



Posts and Telecommunication Institute of Technology
Faculty of Information Technology 1

Introduction to Artificial Intelligence

Informed search

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Blind search & Informed search

▶ Blind search

- Expands nodes according to a **pre-defined rule**
- Moves in the state space **without orientation**; many states must be considered
- Not suitable for problems with large state spaces

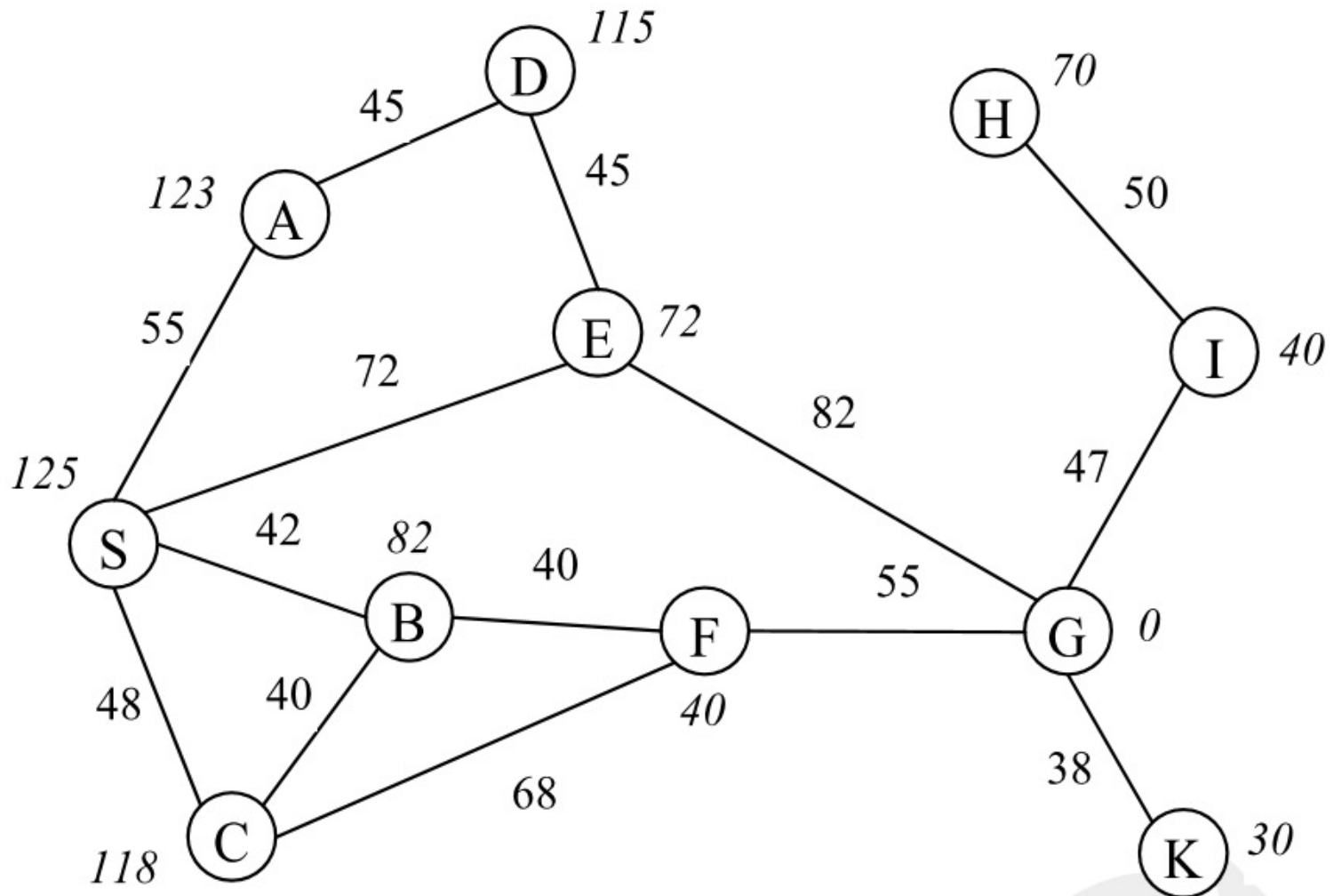
▶ Informed search

- Uses additional information from the problem to guide the search
- Uses a function $f(n)$ to evaluate the potential of node n , then choose the best node to expand first
 - Best-first search
 - **How to construct function $f(n)$?**

Outline

- ▶ Greedy search
- ▶ A* search
- ▶ Heuristic functions
- ▶ Iterative deepening A* (IDA*)

