1)

a) $fl(4.4) \Rightarrow 100,\overline{0110} = 1,000\overline{110} \times 2^2 = 1,000110\ldots$

$01\;100110 \times 2^2$

52 53 54 round up

$= 4,4 - (0,1001 \cdot 2^{-52} \cdot 2^2) + 2^{-52} \cdot 2^2$

$16x - x = 15x = (1001)_2 \Rightarrow x = \frac{9}{15}$

$= 0,6$

$= 4,4 - 0,6 \cdot 2^{-50} + 2^{-50}$

$fl(4,4) = 4,4 + 0,4 2^{-50} \Rightarrow \underline{0,4 2^{-50}}$

Rounding Error

$fl(3,4) \Rightarrow 11.\overline{0110} = 1,1\overline{0110} \times 2$

$= 1,10110\ldots01100110 \times 2$

52 53 round down

$= 3,4 - \underbrace{0.00\overline{11}}_{x} \cdot 2^{-52} \cdot 2$

$16x - x = 15x = (0011)_2 \Rightarrow x = \frac{3}{15}$

$= 0,2$

$fl(3.4) = 3,4 - 0,2 \cdot 2^{-51}$

$= 3,4 - 0,1 \cdot 2^{-50} \Rightarrow \underline{|-0,1 \cdot 2^{-50}|}$

Rounding Error

$fl(1.0) \Rightarrow \underline{1.0}$

No Rounding Error

$4.4 - 3.4 - 1 \Rightarrow 4.4 + 0,4 \cdot 2^{-50} - (3.4 - 0,1\, 2^{-50}) - 1$

$\Rightarrow 2^{-50}(0,4 + 0,1) = 0,5 \cdot 2^{-50}$

$= \underline{4.440892098\ldots\ldots}$

$fl(4.4) = 4.4 + 0.4 \times 2^{-50} = 4.4 + 0,1 \times 2^{-48} = 4.4 + 0,1 \times 16^{-12}$

hexadecimal$(4.4) = \underline{(4,6 + 0.19 \times 10^{-c})}_{16}$

$fl(3.4) = 3.4 - 0,1 \cdot 2^{-50} = 3,4 - 0,025 \times 2^{-48} = 3,4 - 0,025 \times 16^{-12}$

hexadecimal$(3.4) = \underline{(3,6 - 0,06 \times 10^{-c})}_{16}$

$fl(1.0) = 1.0$

hexadecimal$(1.0) = (1.0)_{16}$

b) $\frac{1}{1+x} - \frac{1}{1-x}$ If x is too small, the expression involves.

$$f(10^{-100}) = \frac{1}{1+10^{-100}} - \frac{1}{1-10^{-100}} = 1-1=0$$

We will combine them.

$$f(x) = \frac{1}{\underset{(1-x)}{1+x}} - \frac{1}{\underset{(1+x)}{1-x}} \Rightarrow \frac{(1-x)-(1+x)}{(1+x).(1-x)} = \frac{-2x}{1-x^2}$$

$$f(10^{-100}) = \frac{-2.10^{-100}}{1-10^{-200}} = -2.10^{-100}$$

2) a) Bisection method $f(x) = \ln x + x^2 - 3 = 0$ $[1,2]$

$f(1) = -2.0$ $f(2) = 1,69314$ $f(1,5) = -0,34453$

$(b-a)/2 = 0,5$

$f(1,5) = -0,34453$ $f(2) = 1,69314$ $f(1,75) = 0,62211$

$(b-a)/2 = 0,25$

$f(1,5) = -0,34453$ $f(1,75) = 0,62211$ $f(1,625) = 0,12613$

$(b-a)/2 = 0,125$

⋮

$f(1,59214) = -0,00000000046$ $f(1,5921429377)$

$\Downarrow$ $\Downarrow$

$1,5921429358$ $0,0000000025$

$1,5921429377$

Answer $= 1,5921429368$

b) $f(x) = \frac{x^2-5x}{x^2+x-6}$

$$f(-1) = \frac{1+5}{1-1-6} = -1$$

$$f(0) = \frac{0-0}{0-0-6} = 0$$

$$f(1) = \frac{1-5}{1+1-6} = 1$$

c) $f(x) = \frac{1}{2}x + \frac{1}{x}$ $\quad f(\sqrt{2}) = \frac{\sqrt{2}}{2} + \frac{1}{\sqrt{2}} = \sqrt{2}$ fixed point

$f'(x) = \frac{1}{2} - \frac{1}{x^2}$ $\quad |f'(\sqrt{2})| = \left|\frac{1}{2} - \frac{1}{2}\right| = 0$

$f(x) = \frac{2x}{3} + \frac{2}{3x}$ $\quad f(\sqrt{2}) = \frac{2\sqrt{2}}{3} + \frac{2}{3\sqrt{2}} = \sqrt{2}$ fixed point

$f'(x) = \frac{2}{3} - \frac{2}{3x^2}$ $\quad |f'(\sqrt{2})| = \left|\frac{2}{3} - \frac{2}{3.2}\right| = \frac{1}{3}$

$f(x) = \frac{3x}{4} + \frac{1}{2x}$ $\quad f(\sqrt{2}) = \frac{3\sqrt{2}}{4} + \frac{1}{2\sqrt{2}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$ fixedpoint

$f'(x) = \frac{3}{4} - \frac{1}{2x^2}$ $\quad |f'(\sqrt{2})| = \left|\frac{3}{4} - \frac{1}{4}\right| = \frac{1}{2}$

eq 1 , eq 2 , eq 3
Fastest Slowest

d) $\ln x + x^2 = 3$ $\quad g(x) = \sqrt{3 - \ln x} = x$

0	1,0
1	1,73205081
2	1,56548921
⋮	⋮
14	1,59214294
15	1,59214294

3) $y = -x^2 + 4$ $\quad y = 4x - 1$ $\Rightarrow -x^2 + 4 = 4x - 1 \Rightarrow 0 = x^2 + 4x - 5$

a) $x^2 + 4x - 5 = 0$

$x(x+4) = 5$

$g(x) \Rightarrow \frac{5}{x+4} = x$ $\quad h x \Rightarrow \frac{5}{x} = x + 4$

$\Rightarrow \frac{5}{x} - 4 = x$

n	$g(x) = \frac{5}{x+4}$	$h(x) = \frac{5}{x} - 4$
0	1,5	1,5
1	0,90901	−0,666667
2	1,018579	−115
3	0,996310	−4,434783
4	1,000739	−5,127457
5	0,999852	−4,975143
6	1,000030	−5,004996
7	0,999994	−4,999002
8	1,000001	−5,000200
9	1,0	−4,999960
10	1,0	−5,000007

b) $0 = x^2 + 4x - 5$

$f'(x) = 2x + 4$

$$x_{i+1} = x_i - \frac{x_i^2 + 4x_i - 5}{2x_i + 4}$$

n	$N(x) = x - \frac{x^2+4x-5}{2x+4}$
0	1,5
1	1,035714
2	1,000210
3	1,000000

1,0

⋮