Haley Hill
Linear Algebra - lecture 1
Prof. in training Andrew "#1teach" Evans

Physics

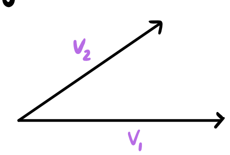
Vectors have magnitude and direction (m, θ)
(Kinda like arrows of some length pointing somewhere)

magnitude: the length of the vector
↳ ex: the pythagorean theorem for 2D vectors
$$\sqrt{x^2 + y^2}$$

direction: the angle the vector is pointing
↳ ex: $\tan^{-1}(y/x)$

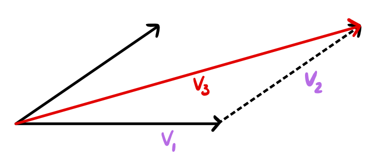
$$\vec{v} = (3, -1)$$
↑ vector (x, y)

adding vectors



$V_1 + V_2 = ?$

1. slide V_2 up so at tip of V_1
2. draw new vector V_3 that starts where V_1 and V_2 meet and stops where V_2 ends



3. $V_1 + V_2 = V_3$

NOTE: vector addition is done component-wise

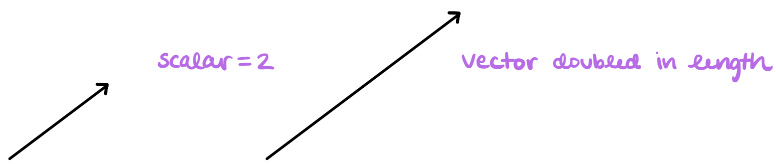
$$V_3 = \left(\text{x-components added}, \text{y-components added}\right)$$

$$V_3 = (\#, \#)$$

*Sketchy definition, works for now!

Scalars

A scalar is a number you multiply a vector by that changes scale*



Scalars are usually real numbers in physics.
Here, a is used to represent one

$$a \in \mathbb{R}$$

the scalar

is an element of

Scaling a vector

$$aV = V_{\text{scaled}}$$

Set-Builder Notation

write a set and the things inside the set follow those rules

$$\{\, x \mid \underbrace{\qquad}_{\text{condition}} \,\}$$

some element

such that

condition

ex:

$$\{a,n,d,r,e,w\} / a,n \rightarrow \{d,r,e,w\}$$

remove

ex: Python list comprehension

$$[x \text{ for } x \text{ in range}(n)] \rightarrow [0,1,\ldots n]$$

Computer Science

vectors are like rules you can put on lists and arrays
- ↳ addition
- ↳ same length
- ↳ scalar multiplication

IN MATH: vectors must have the same length to add them together.

IN CS: it is possible to be kinda squirrely about this, since vectors are more abstract.

$$[0^{th}, 1^{st}, \ldots n^{th}]$$

addition rules

length = n+1

$$V_1 = [\ ]$$

$$V_2 = [\ ]$$

starts at 0, goes to input-1

```
for i in range(n+1):
    new_vector[i] = V1[i] + V2[i]
    scaled_vector[i] = scalar • vector[i]
```

Math

a vector x is an element of a vectorspace

a vectorspace (V) over a field (F) is a set with 2 operations such that the following 8 rules hold.

COMMUTATIVE PROPERTY (order added doesn't matter)

$x, y \in V \implies x + y = y + x$

"if x, y are elements of vector space"

"then"

vector addition

ASSOCIATIVE PROPERTY (group addition however you want)

$x, y, z \in V \implies x + (y + z) = (x + y) + z$

IDENTITY (aka the zero vector)

$\exists$ 0 such that $\forall x \in V$ $x + 0 = 0 + x = x$

"there exists"

an element 0

"where"

"for all"

"x in the Vectorspace"

adding 0 to x gives back x

INVERSE (every element has one!)