Example

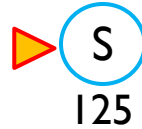


Greedy search

- ▶ **Principle:** expand the node with the cheapest path to a goal first
 - $f(n) = h(n)$: *heuristic* function, estimated cost of the path from n to a goal node
 - Example: $h(n)$ = straight-line distance from n to a goal node

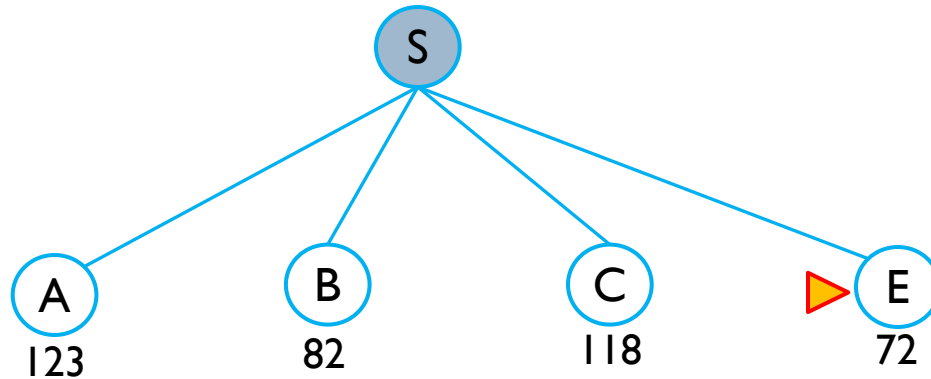
- ▶ “**Greedy**”: expands the node that **appears** to be closest to goal, regardless of the future

Example of greedy search (1 / 4)



Example of greedy search (2/4)

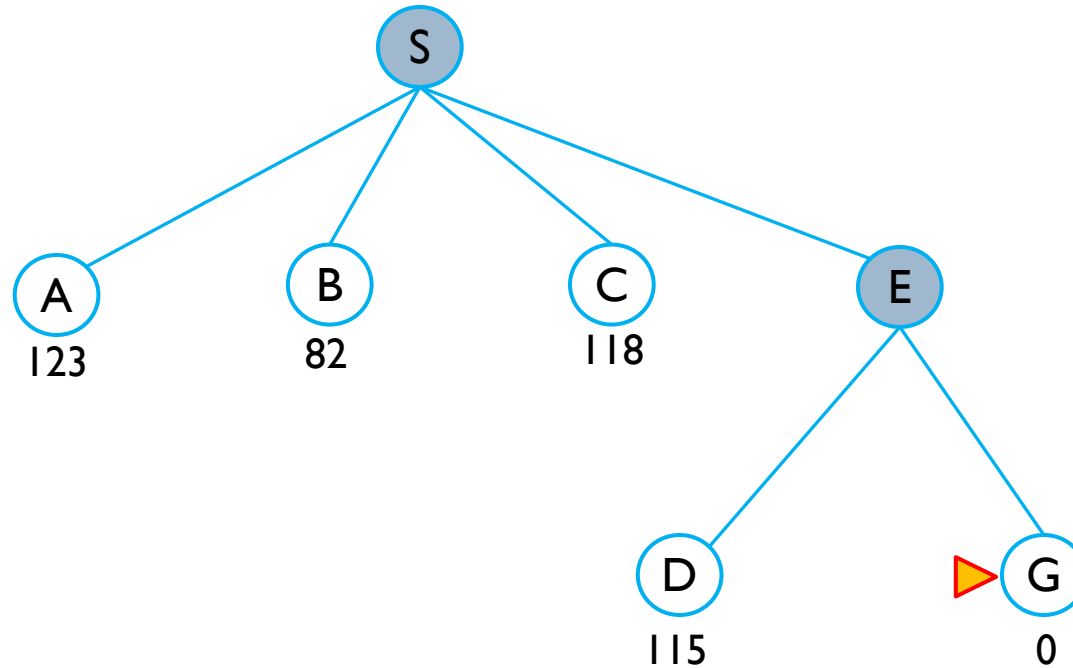
Expand S



Example of greedy search (3/4)

