CENG 216 – NUMERICAL COMPUTATION
Homework 2

14 April 2021

Due Date: 03 May 2021

Note: You are required to submit the homework solutions as handwritten text that belongs personally to you. Typesetting and printing are not allowed.

Exercise 1 Solving Linear Systems of Equations

We are given the following linear system of equations

$$\begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix}$$

a. Find the solution to this system using LU decomposition.

b. Find the solution to this system using PA=LU decomposition and check if you obtain the same solution.

c. For the approximate solution $\mathbf{x}_a = [-1.0, 1.0, 1.0]^\top$, calculate the relative forward and relative backward errors, and the error magnification factor.

Exercise 2 Interpolation

Given the points $P_0 = (0.0, 1.0)$, $P_1 = (2.0, 2.0)$, and $P_2 = (3.0, -1.0)$. Write down **the coefficients** a_0, a_1, and a_2 of the second degree polynomial interpolating the points P_0, P_1, and P_2 using Newton's divided differences.

Exercise 3 Ball Throwing Robot Arm

We want to design a ball throwing robot arm that will throw a ball with initial velocity V_0 at an angle θ to the ground plane. Assuming that the robot arm is at coordinates $x = 0$ the ball will hit the ground at

$$x_f = \frac{V_0^2 \sin 2\theta}{g}$$

where $g = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}$ is the gravitational constant.

We want to ensure that the ball hits the ground at $x_f = 0.1730861$ meters. Unfortunately, we are not free to choose the necessary V_0 and θ since the robot design constrains the speed to be a function of the angle as

$$V_0 = k(1 + \cos\theta)$$

where $k = 0.75$ is a design parameter.

a. Write down a single constraint in terms of θ that ensures that the ball hits the ground at the given value of x_f.

b. Convert the solution of θ into a root finding problem, then solve this problem by searching for θ in the interval $[0°, 40°]$.

c. Calculate the necessary V_0 for the θ you have found.

Hint: Although the θ range is given in degrees, perform the calculations in radians. Note that $360° = 2\pi$ radians.