

CENG 216 – NUMERICAL COMPUTATION

Homework 1

24 March 2021

Due Date: 06 April 2021

Note: You are required to submit the homework solutions as handwritten text that belongs personally to you. Typesetting and printing are not allowed.

Exercise 1 Fundamentals

- a. Compute the result of the following arithmetic expression in double-precision IEEE floating-point representation

$$4.4 - 3.4 - 1.0$$

as was done in class for the computation $9.4 - 9.0 - 0.4$. Show the steps of your work. For each number, write the hexadecimal and binary representations in the computer's memory. Write the rounding error for each number. Check your result using a computer.

- b. Write for which values of x the following expression involves loss of significance:

$$\frac{1}{1+x} - \frac{1}{1-x}$$

and write an alternative formula for the same expression that does not have the same problem.

Exercise 2 Root Finding I

- a. Use the Bisection Method to find the root to eight correct decimal places of the equation

$$\ln x + x^2 = 3.$$

You will need to guess the starting interval. Write down the interval values for the first three and the last iterations.

- b. Show that $-1, 0$, and 1 are fixed points of

$$\frac{x^2 - 5x}{x^2 + x - 6}.$$

- c. Which of the following three Fixed-Point Iterations converge to $\sqrt{2}$? Rank the ones that converge from fastest to slowest.

- $x \rightarrow \frac{1}{2}x + \frac{1}{x}$
- $x \rightarrow \frac{2}{3}x + \frac{2}{3x}$
- $x \rightarrow \frac{3}{4}x + \frac{1}{2x}$

- d. Apply Fixed-Point Iteration to find the solution of the equation

$$\ln x + x^2 = 3$$

to eight correct decimal places. Write down the x values for the first three and the last iterations.

Exercise 3 Root-Finding II

Calculate the x -coordinate of the intersection of the parabola $y = -x^2 + 4.0$ with the line $y = 4x - 1.0$ starting from an estimate of $x_0 = 1.5$ using

- a. Ten iterations of the Fixed Point Iterations method.
- b. Three iterations of the Newton's method.

When writing down the iteration results, it is OK to write down only six decimal digits after the dot.