Expand S

Expand E

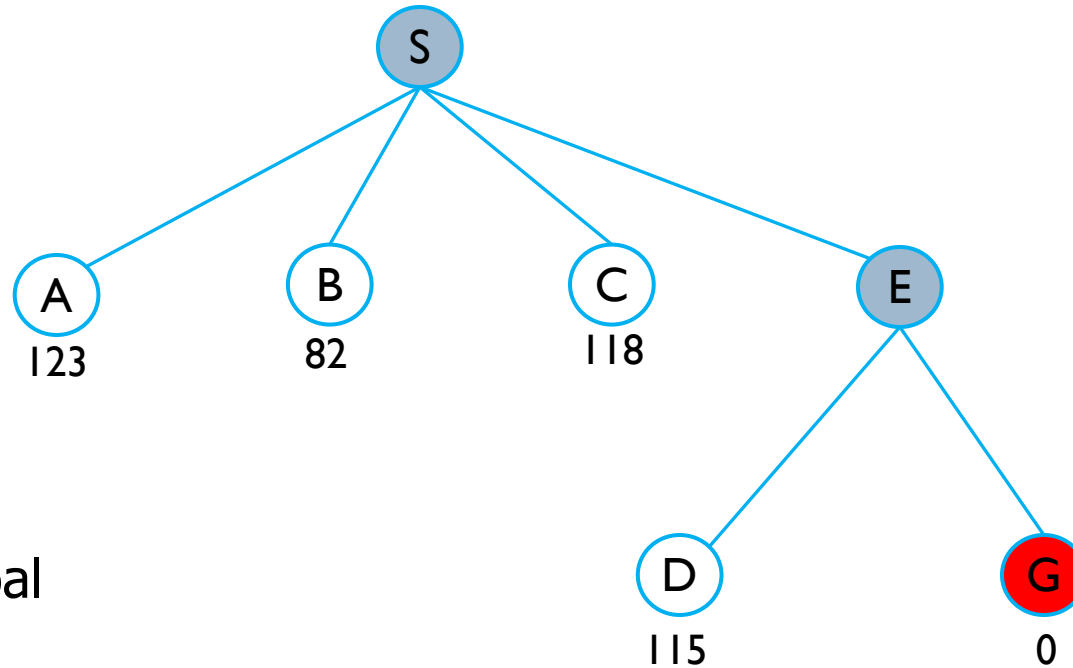


Example of greedy search (4/4)

Expand S

Expand E

Expand G: Goal



Properties of greedy search

▶ Completeness?

- No (can have a loop, or have a branch of infinite nodes with small value of function h but do not lead to a goal)

▶ Optimality?

- No

▶ Time?

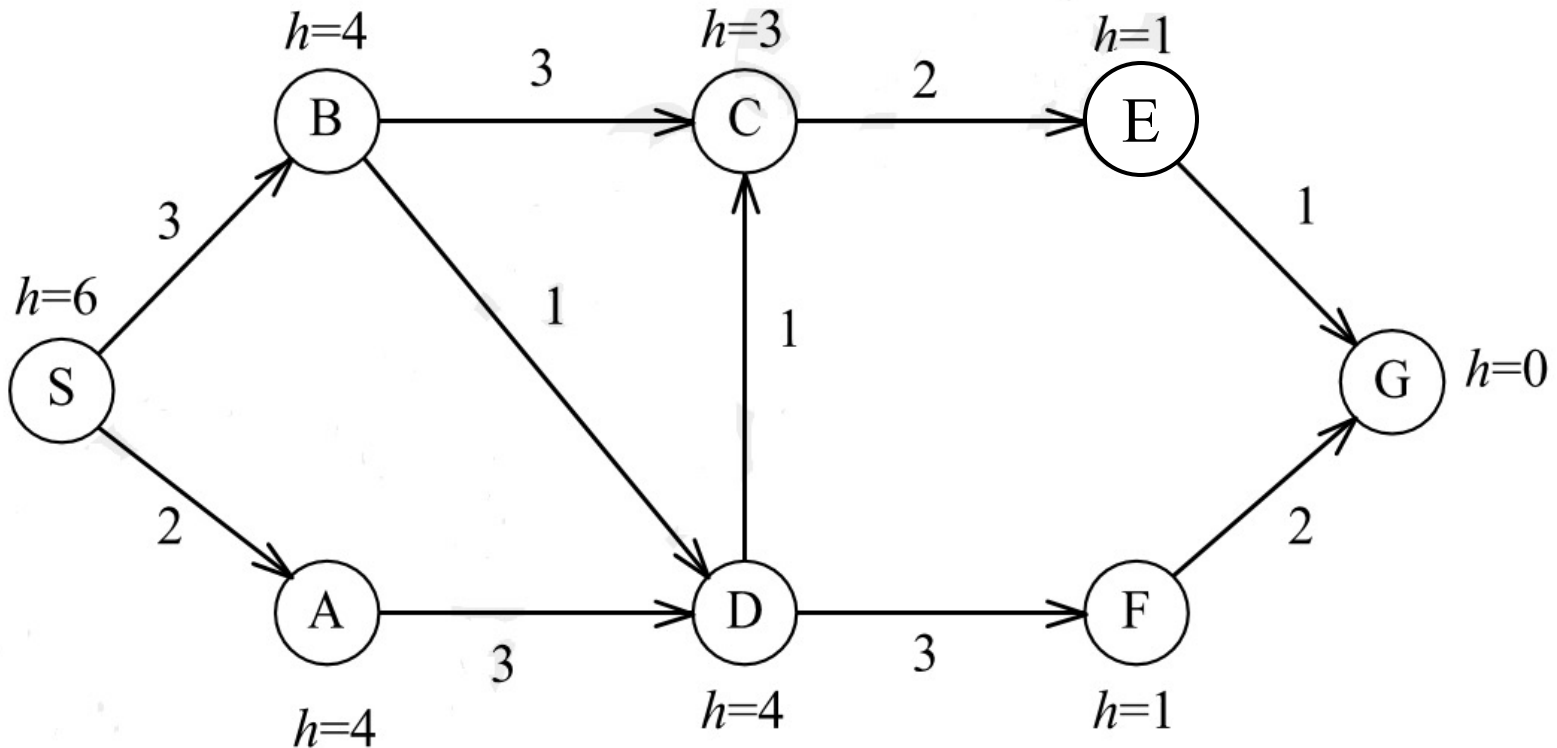
- $O(b^m)$
- If the *heuristic* function is good, the algorithm can be much faster

