# A Strong Baseline for Weekly Time Series Forecasting

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# Abstract

Many businesses and industries require accurate forecasts for weekly time series nowadays. The forecasting literature however does not currently provide easy-to-use, automatic, reproducible and accurate approaches dedicated to this task. We propose a forecasting method that can be used as a strong baseline in this domain, leveraging state-of-the-art forecasting techniques, forecast combination, and global modelling. Our approach uses four base forecasting models specifically suitable for forecasting weekly data: a global Recurrent Neural Network model, Theta, Trigonometric Box-Cox ARMA Trend Seasonal (TBATS), and Dynamic Harmonic Regression ARIMA (DHR-ARIMA). Those are then optimally combined using a lasso regression stacking approach. We evaluate the performance of our method against a set of state-of-the-art weekly forecasting models on six datasets. Across four evaluation metrics, we show that our method consistently outperforms the benchmark methods by a considerable margin with statistical significance. In particular, our model can produce the most accurate forecasts, in terms of mean sMAPE, for the M4 weekly dataset.

Keywords: Weekly Forecasting, Global Models, Ensembling, Lasso Regression

## 1. Introduction

As storing and processing large amounts of data is almost trivial nowadays, and business processes become more and more automated, many businesses that have traditionally operated on quarterly or monthly bases are now operating on a weekly time scale, and weekly data become an important forecasting use case. Despite their relevance in practice, forecasters have traditionally focused more on series with lower or higher granularity. The M3 and M4 competitions (Makridakis and Hibon, 2000; Makridakis et al., 2018) were dominated by monthly series, besides strong presences of quarterly and yearly series. On the other hand, daily and sub-daily series also gain considerable attention as they often show multiple seasonalities which makes them an interesting field of study (Bandara et al., 2019a; O'Hara-Wild and Hyndman, 2018). Consequently, in the M4, the best-performing methods in the weekly category were quite different to the overall winning methods. The method that won the M4-weekly category was a solution based on the commercial Forecast Pro software, discussed in detail by Darin and Stellwagen (2020), where notably those authors also argue that the M4 in general is not very representative for a usual business situation but "the weekly data are perhaps the most similar to business data." Though the method

of Darin and Stellwagen (2020) is very accurate, it requires a significant amount of manual intervention and thorough domain knowledge to retrieve good forecasts, and it is based on a proprietary software.

On the other hand, the methods that performed overall well in the competition popularised important methodological innovations in the forecasting field. Namely 1) forecast combination (ensembling) with meta-learning, as used in the second-winning method from Montero-Manso et al. (2020) that combines the forecasts of a set of base algorithms, and 2) the use of global forecasting models (Januschowski et al., 2020), as in the winning method from Smyl (2020), which is a global forecasting method (that also uses ensembling at different stages of the algorithm).

We hypothesise that the M4 winners did not achieve winning accuracies on the weekly series due to certain particularities of such series. They generally have only a single seasonality with a long cycle, i.e., a yearly seasonality with a seasonal cycle of length approx. 52.18 which is a large and a non-integer value compared to the other commonly considered seasonalities such as quarterly and monthly. On the other hand, the actual seasonal period of weekly series has two possible values: 52 and 53 depending on the year. Selection between these integer and non-integer values for the seasonal period depends on the situation. This particular seasonality needs to be properly accounted for in the modelling, e.g., by the choice of base learners in a meta-learning approach.

Thus, the main aim of our research is to develop a fully automated, publicly available, and reproducible benchmark method using state-of-the-art forecasting practices of global forecasting models and forecast combination. In particular, our paper has the following main contributions.

- We propose an automated meta-learning ensemble forecasting model for weekly time series forecasting. We implement two models for forecast combination: 1) a modified version of Feature-based Forecast Model Averaging (FFORMA, Montero-Manso et al., 2020), the second-winning approach of the M4 forecasting competition, and 2) lasso regression (Tibshirani, 1994) to optimally combine the base model forecasts using a stacking methodology.
- The ensembles use four base models, all selected and designed to be suitable for weekly data: a globally trained Recurrent Neural Network (RNN, Hewamalage et al., 2020) and three univariate forecasting models: Theta (Assimakopoulos and Nikolopoulos, 2000), Trigonometric Box-Cox ARMA Trend Seasonal (TBATS, Livera et al., 2011) and Dynamic Harmonic Regression Auto-Regressive Integrated Moving Average (DHR-ARIMA, Hyndman and Athanasopoulos, 2018), leveraging the strengths of both global and local models.
- We create five experimental weekly benchmark datasets by aggregating series with higher granularities. All the aggregated weekly datasets are publicly available for further research use<sup>1</sup>.

<sup>1</sup>https://drive.google.com/drive/folders/109-ZYZAHQU1YLQfVLDnpgT4MRX\_CqINH?usp=sharing

- We evaluate our proposed weekly forecasting method against a series of state-of-the-art forecasting models using six benchmark datasets across four evaluation metrics. Overall, our model outperforms all benchmark models with statistical significance. Furthermore, our model produces the most accurate forecasts for the M4 weekly dataset based on the mean of the Symmetric Mean Absolute Percentage Error (sMAPE).
- All implementations related to the proposed baseline model are publicly available at: https://github.com/rakshitha123/WeeklyForecasting

The remainder of this paper is organised as follows: Section 2 reviews the related work. Section 3 explains our proposed weekly forecasting models including the details of the base forecasting models and the combination techniques used for aggregating forecasts. Section 4 explains our experimental framework, and presents the model evaluation results. Finally, Section 5 concludes the paper.

## 2. Related Work

In the following, we discuss the related literature in the field of weekly time series forecasting and summarise the state of the art in the fields of meta-learning for forecast combination, and global modelling.

# 2.1. Weekly forecasting

The main challenge when forecasting weekly data is the long and non-integer yearly seasonality. The most popular general forecasting techniques Exponential Smoothing State Space Models (ETS, Hyndman, 2008) and Auto-Regressive Integrated Moving Average (ARIMA, Box and Jenkins, 1990) are not able to deal well with such a long seasonal cycle, and hence, their applicability in weekly forecasting is limited (Hyndman and Athanasopoulos, 2018). There are four main ways in the literature to deal with the seasonality in weekly data. The seasonality 1) can be neglected and a non-seasonal model be built, it 2) can be addressed with seasonal lags, it 3) can be addressed with seasonal indicator variables such as Fourier terms or seasonal dummy variables, or 4) the series can be deseasonalised before forecasting.

Building non-seasonal models can be a good option if the yearly seasonality is not strong or if less than a full year of data is available. In that case, non-seasonal standard models such as ETS, ARIMA, or Theta (Assimakopoulos and Nikolopoulos, 2000) can be fitted. For example, Padt and Bergmeir (2017) use simple exponential smoothing in such a situation. Also commonly employed in the literature are other (non-seasonal) non-linear autoregressive models, e.g., from a machine learning domain. Al-qaness et al. (2020) propose an improved version of an Adaptive Neuro-Fuzzy Inference System (ANFIS) to forecast the weekly confirmed influenza cases in China and the USA which can be used to support health policy-making. Researchers have also proposed models for weekly load forecasting (Barakat and Al-Qasem, 1998), weekly crude oil forecasting (Oussalah and Zaidi, 2018) and weekly groundwater level forecasting (Mohanty et al., 2015) where NNs and regression models are heavily used with the proposed approaches.

Seasonality can also be modelled by incorporating seasonal lags. For example, Landeras et al. (2009) use seasonal ARIMA and Neural Network (NN) models to obtain weekly evapotranspiration forecasts which are then used in planning and designing water resource systems. However, this approach is hindered by the long seasonal cycle and the non-integer length of the cycle in weekly series, and a lag selection mechanism may be necessary to obtain good results.

Fourier terms are also commonly employed nowadays with machine learning models more broadly to model other types of seasonalities (Bandara et al., 2019a). They are commonly used to model the long seasonal cycle of weekly data. TBATS (Livera et al., 2011) and DHR-ARIMA are advocated as suitable methods to be used instead of ETS and ARIMA (Hyndman and Athanasopoulos, 2018) where both methods use Fourier terms to model the seasonality. Pan and Yang (2017) use Autoregressive Moving Average eXogenous (ARMAX) models along with search engine queries, website traffic data and weather information to forecast weekly hotel occupancy for a particular destination. Those authors use Fourier terms and weekly dummy variables to model the seasonality.

Deseasonalisation is another possible approach to handle seasonality. Guttormsen (1999) uses six models to forecast weekly prices for salmon where the forecasts can then be used to reduce the price fluctuation risks. Here, the time series are deseasonalised before forecasting. That author concludes that Classical Additive Decomposition (CAD) and Vector Auto Regression (VAR) models provide the best price forecasts.

