TDS3651
Visual Information Processing

Filtering
Lecture 3

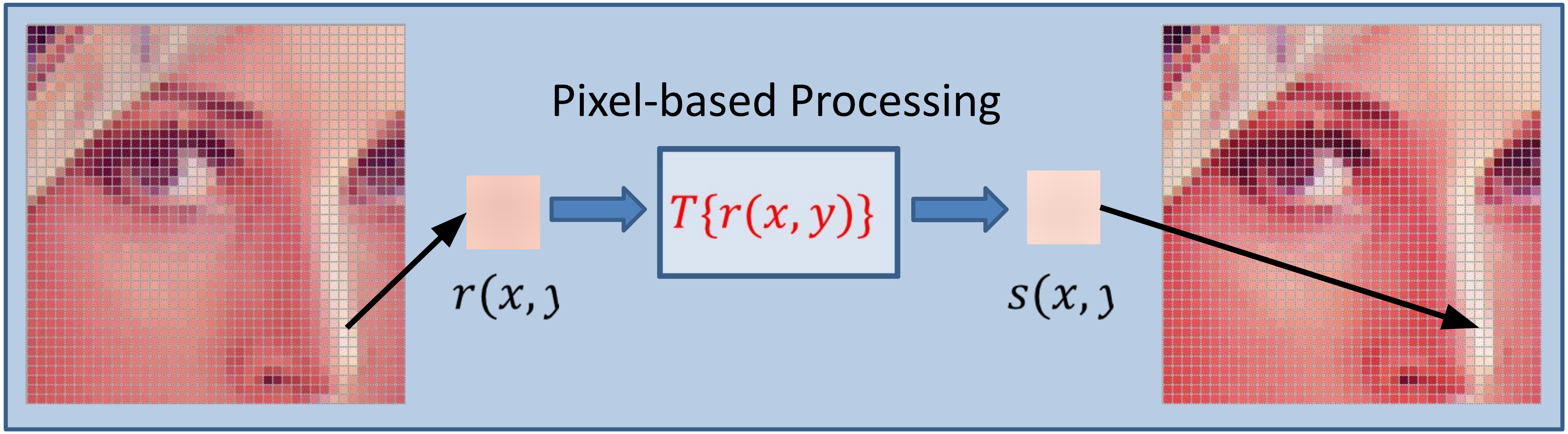
Faculty of Computing and Informatics
Multimedia University
prepared by Lai-Kuan, Wong
modified by Yuen Peng, Loh

Some graphics and materials are taken from UTexas at Austin CS376 & CS381V notes, Stanford Uni's CS131 notes and Gonzalez & Wood's DIP textbook

Lecture Outline

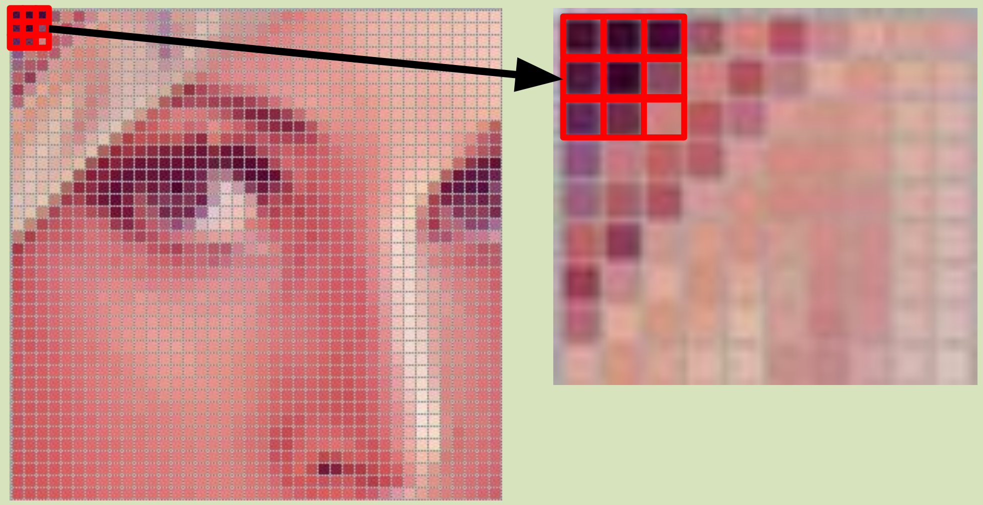
- Image Filtering
- Correlation and Convolution
- Image Sharpening
- Non-linear filters

Previously



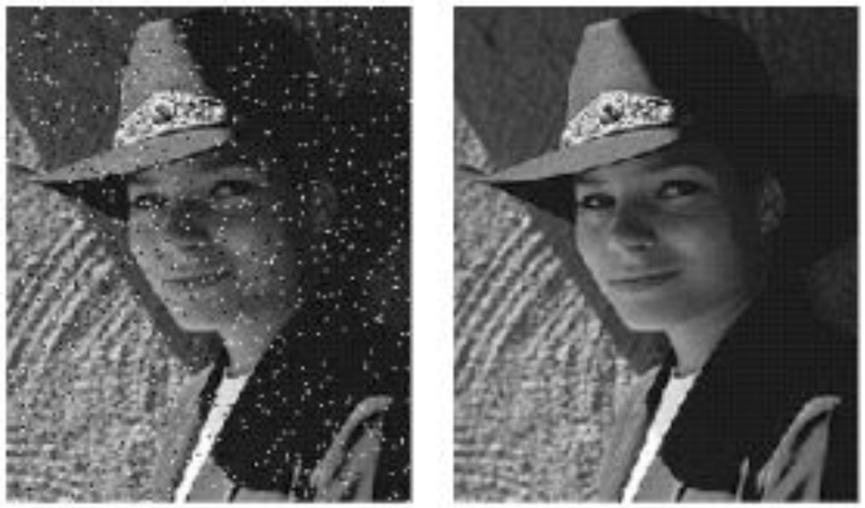
This lecture:

Neighborhood-based Processing



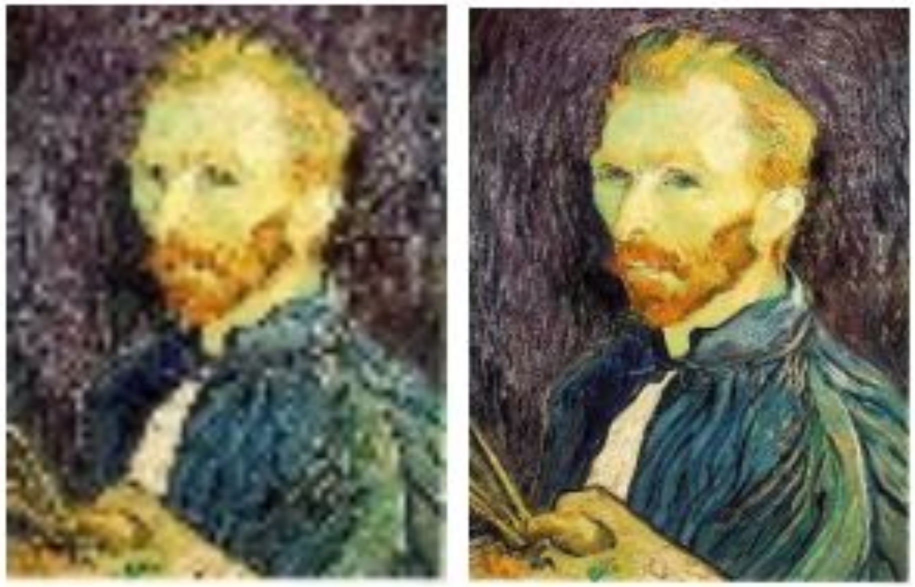
Applications of Filtering

De-noising



Salt and pepper noise

Super-resolution



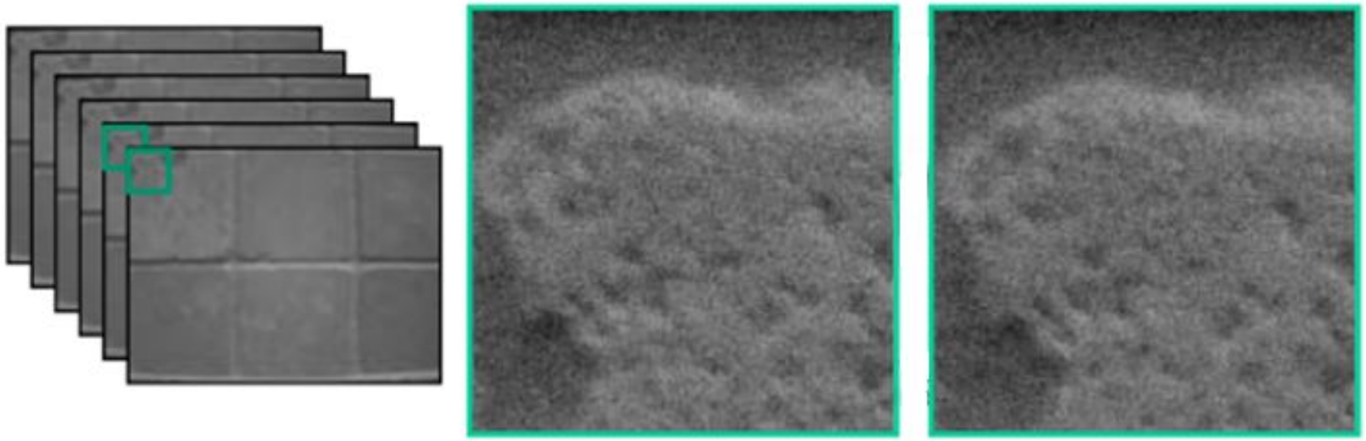
Moving beyond pixel□pixel

- What if there is insufficient knowledge of how to transform a pixel?

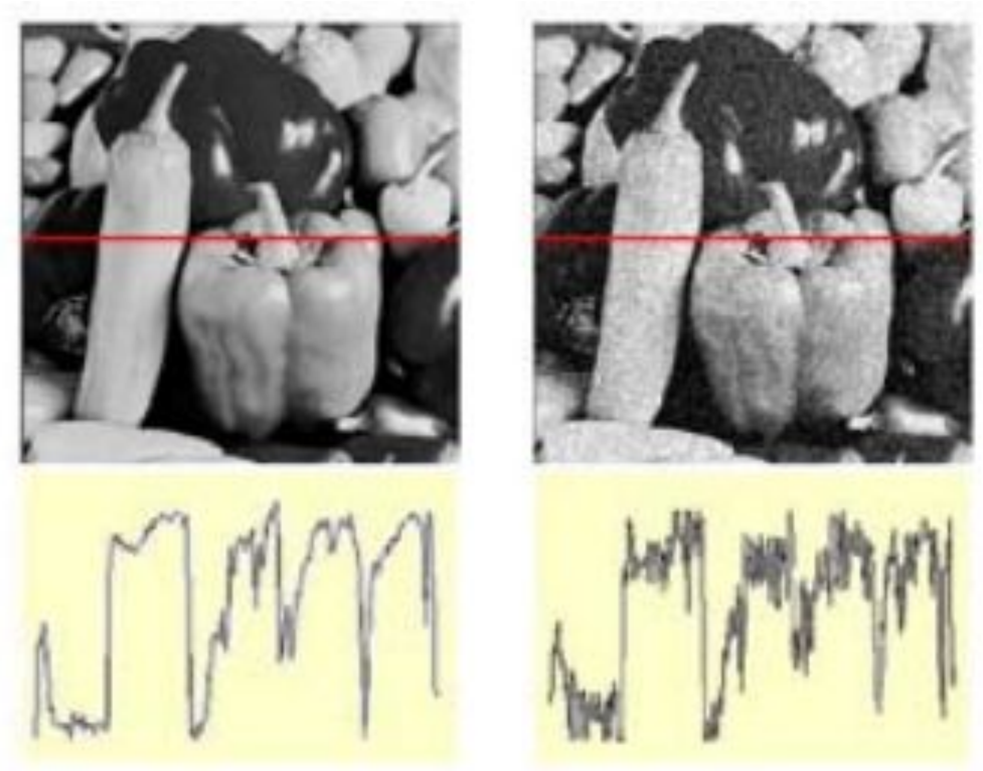
Depending on what we want to achieve, we can use **neighboring pixels** to help provide more information

Motivation: Noise reduction

- Capturing multiple images of the same static scene will not result in identical images
  - Likely: Environmental changes, sensor noise, camera shake, etc.



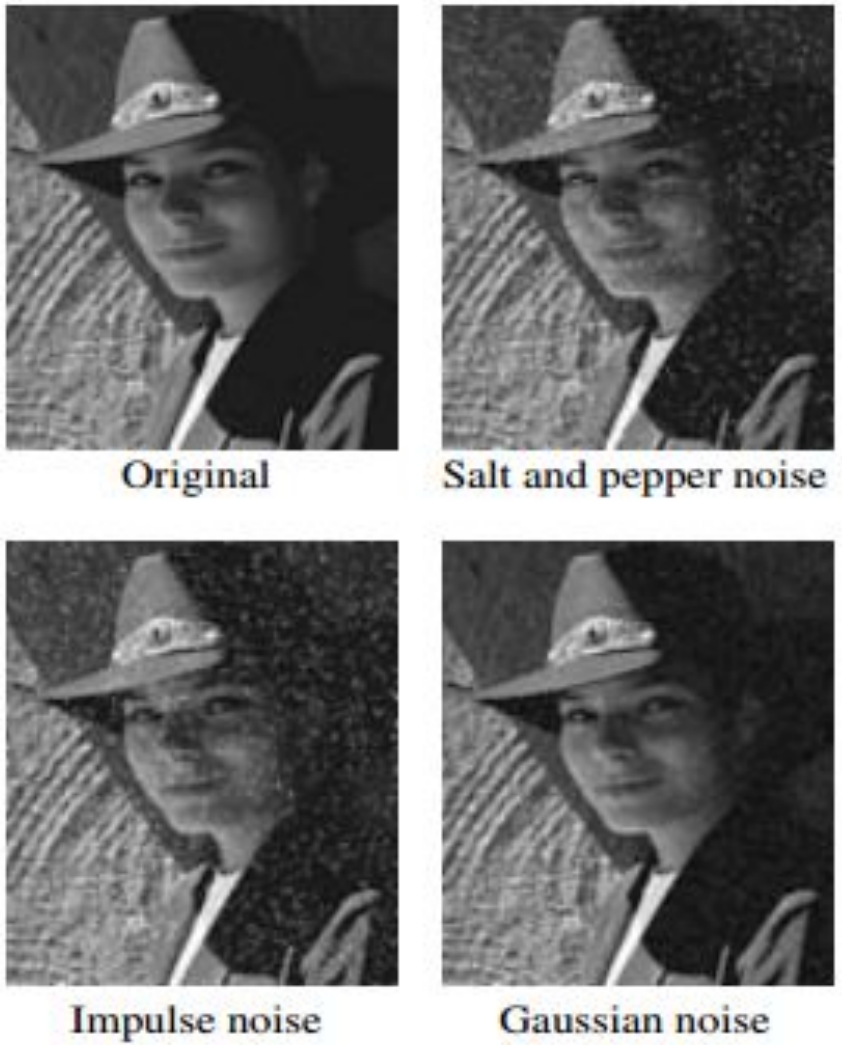
Motivation: Noise reduction



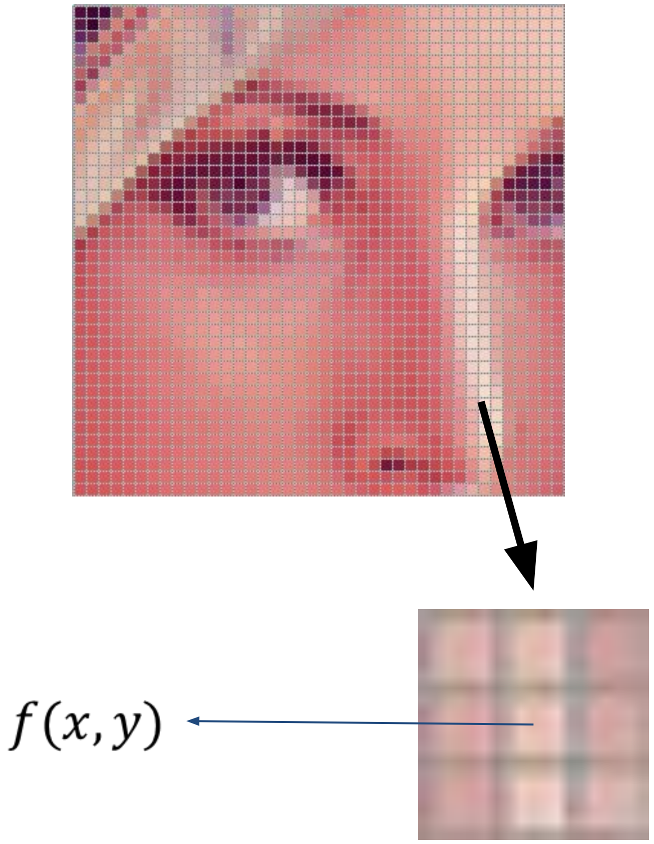
- Sometimes noises can be introduced during transmission of signals or compression of data