▶ Space?

- $O(b^m)$: store all nodes in the memory
- If the *heuristic* function is good, the number of nodes to store can be reduced significantly

Exercise 1

- Use **greedy search** algorithm to find the path from S to G ?



(Phuong TM, 2016)

Outline



- ▶ Greedy search
- ▶ A* search
- ▶ Heuristic functions
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A* search: idea

- ▶ Overcome the disadvantage of greedy search
 - Greedy: only care about the path to the goal
 - A*: also care about the path from the initial node to the current node
 - avoid expanding paths that are already expensive

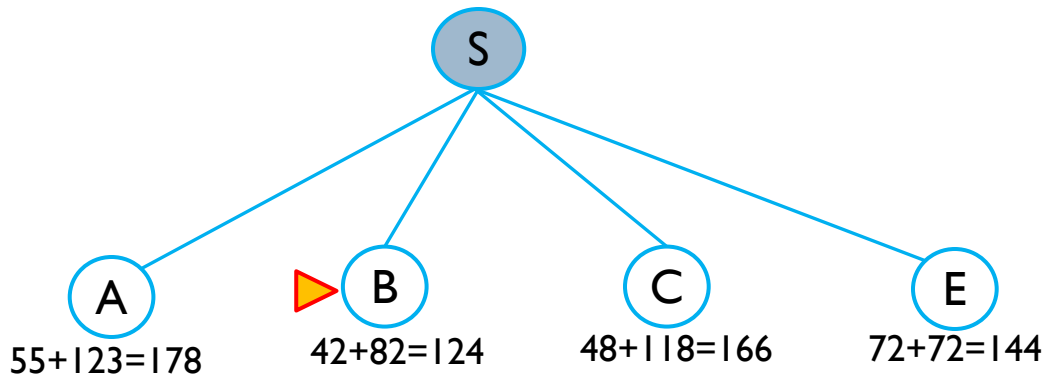
- ▶ **Method:** $f(n) = g(n) + h(n)$
 - $g(n)$: cost so far to reach n
 - $h(n)$: heuristic function, estimated cost from n to a goal node
 - $f(n)$: estimated total cost of the path from the initial node, through n , to a goal node

Example of A* search (1 / 5)

  S
 $0 + 125 = 125$

Example of A* search (2/5)