The winning approach of the M4 weekly competition proposed by Darin and Stellwagen (2020) uses a set of experts which is chosen from a pool of baseline models (model families), mostly variations of exponential smoothing and ARIMA, to forecast each series. Furthermore, the baseline models include naïve2, seasonal naïve and dynamic regression models incorporating seasonal lags and Fourier terms to capture the seasonal effects. Short series that seem to be seasonal are customised to properly capture the seasonal effects. For each series, one or more models are selected from the pool of baseline models to obtain forecasts based on the series' characteristics. If more models are selected, then the approach uses an out-of-sample testing procedure to select the most appropriate base model to obtain forecasts for a given series. Further investigations are performed to identify the series where the selected experts are not adequate and finally, the forecasts of such series are modified using domain knowledge as required. Although this model is highly accurate, it requires considerable manual intervention during the forecasting process.

#### 2.2. Meta-learning for forecast combination

Combining forecasts of several heterogeneous models is known to improve the forecasting accuracy on average over the individual models especially by reducing bias and/or variance (Wolpert, 1992; Cerqueira et al., 2017). In the machine learning space, forecast combinations are known as ensembling. Simple averaging is considered as one of the most efficient and accurate forecast combination methods which tends to be very competitive (Timmermann, 2006). Several weighted averaging combination methods have also been proposed (Sanchez, 2008; Cerqueira et al., 2017).

Most of the top solutions of the M4 forecasting competition use advanced forecast combination approaches in different ways. The winning approach proposed by Smyl (2020) uses a so-called "ensemble of specialists." Hybrid models between exponential smoothing and RNNs are fitted as an ensemble, where the top RNNs are identified for each series and their forecasts are averaged to obtain the final forecasts. The second winning approach, FFORMA (Montero-Manso et al., 2020) combines the forecasts provided by a set of base models using an optimal set of weights obtained using a meta-learner trained using series features (for details see Section 3.2.1). The third winning approach proposed by Pawlikowski and Chorowska (2020) also incorporates forecast combinations where the forecasts that are provided by a set of selected statistical models including ETS, ARIMA, Theta and Naïve are optimally combined for each series to obtain the final forecasts.

# 2.3. Global models in forecasting

Another relatively recent trend in forecasting, employed in the M4 successfully by the winning solution of Smyl (2020), is the use of global forecasting models (Januschowski et al., 2020). Global models build a single model with a set of global parameters across many series. In contrast to local models, they are capable of learning the cross-series information during model training with a fewer amount of parameters. This type of models has been pioneered, e.g., by the works of Duncan et al. (2001); Trapero et al. (2015); Smyl and Kuber (2016); Flunkert et al. (2017); Bandara et al. (2020); Montero-Manso and Hyndman (2020). Many researchers have used RNNs in the context of global modelling (Smyl, 2020; Hewamalage et al., 2020; Bandara et al., 2019a,b, 2020).

In this paper, we examine the capabilities of forecast combinations containing both global and traditional univariate forecasting models to improve the accuracy of weekly time series forecasting without manual intervention.

## 3. Methodology

This section details the base models and combination techniques used for aggregating the forecasts in our proposed models.

#### 3.1. Base Models

Forecast combination models contain a series of base models where the corresponding base-model forecasts are aggregated to obtain the final forecasts. As an example, FFORMA (Montero-Manso et al., 2020) uses eight base models: ETS, ARIMA, naïve, snaïve, random walk with drift, TBATS, Theta, a locally trained NN and an AR model fitted for the seasonally adjusted series. Forecast combinations are expected to produce better results when the base models are heterogeneous and produce relatively uncorrelated forecasts (de Menezes et al., 2000).

Therefore, we choose the base models on the one hand taking into account their accuracy and performance on weekly data, and on the other hand their diversity. In particular, our

proposed forecasting procedure contains four base models, all suitable for weekly forecasting: TBATS, DHR-ARIMA, Theta, and a globally trained RNN. They are detailed in the following.

## 3.1.1. Dynamic Harmonic Regression ARIMA

Dynamic harmonic regression (Hyndman and Athanasopoulos, 2018) is a technique especially suitable to forecast series with long seasonal periods such as weekly series. The seasonality is considered to be fixed during modelling. Here, Fourier terms and ARMA errors are used to model the seasonality and short-term time series dynamics, respectively.

Fourier terms are a set of sine and cosine pairs which are useful in modelling periodic effects in time series (Harvey and Shephard, 1993), especially to model the long seasonal periods presented in weekly data. The Fourier terms related to a particular time point of a series can be obtained using the formula shown in Equation 1, where t is the time point, s is the seasonal periodicity of the time series and k is the number of sine cosine pairs used with the transformation.

$$sin\left(\frac{2\pi kt}{s}\right), cos\left(\frac{2\pi kt}{s}\right)$$
 (1)

The number of Fourier terms controls the smoothness of the seasonal pattern. We find the optimal number of Fourier terms to be considered with a particular set of series using a grid-search approach considering the range from 1 to 25.

We use the *auto.arima* function from the *forecast* package (Hyndman et al., 2015) along with the Fourier terms generated by using the *fourier* function to obtain DHR-ARIMA forecasts for a given weekly series.

## 3.1.2. Trigonometric Box-Cox ARMA Trend Seasonal Model

TBATS (Livera et al., 2011) is capable of dealing with complex seasonal patterns in time series. It models the seasonality using a Fourier-series-based trigonometric representation. Its capability of modelling long and non-integer periodic effects make it one of the state-of-the-art methods used in weekly time series forecasting, where the length of a seasonal period is approximately 52.18. The TBATS method is an automatic procedure that considers different alternatives during modelling such as Box-Cox transformation, trend, trend damping, and the usage of ARMA processes to model residuals. When the type of a particular alternative is not explicitly provided, all possibilities of that alternative are considered. The final model with the best set of alternatives is chosen based on the Akaike information criterion (AIC). For further details of TBATS, we refer to Livera et al. (2011). We use the tbats function in the R package forecast (Hyndman et al., 2015) to obtain the TBATS forecasts for a given weekly series.

## 3.1.3. Theta

Theta (Assimakopoulos and Nikolopoulos, 2000) was the best-performing method in the M3 forecasting competition (Makridakis and Hibon, 2000). The original FFORMA

(Montero-Manso et al., 2020) also uses Theta as one of its base models, and we include it here as a good representative for relatively simple non-seasonal models.

Theta decomposes a series into a set of new series known as *Theta-lines* with the same mean and slope as the original series, but with different curvatures (Assimakopoulos and Nikolopoulos, 2000). The Theta-lines are then separately extrapolated using any forecasting method to obtain their corresponding forecasts. The most common method used for extrapolation, which was also used in the M3, is exponential smoothing. The final forecasts are obtained by combining the individual forecasts of Theta-lines either using equal weights or a set of optimised weights. Theta was initially viewed as a quite complex method, but later Hyndman and Billah (2003) proved that the model used in the M3 forecasting competition is equal to simple exponential smoothing with drift.

Theta does not model seasonality, but due to the long seasonal cycle of weekly series, we often deal with short series that do not contain even one full seasonal cycle, and non-seasonal models are a good option for such time series. We use the *thetaf* function from the R package *forecast* (Hyndman et al., 2015) to obtain the Theta forecasts for a given weekly series.

# 3.1.4. Recurrent Neural Networks with Long Short-Term Memory Cells

An RNN is a type of NN that is especially suitable for sequence modelling problems (Elman, 1990) due to feedback loops in its architecture. The winning approach of the M4 forecasting competition (Smyl, 2020) in 2018 was based on RNNs, which has given them considerable attention in the forecasting community lately. We include this model due to heterogeneity considerations. Combining the forecasts of a globally trained RNN with the forecasts obtained from the other locally trained univariate forecasting models incorporates the strengths of both global and local models while mitigating the weaknesses of each other (Section 4.5). Extracting seasonal patterns through globality is another benefit of using a global RNN (Montero-Manso and Hyndman, 2020).

We use a Long Short-Term Memory (LSTM) with peephole connections (Gers et al., 2000), following the methodology proposed by Hewamalage et al. (2020). As preprocessing steps, series are normalised by dividing them by their corresponding mean values. Then, the normalised series are transformed to a log scale for variance stabilisation.

We use two techniques to deal with periodic/seasonality effects in the time series. If the corresponding timestamps are available with each data point in all series and if it is possible to align series across time, then we use Fourier terms (Hyndman and Athanasopoulos, 2018) to capture the periodic effects of the weekly time series, where a set of sine and cosine terms are used together with the preprocessed time series when training the RNN model (Bandara et al., 2019a). The fourier function in the R package forecast (Hyndman et al., 2015) is used for Fourier term calculations. If the corresponding timestamps are not available or it is not possible to align series across time, which is the case, e.g., in the M4 weekly dataset (Makridakis et al., 2018), then we use a seasonal lag such as 53 to train the RNN model. Our RNN model uses a stacked architecture (Hewamalage et al., 2020; Bandara et al., 2019b; Godahewa et al., 2020), with input and output windows for multi-step-ahead forecasting (Hewamalage et al., 2020).