Common types of noise

- **Salt and pepper noise:** random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise:** variations in intensity drawn from a Gaussian normal distribution



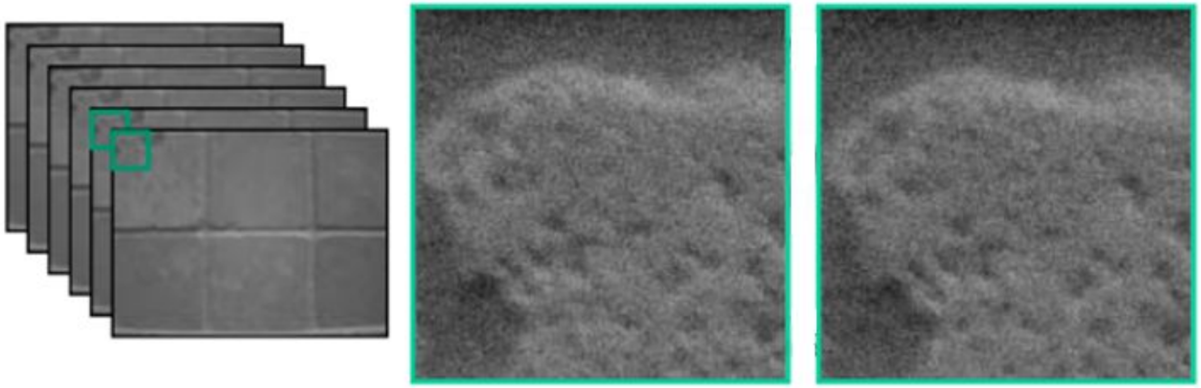
Neighborhood Processing a.k.a. Filtering



- Idea: Modify the value of the pixel f(x,y) based on a small neighbourhood of pixels surrounding it
- If we wish to "soften" the noise in the image, how should we modify?

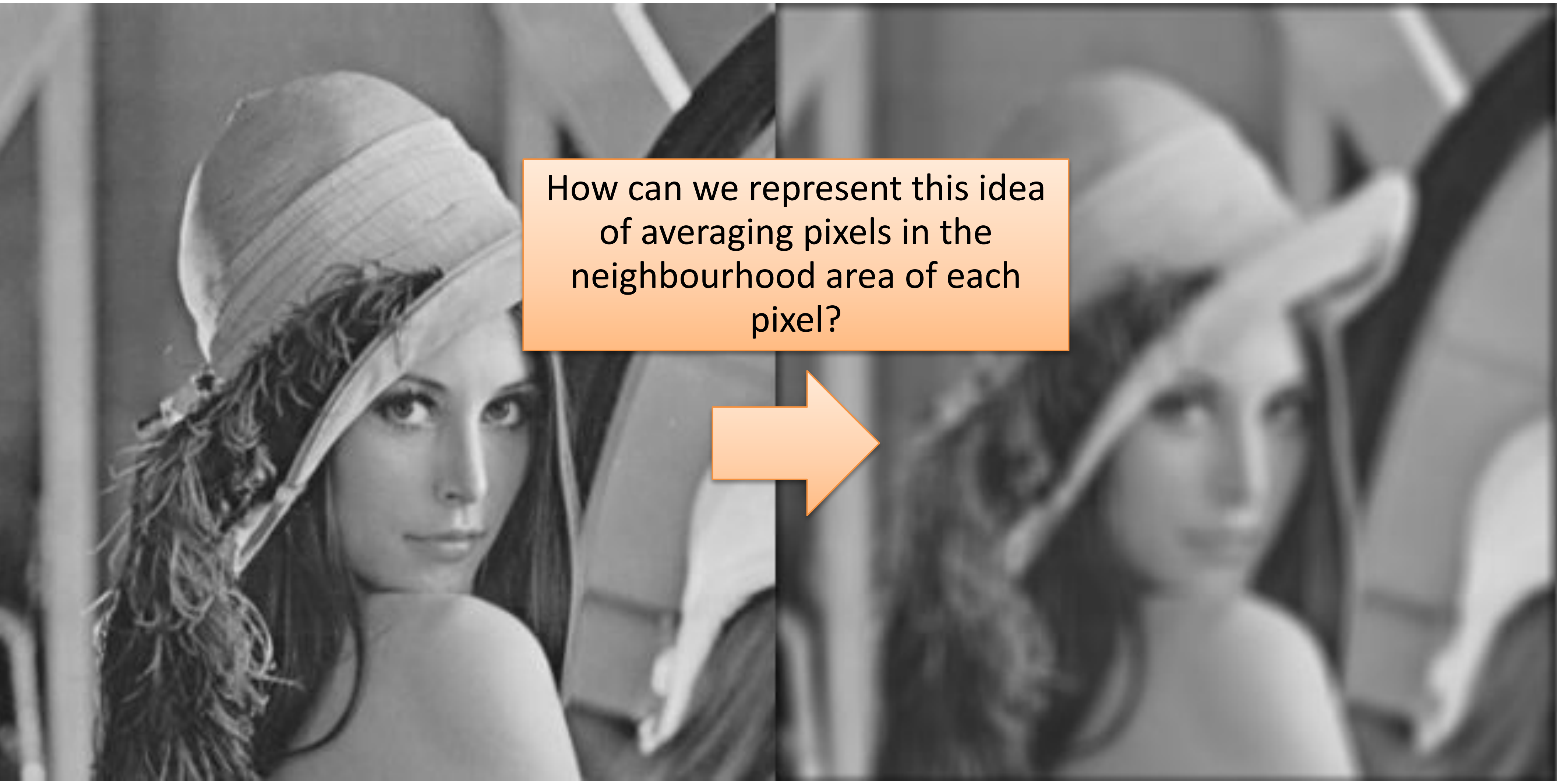
    A. Get minimum pixel
    B. Get maximum pixel
    C. Get average pixel
    D. Use a preset value

To try solve this…



- Idea: Let’s replace each pixel with an average of all the values in its neighbourhood
- Assumptions:
  - Expect pixels to be “like” their neighbours
  - Expect noise processes to be independent from pixel to pixel

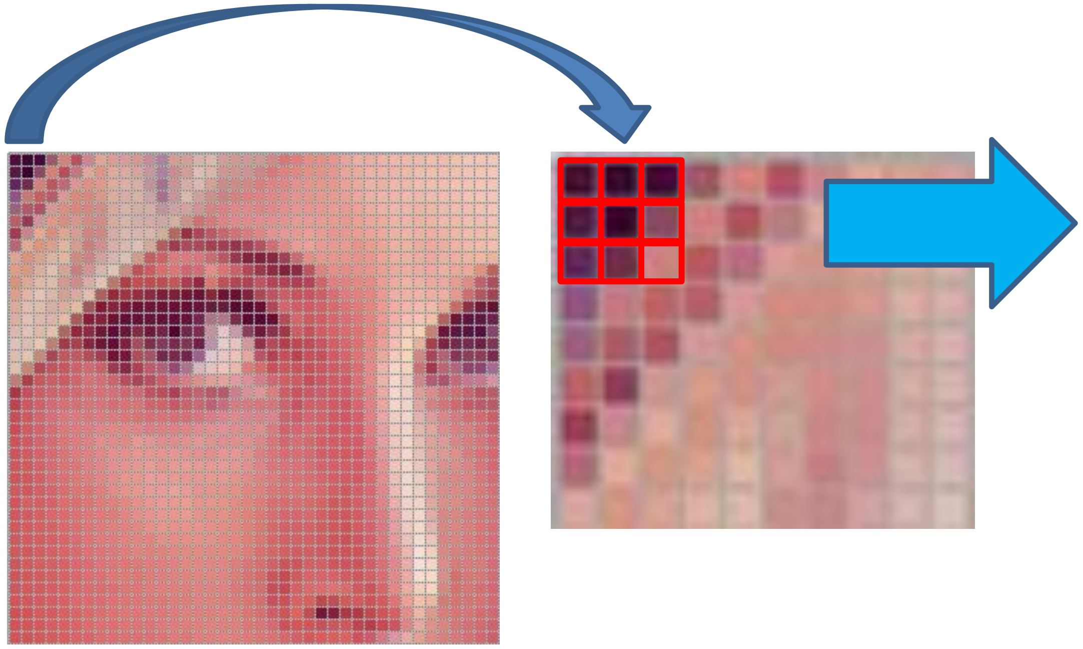
Result

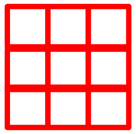


How can we represent this idea of averaging pixels in the neighbourhood area of each pixel?

"Masking" or Filtering

- Run a "mask" or "filter" across the entire image
- Mask corresponds to the neighbourhood that we wish to process

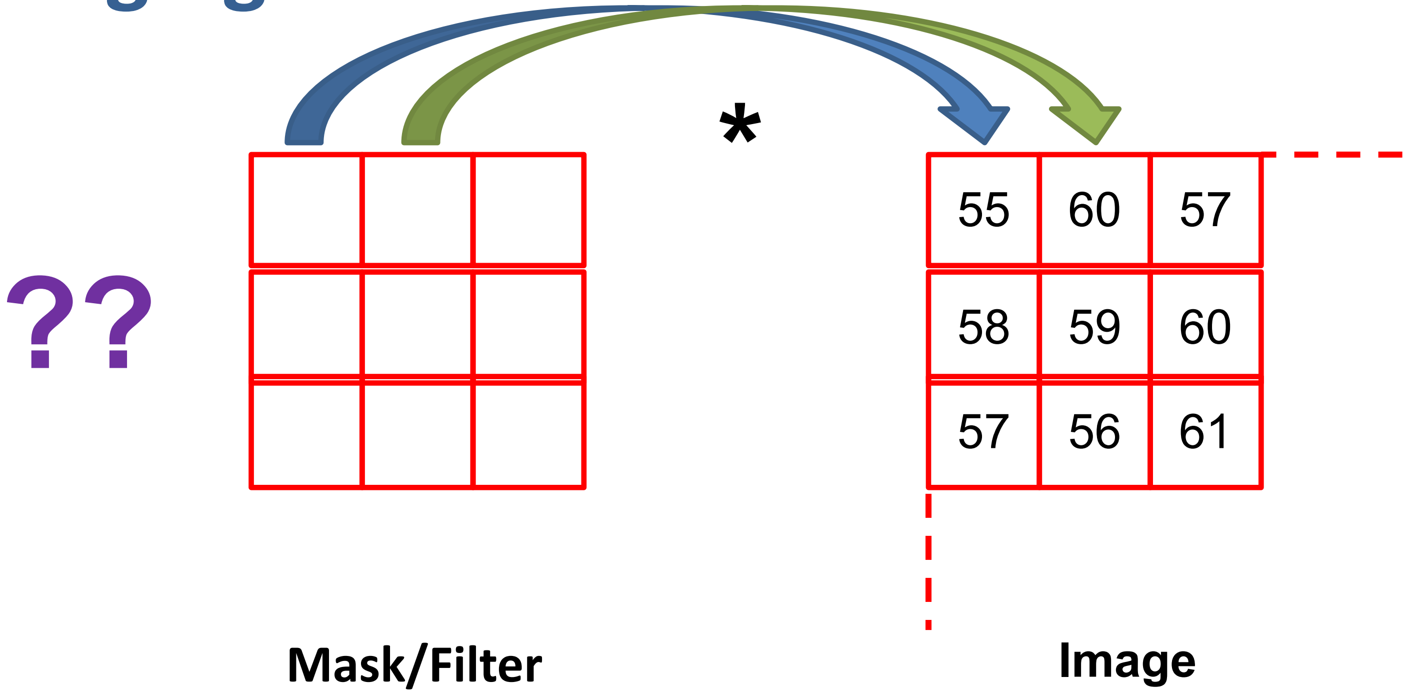




1. Find the average of all 9 pixels
2. Store the output
3. Slide the mask horizontally along the row.
4. Repeat

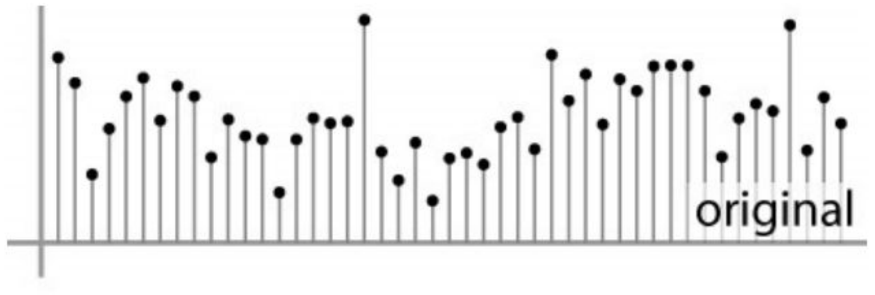
“Masking” or Filtering

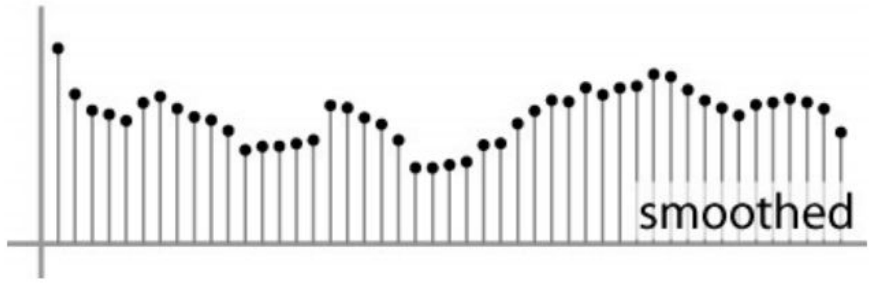
- Let’s say we multiply element-by-element the 3x3 mask/filter against a 3x3 part of the image, what are the values in the mask to achieve the **averaging** effect?



