SM-4331 Exercise 5

1. Let a_i, b_j, c and d be any real numbers. Show that

$$\sum_{i=1}^{n}(a_i - c)(b_i - d) = \sum_{i=1}^{n}(a_i - \bar{a})(b_i - \bar{b}) + n(\bar{a} - c)(\bar{b} - d),$$

where $\bar{a} = n^{-1}\sum_{i=1}^{n} a_i$ and $\bar{b} = n^{-1}\sum_{i=1}^{n} b_i$.

Solution:

$$\begin{aligned}
\sum_{i=1}^{n}(a_i - c)(b_i - d) &= \sum_{i=1}^{n}(a_i - \bar{a} + \bar{a} - c)(b_i - \bar{b} + \bar{b} - d) \\
&= \sum_{i=1}^{n}\Big\{(a_i - \bar{a})(b_i - \bar{b}) + (a_i - \bar{a})(\bar{b} - d) + (b_i - \bar{b})(\bar{a} - c) \\
&\qquad\qquad + (\bar{a} - c)(\bar{b} - d)\Big\} \\
&= \sum_{i=1}^{n}(a_i - \bar{a})(b_i - \bar{b}) + n(\bar{a} - c)(\bar{b} - d),
\end{aligned}$$

since $\sum_{i=1}^{n}(a_i - \bar{a}) = \sum_{i=1}^{n} a_i - n\bar{a} = 0$ and $\sum_{i=1}^{n}(b_i - \bar{b}) = 0$.

2. For the simple linear regression $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $i = 1, \ldots, n$, the ordinary least squares estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are the solutions to

$$\underset{\beta_0,\beta_1}{\arg\min} \sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)^2.$$

Use calculus (i.e. differentiate the sum of squared errors with respect to β_0 and β_1) to derive the LSE solutions.

Solution: Let $L(\beta_0, \beta_1) = \sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)^2$. Then, differentiating wrt β_0, we obtain

$$\frac{\partial L}{\partial \beta_0} = -2\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i) \tag{1}$$

and differentiating wrt β_1 we obtain

$$\frac{\partial L}{\partial \beta_1} = -2\sum_{i=1}^{n} x_i(y_i - \beta_0 - \beta_1 x_i) \tag{2}$$

Setting (2) equal to zero yields

$$\begin{aligned}
\sum_{i=1}^{n} x_i(y_i - \beta_0 - \beta_1 x_i) = 0 \\
\Rightarrow \sum_{i=1}^{n} x_i y_i - n\bar{x}\beta_0 - \beta_1 \sum_{i=1}^{n} x_i^2 = 0 \\
\Rightarrow \hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\hat{\beta}_0}{\sum_{i=1}^{n} x_i^2} \qquad (3)
\end{aligned}$$

Setting (1) to zero yields

$$\begin{aligned}
\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0 \\
\Rightarrow \sum_{i=1}^{n} y_i - n\beta_0 - n\bar{x}\beta_1 = 0 \\
\Rightarrow n\hat{\beta}_0 = \sum_{i=1}^{n} y_i - n\bar{x}\hat{\beta}_1 \qquad (4) \\
\Rightarrow \hat{\beta}_0 = \bar{y} - \bar{x}\hat{\beta}_1
\end{aligned}$$

Putting (4) into (3) we get

$$\begin{aligned}
\hat{\beta}_1 &= \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}\sum_{i=1}^{n} y_i + n\bar{x}^2\hat{\beta}_1}{\sum_{i=1}^{n} x_i^2} \\
\Rightarrow \hat{\beta}_1 - \hat{\beta}_1 \frac{n\bar{x}^2}{\sum_{i=1}^{n} x_i^2} &= \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2} \\
\hat{\beta}_1 \left(\frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{\sum_{i=1}^{n} x_i^2} \right) &= \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2} \\
\Rightarrow \hat{\beta}_1 &= \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2} \\
&= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \qquad (5)
\end{aligned}$$

3. Let the observations $\{(y_i, x_i) | i = 1, \ldots, n\}$ be taken from the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. Suppose n is a large integer.
   (a) Construct a Wald test for $H_0 : \beta_1 = 2\beta_0$ against $H_1 : \beta_1 \neq 2\beta_0$.