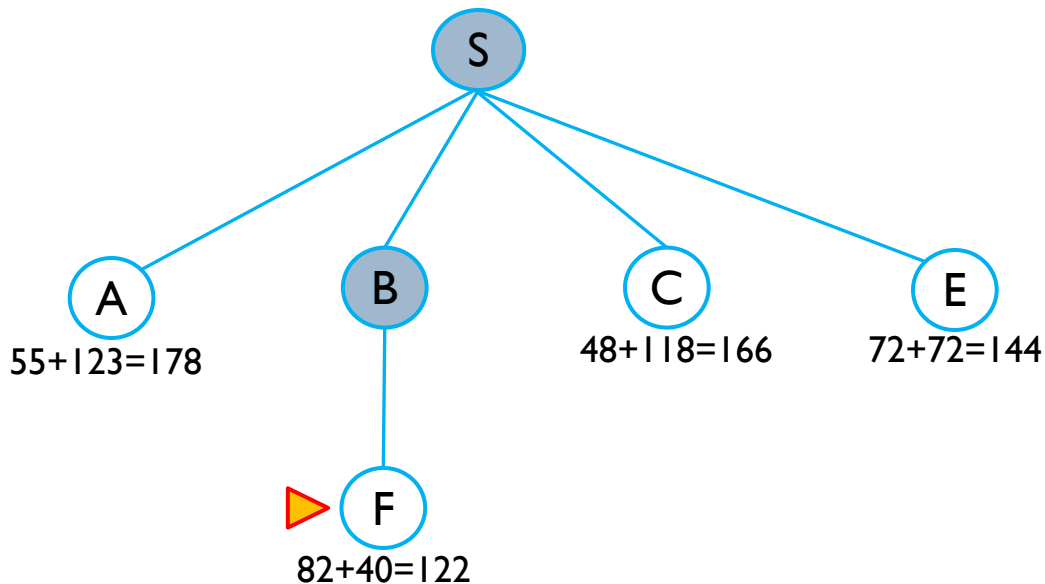
Expand S



Example of A* search (3/5)

Expand S

Expand B

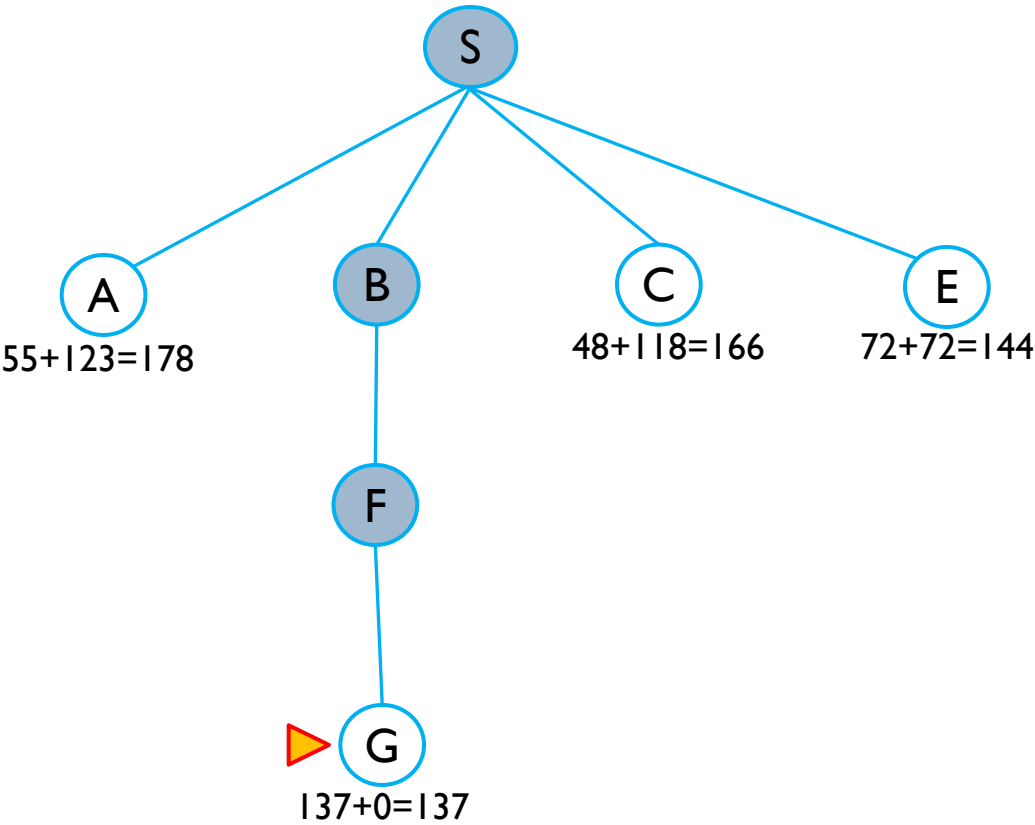


Example of A* search (4/5)