Eight hyperparameters need to be chosen during our RNN model training, namely: cell dimension, mini-batch size, maximum number of epochs, epoch size, number of hidden layers, L2-regularisation weight, standard deviation of random normal initialiser and standard deviation of the Gaussian noise. We perform automatic hyperparameter tuning using the Sequential Model based Algorithm Configuration (SMAC) optimisation method (Hutter et al., 2011) as described by Hewamalage et al. (2020). The COntinuous COin Betting (COCOB, Orabona and Tommasi, 2017) algorithm is used as the learning algorithm due to its capability of selecting an optimal learning rate during model training.

#### 3.2. Forecast Combination Models

We explore the use of two forecast combination models for weekly forecasting as discussed in the following.

# 3.2.1. Feature-based Forecast Model Averaging for Weekly Time Series

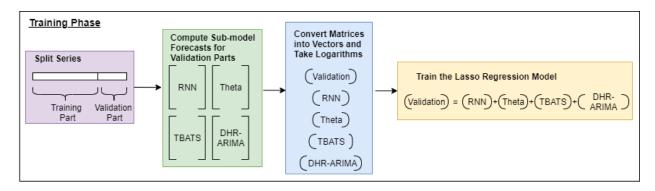
We use the meta-learning combination approach of FFORMA (Montero-Manso et al., 2020) in our work. It optimally combines forecasts produced by a set of sub-forecasting models based on a set of weights obtained by a gradient boosted tree trained using 42 features, calculated from the original time series including trend, seasonality, linearity, curvature, and correlation features calculated using the R package *tsfeatures* (Hyndman et al., 2019). Most of the seasonal features cannot be calculated for short series with less than 2 full periods and hence for those, we calculate the features considering the frequency as one.

In the training phase, FFORMA splits each series into a training period, and a validation period taken from the end of the series, with a length equal to the forecasting horizon. The features are calculated using the training period. Then, for each base model, the forecasts and the respective errors are calculated for the validation periods of all series. A gradient boosted tree is then trained as a meta-learning model over all series to find a function that maps the features to a set of optimal weights that can be used to combine the base model forecasts in a way that the total loss over the validation periods gets minimised. In the testing phase, the features are calculated for the full series and base model forecasts are obtained for the expected forecast horizon. Finally, the base model forecasts are combined using the optimal set of weights provided by the meta-learner given the calculated features.

In the M4 weekly competition, FFORMA achieved second rank according to MASE. Our aim is to transfer the main idea of FFORMA focusing on weekly time series. We modify the FFORMA method to use base models that are especially suitable to forecast weekly series, as discussed in Section 3.1, instead of the base models in the original FFORMA.

## 3.2.2. Combining Forecasts Using Lasso Regression

A potential drawback of FFORMA is that, when determining the combination weights, it only considers information about the time series, and not information about the loss of the combined forecasts on particular series. We address this shortcoming in our second forecast combination approach by using a stacking methodology (Wolpert, 1992) which is a widely used meta-learning approach in the forecast combination space (Divina et al., 2018;



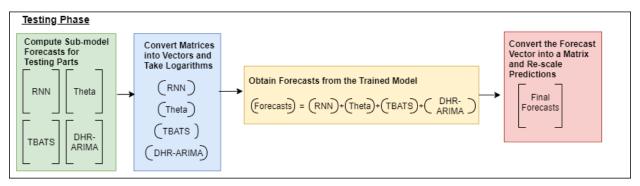


Figure 1: Training and Testing Stages of the Proposed Lasso Regression Model

Khairalla et al., 2018; Zhai and Chen, 2018). Stacking trains a meta-model by using the forecasts provided by the base models and directly outputs the final forecasts.

Model combination generally can be considered more accurate than model selection (Kolassa, 2011), and model selection can in fact be seen as a special case of model combination. We use an approach that uses lasso regression (Tibshirani, 1994) and therewith performs both model combination and selection, excluding the worst performing models from the combinations. Prior works of using local lasso regression models for combining forecasts have shown good results (Diebold and Shin, 2019; Wilms et al., 2016). We use a global lasso regression model to optimally combine the forecasts provided by the four base models: global RNN, Theta, TBATS, and DHR-ARIMA. Lasso regression prevents model overfitting that can occur with the simple linear regression by adding a regularisation term (L1) to the cost function of linear regression. Hence, the coefficients corresponding with the worst performing models shrink towards zero. The main reason to choose lasso regression as our ensembling technique is its ability to select the required base model forecasts when computing the final forecasts, as it can shrink coefficients to zero for models that are performing badly. Other state-of-the-art forecast combination approaches such as simple averaging and FFORMA do not have this capability. Furthermore, a linear model such as lasso regression can be more suitable as a forecast combination method than FFORMA, if there is only a small amount of data available for model training.

Figure 1 represents training and testing stages of our proposed lasso regression model.

We model the relationship between the different forecasting models across the different series and horizons jointly, with a single lasso regression model. The model is constrained to have positive weights, and takes as input the base model forecasts for a particular series and horizon and outputs the corresponding final forecast. In particular, we consider the last sequence of values in a given time series equal to the size of the forecast horizon as its validation part. The base model forecasts are separately computed for this validation part and the lasso regression model is trained with them considering the validation part as the true output. Then, the base model forecasts are again separately calculated using the whole time series corresponding with the actual test period. The base model forecasts are provided as the test inputs, and the final forecasts corresponding with the test period are obtained using the previously trained lasso regression model. Furthermore, we apply a logarithm to the base model forecasts and their corresponding validation outputs before training the lasso regression model to stabilise the variance of the forecasts (Bandara et al., 2020; Smyl, 2020). All time series in our experimental datasets are non-negative. If the base model forecasts or the corresponding validation outputs contain zeros, then we add a constant c=1 before transforming them using the logarithm. Furthermore, any negative forecasts are set to zero to make the final forecasts positive. We use the *qlmnet* function in the R package, glmnet (Friedman et al., 2010) to implement the lasso regression model. The regularisation parameter  $\lambda$  is tuned as a hyperparameter with 10-fold cross validation.

# 4. Experimental Framework and Results

In this section, we present our experimental setup and results on six benchmark datasets for the weekly forecasting models.

# 4.1. Datasets

We use six publicly available datasets to evaluate the performance of our proposed weekly forecasting models. One dataset originally contains weekly series and the series in the remaining five datasets are aggregated from lower granularities accordingly to make them weekly. Table 1 provides a summary of the datasets, in their weekly aggregated versions used in our work. A brief overview of the datasets is as follows.

- M4 Weekly Dataset: The weekly dataset of the M4 forecasting competition (Makridakis et al., 2018).
- NN5 Dataset: The dataset of the NN5 forecasting competition contains 111 daily time series of daily cash withdrawals from Automatic Teller Machines (ATM) in the UK (Taieb et al., 2012). The dataset has missing values which we replace by the median before the temporal aggregation.
- Reduced Kaggle Wikipedia Web Traffic Dataset: We use the first 1000 time series from the Kaggle Wikipedia Web Traffic forecasting competition (Google, 2017). The time series show the number of hits/traffic for a given set of Wikipedia web pages per day. We replace missing values of the dataset by zeros before aggregation.

| Dataset     | No. of Series | Forecast Horizon | Min. Length | Max. Length |
|-------------|---------------|------------------|-------------|-------------|
| M4          | 359           | 13               | 80          | 2597        |
| NN5         | 111           | 8                | 105         | 105         |
| Web Traffic | 1000          | 8                | 106         | 106         |
| Ausgrid     | 299           | 8                | 148         | 148         |
| Traffic     | 862           | 8                | 96          | 96          |
| Solar       | 137           | 5                | 44          | 44          |

Table 1: Summary of the Used Datasets

- Ausgrid Energy Dataset: A half-hourly dataset that contains 300 time series representing the general energy consumption of Australian households (AusGrid, 2019). One series was omitted before aggregation as it has missing values for more than 8 consecutive months.
- San Francisco Traffic Dataset: An hourly dataset that contains 862 time series representing the road occupancy rates on San Francisco Bay area freeways from 2015 to 2016 (Caltrans, 2020; Lai, 2017).
- Solar Dataset: A dataset that contains 137 time series representing the solar power production records per every 10 minutes in the state of Alabama in 2006 (Solar, 2020; Lai, 2017).