Solution: The Wald test statistic to use is

$$Z = \frac{\hat{\beta}_1 - 2\hat{\beta}_0}{\mathrm{SE}(\hat{\beta}_1 - 2\hat{\beta}_0)}$$

which, under H_0 is distributed according to $\mathrm{N}(0,1)$. Here, $\hat{\beta}_0$ and $\hat{\beta}_1$ are the LSE. Now the variance of this estimator $\hat{\beta}_1 - 2\hat{\beta}_0$ is

$$\begin{aligned}\mathrm{Var}(\hat{\beta}_1 - 2\hat{\beta}_0) &= \mathrm{Var}(\hat{\beta}_1) + 4\,\mathrm{Var}(\hat{\beta}_0) - 4\,\mathrm{Cov}(\hat{\beta}_0, \hat{\beta}_1) \\ &= \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2} + \frac{4\sigma^2 \sum_{i=1}^n x_i^2}{n \sum_{i=1}^n (x_i - \bar{x})^2} + \frac{4\sigma^2 \bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2}\end{aligned}$$

Hence,

$$\begin{aligned}\mathrm{SE}(\hat{\beta}_1 - 2\hat{\beta}_0) &= \sqrt{\frac{\hat{\sigma}^2 + 4\bar{x}\hat{\sigma}^2 + 4\hat{\sigma}^2 \sum_{i=1}^n x_i^2/n}{\sum_{i=1}^n (x_i - \bar{x})^2}} \\ &= \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}} \sqrt{1 + 4\bar{x} + \frac{4}{n}\sum_{i=1}^n x_i^2}\end{aligned}$$

where $\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$.

(b) For a given x, construct a confidence interval for $\mu(x) := \mathrm{E}(y) = \beta_0 + \beta_1 x$.

Solution: Let $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$. Thus $\mathrm{E}(y) = \beta_0 + \beta_1 x =: \mu(x)$. For large n, we can construct an interval using the distribution

$$\frac{\hat{\beta}_0 + \hat{\beta}_1 x}{\mathrm{SE}(\hat{\beta}_0 + \hat{\beta}_1 x)} \sim \mathrm{N}(0,1).$$

The variance of the estimator $\hat{\beta}_0 + \hat{\beta}_1 x$ is

$$\begin{aligned}\mathrm{Var}(\hat{\beta}_0 + \hat{\beta}_1 x) &= \mathrm{Var}(\hat{\beta}_0) + x^2\,\mathrm{Var}(\hat{\beta}_1) + 2x\,\mathrm{Cov}(\hat{\beta}_0, \hat{\beta}_1) \\ &= \frac{\sigma^2 \sum_{i=1}^n x_i^2}{n\sum_{i=1}^n (x_i - \bar{x})^2} + \frac{x^2\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2} - \frac{2x\sigma^2\bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2} \\ &= \frac{\sigma^2 \sum_{i=1}^n x_i^2 + nx^2\sigma^2 - 2nx\sigma^2\bar{x}}{n\sum_{i=1}^n (x_i - \bar{x})^2} \\ &= \frac{\sigma^2 \sum_{i=1}^n (x_i - x)^2}{n\sum_{i=1}^n (x_i - \bar{x})^2}\end{aligned}$$

So the standard error is $\mathrm{SE}(\hat{\beta}_0 + \hat{\beta}_1 x) = \hat{\sigma}\sqrt{\frac{\sum_{i=1}^n (x_i - x)^2}{n\sum_{i=1}^n (x_i - \bar{x})^2}}$. Consequently, the approximate $(1-\alpha)$ interval for $\mu(x)$ is given by

$$\hat{\beta}_0 + \hat{\beta}_1 x \pm z(\alpha/2) \cdot \hat{\sigma}\sqrt{\frac{\sum_{i=1}^n (x_i - x)^2}{n\sum_{i=1}^n (x_i - \bar{x})^2}}$$

Remark: It can be proved that if ϵ_i are normal, we replace $z(\alpha/2)$ by $t_{n-2}(\alpha/2)$ and we then have the exact t-interval for $E(y) = \mu(x)$. Of course when n is large, $t_{n-2}(\alpha/2) \approx z(\alpha/2)$.