Let’s take a look in 1-D

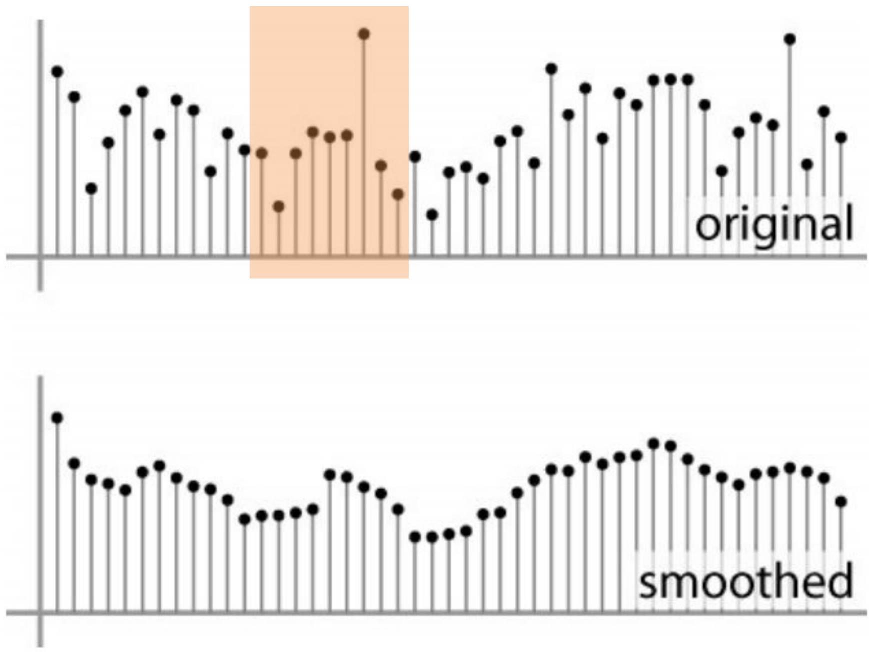
- Let’s replace each pixel with an average of all the values in its neighbourhood (also called “moving average”)
- 1D example





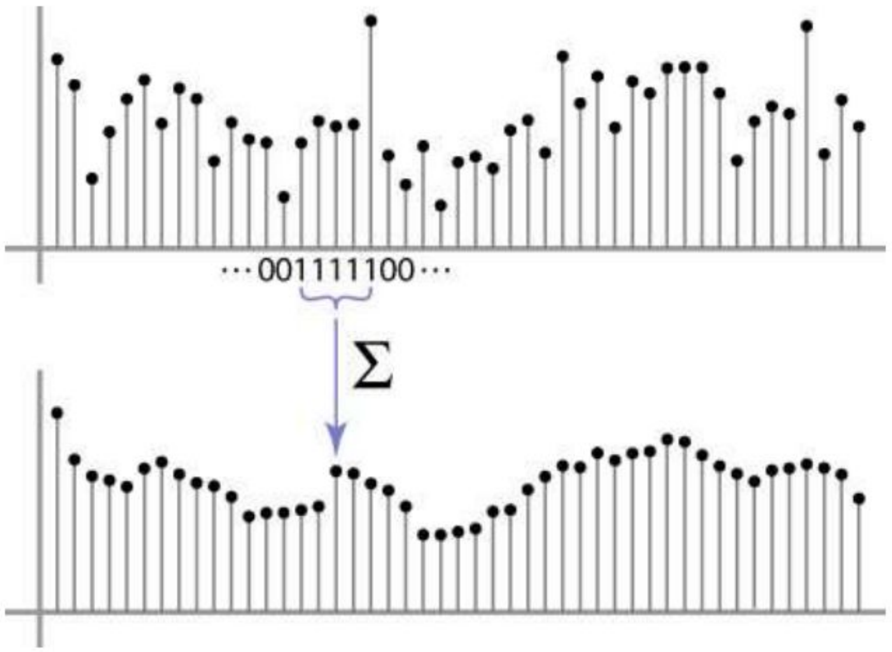
Moving average

- Let’s replace each pixel with an average of all the values in its **neighbourhood**
- What’s this neighbourhood?



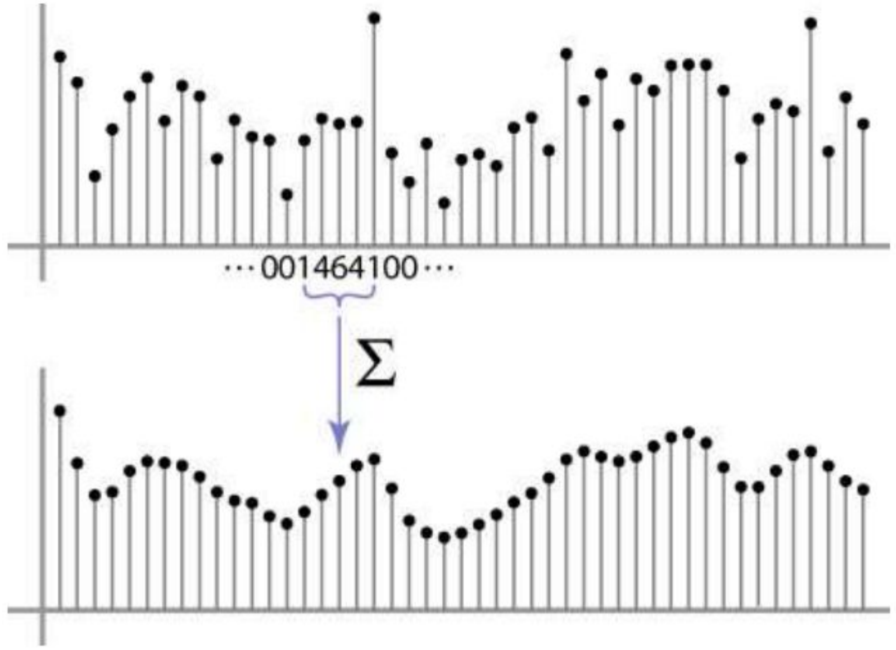
Moving average

- Moving average has equal uniform weights in the neighborhood
- Example: [1 1 1 1 1] / 5

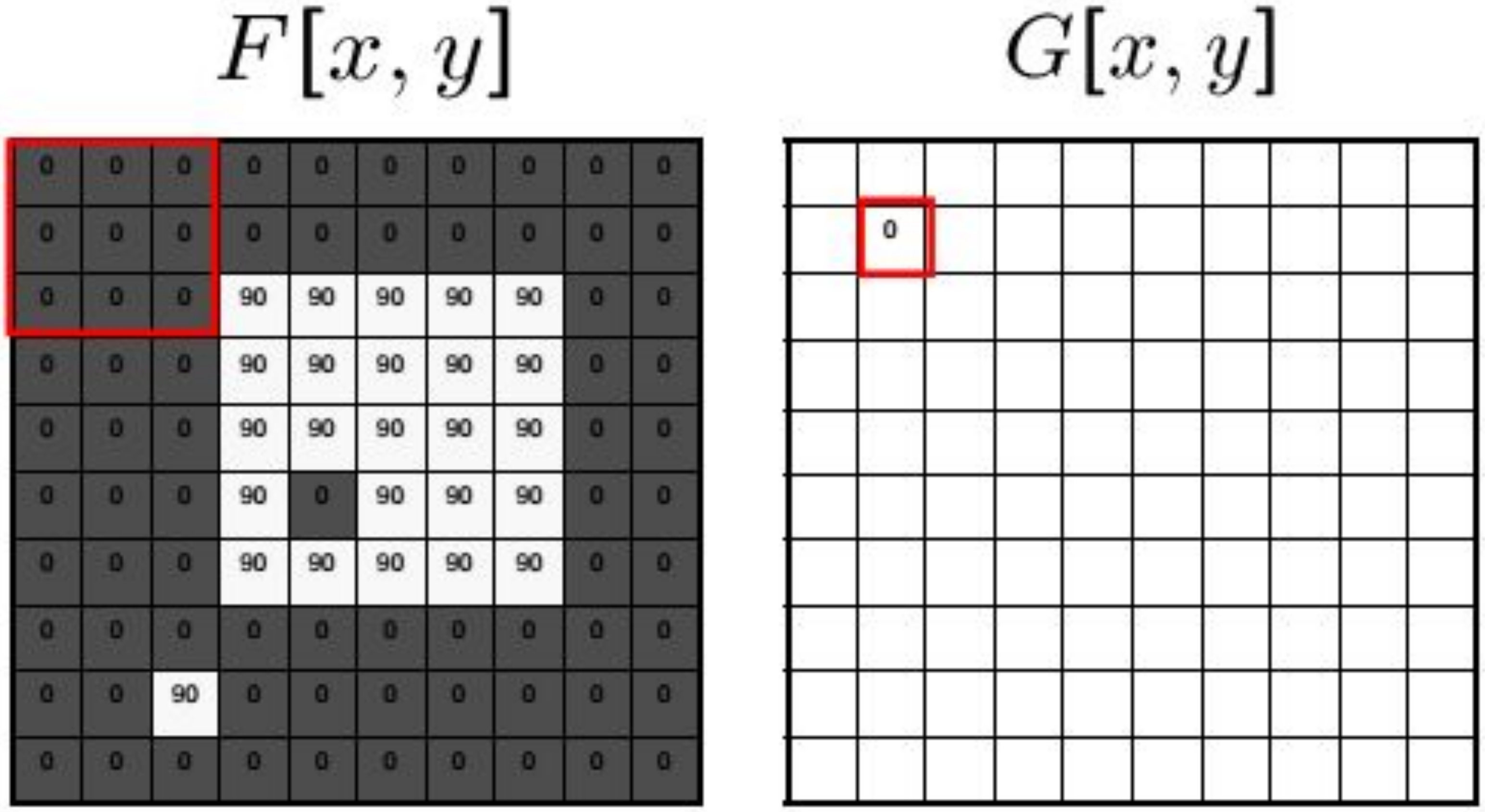


Weighted moving average

- Moving average with non-uniform weights
- Example: [1 4 6 4 1] / 16



Moving average in 2D



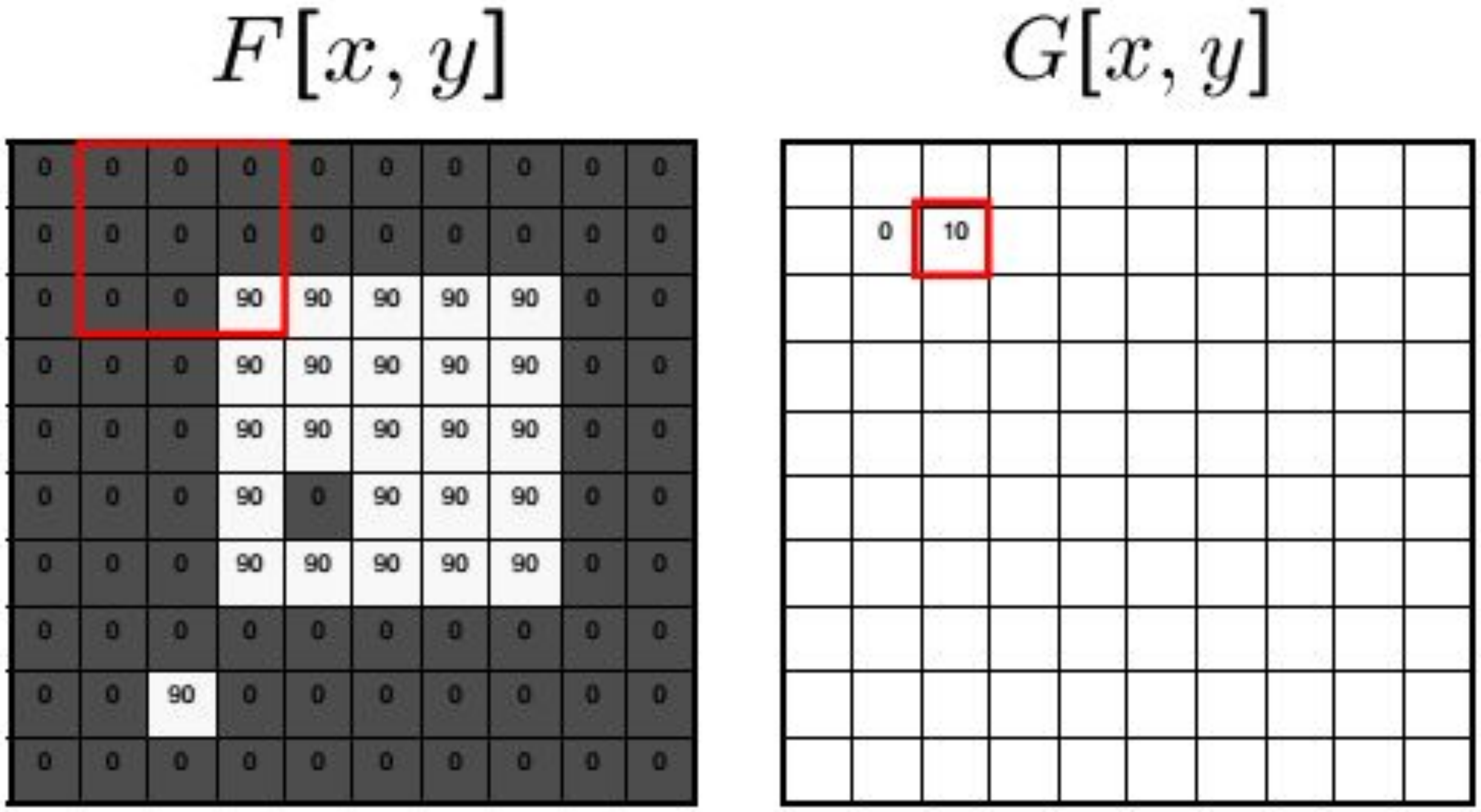
$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0								

Moving average in 2D



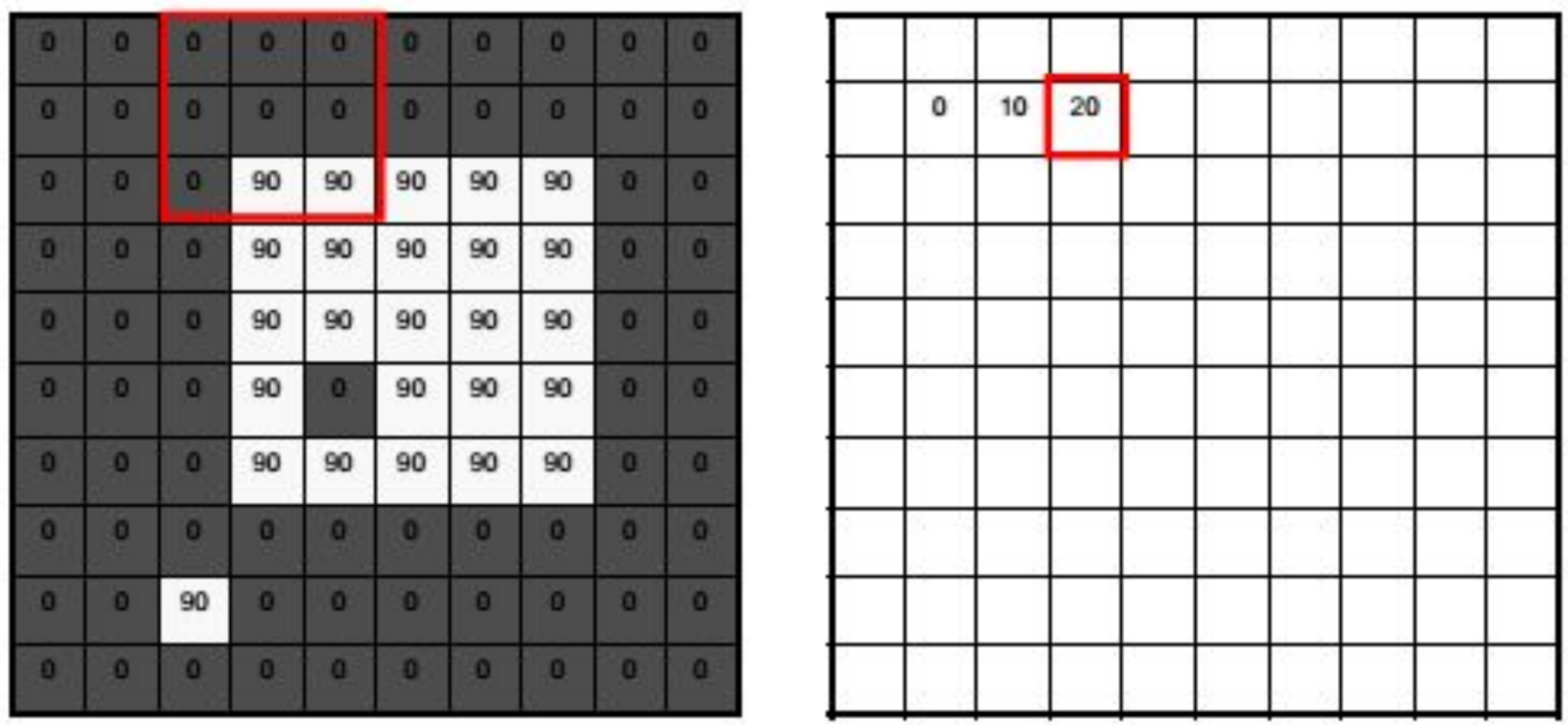
$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10							