## 4.2. Error Metrics

We use two error measures that are common in the forecasting research space: sMAPE and Mean Absolute Scaled Error (MASE, Hyndman and Koehler, 2006), to measure the performance of our models. They are defined in Equations 2 and 3, where N is the number of data points in the test set, M is the number of instances in the training set, S is the length of the seasonal cycle of the dataset,  $F_k$  are the generated forecasts and  $Y_k$  are the actual values corresponding to the required forecast horizon.

$$sMAPE = \frac{100\%}{N} \sum_{k=1}^{N} \frac{|F_k - Y_k|}{(|Y_k| + |F_k|)/2}$$
 (2)

$$MASE = \frac{\sum_{k=1}^{N} |F_k - Y_k|}{\frac{N}{M-S} \sum_{k=S+1}^{M} |Y_k - Y_{k-S}|}$$
(3)

For datasets containing zeros, namely the Kaggle web traffic and San Francisco traffic datasets in our experiments, we use the variant of the sMAPE proposed by Suilin (2017), which eliminates problems with small values and division by zero by changing  $(|Y_k| + |F_k|)$  in the denominator of the sMAPE as defined in Equation 2 to  $max(|Y_k| + |F_k| + \epsilon, 0.5 + \epsilon)$ . We set the parameter  $\epsilon$  to its proposed default of 0.1.

The MASE measures the performance of a model compared to the in-sample average performance of a one-step-ahead naïve or snaïve benchmark. For the M4 dataset, we calculate the MASE using the naïve benchmark to compare our results with the original competition results. For the remaining datasets, we choose the snaïve or naïve benchmarks depending on the series length. If the dataset is seasonal and has at least one full cycle of data points (52 data points for weekly data), then we calculate the MASE using the snaïve benchmark, otherwise using the naïve benchmark. In particular, the maximum series length of the solar dataset is 44, so that we use the naïve benchmark here. The remaining 4 datasets have series with more than one full cycle of data points, and the snaïve benchmark is used.

To measure the performance of the models across series, we further calculate the mean and median values of sMAPE and MASE across the series. Thus, each model is evaluated using four error metrics: mean sMAPE, median sMAPE, mean MASE, and median MASE, across a dataset.

# 4.3. Benchmarks and Variants of the Proposed Models

We use the four base models RNN, Theta, TBATS, and DHR-ARIMA as the main benchmarks. Furthermore, the following benchmarks and model variants are used during our experiments.

- **Average** Average of the forecasts provided by the four base models, as a forecast combination approach.
- FFORMA\_Original We benchmark our proposed weekly forecasting models against the original version of FFORMA (Section 3.2.1, Montero-Manso et al., 2020).
- **FFORMA\_Modified** The modified version of FFORMA which uses our four base models as its base models.
- LR\_S\_Log\_Forecasts Trains a single lasso regression model by using the logarithmic values of the four base model forecasts as inputs.
- LR\_S\_Log\_Forecasts\_Features As a variant, trains a single lasso regression model by using the logarithmic values of the four base model forecasts and the 42 features used in FFORMA as inputs. The R package tsfeatures (Hyndman et al., 2019) is used for feature extraction following the same procedure used in FFORMA.
- LR\_S\_Forecasts\_Features Trains a single lasso regression model by using the original forecasts produced by the four base models and the above-considered 42 features.
- LR\_PH\_Log\_Forecasts Trains separate lasso regression models per each horizon in the required test period where the models are trained using the logarithmic values of the four base model forecasts.
- LR\_PH\_Log\_Forecasts\_Features Trains separate lasso regression models per each horizon where the models are trained using the logarithmic values of the four base model forecasts and the above considered 42 features.

LR\_PH\_Forecasts\_Features Trains separate lasso regression models per each horizon where the models are trained using the original forecasts of the four base models and the above considered 42 features.

# 4.4. Statistical Tests of the Results

We use the non-parametric Friedman rank-sum test to assess the statistical significance of the results provided by different forecasting methods across time series. The sMAPE error measures provided for each series by different forecasting models are used with the statistical testing considering a significance level of  $\alpha=0.05$ . Furthermore, Hochberg's post-hoc procedure is used to further characterise the statistical differences (García et al., 2010).

#### 4.5. Results and Discussion

This section details the results in terms of main accuracy and statistical significance, and later also gives some results that provide more insights into the modelling.

## 4.5.1. Main Accuracy Results

Table 2 presents the results of all experimental datasets for mean sMAPE, median sMAPE, mean MASE, and median MASE. The results of the best-performing models are highlighted in boldface. We see that on mean sMAPE, LR\_S\_Log\_Forecasts outperforms all four individual base models for all experimental datasets except for the solar dataset. None of the other ensemble benchmark models and variants are able to outperform all individual base model results for five datasets. Furthermore, it outperforms all benchmarks and variants on the M4, NN5 and Kaggle web traffic datasets. For the M4 weekly dataset, LR\_S\_Log\_Forecasts produces a mean sMAPE of 6.42, which places it at the first position in the original results of the M4 weekly competition (Mcompetitions, 2018), with an sMAPE result of 6.58 of the original top solution.

On mean MASE, LR\_S\_Log\_Forecasts outperforms the four individual base models for all experimental datasets except the traffic dataset. Furthermore, it is the best-performing model on the M4, NN5, and Kaggle web traffic datasets. For the M4 weekly dataset, LR\_S\_Log\_Forecasts produces a mean MASE of 2.112, which places it at the third position in the original results of the M4 weekly competition. In the original results, FFORMA\_Original was at the second position with a mean MASE of 2.108. A difference between the original participating solution of FFORMA in the M4 competition and FFORMA\_Original in our experiments is that those authors use the full M4 dataset containing 100000 series for model training. But in our case, as our focus is on weekly series, we only use the 359 weekly series in the M4 dataset for FFORMA\_Original model training which produces a mean MASE of 2.156. Thus, FFORMA\_Original in this configuration does not outperform LR\_S\_Log\_Forecasts.

Overall, LR\_S\_Log\_Forecasts demonstrates a better performance on median sMAPE compared to the other benchmark models and variants where it is the best performing model on M4, NN5, and Kaggle web traffic datasets.