4. Consdider a linear model $y_i = \beta x_i + \epsilon_i$, where $\mathrm{E}(\epsilon_i) = 0$, $\mathrm{Var}(\epsilon_i) = \sigma^2 > 0$, $\mathrm{Cov}(\epsilon_i, \epsilon_j) = 0$ for $i \neq j$, and $x_1, \ldots, x_n$ are constants.

   (a) Find the LSE $\hat{\beta}$. Suggest an estimator for σ^2.

Solution: Similar solution to the lecture slides if using LSE estimates. However, let's try differentiation. $\hat{\beta}$ is the solution to

$$\arg\min_{\beta} \left\{ L(\beta) := \sum_{i=1}^{n} (y_i - \beta x_i)^2 \right\}.$$

Differentiating $L(\beta)$ we obtain

$$L'(\beta) = -\sum_{i=1}^{n} x_i (y_i - \beta x_i).$$

Setting this to zero yields

$$\sum_{i=1}^{n} x_i y_i - \beta \sum_{i=1}^{n} x_i^2 = 0$$
$$\Rightarrow \hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$$

We may estimate σ^2 by

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \hat{\beta} x_i)^2.$$

   (b) Show that the LSE $\hat{\beta}$ is unbiased, and find $\mathrm{SE}(\hat{\beta})$.

Solution: Note that since $y_i = \beta x_i + \epsilon_i$, then $\mathrm{E}(y_i) = \beta x_i$, while $\mathrm{Var}(y_i) = \sigma^2$

and each y_i are independent of each other. Thus,

$$\begin{aligned}
\mathrm{E}(\hat{\beta}) &= \mathrm{E}\left(\frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}\right) \\
&= \frac{\sum_{i=1}^n x_i \,\mathrm{E}(y_i)}{\sum_{i=1}^n x_i^2} \\
&= \frac{\sum_{i=1}^n x_i \cdot \beta x_i}{\sum_{i=1}^n x_i^2} \\
&= \frac{\beta \cancel{\sum_{i=1}^n x_i^2}}{\cancel{\sum_{i=1}^n x_i^2}} = \beta.
\end{aligned}$$

The variance of $\hat{\beta}$ is

$$\begin{aligned}
\mathrm{Var}(\hat{\beta}) &= \mathrm{Var}\left(\frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}\right) \\
&= \frac{\sum_{i=1}^n x_i^2 \,\mathrm{Var}(y_i)}{(\sum_{i=1}^n x_i^2)^2} \\
&= \frac{\sigma^2 \cancel{\sum_{i=1}^n x_i^2}}{(\sum_{i=1}^n x_i^2)^{\cancel{2}}}
\end{aligned}$$

Hence the standard error for $\hat{\beta}$ is $\mathrm{SE}(\hat{\beta}) = \hat{\sigma}/\sqrt{\sum_{i=1}^n x_i^2}$.

(c) If in addition $\epsilon_i \overset{\text{iid}}{\sim} \mathrm{N}(0, \sigma^2)$ find a confidence interval for β.

Solution: If $\epsilon_i \overset{\text{iid}}{\sim} \mathrm{N}(0, \sigma^2)$, then $y_i \sim \mathrm{N}(\beta x_i, \sigma^2)$ independently. Therefore, $\hat{\beta} = \sum_{i=1}^n x_i y_i / \sum_{i=1}^n x_i^2 \sim \mathrm{N}(\beta, \sigma^2/\sum_{i=1}^n x_i^2)$. We can use this fact to build an approximate $(1-\alpha)$ confidence interval for β as follows:

$$\hat{\beta} \pm z(\alpha/2) \cdot \hat{\sigma}/\sum_{i=1}^n x_i^2.$$

On the other hand, suppose we know that

$$(n-1)\hat{\sigma}^2/\sigma^2 \sim \chi^2_{n-1},$$

then

$$\frac{\hat{\beta} - \beta}{\mathrm{SE}(\hat{\beta})} \sim t_{n-1}.$$

An exact $(1-\alpha)$ confidence interval would be

$$\hat{\beta} \pm t_{n-1}(\alpha/2) \cdot \hat{\sigma}/\sum_{i=1}^n x_i^2.$$

(d) Based on the interval for β, find a confidence interval for $\mu(x) = \mathrm{E}(y)$, where $y = \beta x + \epsilon$.

