

Invariant Annuli and Singular Fibers

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1 Idea of writeup

This writeup is the first step toward showing that one dimensional orbits of $\text{Homeo}_0(S^1)$ actions on compact three-manifolds are regular fibers of the associated Seifert fibration. In particular, the goal here is to prove that when there is an invariant annulus accumulating onto a given 1-dimensional orbit, it must be regular. The proof of this fact proceeds by making use of a simple observation about sequences of points in invariant surfaces of $\text{Homeo}_0(S^1)$ actions on spaces.

2 Main trick

We have the following fact which more or less amounts to a restatement of continuity.

Remark 1. Suppose that $\text{Homeo}_0(S^1)$ acts on a compact three-manifold M , and A is an invariant annulus with standard model coordinates $\varphi : S^1 \times (0, 1) \rightarrow A$ and σ a 1-dimensional orbit in $\overline{A} - A$. Then, if two sequences of points in A with constant S^1 coordinate under φ converge in M to $X \in \sigma$, then their S^1 coordinate must be the same.