| Mean sMAPE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                   |       |       |        |         |         |       |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------|-------|-------|--------|---------|---------|-------|
| Theta   Rank                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                   | M4    | NN5   | Kaggle | Ausgrid | Traffic | Solar |
| TBATS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Mean sMAPE                        |       |       |        |         |         |       |
| DHR-ARIMA   RNN   7.77   10.38   26.86   25.56   12.24   27.78   RNN   Average   6.84   10.38   29.58   22.66   11.67   22.81   FFORMA_Original   7.91   13.34   34.30   26.45   12.41   24.96   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24.91   24. | Theta                             | 8.70  | 12.06 | 30.42  | 29.99   | 12.48   | 24.76 |
| RNN   7.77   10.38   26.86   25.56   12.24   27.78     Average   6.84   10.38   29.58   22.66   11.67   22.81     FFORMA.Original   7.91   13.34   34.30   26.45   12.41   24.98     FFORMA.Modified   6.96   10.66   32.43   24.46   12.01   23.50     LR.PH.Forecasts.Features   7.98   12.26   38.49   23.31   11.62   26.85     LR.PH.Log.Forecasts   7.98   12.44   27.28   26.00   11.52   22.52     LR.PH.Log.Forecasts   6.93   12.04   26.82   25.85   11.26   20.57     LR.S.Forecasts.Features   6.87   10.65   27.19   22.87   11.99   18.92     LR.S.Log.Forecasts   6.42   10.16   26.73   24.45   11.96   18.77     LR.S.Log.Forecasts   6.42   10.16   26.73   24.45   11.76   19.02      Median sMAPE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                   |       |       |        |         |         |       |
| Average   FFORMA_Original   7.91   13.34   34.30   26.45   12.41   24.98   FFORMA_Modified   6.96   10.66   32.43   24.46   12.01   23.50   12.PH_Forecasts_Features   7.98   12.26   38.49   23.31   11.62   26.85   12.PH_Forecasts_Features   7.08   12.44   27.28   26.00   11.52   22.52   12.PH_Log_Forecasts_Features   7.07   10.55   27.19   22.87   11.96   20.57   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85   12.85  |                                   |       |       |        |         |         |       |
| FFORMA_Modified   LR_PH_Log_Forecasts   Features   FFORMA_Modified   LR_PH_Log_Forecasts   Features   FFORMA_Modified   LR_PH_Log_Forecasts   Features   FFORMA_Modified   LR_PH_Log_Forecasts   Features   LR_PH_Log_Forecasts   LR_PH_Log_Fo |                                   |       |       |        |         |         |       |
| LR_PH_Forecasts_Features                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                   |       |       |        |         |         |       |
| LR_PH_Log_Forecasts   7.08   12.44   27.28   26.00   11.52   22.52     LR_PH_Log_Forecasts   6.93   12.04   26.82   25.85   11.26   20.57     LR_S_Forecasts_Features   7.07   10.55   27.19   22.87   11.99   18.92     LR_S_Log_Forecasts   6.42   10.16   26.73   24.45   11.76   19.02     Median sMAPE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                   |       |       |        |         |         |       |
| LR.PH.Log.Forecasts         6.93         12.04         26.82         25.85         11.26         20.57           LR.S.Log.Forecasts.Features         6.87         10.65         27.19         22.87         11.99         18.92           LR.S.Log.Forecasts         6.42         10.16         26.73         24.45         11.76         19.02           Median sMAPE           That         5.40         10.97         26.20         24.31         9.72         24.90           TBATS         4.94         11.15         27.28         17.83         10.00         18.02           DHR-ARIMA         5.12         11.07         34.34         18.98         8.39         19.60           RNN         5.09         9.94         24.25         19.15         9.60         27.69           Average         4.51         9.86         25.61         16.13         8.75         22.50           FFORMA.Original         4.52         11.92         27.93         19.78         8.63         25.01           FFORMA.Modified         4.69         10.10         26.46         17.01         9.50         23.54           LR.PH.Log.Forecasts Features         4.81         12.30         24.80                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                   |       |       |        |         |         |       |
| LR.S. Forecasts Features         7.07         10.55         27.19         22.87         11.96         18.72           LR.S. Log. Forecasts         6.87         10.65         27.27         25.34         11.96         18.77           Median sMAPE           Theta TBATS         4.94         11.15         27.28         17.83         10.00         18.02           DHR-ARIMA         5.12         11.07         34.34         18.98         8.39         19.60           RNN         5.09         9.94         24.25         19.15         9.60         27.69           Average         4.51         9.86         25.61         16.13         8.75         22.54           FFORMA Modified         4.69         11.192         27.93         19.78         9.60         27.69           LR.PH Forecasts Features         5.82         11.86         31.83         17.16         8.65         25.30           LR.PHLog Forecasts Features         4.81         12.30         24.80         19.62         8.51         21.54           LR.S Log-Forecasts Features         4.28         9.70         24.69         16.62         9.16         18.91           LR.S                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |       |       |        |         |         |       |
| LR_SLog_Forecasts                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                   |       |       |        |         |         |       |
| Median sMAPE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                   | 6.87  |       | 27.27  |         |         | 18.77 |
| Theta TBATS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | LR_S_Log_Forecasts                | 6.42  | 10.16 | 26.73  | 24.45   | 11.76   | 19.02 |
| TBATS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | ${\bf Median~sMAPE}$              |       |       |        |         |         |       |
| DHR-ARIMA   5.12   11.07   34.34   18.98   8.39   19.60   RNN   5.09   9.94   24.25   19.15   9.60   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   27.69   |                                   |       |       |        |         |         |       |
| RNN   5.09   9.94   24.25   19.15   9.60   27.69   Average   4.51   9.86   25.61   16.13   8.75   22.54   EFFORMA_Original   4.52   11.92   27.93   19.78   9.63   25.01   EFFORMA_Modified   4.69   10.10   26.46   17.01   9.50   23.54   LR_PH_Forecasts_Features   5.82   11.86   31.83   17.16   8.65   25.30   LR_PH_Log_Forecasts   4.81   12.30   24.80   19.62   8.51   21.54   LR_PH_Log_Forecasts   4.60   11.52   24.42   19.37   8.31   19.66   LR_S_Forecasts_Features   4.74   10.03   24.51   18.32   9.25   18.71   LR_S_Log_Forecasts   4.08   9.56   24.13   18.26   9.03   19.03   19.03   19.03   19.03   19.03   19.04   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64   19.64 |                                   |       |       |        |         |         |       |
| Average   4.51   9.86   25.61   16.13   8.75   22.54     FFORMA_Original   4.52   11.92   27.93   19.78   9.63   25.01     FFORMA_Modified   4.69   10.10   26.46   17.01   9.50   23.54     LR_PH_Forecasts_Features   5.82   11.86   31.83   17.16   8.65   25.30     LR_PH_Log_Forecasts   4.81   12.30   24.80   19.62   8.51   21.54     LR_S_H_Log_Forecasts   4.60   11.52   24.42   19.37   8.31   19.66     LR_S_Forecasts_Features   4.28   9.70   24.69   16.62   9.16   18.91     LR_S_Log_Forecasts   4.74   10.03   24.51   18.32   9.25   18.71     LR_S_Log_Forecasts   4.08   9.56   24.13   18.26   9.03   19.03      Mean MASE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                   |       |       |        |         |         |       |
| FFORMA_Modified                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                   |       |       |        |         |         |       |
| LR_PH_Forecasts_Features                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | FFORMA_Original                   |       |       |        |         |         |       |
| LR_PH_Log_Forecasts                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                   |       |       |        |         |         |       |
| LR_PH_Log_Forecasts         4.60         11.52         24.42         19.37         8.31         19.66           LR_S_Forecasts_Features         4.28         9.70         24.69         16.62         9.16         18.91           LR_S_Log_Forecasts_Features         4.74         10.03         24.51         18.32         9.25         18.71           LR_S_Log_Forecasts         4.08         9.56         24.13         18.26         9.03         19.03           Mean MASE           Theta         2.734         0.898         0.688         1.206         1.122         1.224           TBATS         2.249         0.864         0.692         1.045         1.149         0.910           DHR-ARIMA         2.418         0.854         0.834         1.068         1.023         0.991           RNN         2.482         0.768         0.608         1.045         1.128         1.422           Average         2.142         0.777         0.653         0.930         1.020         1.119           FFORMA_Original         2.156         0.944         0.723         1.062         1.683         1.249           FFORMA_Figure         2.688         0.924         1.006                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                   |       |       |        |         |         |       |
| LR_S_Forecasts_Features         4.28         9.70         24.69         16.62         9.16         18.91           LR_S_Log_Forecasts_Features         4.74         10.03         24.51         18.32         9.25         18.71           LR_S_Log_Forecasts         4.08         9.56         24.13         18.26         9.03         19.03           Mean MASE           Theta         2.734         0.898         0.688         1.206         1.122         1.224           TBATS         2.249         0.864         0.692         1.045         1.149         0.910           DHR-ARIMA         2.418         0.854         0.834         1.068         1.023         0.991           RNN         2.482         0.768         0.608         1.042         1.128         1.422           Average         2.142         0.777         0.653         0.930         1.020         1.119           FFORMA_Original         2.156         0.944         0.723         1.062         1.683         1.249           FFORMA_Forecasts_Features         2.688         0.924         1.006         0.968         1.069         1.414           LR_PH_Log_Forecasts_Features         2.599                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |       |       |        |         |         |       |
| Mean MASE         Theta         2.734         0.898         0.688         1.206         1.122         1.224           TBATS         2.249         0.864         0.692         1.045         1.149         0.910           DHR-ARIMA         2.418         0.854         0.834         1.068         1.023         0.991           RNN         2.482         0.768         0.608         1.042         1.128         1.422           Average         2.142         0.777         0.653 <b>0.930 1.020</b> 1.119           FFORMA_Original         2.156         0.944         0.723         1.062         1.683         1.249           FFORMA_Modified         2.194         0.796         0.659         0.994         1.541         1.159           LR_PH_Forecasts_Features         2.688         0.924         1.006         0.968         1.069         1.441           LR_PH_Log_Forecasts_Features         2.434         0.941         0.616         1.080         1.064         1.126           LR_SLog_Forecasts_Features         2.340         0.784         0.617         0.948         1.111         0.901           LR_S_Log_Forecasts_Features         2.259         0.794         0.617                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | LR_S_Forecasts_Features           | 4.28  |       |        |         | 9.16    |       |
| Mean MASE           Theta TBATS         2.734         0.898         0.688         1.206         1.122         1.224           TBATS         2.249         0.864         0.692         1.045         1.149         0.910           DHR-ARIMA         2.418         0.854         0.834         1.068         1.023         0.991           RNN         2.482         0.768         0.608         1.042         1.128         1.422           Average         2.142         0.777         0.653 <b>0.930 1.020</b> 1.119           FFORMA_Original         2.156         0.944         0.723         1.062         1.683         1.249           FFORMA_Modified         2.194         0.796         0.659         0.994         1.541         1.159           LR_PH_Forecasts_Features         2.688         0.924         1.006         0.968         1.069         1.441           LR_PH_Log_Forecasts_Features         2.434         0.941         0.616         1.080         1.064         1.126           LR_S_Log_Forecasts_Features         2.340         0.784         0.617         0.948         1.111         0.901           LR_S_Log_Forecasts_Features         2.259 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                   |       |       |        |         |         |       |