Solution: Based on the confidence interval above, we have that

$$\mathrm{P}\left(\left|\frac{\hat{\beta}-\beta}{\mathrm{SE}(\hat{\beta})}\right| < t_{n-1}(\alpha/2)\right) = 1-\alpha$$

$$\Rightarrow \mathrm{P}\left(\hat{\beta} - t_{n-1}(\alpha/2)\cdot\mathrm{SE}(\hat{\beta}) < \beta < \hat{\beta} + t_{n-1}(\alpha/2)\cdot\mathrm{SE}(\hat{\beta})\right) = 1-\alpha$$

$$\Rightarrow \mathrm{P}\left(\hat{\beta}x - xt_{n-1}(\alpha/2)\cdot\mathrm{SE}(\hat{\beta}) < \overbrace{\beta x}^{\mu(x)} < \hat{\beta}x + xt_{n-1}(\alpha/2)\cdot\mathrm{SE}(\hat{\beta})\right) = 1-\alpha$$

which implies that a $(1-\alpha)$ confidence interval for $\mu(x)$ is

$$\hat{\beta}x \pm xt_{n-1}(\alpha/2)\cdot\mathrm{SE}(\hat{\beta}).$$

5. The table below lists the USA social security costs in 7 years between 1965 to 1992.

Year	1965	1970	1975	1980	1985	1990	1992
x = number of years from 1960	5	10	15	20	25	30	32
y = social security cost ($ billions)	17.1	29.6	63.6	117.1	186.4	246.5	285.1

(a) Plot the data y against x.

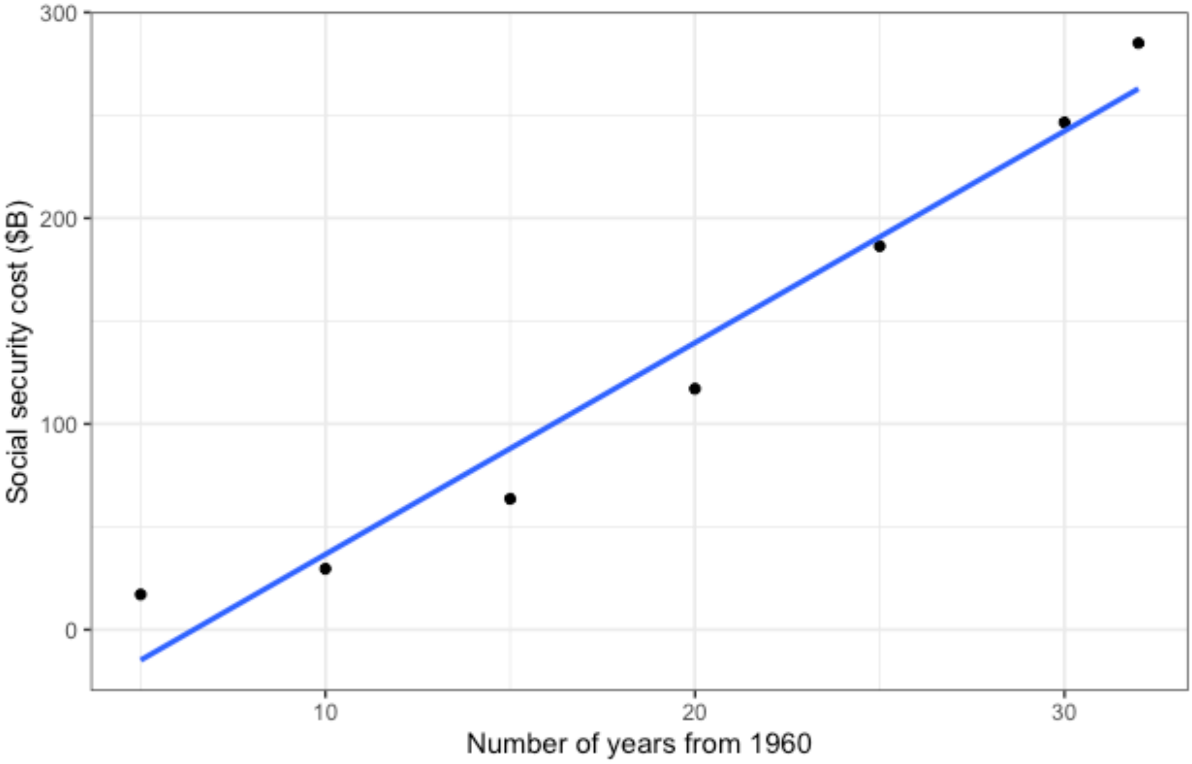
(b) Compute $\sum_i x_i$, $\sum_i y_i$, $\sum_i x_i^2$, $\sum_i y_i^2$, and $\sum_i x_i y_i$, and therefore fit the data to a simple linear regression model $y = \beta_0 + \beta_1 x + \epsilon$. Superimpose the fitted regression line onto the plot in (a).