Moving average in 2D



$F[x, y]$

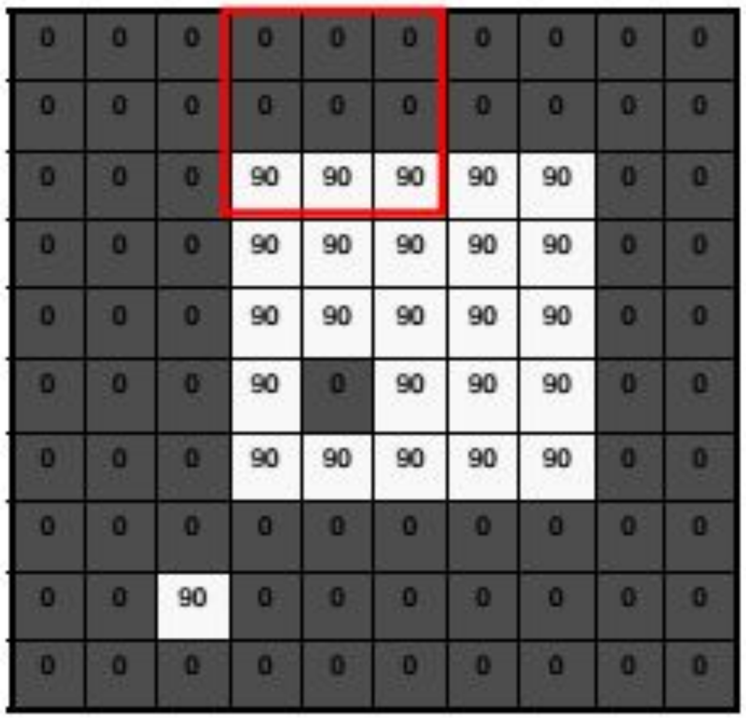
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20						

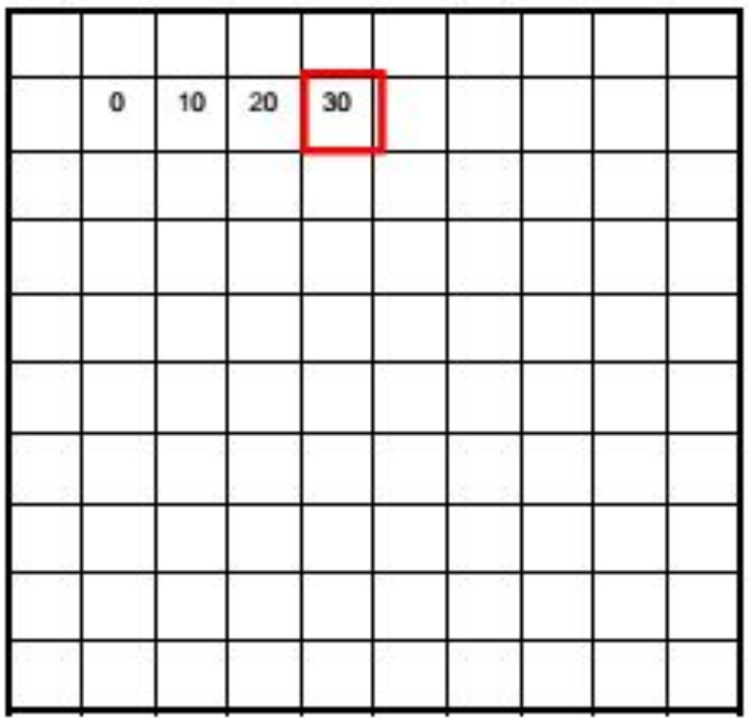
Moving average in 2D

$F[x, y]$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

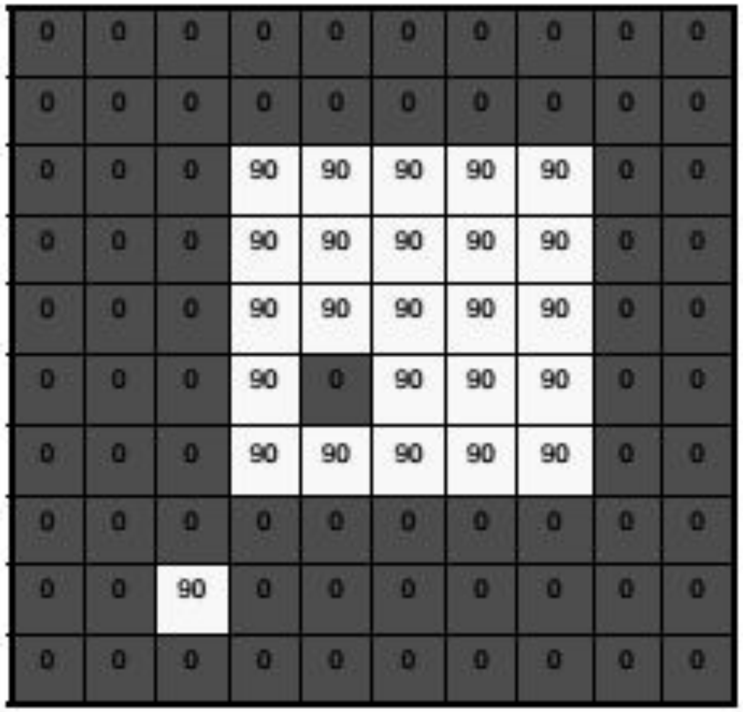
$G[x, y]$



	0	10	20	30					

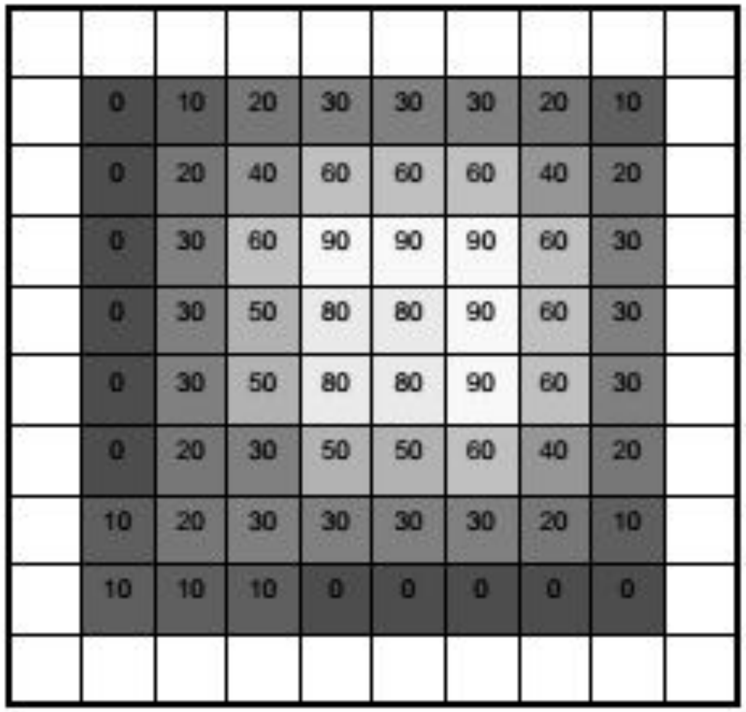
Finish: Moving average in 2D

$F[x, y]$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

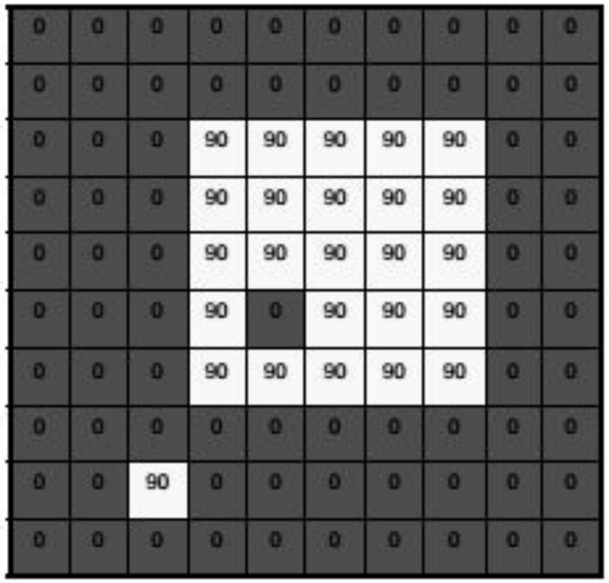
$G[x, y]$



	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

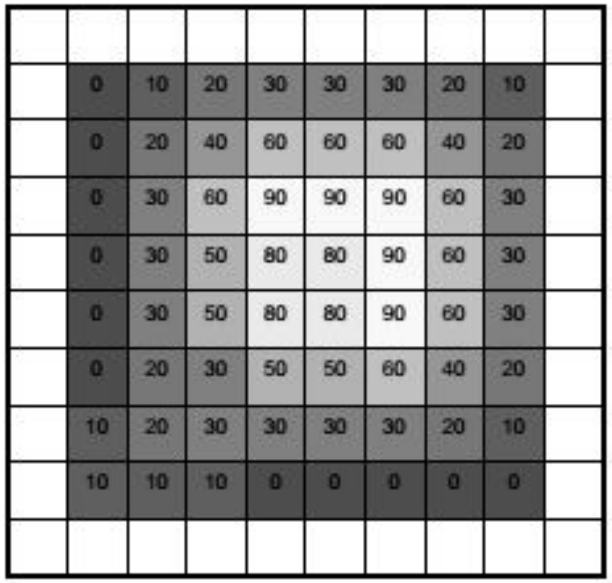
Finish: Moving average in 2D

$F[x, y]$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$



	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Correlation
and
Convolution

Correlation filtering

- Say the averaging window size is $2k+1 \times 2k+1$:

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\text{Attribute uniform weight to each pixel}} \underbrace{\sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]}_{\text{Loop over all pixels in neighborhood around image pixel F[i,j]}}$$

- Generalize to allow **different weights** depending on neighbouring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \underbrace{H[u,v]}_{\text{Non-uniform weights}} F[i+u, j+v]$$

Correlation filtering

$$G[i,j] = \sum_{u=-k}^{k} \underline{\sum_{v=-k}^{k}} H[u,v]F[i+u, j+v]$$

Move v (column) first

Steps:

1. Filter size = $(2k+1) \times (2k+1)$

   If filter size = 3×3, then $k = 1$

2. When $i = 2, j = 9, F[2,9]$ is the center pixel

   From $u = -k = -1, v = -k = -1$

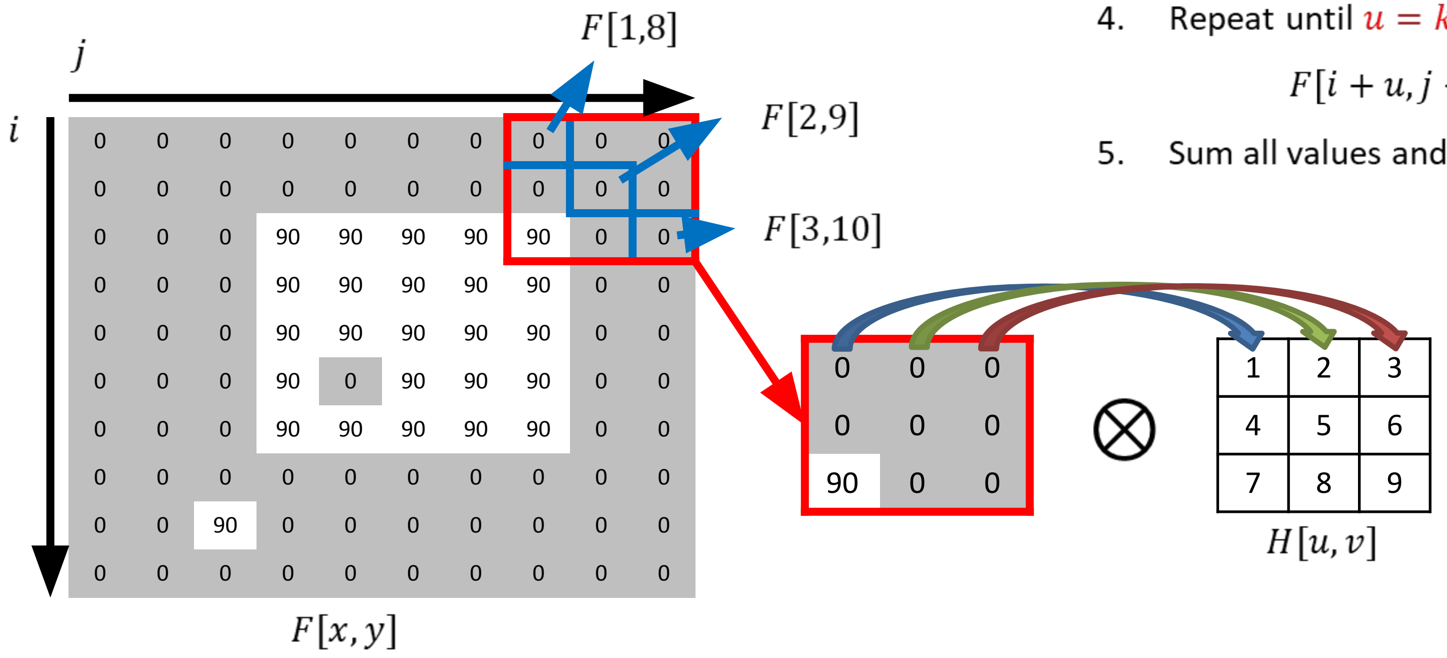
   $F[i+u, j+u] = F[2-1, 9-1] = F[1,8]$

3. Multiply with corresponding filter value

4. Repeat until $u = k = 1, v = k = 1$

   $F[i+u, j+u] = F[2+1, 9+1] = F[3,10]$

5. Sum all values and place in $G[i,j] = G[2,9]$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

0	0	0
0	0	0
90	0	0

$\otimes$

1	2	3
4	5	6
7	8	9

$$= (0\times1)+(0\times2)+(0\times3)+ (0\times4)+(0\times5)+(0\times6)+ (90\times7)+(0\times8)+(0\times9)$$

$$= 630 = G[2,9]$$

Correlation filtering

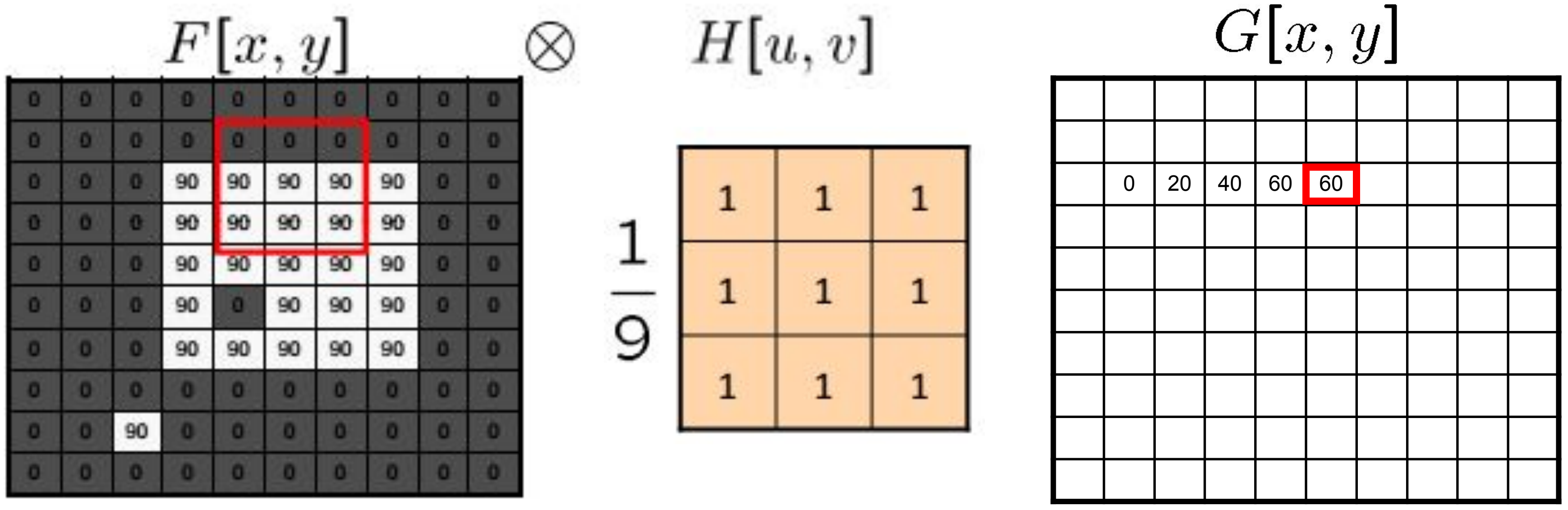
$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

- Cross-correlation: $G = H \otimes F$

- Summary:
  - Filtering an image: Replace each pixel with a linear combination of its neighbors
  - The filter "kernel" or "mask" $H(u,v)$ is the prescription for the weights in the linear combination

Correlation filtering

- What values belong in the kernel *H* for the moving average example?