| Theta 2.734 0.898 0.688 1.206 1.122 1.224 TBATS 2.249 0.864 0.692 1.045 1.149 0.910 DHR-ARIMA 2.418 0.854 0.834 1.068 1.023 0.991 RNN 2.482 0.768 0.608 1.042 1.128 1.422 Average 2.142 0.777 0.653 0.930 1.020 1.119 FFORMA_Original 2.156 0.944 0.723 1.062 1.683 1.249 FFORMA_Modified 2.194 0.796 0.659 0.994 1.541 1.159 LR_PH_Forecasts_Features 2.688 0.924 1.006 0.968 1.069 1.441 LR_PH_Log_Forecasts_Features 2.434 0.941 0.616 1.080 1.064 1.126 LR_PH_Log_Forecasts_2.599 0.909 0.608 1.074 1.024 1.013 LR_S_Forecasts_Features 2.340 0.784 0.617 0.948 1.111 0.901 LR_S_Log_Forecasts_Features 2.259 0.794 0.617 0.948 1.111 0.901 LR_S_Log_Forecasts_2.210 0.761 0.606 1.004 1.053 0.905  Median MASE  Theta 1.969 0.805 0.549 0.981 0.983 1.241 TBATS 1.414 0.874 0.553 0.848 0.997 0.892 DHR-ARIMA 1.640 0.826 0.686 0.903 0.793 0.967 RNN 1.582 0.680 0.483 0.867 0.924 1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                   | 4.08  | 9.56  | 24.13  | 18.26   | 9.03    | 19.03 |
| TBATS         2.249         0.864         0.692         1.045         1.149         0.910           DHR-ARIMA         2.418         0.854         0.834         1.068         1.023         0.991           RNN         2.482         0.768         0.608         1.042         1.128         1.422           Average         2.142         0.777         0.653 <b>0.930 1.020</b> 1.119           FFORMA_Original         2.156         0.944         0.723         1.062         1.683         1.249           FFORMA_Modified         2.194         0.796         0.659         0.994         1.541         1.159           LR_PH_Forecasts_Features         2.688         0.924         1.006         0.968         1.069         1.441           LR_PH_Log_Forecasts_Features         2.434         0.941         0.616         1.080         1.064         1.126           LR_PH_Log_Forecasts_Features         2.599         0.909         0.608         1.074         1.024         1.013           LR_S_Log_Forecasts_Features         2.259         0.794         0.617         1.051         1.108 <b>0.895</b> LR_S_Log_Forecasts <b>2.112 0.761 0.606</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Mean MASE                         |       |       |        |         |         |       |
| DHR-ARIMA         2.418         0.854         0.834         1.068         1.023         0.991           RNN         2.482         0.768         0.608         1.042         1.128         1.422           Average         2.142         0.777         0.653 <b>0.930 1.020</b> 1.119           FFORMA_Original         2.156         0.944         0.723         1.062         1.683         1.249           FFORMA_Modified         2.194         0.796         0.659         0.994         1.541         1.159           LR_PH_Forecasts_Features         2.688         0.924         1.006         0.968         1.069         1.441           LR_PH_Log_Forecasts_Features         2.434         0.941         0.616         1.080         1.064         1.126           LR_S_Log_Forecasts_Features         2.340         0.784         0.617         0.948         1.111         0.901           LR_S_Log_Forecasts_Features         2.259         0.794         0.617         1.051         1.108 <b>0.895</b> LR_S_Log_Forecasts <b>2.112 0.761 0.606</b> 1.004         1.053         0.905           Median MASE    That  1.969  0.805  0.805  0.549  0.981                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                   |       |       |        | 1.206   |         |       |
| RNN         2.482         0.768         0.608         1.042         1.128         1.422           Average         2.142         0.777         0.653 <b>0.930 1.020</b> 1.119           FFORMA_Original         2.156         0.944         0.723         1.062         1.683         1.249           FFORMA_Modified         2.194         0.796         0.659         0.994         1.541         1.159           LR_PH_Forecasts_Features         2.688         0.924         1.006         0.968         1.069         1.441           LR_PH_Log_Forecasts_Features         2.434         0.941         0.616         1.080         1.064         1.126           LR_PH_Log_Forecasts         2.599         0.909         0.608         1.074         1.024         1.013           LR_S_Log_Forecasts_Features         2.340         0.784         0.617         0.948         1.111         0.901           LR_S_Log_Forecasts_Features         2.259         0.794         0.617         1.051         1.108 <b>0.895</b> LR_S_Log_Forecasts <b>2.112 0.761 0.606</b> 1.004         1.053         0.905           Median MASE    Theta  That  1.969  0.805  0.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                   |       |       |        |         |         |       |
| Average         2.142         0.777         0.653         0.930         1.020         1.119           FFORMA_Original         2.156         0.944         0.723         1.062         1.683         1.249           FFORMA_Modified         2.194         0.796         0.659         0.994         1.541         1.159           LR_PH_Forecasts Features         2.688         0.924         1.006         0.968         1.069         1.441           LR_PH_Log_Forecasts Features         2.434         0.941         0.616         1.080         1.064         1.126           LR_PH_Log_Forecasts         2.599         0.909         0.608         1.074         1.024         1.013           LR_S_Forecasts_Features         2.340         0.784         0.617         0.948         1.111         0.901           LR_S_Log_Forecasts         2.259         0.794         0.617         1.051         1.108         0.895           LR_S_Log_Forecasts         2.112         0.761         0.606         1.004         1.053         0.905           Median MASE    Theta  Theta  1.969  0.805  0.805  0.549  0.981  0.981  0.983  1.241  TBATS  1.414  0.874  0.553  0.848  0.997  0.892  DHR-ARIMA  1.640  0.826  0.686  0.903  0.793  0.967  RNN< 1.582  0.686  0.686  0.903  0.793  0.924  1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                   |       |       |        |         |         |       |
| FFORMA_Original FFORMA_Modified 2.194 0.796 0.659 0.994 1.541 1.159   LR_PH_Forecasts_Features 2.688 0.924 1.006 0.968 1.069 1.441   LR_PH_Log_Forecasts_Features 2.434 0.941 0.616 1.080 1.064 1.126   LR_PH_Log_Forecasts_Seatures 2.599 0.909 0.608 1.074 1.024 1.013   LR_S_Forecasts_Features 2.340 0.784 0.617 0.948 1.111 0.901   LR_S_Log_Forecasts_Seatures 2.259 0.794 0.617 1.051 1.108 0.895   LR_S_Log_Forecasts_Seatures 2.112 0.761 0.606 1.004 1.053 0.905    Median MASE  Theta 1.969 0.805 0.549 0.981 0.983 1.241   TBATS 1.414 0.874 0.553 0.848 0.997 0.892   DHR-ARIMA 1.640 0.826 0.686 0.903 0.793 0.967   RNN 1.582 0.680 0.483 0.867 0.924 1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                   |       |       |        |         |         |       |
| LR_PH_Forecasts_Features       2.688       0.924       1.006       0.968       1.069       1.441         LR_PH_Log_Forecasts_Features       2.434       0.941       0.616       1.080       1.064       1.126         LR_PH_Log_Forecasts       2.599       0.909       0.608       1.074       1.024       1.013         LR_S_Forecasts_Features       2.340       0.784       0.617       0.948       1.111       0.901         LR_S_Log_Forecasts_Features       2.259       0.794       0.617       1.051       1.108 <b>0.895</b> LR_S_Log_Forecasts <b>2.112 0.761 0.606</b> 1.004       1.053       0.905         Median MASE         Theta       1.969       0.805       0.549       0.981       0.983       1.241         TBATS       1.414       0.874       0.553       0.848       0.997 <b>0.892</b> DHR-ARIMA       1.640       0.826       0.686       0.903 <b>0.793</b> 0.967         RNN       1.582 <b>0.680 0.483</b> 0.867       0.924       1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                   |       |       |        |         |         |       |
| LR_PH_Log_Forecasts Features       2.434       0.941       0.616       1.080       1.064       1.126         LR_PH_Log_Forecasts       2.599       0.909       0.608       1.074       1.024       1.013         LR_S_Forecasts_Features       2.340       0.784       0.617       0.948       1.111       0.901         LR_S_Log_Forecasts_Features       2.259       0.794       0.617       1.051       1.108 <b>0.895</b> LR_S_Log_Forecasts <b>2.112 0.761 0.606</b> 1.004       1.053       0.905         Median MASE         Theta       1.969       0.805       0.549       0.981       0.983       1.241         TBATS <b>1.414</b> 0.874       0.553       0.848       0.997 <b>0.892</b> DHR-ARIMA       1.640       0.826       0.686       0.903 <b>0.793</b> 0.967         RNN       1.582 <b>0.680 0.483</b> 0.867       0.924       1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                   |       |       |        |         |         |       |
| LR_PH_Log_Forecasts       2.599       0.909       0.608       1.074       1.024       1.013         LR_S_Forecasts_Features       2.340       0.784       0.617       0.948       1.111       0.901         LR_S_Log_Forecasts_Features       2.259       0.794       0.617       1.051       1.108 <b>0.895</b> LR_S_Log_Forecasts <b>2.112 0.761 0.606</b> 1.004       1.053       0.905         Median MASE         Theta       1.969       0.805       0.549       0.981       0.983       1.241         TBATS <b>1.414</b> 0.874       0.553       0.848       0.997 <b>0.892</b> DHR-ARIMA       1.640       0.826       0.686       0.903 <b>0.793</b> 0.967         RNN       1.582 <b>0.680 0.483</b> 0.867       0.924       1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | LR_PH_Forecasts_Features          |       |       |        |         |         |       |
| LR_S_Forecasts_Features       2.340       0.784       0.617       0.948       1.111       0.901         LR_S_Log_Forecasts_Features       2.259       0.794       0.617       1.051       1.108 <b>0.895</b> LR_S_Log_Forecasts <b>2.112 0.761 0.606</b> 1.004       1.053       0.905       Median MASE  Theta <ul> <li>Theta TBATS</li> <li>1.414</li> <li>0.874</li> <li>0.553</li> <li>0.848</li> <li>0.997</li> <li><b>0.892</b></li> </ul> DHR-ARIMA T.640       0.826       0.686       0.903 <b>0.793</b> 0.967         RNN       1.582 <b>0.680 0.483</b> 0.867       0.924       1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                   |       |       |        |         |         |       |
| LR_S_Log_Forecasts_Features       2.259       0.794       0.617       1.051       1.108       0.895         LR_S_Log_Forecasts       2.112       0.761       0.606       1.004       1.053       0.905         Median MASE         Theta       1.969       0.805       0.549       0.981       0.983       1.241         TBATS       1.414       0.874       0.553       0.848       0.997       0.892         DHR-ARIMA       1.640       0.826       0.686       0.903       0.793       0.967         RNN       1.582       0.680       0.483       0.867       0.924       1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                   |       |       |        |         |         |       |
| Median MASE           Theta 1.969 0.805 0.549 0.981 0.983 1.241 TBATS 1.414 0.874 0.553 0.848 0.997 0.892 DHR-ARIMA 1.640 0.826 0.686 0.903 0.793 0.967 RNN 1.582 0.680 0.483 0.867 0.924 1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $LR\_S\_Log\_Forecasts\_Features$ |       | 0.794 | 0.617  |         |         |       |
| Theta 1.969 0.805 0.549 0.981 0.983 1.241<br>TBATS <b>1.414</b> 0.874 0.553 0.848 0.997 <b>0.892</b><br>DHR-ARIMA 1.640 0.826 0.686 0.903 <b>0.793</b> 0.967<br>RNN 1.582 <b>0.680 0.483</b> 0.867 0.924 1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | LR_S_Log_Forecasts                | 2.112 | 0.761 | 0.606  | 1.004   | 1.053   | 0.905 |
| TBATS <b>1.414</b> 0.874 0.553 0.848 0.997 <b>0.892</b> DHR-ARIMA 1.640 0.826 0.686 0.903 <b>0.793</b> 0.967 RNN 1.582 <b>0.680 0.483</b> 0.867 0.924 1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Median MASE                       |       |       |        |         |         |       |
| DHR-ARIMA 1.640 0.826 0.686 0.903 <b>0.793</b> 0.967<br>RNN 1.582 <b>0.680 0.483</b> 0.867 0.924 1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                   |       |       |        |         |         |       |
| RNN 1.582 <b>0.680 0.483</b> 0.867 0.924 1.426                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                   |       |       |        |         |         |       |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                   |       |       |        |         |         |       |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                   |       |       |        |         |         |       |
| FFORMA_Original 1.569 0.899 0.553 0.874 1.372 1.272                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | FFORMA_Original                   | 1.569 |       | 0.553  | 0.874   | 1.372   | 1.272 |
| FFORMA_Modified 1.560 0.696 0.528 0.775 1.125 1.155                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                   |       |       |        |         |         |       |
| LR_PH_Forecasts_Features 1.819 0.871 0.624 0.766 0.858 1.388                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                   |       |       |        |         |         |       |
| LR_PH_Log_Forecasts                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                   |       |       |        |         |         |       |
| LR_S_Forecasts_Features 1.607 0.753 0.501 0.600 0.980 0.980                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                   |       |       |        |         |         |       |
| LR_S_Log_Forecasts_Features 1.492 0.739 0.496 0.819 0.888 0.922                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | LR_S_Log_Forecasts_Features       | 1.492 | 0.739 | 0.496  | 0.819   | 0.888   | 0.922 |
| LR_S_Log_Forecasts 1.512 0.691 0.485 0.799 0.879 0.936                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | LR_S_Log_Forecasts                | 1.512 | 0.691 | 0.485  | 0.799   | 0.879   | 0.936 |