Expand S

Expand B

Expand F



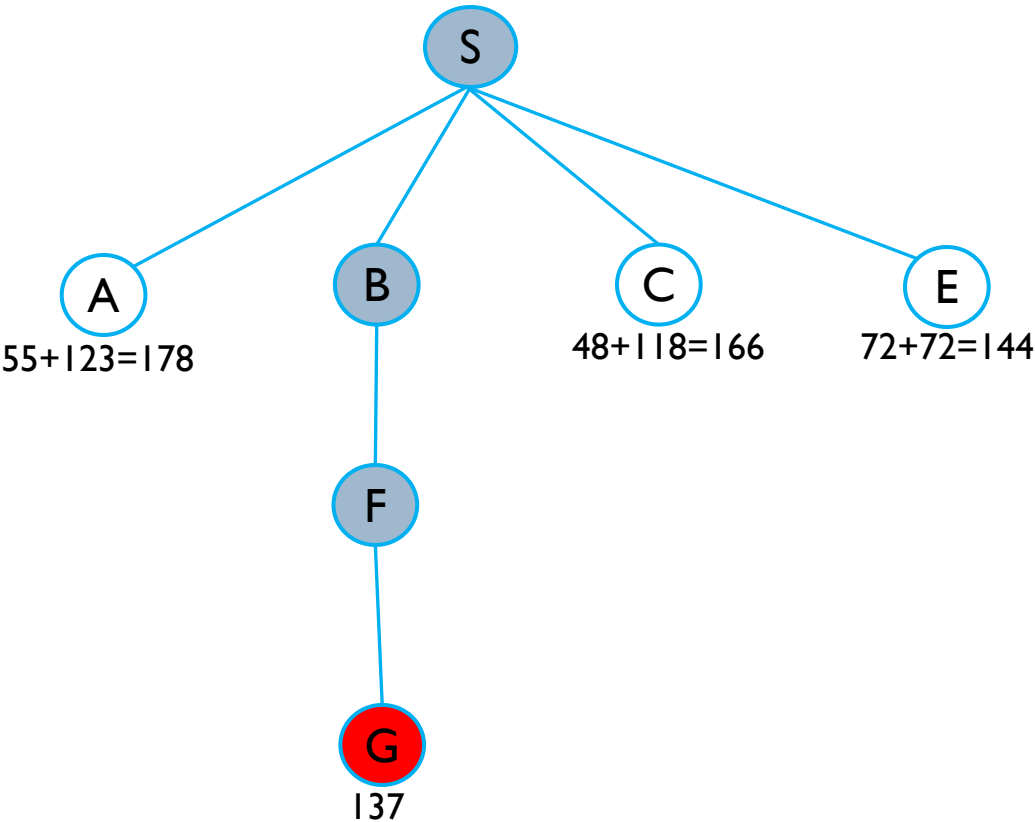
Example of A* search (5/5)

Expand S

Expand B

Expand F

Expand G: Goal



A* search algorithm

$A^* (Q, S, G, P, c, h)$

(Q : state space, S : initial state, G : goals, P : successor function, c : cost, h : heuristic)

Input: search problem, heuristic function h

Output: goal state (path to the goal state)

Initialize: $O \leftarrow S$ (O : the open node list)

while ($O \neq \emptyset$) **do**

1. take node n whose $f(n)$ is the smallest from O
2. **if** $n \in G$, **return** (path to n)
3. **for each** $m \in P(n)$
 - a) $g(m) = g(n) + c(n, m)$
 - b) $f(m) = g(m) + h(m)$
 - c) add m along with $f(m)$ to O

return no solution

Properties of A* search

▶ Complete?

- Yes (unless there are infinite nodes n with $f(n) \leq f(G)$)

▶ Optimality?

- Yes (if heuristic function h is admissible)

▶ Time?

- $O(b^m)$
- If the *heuristic* function is good, the algorithm can be much faster

▶ Space?

- $O(b^m)$: store all nodes in the memory
- If the *heuristic* function is good, the number of nodes to store can be reduced significantly