Solution:

- $\sum_i x_i = 137$
- $\sum_i y_i = 945.4$
- $\sum_i x_i^2 = 3299$
- $\sum_i y_i^2 = 195715.2$
- $\sum_i x_i y_i = 24855.7$

Using the LSE formula, we have that the fitted line is $\hat{y} = -66.22 + 10.28x$.

(c) Test the hypothesis $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 > 0$. What can be concluded on the social security costs from the test?



Solution: We use the following distribution for $\hat{\beta}_1$:

$$\hat{\beta}_1 \sim \mathrm{N}\left(\beta_1, \sigma^2 / \sum_i (x_i - \bar{x})^2\right),$$

and with the additional calculation that

$$\hat{\sigma}^2 = \frac{1}{7-2} \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2 = 23.22$$

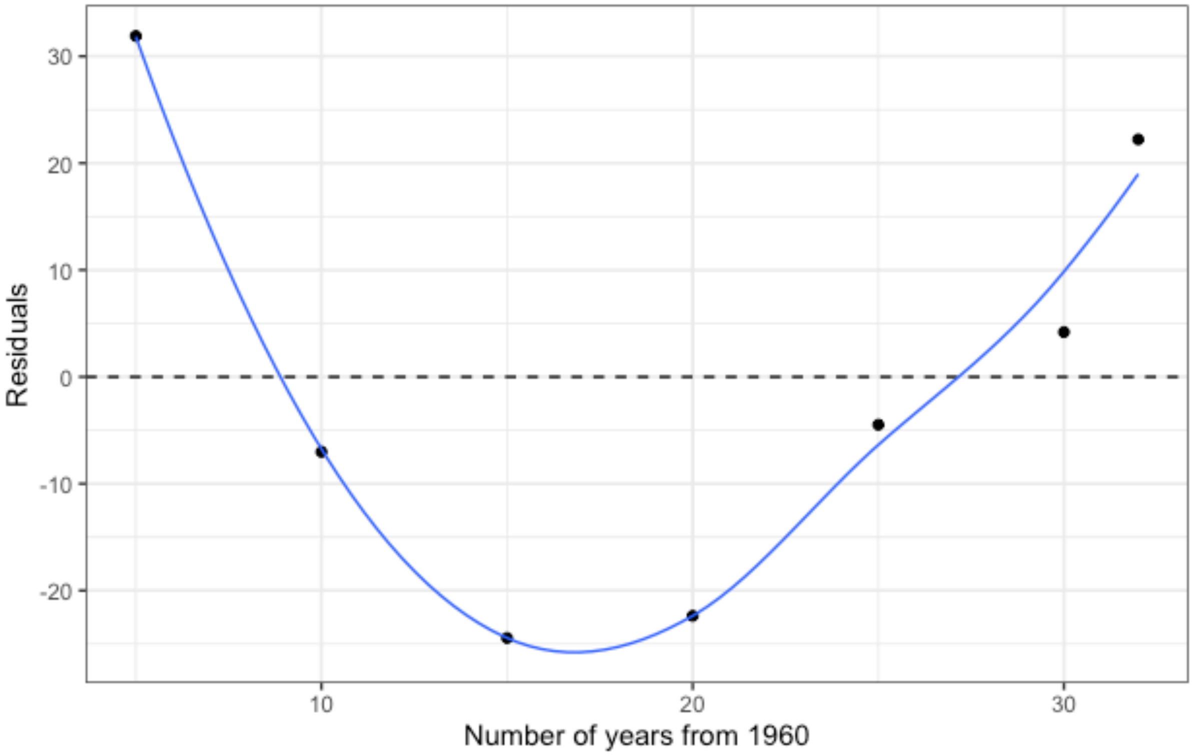
we get that $\mathrm{SE}(\hat{\beta}_1) = \hat{\sigma} \big/ \sqrt{\sum_i (x_i - \bar{x})^2 = 0.93}$. The test statistic for testing $H_0 : \beta_1 = 0$ is