Table 2: Results Across All Experimental Datasets

On median MASE, the performance is slightly different. The best method on median MASE is a single base model for all datasets, except the Ausgrid dataset, which also shows the suitability of the considered pool of baseline models to forecast weekly series.

As lasso regression optimally combines the base model forecasts and is also able to assign base model coefficients of zero if their effect on the final results is small, it is able to effectively select the best-performing base models for a given set of series and combine the forecasts of only the selected base models when producing the final forecasts. Furthermore, LR\_S\_Log\_Forecasts contains the strengths of both global and local models while mitigating the weaknesses of each other. As a result, it demonstrates a better performance than the other benchmark models.

FFORMA\_Modified has performed better compared to FFORMA\_Original for all experimental datasets on all error metrics except the median sMAPE and mean MASE of the M4 weekly dataset. This indicates that our four base models are more suitable to forecast weekly data than the sub-forecasting models used in the original FFORMA approach. However, on the NN5, Kaggle web traffic, and solar datasets, FFORMA\_Modified has not been able to outperform some of the base models, underlining the need to explore other combination approaches.

| Model                             | $p_{Hoch}$              |
|-----------------------------------|-------------------------|
| LR_S_Log_Forecasts                | -                       |
| LR_PH_Log_Forecasts               | $6.14 \times 10^{-3}$   |
| Average                           | $2.56\times10^{-7}$     |
| $LR\_S\_Forecasts\_Features$      | $1.04 \times 10^{-8}$   |
| $LR\_S\_Log\_Forecasts\_Features$ | $1.86 \times 10^{-12}$  |
| LR_PH_Log_Forecasts_Features      | $1.52 \times 10^{-19}$  |
| $FFORMA\_Modified$                | $3.55 \times 10^{-45}$  |
| RNN                               | $1.30 \times 10^{-49}$  |
| TBATS                             | $7.85 \times 10^{-67}$  |
| FFORMA_Original                   | $2.71 \times 10^{-88}$  |
| DHR-ARIMA                         | $2.36 \times 10^{-95}$  |
| Theta                             | $3.36 \times 10^{-99}$  |
| LR_PH_Forecasts_Features          | $2.45 \times 10^{-107}$ |

Table 3: Results of Statistical Testing

Generally, averaging the base model forecasts is considered an efficient and accurate ensembling technique in the forecasting research space. Simple averaging has outperformed FFORMA\_Modified in many cases even though FFORMA starts optimising weights considering the simple average. Major differences between the validation and test sets and small sample sizes for the flexibility of FFORMA can be the reasons for this phenomenon. But simple averaging has only outperformed LR\_S\_Log\_Forecasts on the Ausgrid and traffic datasets on all error metrics. Averaging does not select the best forecasting models when producing the final forecasts. Furthermore, it assigns equal weights to all base models and hence, if a subset of models provides poor forecasts, then the final result may considerably

be affected by it. This can be a problem especially if there are models that may be clearly unsuitable, as in our case where we use non-seasonal models to forecast seasonal time series. LR\_S\_Log\_Forecasts addresses both of the above-mentioned issues of simple averaging and hence, overall, it has produced more accurate forecasts than the averaging benchmark. LR\_S\_Log\_Forecasts outperforms FFORMA\_Original for all datasets on all error metrics, and outperforms FFORMA\_Modified for all datasets on all error metrics except for 3 cases: median sMAPE, mean MASE and median MASE of Ausgrid dataset.

Next, we analyse whether including time series features as inputs can improve the fore-casting accuracy. The performance of lasso regression models that only take base model fore-casts as inputs, namely LR\_S\_Log\_Forecasts and LR\_PH\_Log\_Forecasts, is generally better than the performance of lasso regression models that take both forecasts and features as inputs: LR\_S\_Log\_Forecasts\_Features, LR\_S\_Forecasts\_Features, LR\_PH\_Log\_Forecasts\_Features and LR\_PH\_Forecasts\_Features. Hence, it is clear that including features as inputs in our case does not increase the accuracy. Furthermore, for datasets with short time series, incorporating features to train models can pose additional problems as seasonality is not taken into account during feature calculation.

LR\_S\_Log\_Forecasts builds a single lasso regression model to provide forecasts for each horizon of all series simultaneously. The results show that using this kind of modelling is considerably better than using separate models for each horizon, in our case. Furthermore, our experiments show that transforming the base model forecasts with a logarithm is beneficial, when training the lasso regression model.