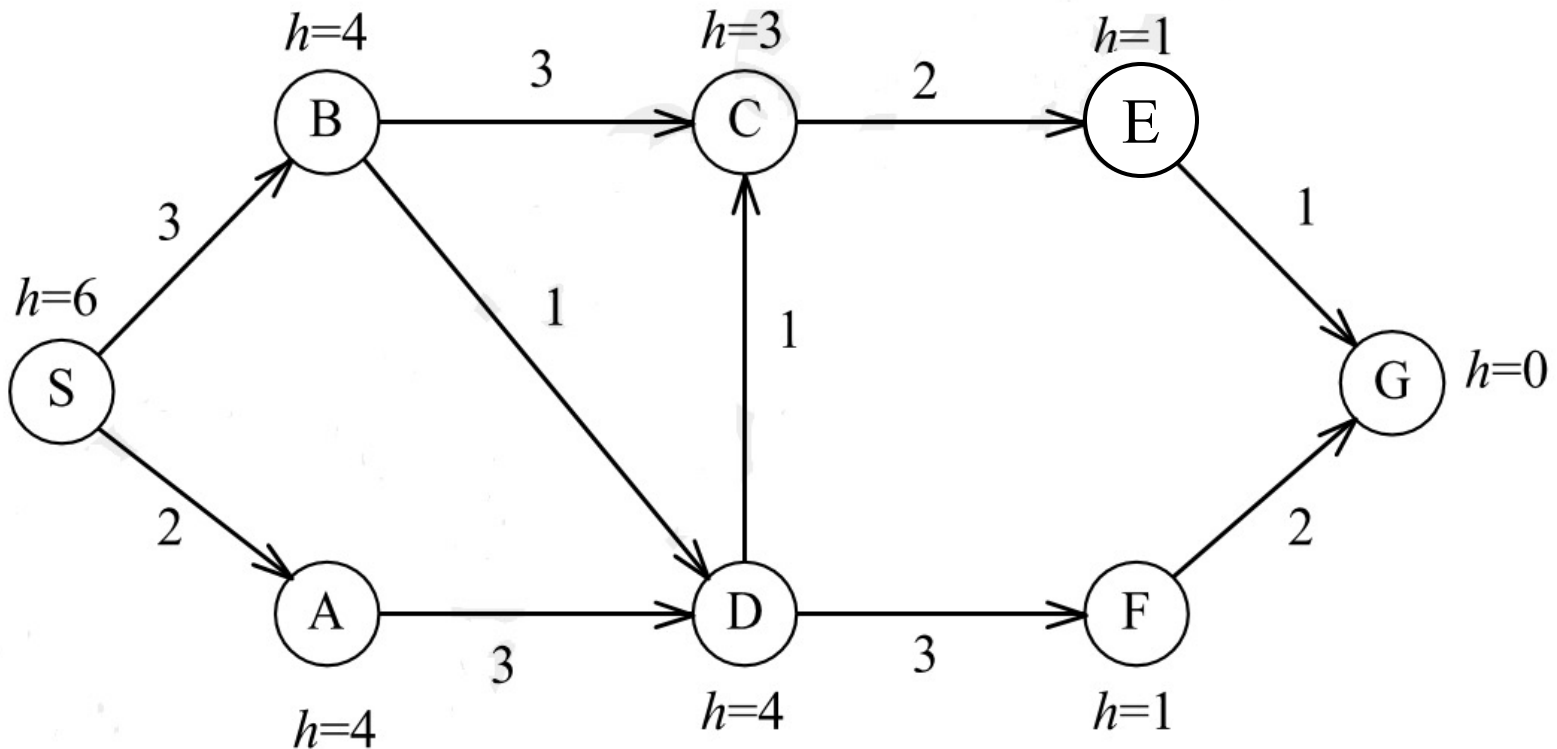
The optimality of A* search

- ▶ Admissible heuristic function
 - For every node n , $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cheapest cost from n to a goal node
 - Example: straight-line distance is an admissible heuristic function

- ▶ **Theorem:** A* algorithm is guaranteed to find an **optimal** solution if $h(n)$ is admissible

Exercise 2

- Use **A* search** to find the path from S to G ?



(Phuong TM, 2016)

Outline

- ▶ Greedy search
- ▶ A* search
- ▶ Heuristic functions
- ▶ Iterative deepening A* (IDA*)

Heuristic functions

- ▶ Heuristic functions are constructed depending on each specific problem
 - A problem may have some heuristic functions
 - The quality of the heuristic function greatly affects the search process

- ▶ Dominance
 - If $h_1(n)$ and $h_2(n)$ are 2 admissible heuristic functions satisfying $h_1(n) \leq h_2(n)$ for all nodes n , then h_2 **dominates** (is better than) h_1

Example of heuristic functions

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- ▶ $h_1(n)$: number of wrong cells
 - $h_1(S) = 8$
- ▶ $h_2(n)$: total of Manhattan distances
 - $h_2(S) = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18$

Outline

- ▶ Greedy search
- ▶ A* search
- ▶ Heuristic functions
- ▶ Iterative deepening A* (IDA*)

Iterative deepening A^* – IDA*

- ▶ **Goal:** deals with the memory problem in A^* search
 - a variant of IDS (uses the heuristic function from A^* search to limit nodes)

- ▶ **Method:** Iterate **DFS** over sub-trees where $f(n)$ is not greater than a threshold
 - The threshold is incremented after each loop so that we have new nodes

IDA* algorithm

$IDA^*(Q, S, G, P, c, h)$

Input: search problem, heuristic function h

Output: goal state (path to the goal state)

Initialize: $O \leftarrow S$ (O : the open node list)

Threshold $i \leftarrow 0$

while (1) **do**

1. **while** ($O \neq \emptyset$) **do**

a) Take the **first node** n from O

b) **if** $n \in G$, **return** (path to n)

c) For each $m \in P(n)$

i) $g(m) = g(n) + c(m, n)$

ii) $f(m) = g(m) + h(m)$

iii) **if** $f(m) \leq i$ **then** add m to the **head** of O

2. $i \leftarrow i + \beta, O \leftarrow S$

Properties of IDA*

▶ Completeness?

