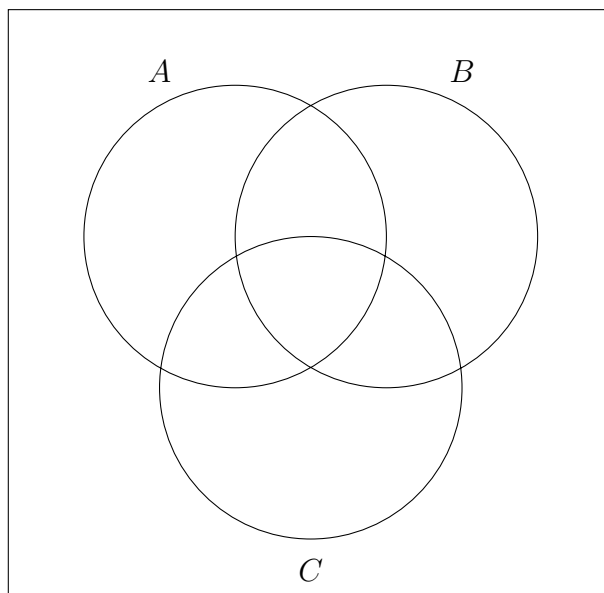


## SM-1402 Exercise 2

1. Three events are shown on the Venn diagram:



Reproduce the figure above and shade the region that corresponds to each of the following events:

- (a)  $A'$
  - (b)  $(A \cap B) \cap C'$
  - (c)  $(A \cap B) \cup C$
  - (d)  $(B \cup C)'$
  - (e)  $(A \cap B)' \cup C'$
2. The sample space of a random experiment is  $\mathcal{S} = \{A, B, C, D, E\}$ , with probabilities

$$\begin{aligned} P(\{A\}) = P(\{B\}) = 0.1, \quad P(\{D\}) = 0.4 \\ P(\{C\}) = P(\{E\}) \end{aligned}$$

Define the events  $\mathcal{A}$  and  $\mathcal{B}$  by  $\mathcal{A} = \{A, B, C\}$   $\mathcal{B} = \{C, D, E\}$ . Calculate the following probabilities:

- (a)  $P(\{C\})$
  - (b)  $P(\mathcal{A})$
  - (c)  $P(\mathcal{B})$
  - (d)  $P(\mathcal{A}')$
  - (e)  $P(\mathcal{A} \cap \mathcal{B})$
  - (f)  $P(\mathcal{A} \cup \mathcal{B})$
3. If  $A$ ,  $B$ , and  $C$  are mutually exclusive events with  $P(A) = 0.2$ ,  $P(B) = 0.3$ , and  $P(C) = 0.4$ , determine the following probabilities:

- (a)  $P(A \cup B \cup C)$   
 (b)  $P(A \cap B \cap C)$   
 (c)  $P(A \cap B)$   
 (d)  $P((A \cup B) \cap C)$   
 (e)  $P(A' \cap B' \cap C')$
4. Discs of polycarbonate plastic from a supplier are analysed for scratch and shock resistance. The results from 100 discs are summarised as follows:

		Shock resistance	
		High	Low
Scratch resistance	High	70	9
	Low	16	5

- Let  $A$  denote the event that a disc has high shock resistance, and let  $B$  denote the event that a disc has high scratch resistance. Determine the following probabilities:
- (a)  $P(A)$   
 (b)  $P(B)$   
 (c)  $P(A|B)$   
 (d)  $P(B|A)$
5. Samples of a cast aluminium part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarised as follows:

		Edge finish	
		Excellent	Good
Surface finish	Excellent	80	2
	Good	10	8

- What is the probability that a part selected at random
- (a) will have an excellent surface finish?  
 (b) will have an excellent edge finish?  
 (c) will not have an excellent surface finish?  
 (d) will have both excellent surface and edge finish?  
 (e) will have either an excellent surface or edge finish?  
 (f) will have not have an excellent surface finish, but an excellent edge finish?
6. You are accused of a crime you did not commit. To make matters worse, during your trial, it was found that you failed a polygraph test. The judge takes this as strong evidence against you is about to convict you for your alleged wrongdoings. Luckily, you remembered what you learnt in statistics class regarding conditional probabilities, and you implore the judge not to fall into the trap of the 'Prosecutor's fallacy'.
- Your argument is as follows:

- From data it is shown that 15 out of 100 people lie on polygraph tests.
- The accuracy of polygraph tests detecting lying people is 0.81.
- On the other hand, polygraph tests have a small chance of ‘false detection’, estimated to be 0.17.

You therefore conclude, using Bayes’ theorem, that the probability of you truly lying on the polygraph test is less than a half (worse than chance). Show your working to save yourself from lifetime in prison.

7. The probability that it is Saturday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Saturday is 0.2. What is the probability that a student is absent given that today is Saturday?
8. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?
9. Find the number of permutations of the letters of the word ‘BRUNEI’ such that the consonants always occur in odd places.