1)

a) $A = LU$

$$\begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix} \longrightarrow A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix} \xrightarrow[R_2 = R_2 - \frac{1}{2}R_1]{R_3 = 2 \times R_1 + R_3} \begin{bmatrix} 4 & 2 & -1 \\ 0 & 5 & -2{,}5 \\ 0 & -5 & 7 \end{bmatrix} \xrightarrow{R_3 = R_3 + R_2} \underbrace{\begin{bmatrix} 4 & 2 & -1 \\ 0 & 5 & -2{,}5 \\ 0 & 0 & 4{,}5 \end{bmatrix}}_{U}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix} \qquad LY = B \text{ then } UX = Y$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix}$$

$L \quad Y \quad B$

$y_1 = -1 \qquad \frac{1}{2} y_1 + y_2 = -8$

$-2y_1 - y_2 + y_3 = 14 \qquad y_2 = \frac{-15}{2}$

$y_3 = 14 - 2 - \frac{15}{2}$

$y_3 = \frac{9}{2}$

$$\begin{bmatrix} 4 & 2 & -1 \\ 0 & 5 & \frac{-5}{2} \\ 0 & 0 & \frac{9}{2} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{-5}{2} \\ \frac{9}{2} \end{bmatrix}$$

$U \quad X \quad Y$

$\frac{9}{2} x_3 = \frac{9}{2}$

$x_3 = 1$

$5x_2 - \frac{5}{2} x_3 = -\frac{15}{2}$

$5x_2 = -5$

$x_2 = -1$

$4x_1 + 2x_2 - x_3 = -1$

$4x_1 - 2 - 1 = -1$

$x_1 = \frac{1}{2}$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{-15}{2} \\ \frac{-9}{2} \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -1 \\ 1 \end{bmatrix}$$

b)

$$L \cdot C = P \cdot b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \frac{1}{4} & -\frac{2}{3} & 1 \\ -\frac{1}{2} & \frac{3}{3} & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 \\ -\frac{1}{2} & -\frac{2}{3} & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -8 \\ -1 \end{bmatrix}$$

$$1 \cdot c_1 = 14$$
$$c_1 = 14$$

$$-\frac{1}{4} c_1 + c_2 = -8$$
$$c_2 = -8 + \frac{14}{4}$$
$$c_2 = -\frac{9}{2}$$

$$-\frac{1}{2} c_1 - \frac{2}{3} c_2 + c_3 = -1$$
$$-7 + 3 + c_3 = -1$$
$$c_3 = 3$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -\frac{9}{2} \\ 3 \end{bmatrix}$$

$$U \cdot X = C$$

$$\begin{bmatrix} -8 & -9 & 9 \\ 0 & \frac{15}{4} & -3/4 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -\frac{9}{2} \\ 3 \end{bmatrix}$$

$$3 X_3 = 3$$
$$X_3 = 1$$

$$\frac{15}{4} X_2 - \frac{3}{4} X_3 = -\frac{9}{2}$$
$$\frac{15}{4} X_2 = -\frac{9}{2} + \frac{3}{4}$$
$$\frac{15}{4} X_2 = -\frac{15}{4}$$
$$X_2 = -1$$

$$-8 X_1 - 9 X_2 + 9 X_3 = 14$$
$$-8 X_1 = 14 - 9 - 9$$
$$X_1 = \frac{4}{8} = \frac{1}{2}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -1 \\ 1 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix}$$

$$\Downarrow$$

$$A = \begin{bmatrix} -8 & -9 & 9 \\ 2 & 6 & -3 \\ 4 & 2 & -1 \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Downarrow \quad R_2 = \tfrac{1}{4}R_1 + R_2, \quad R_3 = \tfrac{1}{2}R_1 + R_3$$

$$\begin{bmatrix} -8 & -9 & 9 \\ 0 & \frac{15}{4} & -\frac{3}{4} \\ 0 & -\frac{5}{2} & \frac{7}{2} \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 \\ -\frac{1}{2} & -\frac{2}{3} & 1 \end{bmatrix}$$

$$\Downarrow \quad R_3 = \tfrac{2}{3}R_2 + R_3$$

$$U = \begin{bmatrix} -8 & -9 & 9 \\ 0 & \frac{15}{4} & -\frac{3}{4} \\ 0 & 0 & 3 \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 \\ -\frac{1}{2} & -\frac{2}{3} & 1 \end{bmatrix} \cdot \begin{bmatrix} -8 & -9 & 9 \\ 0 & \frac{15}{4} & -\frac{3}{4} \\ 0 & 0 & 3 \end{bmatrix}$$

$$P \cdot A = L \cdot U$$

c)