$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$\otimes$

$H[u, v]$

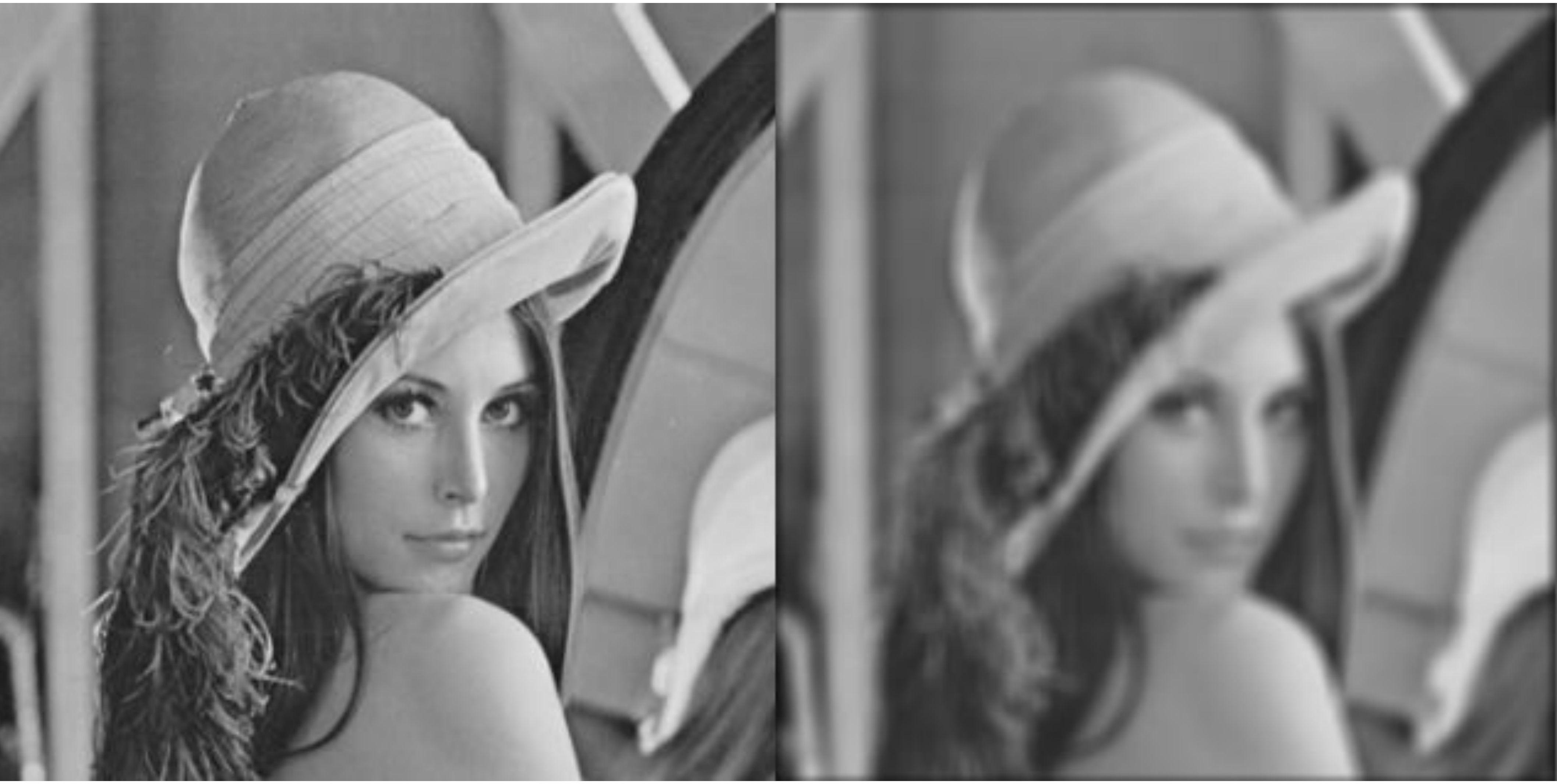
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$G[x, y]$

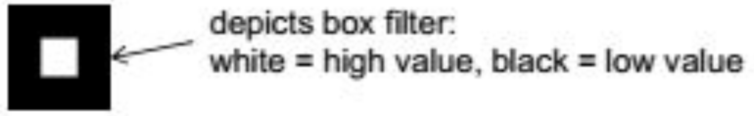
	0	20	40	60	$\boxed{60}$				

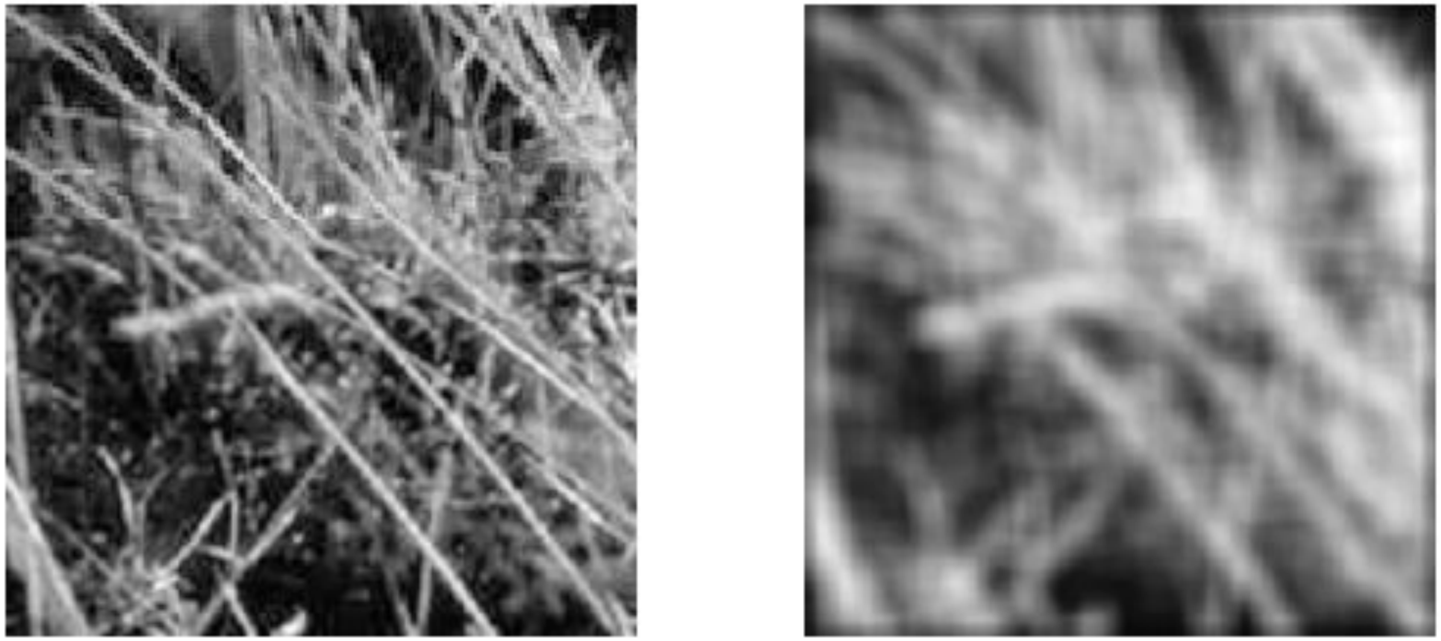
$$G = H \otimes F$$

Filter #1: Moving Average



Correlation filtering





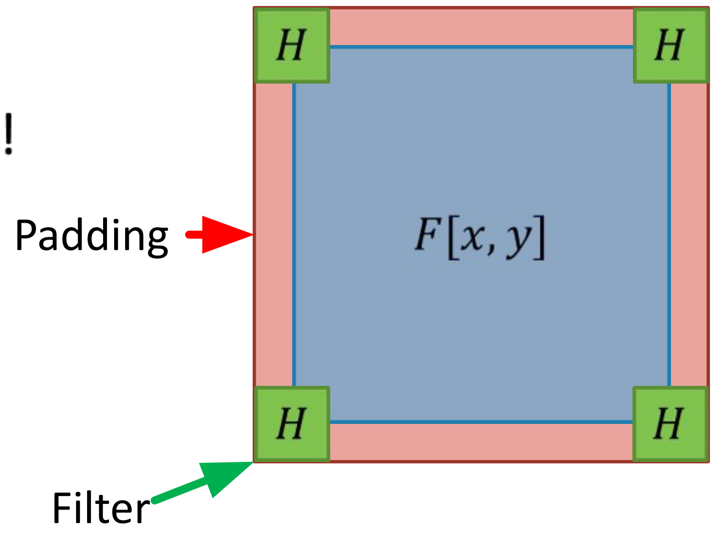
original

filtered

- What can we expect from the output if the filter size is 5 x 5 or 7 x 7 instead of 3 x 3 ?

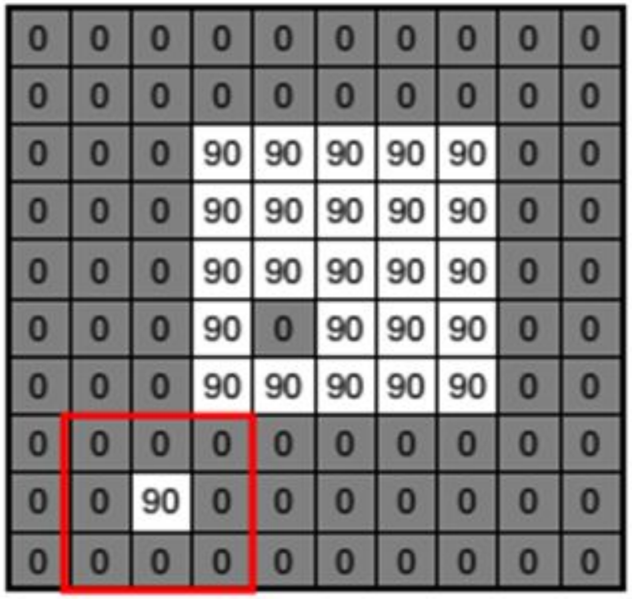
Boundary issues

- What about near the edge?
  - Filter window falls off the edge of the input image, some pixels are not defined ⇒ Need to extrapolate
  - Some common padding methods:
    - Constant value (with 0's we get zero-padding)
    - Wrap around
    - Copy edge
    - **`numpy.pad`** has a lot more options!



Gaussian filter

- What if we want the nearest neighbouring pixels to have **more** influence on the output?



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

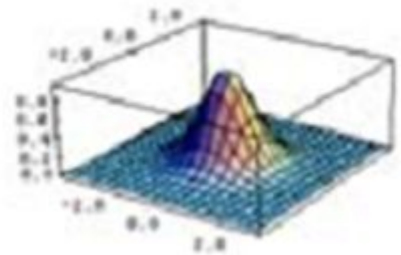
$F[x, y]$

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

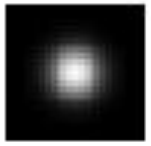
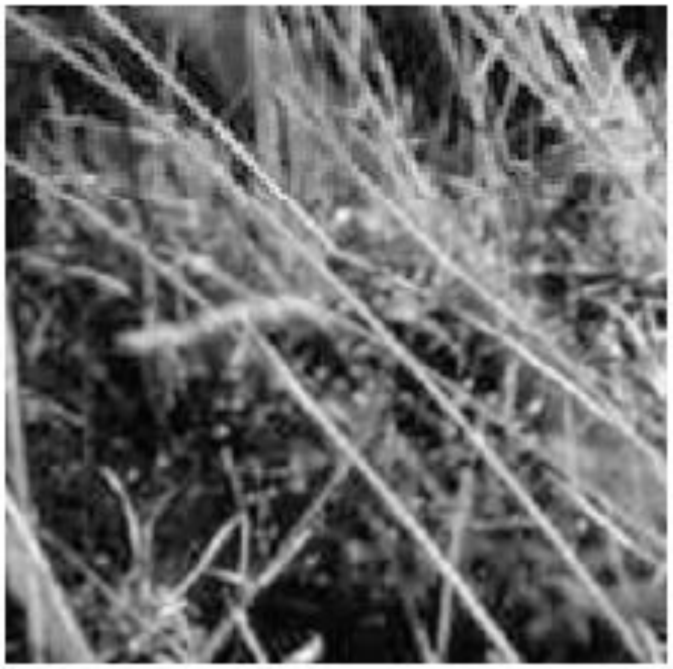
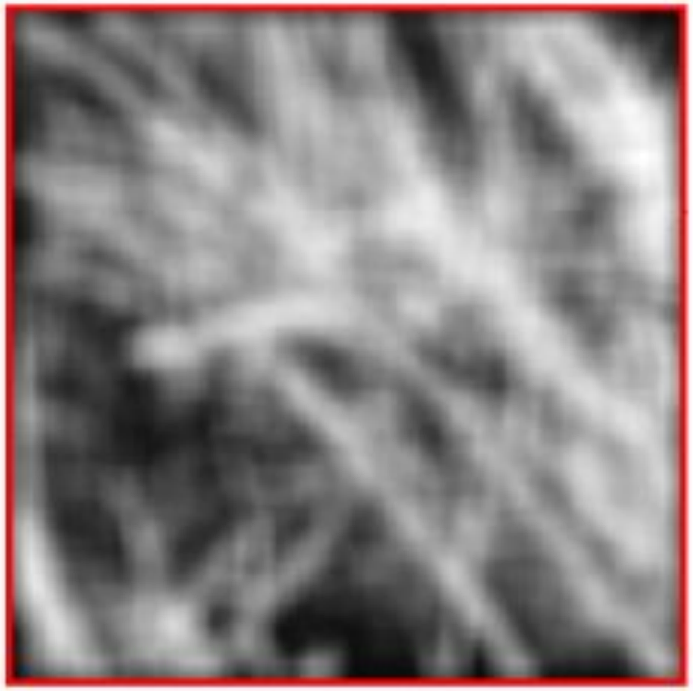
$H[u, v]$

This kernel is an approximation of a 2d Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