$\forall x \in V$ $\exists y \in V$ s.t. $x + y = 0$

"for all x in the Vectorspace"

"there is some element y"

"where"

an element that isn't the inverse

\+

the inverse

gives back zero

SCALING IDENTITY

$\forall x \in V \quad \exists a \in F \quad$ s.t. $\quad ax = x$

"there is an element a in the field"

ASSOCIATIVITY FOR SCALING

$\forall \underbrace{a, b \in F}_{\text{elements of the field are scalars}} \quad \underbrace{x \in V}_{\text{elements of the vectorspace are vectors}} \qquad (ab)x = a(bx)$

$(6)x = \underbrace{2(3x)}_{\text{scaling twice}}$

- scaling vector x by 6
- scaling vector x by 3 then scaling vector x by 2

DISTRIBUTIVE 1

you can distribute a scalar across vector sums

$\forall a \in F \quad x, y \in V \qquad a(x+y) = ax + ay$

DISTRIBUTIVE 2

you can distribute a vector across a sum of scalars

$\forall a, b \in F \quad x \in V \qquad (a+b)x = ax + bx$

$\mathbb{R}^n$

$\mathbb{R}^2$ is a 2-dimensional vectorspace
so!
$\mathbb{R}^n$ is an n-dimensional vectorspace

over the field of real numbers ($\mathbb{R}$)

let $x \in \mathbb{R}^n$ ← # of dimensions

↑ tuple of length n

$\mathbb{R}^2$ ← length 2 tuple
$\mathbb{R}^3$ ← length 3 tuple

$$\mathbb{R}^n = \{x \mid x = (a_1, a_2, \dots a_n) \text{ for some } a_i \in \mathbb{R}\}$$

↑ such that

↑ whatever index

"The set $\mathbb{R}^n$ is the set of real-valued ntuples"

↑ $\mathbb{R}$

↑ $(a_1, a_2, \dots a_n)$
a list of #s in parentheses

$s \in \mathbb{R}$ $\quad s \cdot x = (s \cdot a_1, s \cdot a_2, \dots s \cdot a_n)$

scalar multiplication & vector addition
are done component-wise

Proving a set and 2 operations don't form a vectorspace

- check Identity
  if not true for all x → failed
- check Inverse
  if an element doesn't have one → failed

PROVE $\mathbb{R}^n$ FORMS A VECTORSPACE

Proving Commutative Property for $\mathbb{R}^n$

WTS: $x + y = y + x$

let $x, y \in \mathbb{R}^n$

thus, we can write

$$x = (a_1, a_2, \dots a_n) \text{ for some } a_i \in \mathbb{R}$$

$$y = (b_1, b_2, \dots b_n) \text{ for some } b_i \in \mathbb{R}$$

Suppose we have $x + y$

$$x + y = (a_1, a_2, \dots a_n) + (b_1, b_2, \dots b_n)$$

by the definition of addition of vectors in $\mathbb{R}^n$

$$x + y = (a_1 + b_1, a_2 + b_2 \dots a_n + b_n)$$

by the commutivity of $\mathbb{R}$

$$x + y = (b_1 + a_1, b_2 + a_2 \dots b_n + a_n)$$

by the definition of vectors in $\mathbb{R}^n$

$$x + y = (b_1, b_2, \dots b_n) + (a_1, a_2, \dots a_n)$$

$$x + y = y + x$$

Proving that $\mathbb{R}^n$ has an Identity

let $x \in \mathbb{R}^n$

Consider

$$e = (\underbrace{0, 0, 0, \dots 0}_{n \text{ times}}) \in \mathbb{R}^n$$

Suppose

$$x + e = (x_1 + x_2 + \dots x_n) + (0, 0, 0, \dots 0)$$

$$= (x_1 + 0, x_2 + 0, \dots x_n + 0)$$

$$= (x_1 + x_2 + \dots x_n)$$

$$x + e = x$$

Therefore

e is the zerovector (0)

identity for vector addition

CODING $\mathbb{R}^n$

add vector function
expects two vectors in $\mathbb{R}^n$
return sum

scalar multiplication function
expects one vector and one scalar
return scaled vector

class Vector
add vector method — myVector.add(vector)
expects one vector in $\mathbb{R}^n$
return sum

scalar multiplication method — myVector.scale(scalar)
expects one scalar
return scaled vector

using the 8 rules for a vectorspace,
we can build theorems to solve problems (like facial recognition!)