SM-4331 Advanced Statistics Class Test 2
2019/20 Semester 2
14 April 2020
Time allowed: 60 minutes

Instructions:

- There are three (2) questions totalling 20 marks and one (1) bonus question for 5 marks. The total attainable marks is 20 only.
- Answer **ALL** questions on a separate answer sheet.
- Ensure that you have written your name and student number on your answer sheets that you are submitting.
- The use of calculators is allowed.

Question:	1	2	3	Total
Marks:	10	10	5	20

1. (a) Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Suppose we wish to test the hypothesis that $H_0 : \mu = \mu_0$ against the alternative that $H_1 : \mu \neq \mu_0$.
   i. (3 marks) What test statistic would you suggest to use? State its distribution. *Hint: What is an estimator for μ?*
   ii. (2 marks) Based on your answer to (a), write down the p-value for this test, and explain how you would use it to test the stated hypothesis at the α significance.

   (b) (5 marks) You play a game with your friend which involves a coin toss. Each time the coin lands heads, your friend flicks your forehead. Each time the coin lands tails, you get to flick your friend's forehead in return. You play for 30 rounds, and you get your head flicked 22 times. Obviously, you are angry and reeling from pain, and you accuse your friend of providing a biased coin. Prove your claim statistically.

2. Let $Z_i \overset{\text{iid}}{\sim} \text{N}(0, 1)$ for $i = 1, \ldots, n$.

   (a) (2 marks) Explain how you would construct a random variable X distributed according to χ^2_n from $Z_1, \ldots, Z_n$.

   (b) (3 marks) Prove that $\text{E}(X) = n$ and $\text{Var}(X) = 2n$. *Hint: Use the following `raw moment' result for χ^2-distributions*

$$\text{E}(X^2) = \prod_{k=0}^{1} (n + 2k)$$

   (c) (3 marks) What is the distribution of X/n as $n \to \infty$?

   (d) (2 marks) Deduce the distribution of $F_{n,m}$ as $n, m \to \infty$.

Bonus Question

3. Consider the following function of two variables

$$f_{X,Y}(x,y) = \begin{cases} 4y^2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) (3 marks) Show that $f_{X,Y}$ is a density function

(b) (2 marks) Derive an expression for the marginal density $f_Y(y)$.

End of Paper