$$T = \frac{\hat{\beta}_1}{\mathrm{SE}(\hat{\beta}_1)} = \frac{10.28}{0.93} = 11.05.$$

Under H_0, $T \sim t_5$, and the critical value for this t-distribution is $t_5(0.01) = 3.365$ (one-sided test at the $\alpha = 0.01$ level). Since the observed $T = 11.05 > t_5(0.01)$, we reject the null hypothesis. There is a strong evidence indicating that the social security cost increases over the years.

(d) Plot the residuals against x. Are you happy with the fitted model? If not, discuss what you may try to do to achieve a better fitting.

> **Solution:** The residual plot shows a clear non-random pattern, indicating inadequacy of the linear model. Looking at the original data plot in (a), we would think to apply a log-transformation on $z = \log(y)$ in order to accommodate the nonlinear relationship between x and y (looks like an exponential growth curve).



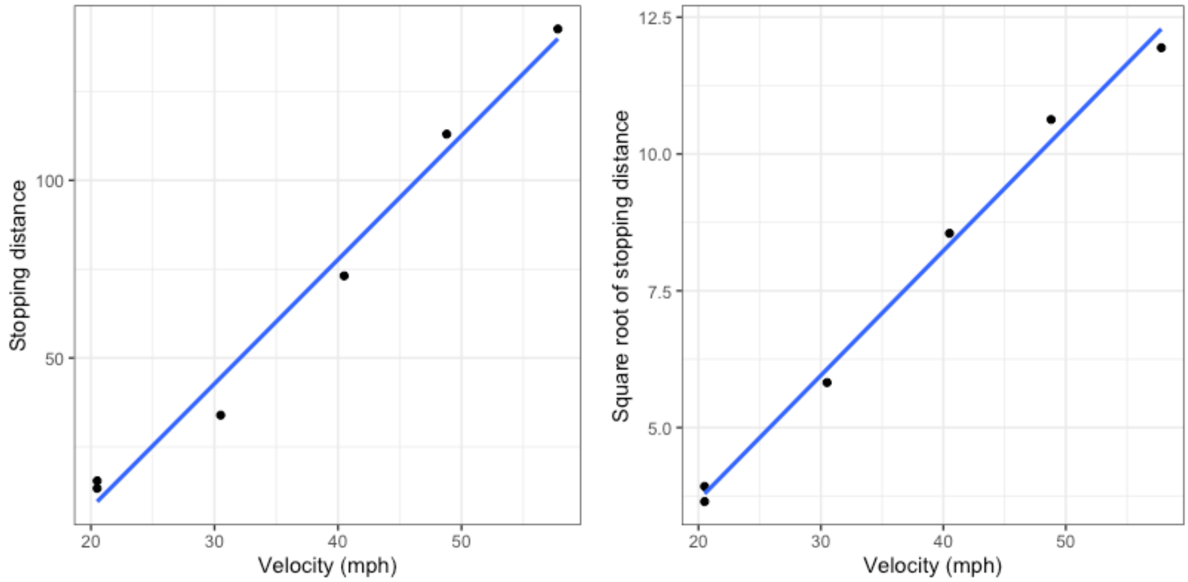
6. The stopping distance (y) of a car was studied in relation to the velocity (x) of the car. The table below lists the stop distances at 6 different velocities.