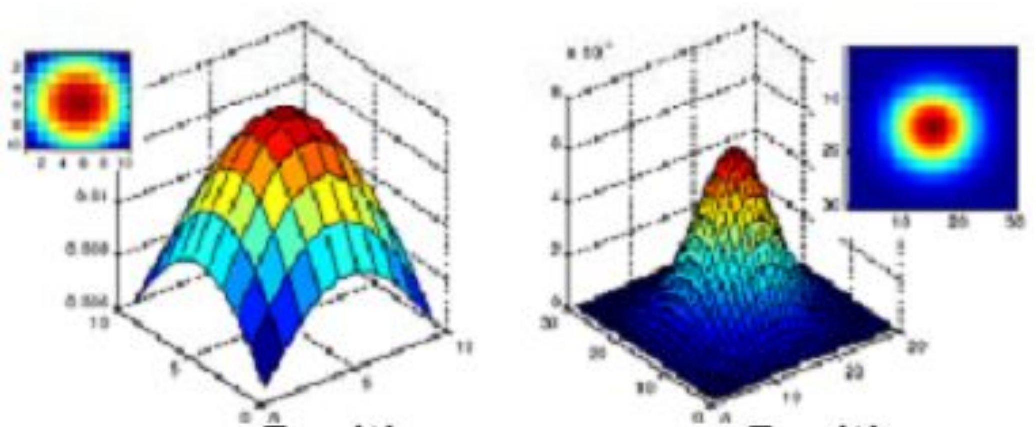


Smoothing with Gaussian filter

Gaussian filter parameters

- What parameters are important?
- **(1) Size** of kernel or mask



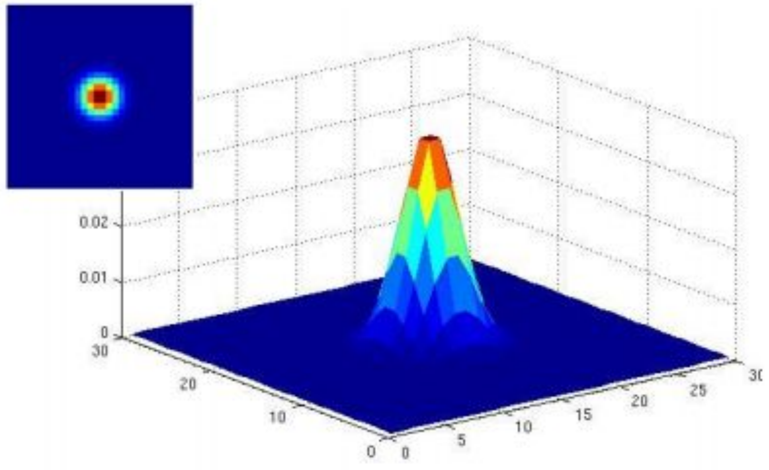
σ = 5 with 10 x 10 kernel

σ = 5 with 30 x 30 kernel

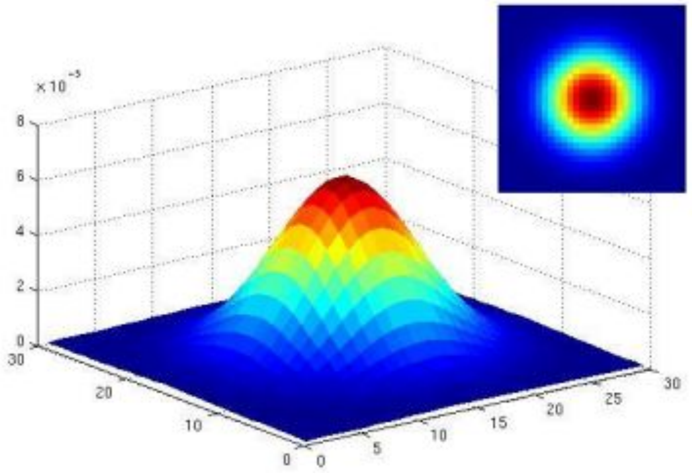
  - Note: Gaussian function has infinite support, but discrete filters use finite kernels

Gaussian filter parameters

- What parameters are important?
- **(2) Variance** of Gaussian: determines extent of smoothing



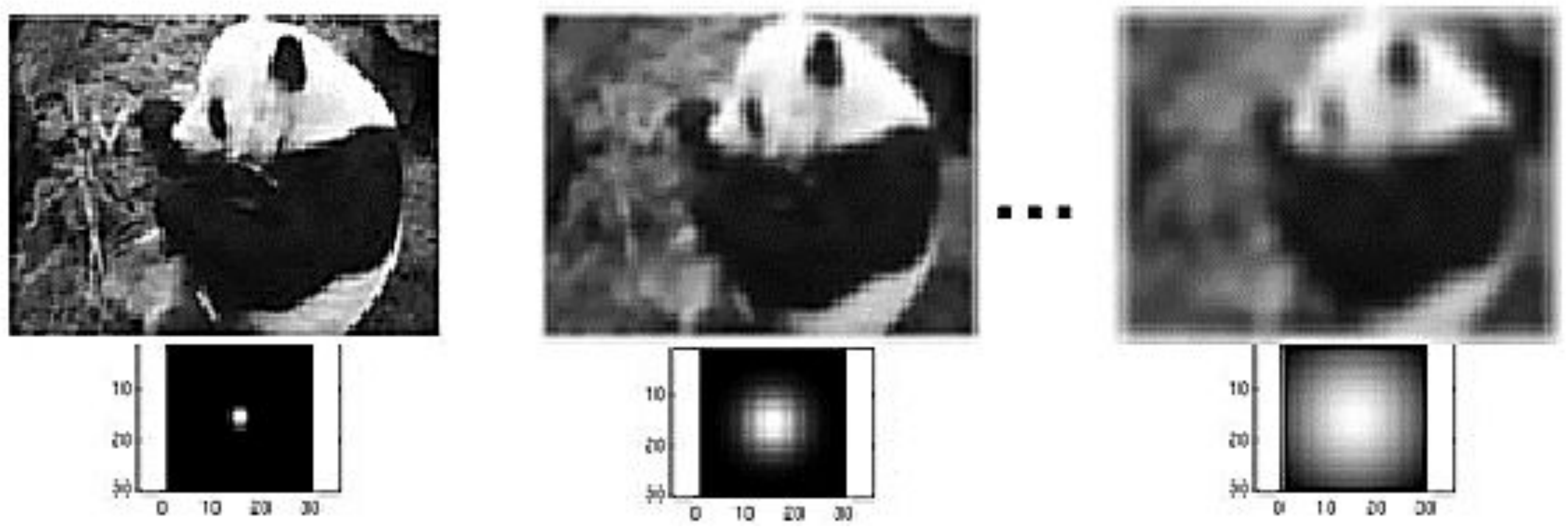
Size = 30×30
$\sigma = 2$



Size = 30×30
$\sigma = 5$

σ parameter

- The σ parameter is the “scale” or “width” or “spread” of the Gaussian kernel $\Rightarrow$ controls the amount of smoothing

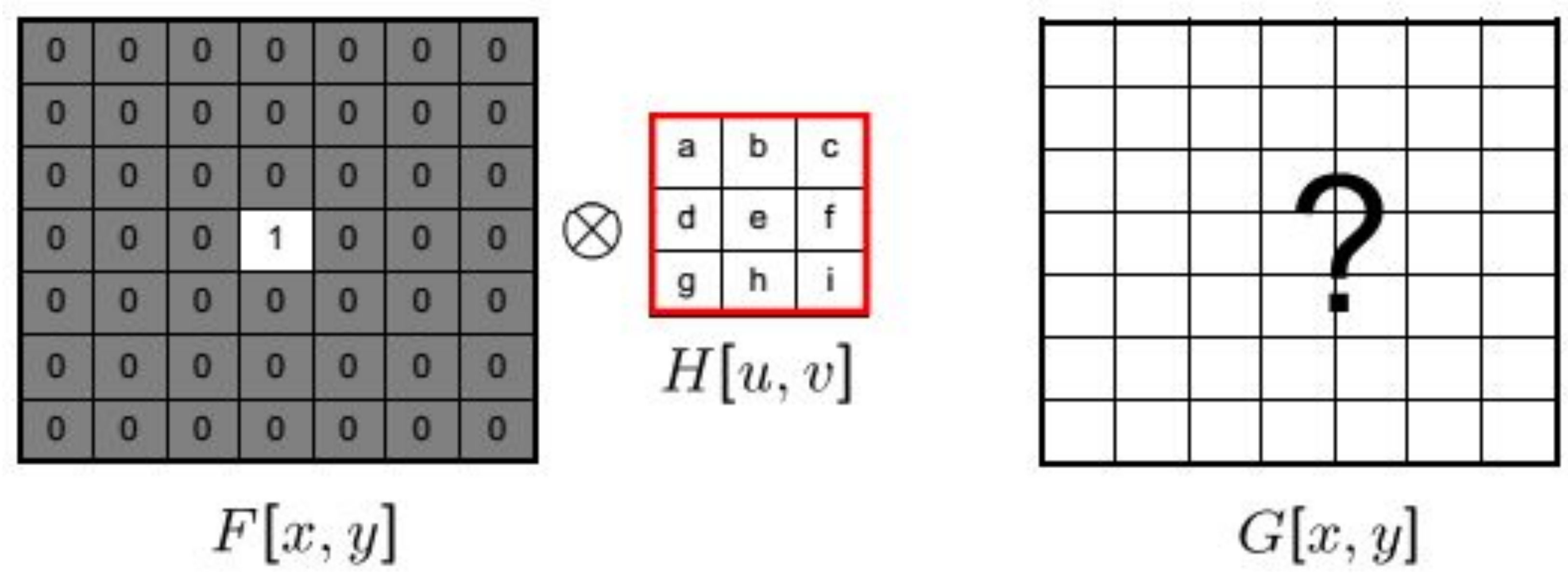


Properties of Smoothing

- Values are **positive**
- **Sum to 1** ⇒ constant regions same as input
- Amount of smoothing proportional to mask size
- "Low-pass" filtering, remove "high-frequency" components

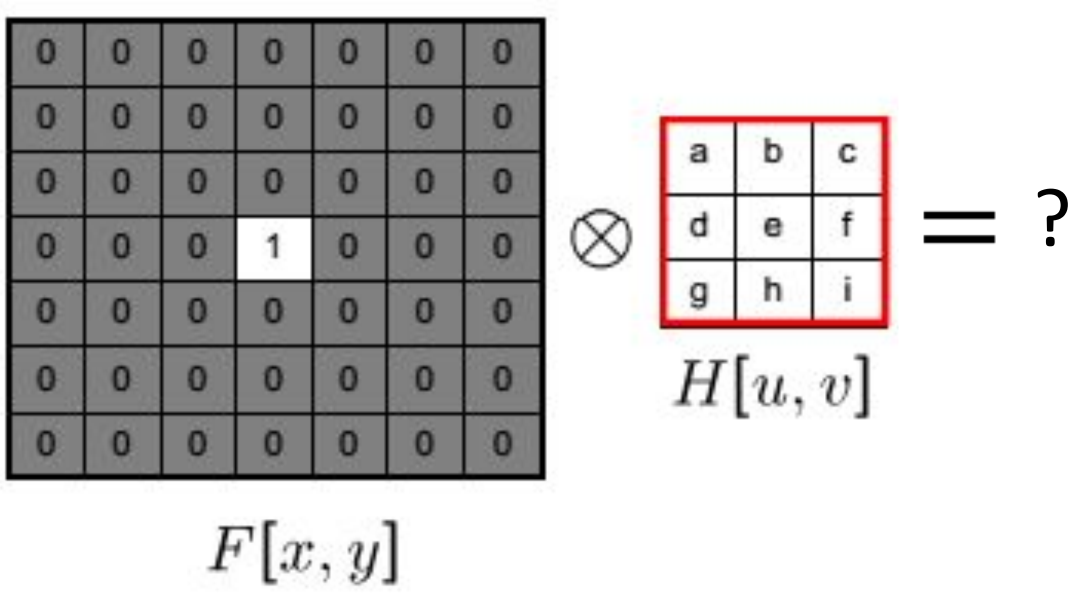
Filtering an impulse signal

- What is the result of filtering the impulse signal (image) *F* with an arbitrary kernel *H* ?



Filtering an impulse signal

- What is the result of filtering the impulse signal (image) *F* with an arbitrary kernel *H* ?



A.

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	**a**	**b**	**c**	0	0
0	0	**d**	**e**	**f**	0	0
0	0	**g**	**h**	**i**	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

B.

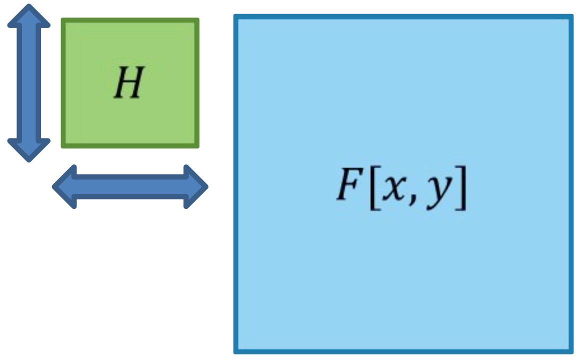
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	**i**	**h**	**g**	0	0
0	0	**f**	**e**	**d**	0	0
0	0	**c**	**b**	**a**	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = \underset{\substack{\uparrow \\ \text{Notation for} \\ \text{convolution} \\ \text{operator}}}{H \star F}$$



Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

Steps:

1. Filter size = $(2k+1) \times (2k+1)$

   If filter size = 3×3, then $k = 1$

2. When $i = 2, j = 9, F[2,9]$ is the center pixel

   From $u = -k = -1, v = -k = -1$

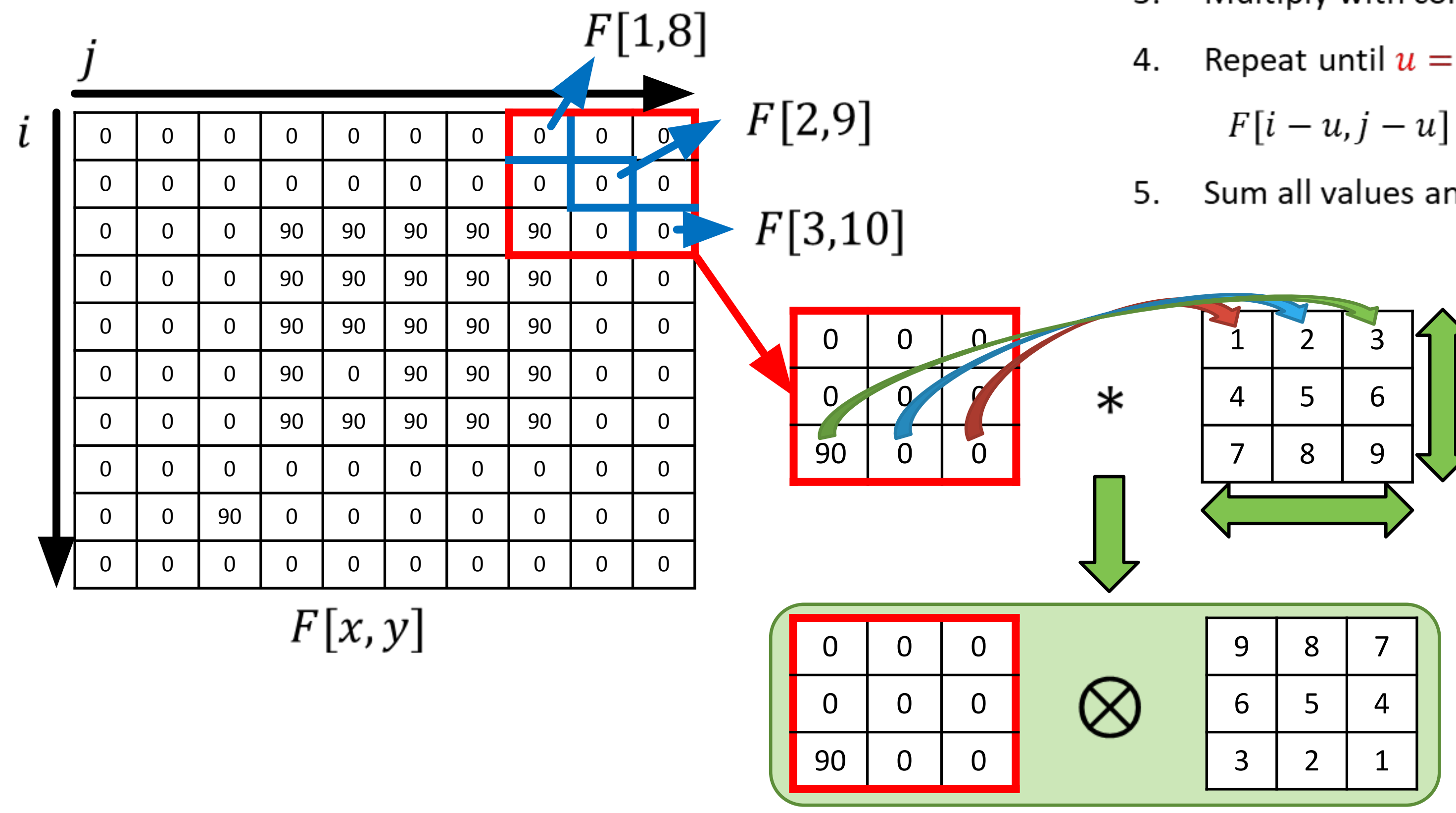
   $F[i-u, j-u] = F[2+1, 9+1] = F[3,10]$

