel{\mathtt{Type}}{\equiv} \tau_3 \oplus \tau_4} \qquad \frac{\Delta;\Phi \vdash \kappa_1 \equiv \kappa_2 \qquad \Delta;\Phi, t::\kappa_1 \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \lambda t::\kappa_1.\tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa}{\equiv} \lambda t::\kappa_2.\tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_3 \qquad \Delta;\Phi \vdash \tau_2 \stackrel{\kappa_1}{\equiv} \tau_4}{\Delta;\Phi \vdash \tau_1\ \tau_2 \stackrel{[\tau_1/t]\kappa_2}{\equiv} \tau_3\ \tau_4}$$

$$\frac{\Delta;\Phi \vdash \tau_1 :: \Pi_{t::\kappa_1}.\kappa_3 \qquad \Delta;\Phi \vdash \tau_2 :: \Pi_{t::\kappa_1}.\kappa_4 \qquad \Delta;\Phi, t::\kappa_1 \vdash \tau_1\ t \stackrel{\kappa_2}{\equiv} \tau_2\ t}{\Delta;\Phi \vdash \tau_1 \stackrel{\Pi_{t::\kappa_1}.\kappa_2}{\equiv} \tau_2}$$

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa_1}{\equiv} \tau_2 \qquad \Delta;\Phi \vdash \kappa_1 \equiv \kappa}{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}$$

$\boxed{\Delta;\Phi \vdash \tau_1 \equiv \tau_2}$ τ_1 is equivalent to τ_2 at “top” kind

$$\frac{\Delta;\Phi \vdash \tau_1 \stackrel{\kappa}{\equiv} \tau_2}{\Delta;\Phi \vdash \tau_1 \equiv \tau_2} \qquad \frac{\Delta;\Phi \vdash \tau :: \kappa}{\Delta;\Phi \vdash \tau \equiv \tau} \qquad \frac{\Delta;\Phi \vdash \tau_2 \equiv \tau_1}{\Delta;\Phi \vdash \tau_1 \equiv \tau_2} \qquad \frac{\Delta;\Phi \vdash \tau_1 \equiv \tau_3 \qquad \Delta;\Phi \vdash \tau_3 \equiv \tau_2}{\Delta;\Phi \vdash \tau_1 \equiv \tau_2}$$

$\boxed{\Delta;\Phi \vdash \tau ::> \kappa}$ τ has principal kind κ

$$\frac{}{\Delta;\Phi \vdash \mathtt{bse} ::> \mathsf{S}_{\mathtt{Type}}(\mathtt{bse})} \qquad \frac{t::\kappa \in \Phi}{\Delta;\Phi \vdash t ::> \mathsf{S}_{\kappa}(t)} \qquad \frac{\Delta;\Phi \vdash \tau_1 :: \mathtt{Type} \qquad \Delta;\Phi \vdash \tau_2 :: \mathtt{Type}}{\Delta;\Phi \vdash \tau_1 \oplus \tau_2 ::> \mathsf{S}_{\mathtt{Type}}(\tau_1 \oplus \tau_2)}$$

$$\frac{\mathtt{u}::\kappa \in \Delta}{\Delta;\Phi \vdash (\!|\,|\!)^{\mathtt{u}} ::> \kappa} \qquad \frac{\mathtt{u}::\kappa \in \Delta \qquad \Delta;\Phi \vdash \tau :: \kappa_1}{\Delta;\Phi \vdash (\!|\tau|\!)^{\mathtt{u}} ::> \kappa} \qquad \frac{\mathtt{u}::\kappa \in \Delta \qquad t \notin \mathsf{dom}(\Phi)}{\Delta;\Phi \vdash (\!|t|\!)^{\mathtt{u}} ::> \kappa}$$

$$\frac{\Delta;\Phi, t::\kappa_1 \vdash \tau ::> \kappa_2}{\Delta;\Phi \vdash \lambda t::\kappa_1.\tau ::> \mathsf{S}_{\Pi_{t::\kappa_1}.\kappa_2}(\lambda t::\kappa_1.\tau)}$$

$$\frac{\Delta;\Phi \vdash \tau_1 ::> \kappa \qquad \Delta;\Phi \vdash \kappa \stackrel{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2 \qquad \Delta;\Phi \vdash \tau_2 :: \kappa_1}{\Delta;\Phi \vdash \tau_1\ \tau_2 ::> [\tau_2/t]\kappa_2}$$

$\boxed{\Delta;\Phi \vdash \kappa \stackrel{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}$ κ has matched Π-kind $\Pi_{t::\kappa_1}.\kappa_2$

$$\frac{}{\Delta;\Phi \vdash \mathtt{KHole} \stackrel{\blacktriangleright}{\Pi} \Pi_{t::\mathtt{KHole}}.\mathtt{KHole}} \qquad \frac{\Delta;\Phi \vdash \kappa \equiv \Pi_{t::\kappa_1}.\kappa_2}{\Delta;\Phi \vdash \kappa \stackrel{\blacktriangleright}{\Pi} \Pi_{t::\kappa_1}.\kappa_2}$$

$\boxed{\Delta; \Phi \vdash \tau :: \kappa}$ τ is well formed at kind κ

$$\frac{\Delta; \Phi \vdash \tau \mathrel{::>} \kappa_1 \qquad \Delta; \Phi \vdash \kappa_1 \lesssim \kappa}{\Delta; \Phi \vdash \tau :: \kappa} \qquad\qquad \frac{\Delta; \Phi \vdash \tau :: \mathtt{S}_{\kappa}(\tau_1)}{\Delta; \Phi \vdash \tau :: \kappa}$$