Velocity (mph)	20.5	20.5	30.5	40.5	48.8	57.8
Stopping distance (ft)	15.4	13.3	33.9	73.1	113.0	142.6

(a) Plot y against x, and $z := \sqrt{y}$ against x.

(b) Compute the sample correlation coefficients of y and x, and z and x.

> **Solution:** The formula for the sample correlation coefficient is
>
> $$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$
>
> We obtain $r(x, y) = 0.992$ and $r(x, z) = 0.997$.



(c) Fit the linear regression model for y on x, and examine the residuals.

> **Solution:** The fitted line is $\bar{y} = -62.045 + 3.493x$ with $\hat{\sigma} = 7.563$.

(d) Fit the linear regression model for z on x, and examine the residuals.

> **Solution:** The fitted line is $\bar{y} = -0.878 + 0.228x$ with $\hat{\sigma} = 0.322$.

(e) For a given x, a predictive interval for $y = \beta_0 + \beta_1 x + \epsilon$ with coverage probability $1 - \alpha$ is given by

$$\hat{\beta}_0 + \hat{\beta}_1 x \pm t_{n-2}(\alpha/2)\hat{\sigma}\sqrt{1 + \frac{\sum_{i=1}^n (x_i - x)^2}{n\sum_{j=1}^n (x_j - \bar{x})^2}}.$$

Based on this formula, compute the predictive intervals with coverage probability 0.95 for y and z when $x = 35$.

> **Solution:** These calculations will be helpful:
>
> - $\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2 = 1168.952.$
> - $\sum_{i=1}^n (x_i - x)^2 = \sum_{i=1}^n (x_i - \bar{x})^2 + n(x - \bar{x})^2 = 1181.273.$
> - With $n = 6$, $t_4(0.025) = 2.776$.
> - $t_4(0.025)\sqrt{1 + \frac{\sum_{i=1}^n (x_i - x)^2}{n\sum_{j=1}^n (x_j - \bar{x})^2}} = 3.001.$
>
> Hence,
>
> - the predictive interval for y is $60.21 \pm 7.563 \times 3.001 = (37.513, 82.907)$;

- the predictive interval for z is $7.102 \pm 7.563 \times 3.001 = (6.136, 8.068)$.

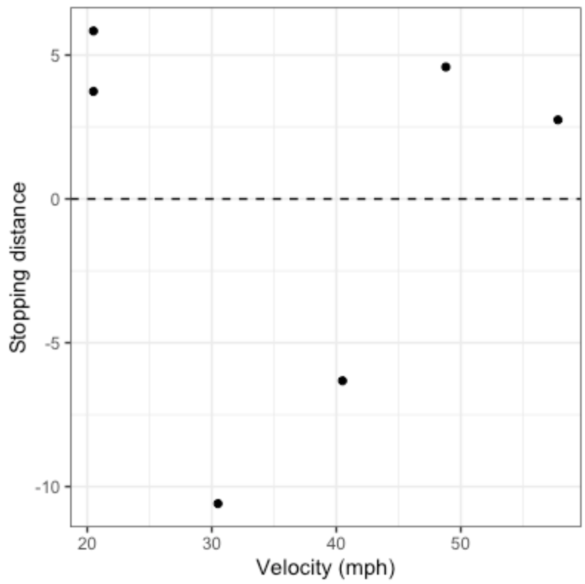
Note that from the interval for z, we get