3. Multiply with corresponding filter value

4. Repeat until $u = k = 1, v = k = 1$

   $F[i-u, j-u] = F[2+1, 9+1] = F[1,8]$

5. Sum all values and place in $G[i,j] = G[2,9]$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

0	0	0
0	0	0
90	0	0

1	2	3
4	5	6
7	8	9

0	0	0
0	0	0
90	0	0

9	8	7
6	5	4
3	2	1

To simplify, the convolution can be converted to correlation by flipping the filter horizontally and vertically

$$= (0\times9)+(0\times8)+(0\times7)+(0\times6)+(0\times5)+(0\times4)+(90\times3)+(0\times2)+(0\times1)$$

$$= 270 = G[2,9]$$

Convolution vs. Cross-correlation

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

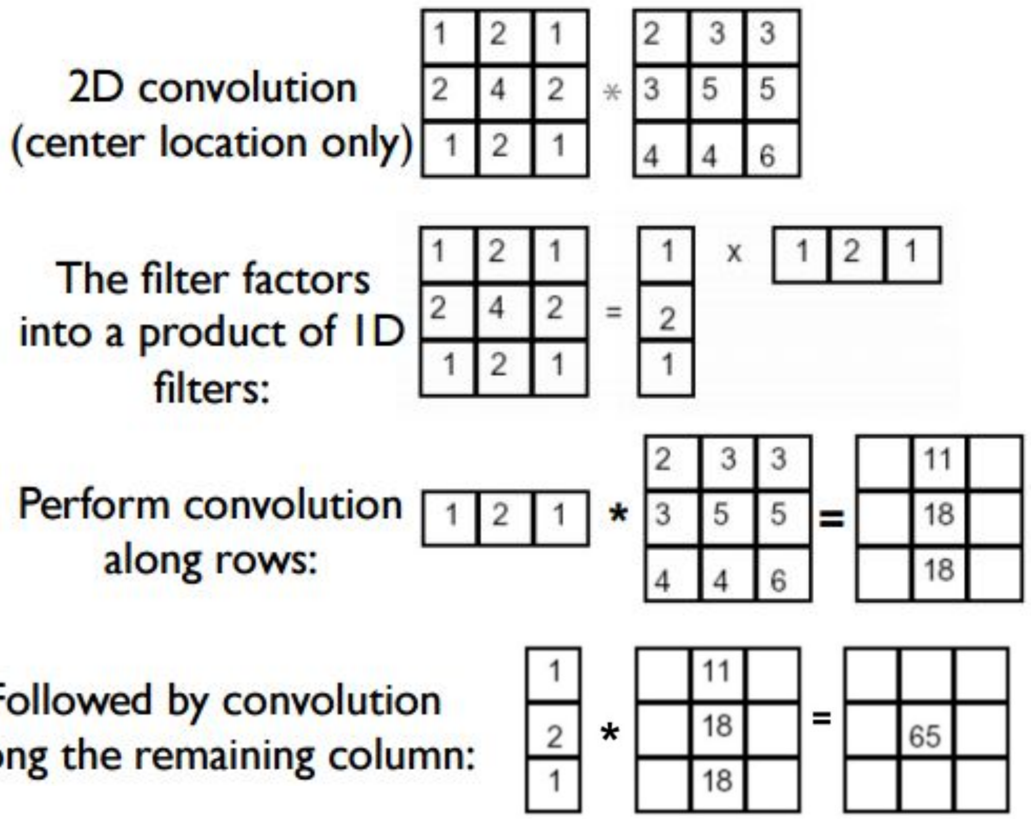
$$G = H \otimes F$$

Important! If the kernel is **symmetric**, **Convolution = Correlation**

Convolution is useful when dealing in the frequency domain (Fourier transform) to enable easy combination of more than 1 filter. Nice explanation on the differences: http://www.cs.umd.edu/~djacobs/CMSC426/Convolution.pdf

Separability of Filters

- In some cases, filters are **separable** ⇒ can be factored into two steps
  - Convolve all rows
  - Convolve all column



2D convolution (center location only)

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix}$$

The filter factors into a product of 1D filters:

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix} \times \begin{bmatrix}1&2&1\end{bmatrix}$$

Perform convolution along rows:

$$\begin{bmatrix}1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix} = \begin{bmatrix}&11&\\&18&\\&18&\end{bmatrix}$$

Followed by convolution along the remaining column:

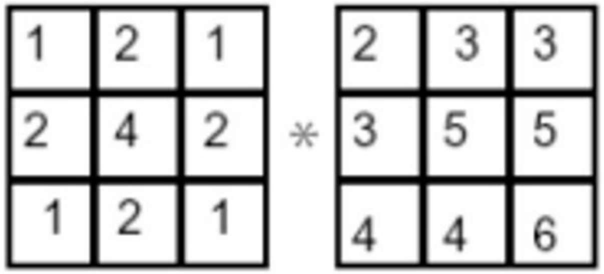
$$\begin{bmatrix}1\\2\\1\end{bmatrix} * \begin{bmatrix}&11&\\&18&\\&18&\end{bmatrix} = \begin{bmatrix}&&\\&65&\\&&\end{bmatrix}$$

Separability of the Gaussian filter

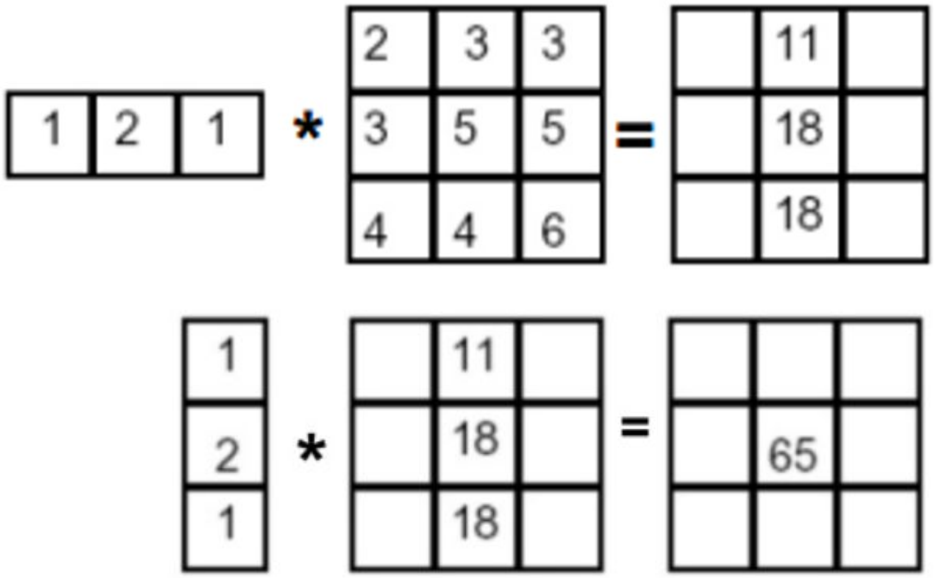
$$\begin{aligned}G_\sigma(x,y) &= \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}\\ &= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)\end{aligned}$$

Separability of Filters

- What is the complexity of filtering an n × n image with an m × m kernel?
  - $O(n^2m^2)$



- What if the kernel is separable?
  - $O(n^2m) = O(2n^2m)$



Separability of Filters

- Is this separable? If yes, what is the separable version?

$$\frac{1}{K^2}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline 1 & 1 & \cdots & 1 \\ \hline \vdots & \vdots & 1 & \vdots \\ \hline 1 & 1 & \cdots & 1 \\ \hline \end{array}$$

$$\frac{1}{K}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline \end{array}$$

- What does this filter do?

Separability of Filters

- Is this separable? If yes, what is the separable version?

$$\frac{1}{256}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 6 & 24 & 36 & 24 & 6 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array}$$

$$\frac{1}{16}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array}$$

- What does this filter do?

Separability of Filters

- Is this separable? If yes, what is the separable version?

$$\frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

$$\frac{1}{2}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline \end{array}$$

- What does this filter do?

Separability of Filters

- Is this separable? If yes, what is the separable version?

$$\frac{1}{4}\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline \end{array}$$

$$\frac{1}{2}\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline \end{array}$$

- What does this filter do?

Image Sharpening

Intuition of Filtering

Original

$$\begin{array}{|c|c|c|}\hline \bullet 0 & \bullet 0 & \bullet 0 \\ \hline \bullet 0 & \bullet 1 & \bullet 0 \\ \hline \bullet 0 & \bullet 0 & \bullet 0 \\ \hline \end{array} =$$

Filtered
(no change)

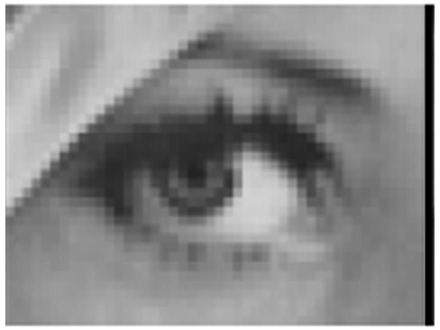
Intuition of Filtering

Original

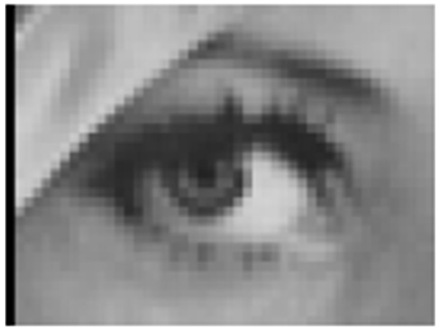
•0	•0	•0
•0	•0	•1
•0	•0	•0

=

Shifted left by 1 pixel

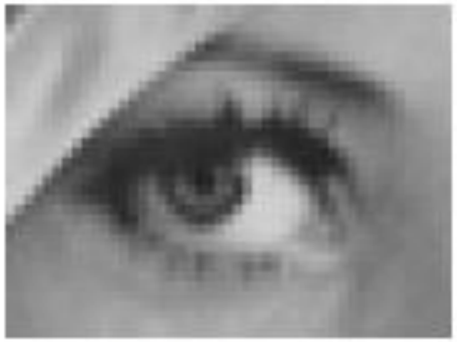


?

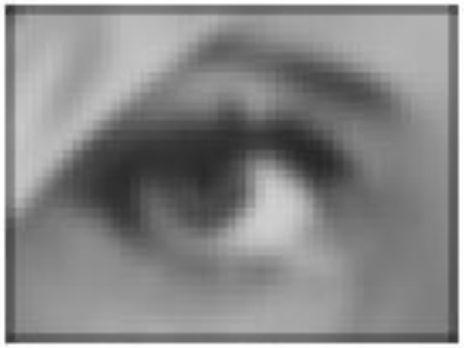


Shifted right by 1 pixel

Intuition of Filtering


Original

$$\frac{1}{9}\begin{array}{|c|c|c|}\hline \bullet 1 & \bullet 1 & \bullet 1 \\ \hline \bullet 1 & \bullet 1 & \bullet 1 \\ \hline \bullet 1 & \bullet 1 & \bullet 1 \\ \hline \end{array} \quad =$$

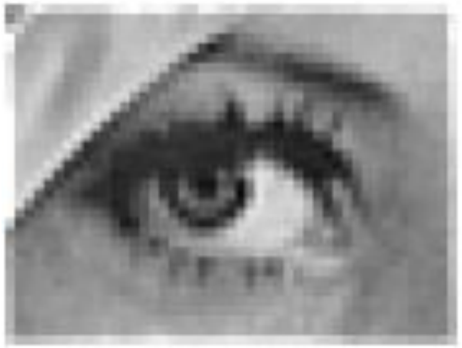

Blur (with a box filter)

Sharpening

- **Sharpening by filtering:** Accentuates differences with local average

Original

$$* \quad \frac{1}{9}\begin{bmatrix}-1 & -1 & -1\\ -1 & 17 & -1\\ -1 & -1 & -1\end{bmatrix} \quad =$$



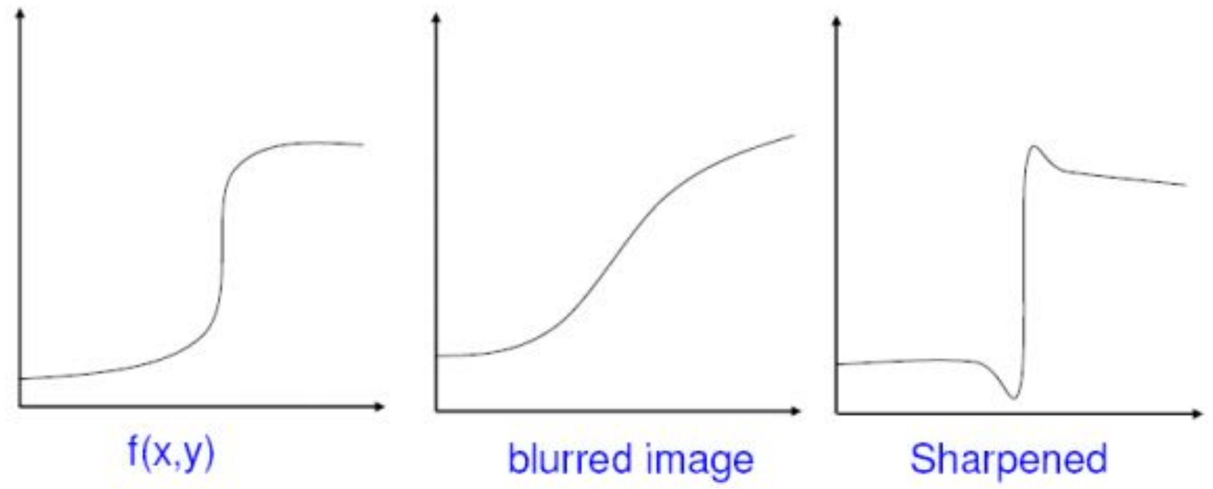
$$\Downarrow$$

$$\boxed{\begin{bmatrix}0 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}}$$

- This is also related to a process called **Unsharp masking**

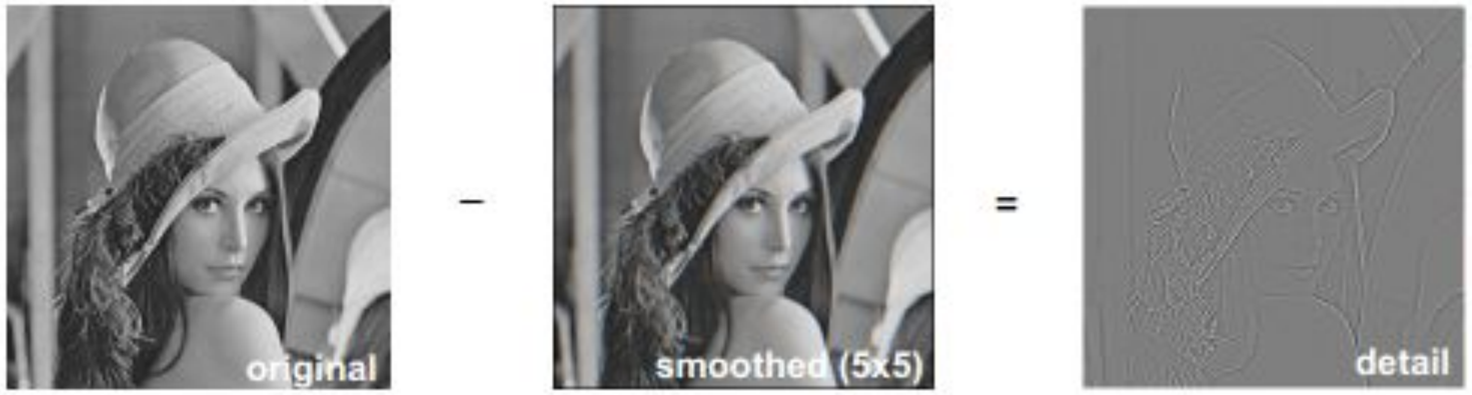
Unsharp masking

- A process used many years in the publishing industry to sharpen images
- **Details = Original image – Smoothened image**
- **Sharpened image = Original image + Details**

