Matrix Multiplication

four nested loops $= n^3$

$A_1 \times A_2 \times A_3 \times A_4$

$5 \times 4 \quad 4 \times 6 \quad 6 \times 2 \quad 2 \times 7$

$5 \times 4 \times 6 = 120$

```
main ( )
 {
   int  n = 5
   int  p[ ] = {5,4,6,2,7};   ← dimension
   int  m[5] [5] = {0};
   int  s[5] [5] = {0};
   int  j, min, q ;
   for(int d = 1; d<n-1; d++)
     {
      for (int i = 1; i<n-d; i++)
          j = i+d
          min = 32767;
      for (int K = 1; K<j-1 ; K++)   ← when distance is greater than 1
          {
          q = m[i][K] + m[K+1][j] + p[i-1] * p[K] * p[j];
      if (q < min)
          {
          min = q ;
          s[i][j] = K   → i ≤ K < j
          }
          }
          m[i][j] = min
          }
          Cout << m[1][n-1];
```

$$C[i,j] = \min_{i \le K < j} \{ C[i,K] + C[K+1][j] + d_{i-1}\, dK\, d_j \}$$

Select array
make matrices

0	1	2	3	4
5	4	6	2	7

i	j
1	2
2	3
3	4

distance 1

i	j
1	3
2	4

distance 2

i	j
1	4

distance 3

between i and j

m (Minimum Cost)

	0	1	2	3	4
0	0	0	0	0	0
1		0	120		
2			0	48	
3				0	84
4					0

S

	0	1	2	3	4
0	0	0	0	0	0
1		0	1		
2			0	2	
3				0	3
4					0

DISTANCE 1 :

$m[1,2] = A_1 \times A_2 = 120$ (i = 1, j = 2)

A_1 A_2 5x4 4x6

$1 \le k < 2$

$m[2,3] = A_2 \times A_3 = 48$

A_2 A_3 4x6 6x2

$2 \le k < 3$

$m[3,4] = A_3 \times A_4 = 84$

A_3 A_4 6x2 2x7

$3 \le k < 4$

DISTANCE 2 :

$q = m[i][k] + m[k+1][j] + p[i-1] * p[k] * p[j]$

$m[1,3] = A_1 \times A_2 \times A_3$

$1 \le k < 3$ — 5×4 (A_1), 4×6 (A_2), 6×2 (A_3); $5\times4\times6 = 120$, $4\times6\times2 = 48$

$$m[1,3] = \min \begin{cases} k=1 & q = m[1][1] + m[2][3] + p[\cancel{1-1}^{0}] \times p[1] \times p[3] = 88 \\ & \quad 0 + 48 + 5 \times 4 \times 2 \\ & \quad (\text{OG array values}) \\ k=2 & q = m[1][2] + m[3][3] + p[0] \times p[2] \times p[3] = 180 \\ & \quad 120 + 0 + 5 \times 6 \times 2 \end{cases}$$

- Choose 88 because min for $m[1,3]$
- Choose $k=1$ for $s[1,3]$ because corresponds to 88

$m[2,4] = A_2 \times A_3 \times A_4$

$2 \le k < 4$ — 4×6 (A_2), 6×2 (A_3), 2×7 (A_4)

$$m[2,4] = \min \begin{cases} k=2 & q = m[2][2] + m[3][4] + p[1] \times p[2] \times p[4] = \\ & \quad 4 \times 6 \times 7 \\ k=3 & q = m[2][3] + m[4][4] + p[1] \times p[2] \times p[4] = 104 \\ & \quad 4 \times 6 \times 7 \end{cases}$$

- Choose 104 because min for $m[2,4]$
- Choose $k=3$ for $s[2,4]$ because corresponds to 104

m (Minimum Cost)

	0	1	2	3	4
0	0	0	0	0	0
1		0	120	88	158
2			0	48	104
3				0	84
4					0

S

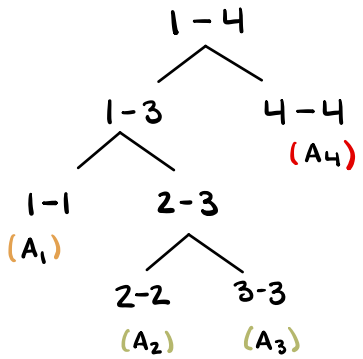
	0	1	2	3	4
0	0	0	0	0	0
1		0	1	1	3
2			0	2	3
3				0	3
4					0

DISTANCE 3 :

$m[\overset{i}{1},\overset{j}{4}] = 158 \quad K = 3$

$A_1 A_2 A_3 A_4$

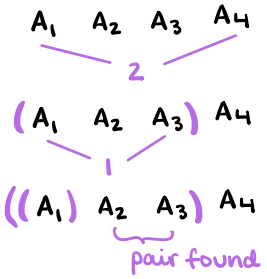
PARENTHESIZATION :



$(A_1 (A_2 A_3))(A_4)$

OR

Stop at pair :



$((A_1) A_2 A_3) A_4$