## 4.5.2. Statistical Testing Results

Table 3 shows the results of the statistical testing evaluation, namely the adjusted p-values calculated from the Friedman test with Hochberg's post-hoc procedure (García et al., 2010). The overall p-value of the Friedman rank sum test is less than  $10^{-30}$  which is highly significant. LR\_S\_Log\_Forecasts performs the best and hence, it is used as the control method. All individual models as well as ensemble benchmarks and variants are significantly worse than the control method. FFORMA\_Modified has a better performance compared to FFORMA\_Original, but it has not outperformed LR\_S\_Log\_Forecasts and a set of benchmarks and variants.

## 4.5.3. Individual Model Contributions in Optimal Oracle Combination Approach

We further conduct a study to explore the contributions of the individual base models to forecasting accuracy. For that, we assume an optimal oracle combination approach, where the base model with the lowest error (sMAPE or MASE) on the test set is identified for each series and selected for forecasting. This procedure is followed considering all four base models, to identify a lower bound of errors. Then, the same procedure is followed by subsequently removing one of the base models, and only using the three remaining ones. This way, we are able to identify base models that perform better than the others on certain series, and are not consistently dominated by other methods. Therewith, these models add to the diversity of forecasting methods. Table 4 shows the results of the study. The results of the lower bound containing all base models, which is by definition the best-performing method

|                                                                                                                     |                                                  |                                                  |                                                                                   |                                                                                   | ~                                                | ~ .                                              |
|---------------------------------------------------------------------------------------------------------------------|--------------------------------------------------|--------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
|                                                                                                                     | M4                                               | NN5                                              | Kaggle                                                                            | Ausgrid                                                                           | Traffic                                          | Solar                                            |
| Mean sMAPE                                                                                                          |                                                  |                                                  |                                                                                   |                                                                                   |                                                  |                                                  |
| TBATS_DHR-ARIMA_Theta<br>TBATS_Theta_RNN<br>Theta_RNN_DHR-ARIMA<br>TBATS_DHR-ARIMA_RNN<br>TBATS_DHR-ARIMA_Theta_RNN | 5.99<br>5.69<br>5.83<br>5.71<br><b>5.47</b>      | 9.95<br>9.40<br>9.37<br>9.37<br><b>9.18</b>      | 25.84<br>24.61<br>24.45<br>24.79<br><b>24.06</b>                                  | 19.63<br>19.59<br>18.54<br>18.39<br><b>17.84</b>                                  | 10.07<br>11.39<br>9.93<br>9.93<br><b>9.83</b>    | 18.41<br>18.89<br>20.17<br>18.43<br><b>18.41</b> |
| ${\bf Median~sMAPE}$                                                                                                |                                                  |                                                  |                                                                                   |                                                                                   |                                                  |                                                  |
| TBATS_DHR-ARIMA_Theta<br>TBATS_Theta_RNN<br>Theta_RNN_DHR-ARIMA<br>TBATS_DHR-ARIMA_RNN<br>TBATS_DHR-ARIMA_Theta_RNN | 3.58<br>3.41<br>3.41<br>3.42<br><b>3.24</b>      | 9.66<br>8.96<br>8.75<br>8.84<br><b>8.66</b>      | 23.27<br>22.49<br>22.36<br>22.68<br><b>22.11</b>                                  | 14.85<br>14.33<br>14.04<br>13.95<br><b>13.66</b>                                  | 7.05<br>8.74<br>7.02<br>7.05<br><b>6.99</b>      | 17.78<br>18.02<br>19.60<br>17.78<br><b>17.78</b> |
| Mean MASE                                                                                                           |                                                  |                                                  |                                                                                   |                                                                                   |                                                  |                                                  |
| TBATS_DHR-ARIMA_Theta<br>TBATS_Theta_RNN<br>Theta_RNN_DHR-ARIMA<br>TBATS_DHR-ARIMA_RNN<br>TBATS_DHR-ARIMA_Theta_RNN | 1.736<br>1.664<br>1.648<br>1.648<br><b>1.561</b> | 0.745<br>0.700<br>0.699<br>0.699<br><b>0.684</b> | $\begin{array}{c} 0.587 \\ 0.564 \\ 0.562 \\ 0.571 \\ \textbf{0.555} \end{array}$ | $\begin{array}{c} 0.805 \\ 0.800 \\ 0.758 \\ 0.761 \\ \textbf{0.732} \end{array}$ | 0.853<br>1.020<br>0.843<br>0.846<br><b>0.837</b> | 0.884<br>0.908<br>0.978<br>0.886<br><b>0.884</b> |
| Median MASE                                                                                                         |                                                  |                                                  |                                                                                   |                                                                                   |                                                  |                                                  |
| TBATS_DHR-ARIMA_Theta<br>TBATS_Theta_RNN<br>Theta_RNN_DHR-ARIMA<br>TBATS_DHR-ARIMA_RNN<br>TBATS_DHR-ARIMA_Theta_RNN | 1.170<br>1.097<br>1.113<br>1.059<br><b>1.043</b> | 0.708<br>0.636<br>0.655<br>0.655<br><b>0.620</b> | 0.465<br>0.447<br>0.446<br>0.448<br><b>0.437</b>                                  | 0.646<br>0.606<br>0.618<br>0.601<br><b>0.571</b>                                  | 0.676 $0.855$ $0.666$ $0.669$ $0.664$            | 0.874<br>0.892<br>0.957<br>0.874<br><b>0.874</b> |

Table 4: Results of the optimal oracle combination study.

|                                    | M4                               | NN5                              | Kaggle                    | Ausgrid                   | Traffic                          | Solar                          |
|------------------------------------|----------------------------------|----------------------------------|---------------------------|---------------------------|----------------------------------|--------------------------------|
| TBATS<br>DHR-ARIMA<br>Theta<br>RNN | 0.712<br>0.009<br>0.044<br>0.238 | 0.408<br>0.042<br>0.049<br>0.477 | 0.006 $0.0$ $0.0$ $0.976$ | 0.219 $0.0$ $0.0$ $0.642$ | $0.226 \\ 0.110 \\ 0.0 \\ 0.633$ | 0.042<br>0.003<br>0.950<br>0.0 |

Table 5: Base model Weights Chosen by LR\_S\_Log\_Forecasts

in this comparison, are highlighted in boldface, and the results of the worst performing combinations are italicized. The combination containing only local models, TBATS\_DHR-ARIMA\_Theta is the worst performing combination for M4, NN5, Kaggle web traffic and Ausgrid datasets, and the second worst combination for the traffic dataset across all error metrics. The combinations with a mixture of local and global models show an overall better performance compared to TBATS\_DHR-ARIMA\_Theta. Thus, we can conclude that the RNN adds diversity to the forecasting pool of methods. Using a mixture of local and global models provides better forecasts as a result of incorporating the strengths of both global and local models while mitigating the weaknesses of each other.

## 4.5.4. Analysis of Combination Weights

Table 5 shows the base model weights chosen by LR\_S\_Log\_Forecasts across all datasets. TBATS is the best performing base model for M4, and RNN is the best performing base

model for NN5 and Kaggle web traffic datasets. LR\_S\_Log\_Forecasts has also assigned the highest weights for the corresponding base models with those datasets and it shows our model has the capability of identifying the best base models that should be used for combining. Furthermore, the method also fully discards some of the base models, when computing the forecasts for Kaggle web traffic, Ausgrid, traffic and solar datasets. Combining only the best base models while discarding the less important ones is a major benefit of using lasso regression for combining forecasts.

Overall, the results indicate that LR\_S\_Log\_Forecasts is a simple, automated and strong baseline which achieves competitive performance compared to state-of-the-art weekly forecasting techniques. Furthermore, it is applicable to any weekly dataset irrespective of the series length as it does not require time series features during model training where, to calculate features with proper seasonal handling, the series length should be at least two seasonal cycles. Hence, LR\_S\_Log\_Forecasts can be used as a strong baseline when forecasting weekly data.

#### 5. Conclusion

Many businesses and industries nowadays deal with weekly data and need accurate forecasts on those. However, the forecasting literature is currently lacking easy-to-use, accurate, and automated approaches dedicated to forecast weekly series. In this paper, we have proposed such an approach that optimally combines the forecasts provided by four base models: RNN, Theta, TBATS and DHR-ARIMA using lasso regression to produce the weekly forecasts for a given set of series. We have also explored a modified version of FFORMA by replacing its original base models by base models dedicated to weekly forecasting. We have compared the prediction accuracy of our proposed models with a series of benchmarks and model variants such as simple averaging, the original FFORMA, lasso regression models that use forecasts and series features as inputs, lasso regression models trained per each horizon and the current state-of-the-art weekly forecasting models: TBATS and DHR-ARIMA. We have shown that our proposed weekly forecasting model based on lasso regression is significantly more accurate than the considered benchmarks and variants across six experimental datasets. The proposed lasso regression model ranks first among the original contenders of the M4 weekly competition, based on mean sMAPE. Hence, we conclude that our proposed weekly forecasting model can be used as an easy-to-implement and automatic, strong baseline in the weekly time series forecasting space.

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