$x_a = [-1.0, 1.0, 1.0]^T$

$$\begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix} \qquad x = \begin{bmatrix} \frac{1}{2} \\ -1 \\ 1 \end{bmatrix}$$

Backward Error: $\|b - Ax_a\|_\infty$

$$= \left\| \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix} - \begin{bmatrix} 4 & 2 & -1 \\ 2 & 6 & -3 \\ -8 & -9 & 9 \end{bmatrix} \cdot \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix} \right\|_\infty$$

$$\left\| \begin{bmatrix} -1 \\ -8 \\ 14 \end{bmatrix} - \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix} \right\|_\infty = \left\| \begin{bmatrix} 2 \\ -9 \\ 6 \end{bmatrix} \right\| = 9$$

Forward Error: $\|x_a - x\|_\infty$

$$\left\| \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} \\ -1 \\ 1 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -\frac{3}{2} \\ 2 \\ 0 \end{bmatrix} \right\| = 2$$

$$\text{Relative Backward Error} = \frac{\text{Backward}}{\text{Forward}} = \frac{9}{2}$$

$$\text{Relative Forward Error} = \frac{\|x - x_a\|_\infty}{\|x\|_\infty} = \frac{\left\| \begin{bmatrix} \frac{1}{2} \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix} \right\|}{\left\| \begin{bmatrix} \frac{1}{2} \\ -1 \\ 1 \end{bmatrix} \right\|}$$

$$= \frac{\left\| \begin{bmatrix} \frac{3}{2} \\ -2 \\ 0 \end{bmatrix} \right\|}{1} = 2$$

$$\text{Error Magnification Error} = \frac{\text{Relative Forward}}{\text{Relative Backward}} = \frac{2}{\frac{9}{2}} = \frac{4}{9}$$

$$= \underline{0,4}$$

2)

$P_0 = (0.0, 1.0)$, $P_1 = (2.0, 2.0)$, $P_2 = (3.0, -1.0)$

$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_0)\dots(x - x_{n-1})$

a	f(a)
0.0	1.0
2.0	2.0
3.0	-1.0

$\frac{2.0 - 1.0}{2.0 - 0.0} = \frac{1}{2}$

$\frac{-1 - 2.0}{3.0 - 2.0} = \frac{-3}{1} = -3$

$\frac{-3 - \frac{1}{2}}{3.0 - 0.0} = \frac{\frac{-7}{2}}{3} = \frac{-7}{6}$

$P_2(x) = 1.0 + \frac{1}{2}(x - 0.0) - \frac{7}{6}(x - 0.0)(x - 2.0)$

$a_0 = 1.0$

$a_1 = \frac{1}{2}$

$a_2 = -\frac{7}{6}$

3) $X_f = \frac{V_0^2 \cdot \sin 2x}{g}$ $\quad g = 9{,}8\ m/s^2$

$X_f = 0{,}1730861$ $\quad V_0 = k(1+\cos x)$ $\quad k = 0{,}75$

$$X_f = \frac{k^2(1+\cos x)^2 \cdot \sin 2x}{g} = \frac{\frac{9}{16} \cdot (\cos^2 x + 2\cos x + 1) \cdot \sin 2x}{9{,}8}$$

$\sin 2x = 2 \cdot \cos x \cdot \sin x$

$$X_f = \frac{9}{16} \cdot \frac{(\cos^2 x + 2\cos x + 1) \cdot (2 \cdot \sin x \cdot \cos x)}{9{,}8}$$

$\frac{9}{16} \cdot \frac{2}{9{,}8} = 0{,}11479 = a$

$\sin x = \sqrt{1-\cos^2 x}$

$$X_f = a \cdot (\cos^2 x + 2\cos x + 1) \cdot \cos x \cdot \sqrt{1-\cos^2 x}$$

$$X_f = a \cdot (\cos^3 x + 2\cos^2 x + \cos x)\sqrt{1-\cos^2 x}$$

$$\frac{0{,}1730861}{0{,}114785} = 1{,}507784 = (\cos^3 x + 2\cos^2 x + \cos x)\sqrt{1-\cos^2 x} - 1{,}507784$$

F

•) With bisection method, I made calculator for calculations.
The program found as $x = 0{,}166987848\,5\pi$ which is $\cong 30°$

$V_0(x) = \frac{3}{4}(1+\cos x) = 1{,}39914034 \cong 1{,}4\ m/s$