SM-1402 Basic Statistics Assignment 2 – Semester 2 2019/20

This assignment counts for 20% of your summative assessment. Attempt all questions and submit your solutions online via Canvas by 12:00 pm on **Tuesday, 21 April 2020**. The total marks attainable is 50. Late submissions will be penalised.

1. In a game of tennis, each point is won by one of the two players A and B. The usual rules of scoring apply. That is, the winner of the game is the player who first scores four points, unless each player has won three points, in which case deuce is called and play proceeds until one player is two points ahead of the other and hence wins the game.

   A is serving and has probability of winning any point of 2/3. The result of each point is assumed to be independent of every other point.

   (a) (4 marks) Show that the probability of A winning the game without deuce being called is 496/729.

   (b) (2 marks) Find the probability of deuce being called for the first time.

   (c) (2 marks) The player that wins a point when deuce is called is said to have the 'advantage'. If the player with the advantage wins the next point, they win the game; otherwise the game returns to deuce. What is the probability of returning to deuce for the first time?

   (d) (4 marks) If deuce is called, show that A's subsequent probability of winning the game is 4/5. *Hint: Use the following formula for an infinite geometric series* $\sum_{k=0}^{\infty} ar^k = a/(1-r)$ *when* $|r| < 1$.

   (e) (3 marks) Hence determine A's overall chance of winning the game.

2. Abu goes fishing every Sunday. The number of fish he catches follows a Poisson distribution. On a proportion π of the days he goes fishing, he does not catch anything. He makes it a rule to take home the first and then every other fish that he catches (i.e. the first, third, fifth, and so on).

   (a) (3 marks) Using a Poisson distribution, find the mean number of fish he catches.

   (b) (3 marks) Show that the probability that he takes home the last fish he catches is $(1-\pi^2)/2$. *Hint: Use the fact that* $\sum_{k=0}^{\infty} \frac{e^{-\lambda}\lambda^{2k+1}}{(2k+1)!} = (1-e^{-2\lambda})/2$.

3. A continuous random variable, X, has the following cdf:

$$F(x) = \begin{cases} 1 - e^{-x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) (1 mark) Calculate $\mathrm{P}(0.5 < X < 1)$.

   (b) (2 marks) Find x such that $\mathrm{P}(X > x) = 0.05$.

   (c) (2 marks) Determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

4. A random variable, X, has the following pdf:

$$f(x) = \begin{cases} 2x/3 & \text{for } 0 < x < 1 \\ (3-x)/3 & \text{for } 1 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) (2 marks) Derive the cdf of X.

(b) (3 marks) Find the mean and standard deviation of X.

5. The number of newspapers sold daily at a kiosk is normally distributed with a mean of 350 and a standard deviation of 30.

   (a) (3 marks) Find the probability that fewer newspapers are sold on Tuesday than on Monday.

   (b) (4 marks) Find the probability that fewer than 1,700 newspapers are sold in a (five-day) week. What assumption have you made in order to answer this?

   (c) (3 marks) How many newspapers should the newsagent stock each day such that the probability of running out on any particular day is 10%?

6. This question is about calculations based on random sample surveys of people.

   (a) (3 marks) It is believed that 40% of the 20 adults in a village are supporters of DPMM FC. If this belief is correct, and four different people are picked at random and asked whether they support DPMM FC, what is the probability that exactly three will be supporters?

   (b) (3 marks) It is believed that 40% of the many thousands of adults in Brunei-Muara district are supporters of DPMM FC. If this belief is correct, and 40 people are picked at random and asked about their allegiance, what is the probability that exactly twenty will be supporters?

   (c) (3 marks) It is believed that 40% of the many thousands of adults in Brunei-Muara are supporters of DPMM FC. If this belief is correct, and 100 people are picked at random and asked about their allegiance, what is the probability that at least thirty will be supporters?

   If you have used a suitable approximation in any of the previous parts, explain why it is appropriate in each case.