- Yes

▶ Optimality?

- β -optimal (cost of the found solution does not exceed β compared to cost of the optimal solution)

▶ Time?

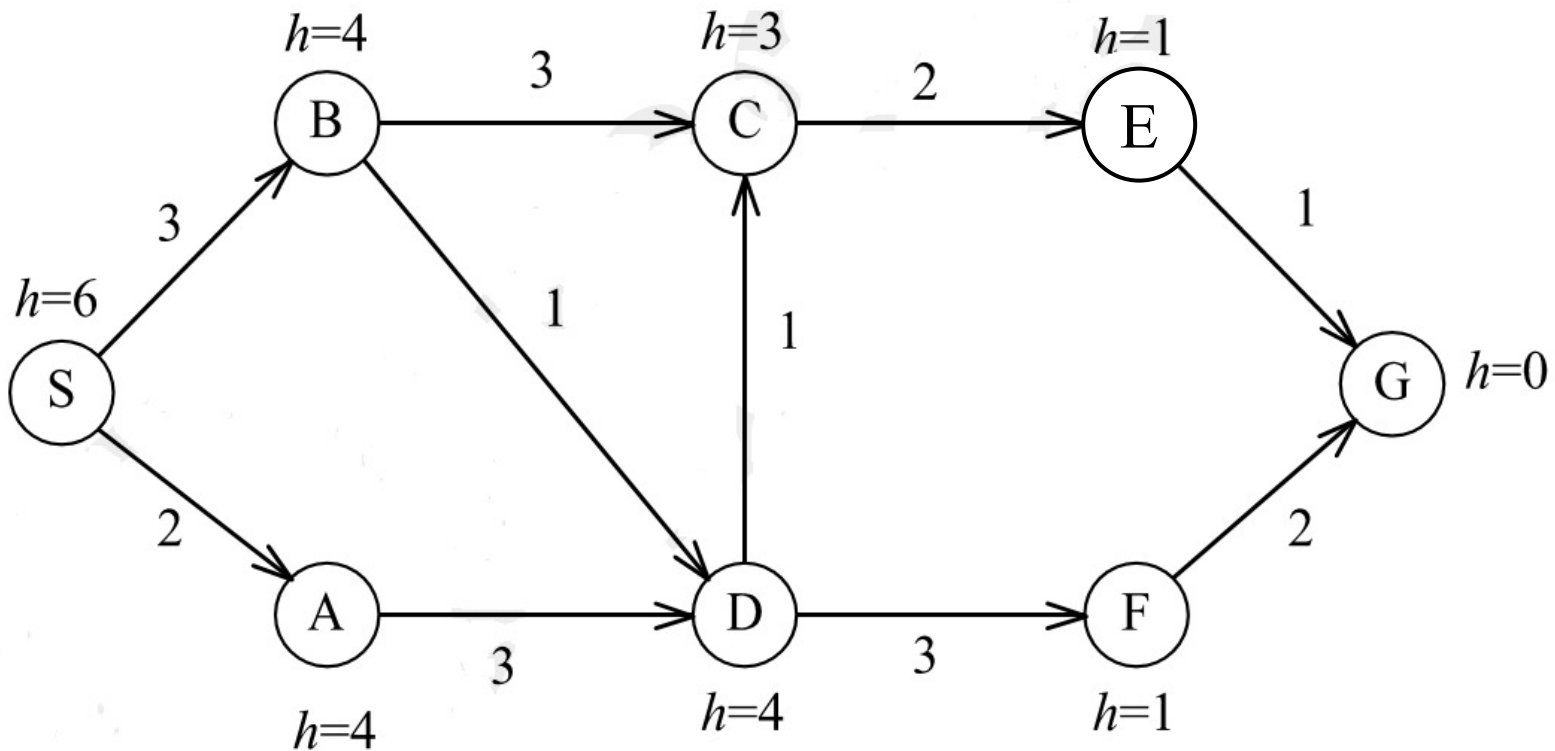
- Computational complexity is greater than that of A* search

▶ Space?

- Requires linear memory

Exercise 3

- Use **IDA* search** to find the path from S to G with $\beta = 2$?



(Phuong TM, 2016)

When to add repeated nodes to the open node list?

▶ Greedy

- **No**: adding repeated nodes does not change the algorithm (may lead to loops)

▶ A*

- In cases the repeated node has better cost, it will be **added to the list** (if it is already expanded) or **updated to replace the old node** (if it is on the list)

▶ IDA*:

- **Yes**