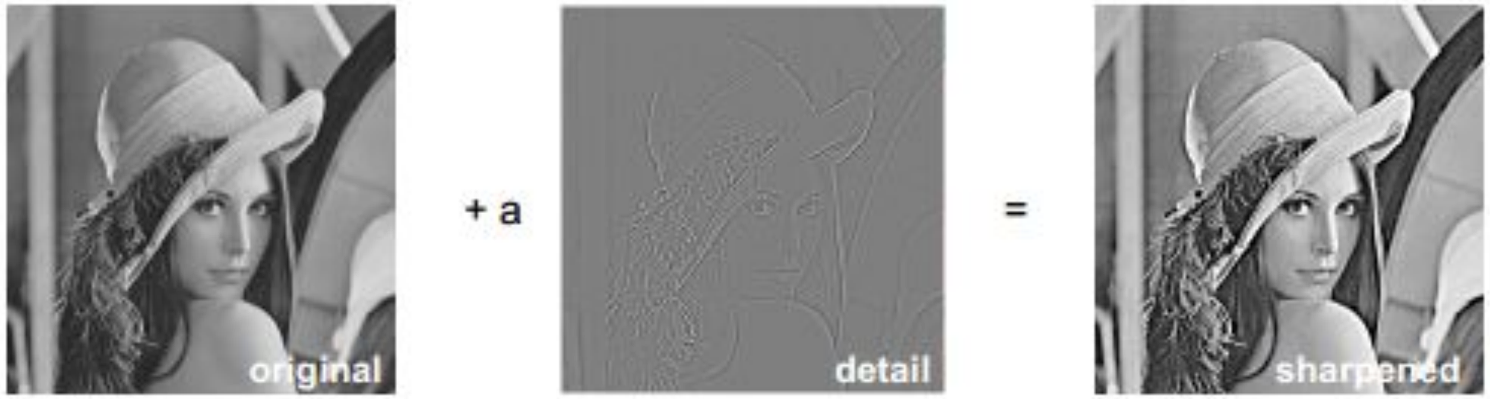


Getting a sharpened image

- What does blurring take away?



- Adding the details back…



Sharpening Filter

Original

$$\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = ?$$

(Note that filter sums to 1)

$$\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} + \overbrace{\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}}^{\text{“details of the image”}}$$

Amount of sharpening

- **Details = Original image – Smoothened image**
- **Sharpened image = Original image + Details**
  - How can we control the amount of sharpening that is applied?

The strength of the details/smoothing filter!

Non-linear Filters

Non-linear filtering

- So far, we look at **linear filtering**

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

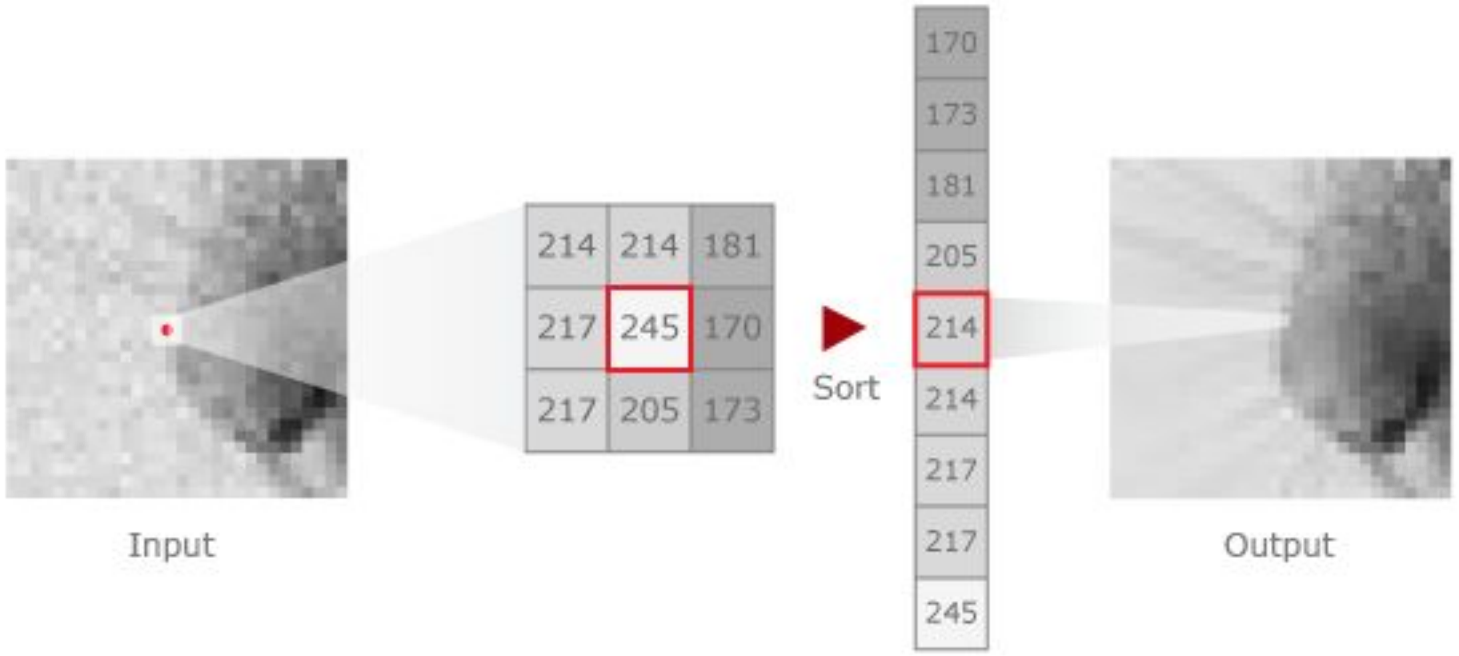
Linear combination of multiplied terms

$$G = H \star F$$

- What about **non-linear filtering?**
- Can the choice of filter H[u,v] produce non-linear filtering?

Median Filter

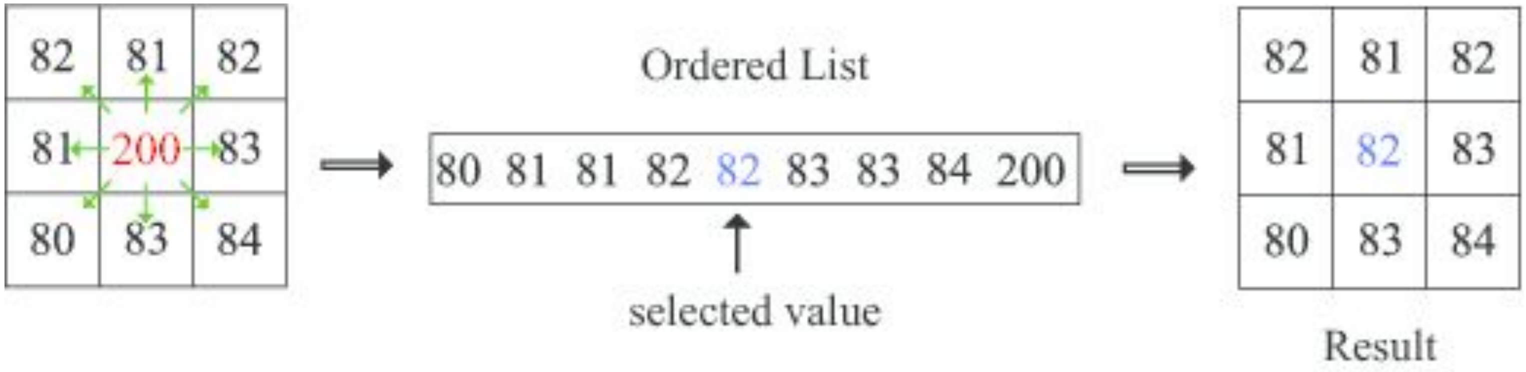
- A type of **order-statistics** filter – “statistical estimator”
- No new pixel values are introduced

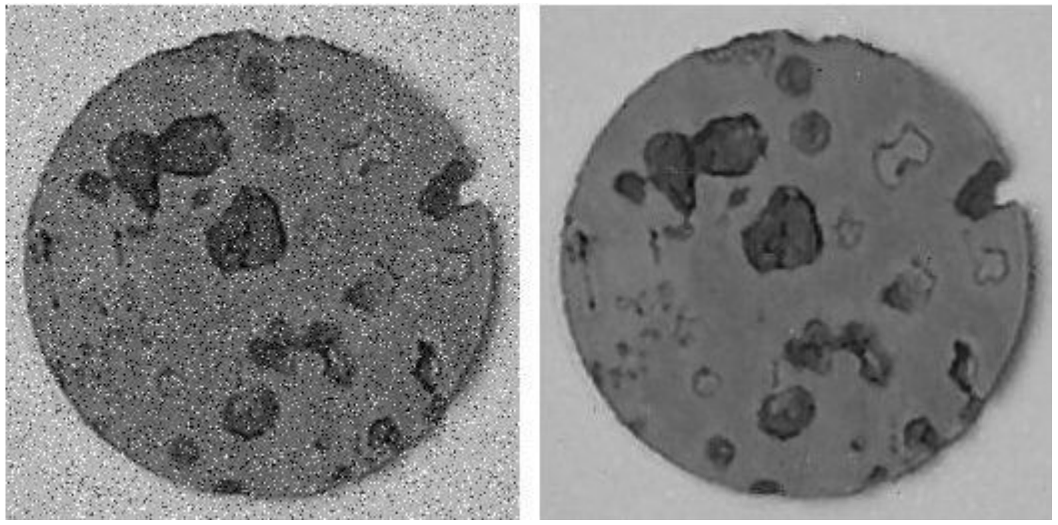


- Removes spikes: Good for impulse, salt & pepper noise

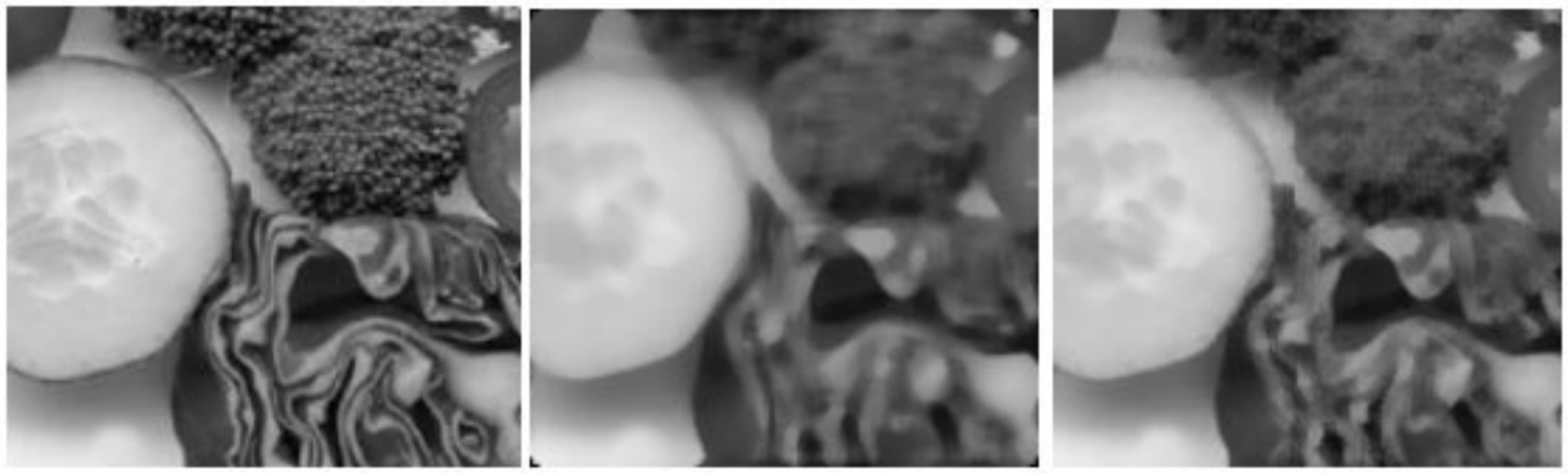
Median Filter

- Stray white pixels (very high values) or stray black pixels (very low values) can be dealt with





Median Filter



- Example:
  - Applying 7x7 median filter (middle pic) : broccoli loses details
  - Applying 7x7 multi-stage median filter : less smoothing occurs, some details are maintained

Median Filter

- **Multi-stage median filter**:
  - Median of a set of different median filters (obtained in different neighbourhoods)

$$\begin{aligned} y_{ij} &= med(z_1, z_2, z_3, z_4) \\ z_1 &= med(\{x_{uv} | x_{uv} \in N^1_{ij}\}) \\ z_2 &= med(\{x_{uv} | x_{uv} \in N^2_{ij}\}) \\ z_3 &= med(\{x_{uv} | x_{uv} \in N^3_{ij}\}) \\ z_4 &= med(\{x_{uv} | x_{uv} \in N^4_{ij}\}) \end{aligned}$$

  - Preserves sharp corners

Median Filter

- Drawback: Tend to be better at rejecting **outliers** but not so good at handling noise without outliers (e.g. Gaussian white noise)

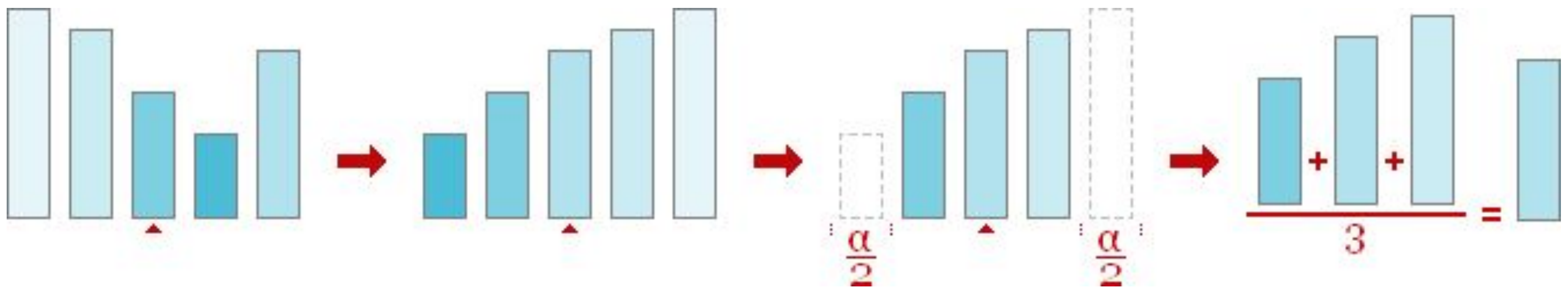
- Averaging filter / weighted average filter
  - too much blurriness, noise remains but smoothened

Alpha-trimmed mean filter

- Select the average of the values within a window which excludes a percent of the largest and smallest values in the neighbourhood
- Delete $\alpha/2$ lowest and $\alpha/2$ highest values in the neighbourhood S_{XY} , find average of remaining pixels:

$$\hat{f}(x,y) = \frac{1}{mn-\alpha} \sum_{(s,t)\in S_{XY}} g_r(s,t)$$

Alpha-trimmed mean filter



- How should *α* be chosen?

Summary

- **Linear filtering – Convolution**
  - Smoothing by filtering
  - Sharpening by filtering
  - Unsharp masking
- **Non-linear filtering**
  - Median filter
  - Alpha-trimmed mean filter

- Next: Edge detection

Recommended Reading

- [Gonzalez & Woods] Chapter 3
- [Forsyth & Ponce] Chapter 8 & 10