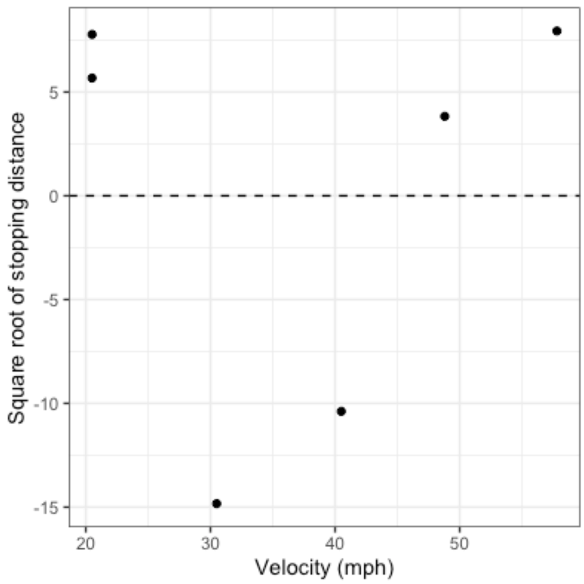
$$6.136 < \sqrt{y} < 8.068$$

which implies that $y \in (6.136^2, 8.068^2) = (37.651, 8.068)$. We notice that this interval is shorter than the interval derived directly from the linear modeal for y.

(f) Which model is better?

Solution: The figures indicate that both the models appear to fit the data well, note the sample size is merely 6. The residuals plots do not give any preference between the two models. However the sample correlation coefficient between z and x is greater than that between y and x, suggesting stronger linear relationship between z and x. The fitted model for z has much smaller errors, as reflected by the smaller value of σ. In turns out that this smaller σ leads to much more accurate predictive interval for the stopping distance. Hence it seems a better option to fit a linear regression model to $z = \sqrt{y}$ instead of y.





7. In a regression analysis, three possible models have been tried:
   - **Model 1**: Regress y on x_1.
   - **Model 2**: Regress y on x_2.
   - **Model 3**: Regress y on x_1 and x_2.

The numerical output of these models are shown below.

(a) Find the missing values **A1–A8**.

(b) What can be concluded from these three fitted regression models?

Model 1: $y = \beta_0 + \beta_1 x_1 + \epsilon$

	Estimate	SE	T	P($t > \|T\|$)
β_0	1.1398	0.1019	11.183	$< 2e^{-16}$
β_1	0.8604	0.1025	**A1**	$1.6e^{-12}$

$\hat{\sigma} = 0.905$ on 78 degrees of freedom.
$R^2 = 0.4746$, $\tilde{R}^2 =$ **A2**

Model 2: $y = \beta_0 + \beta_2 x_2 + \epsilon$

	Estimate	SE	T	P($t > \|T\|$)
β_0	1.04989	0.20152	5.210	$1.5e^{-6}$
β_2	-0.01336	**A3**	-0.092	**A4**

$\hat{\sigma} = 1.248$ on 78 degrees of freedom.

Model 3: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

	Estimate	SE	T	P($t > \|T\|$)
β_0	1.16464	0.14762	7.890	$1.66e^{-11}$
β_1	0.86067	0.10314	8.345	$2.20e^{-12}$
β_2	-0.02493	0.10635	-0.234	0.815

$\hat{\sigma} =$ **A5** on **A6** degrees of freedom.
$R^2 =$ **A7**

ANOVA for Model 3: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

Source	d.f.	SS	Mean SS	T	P($F > T$)
$\sum_{i=1}^{79}(\hat{\beta}_0 + \hat{\beta}_1 x_{i1} - \bar{y})^2$	1	57.695	57.695	**A8**	$2.225e^{-12}$
$\sum_{i=1}^{79}(\hat{\beta}_0 + \hat{\beta}_2 x_{i2} - \bar{y})^2$	1	0.046	0.046	0.055	0.8153
$\sum_{i=1}^{79}(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \hat{\beta}_2 x_{i2})^2$	77	63.833	0.829		

Solution:

- A1 is the t-statistic for testing $H_0 : \beta_1 = 0$ v.s. $H_1 : \beta_1 \neq 0$. Thus,

$$\text{A1} = \frac{\hat{\beta}_1}{\text{SE}(\hat{\beta}_1)} = \frac{0.8604}{0.1025} = 8.3941.$$

- A2 is the adjusted R^2 value, given by

$$A2 = \tilde{R}^2 = 1 - \frac{\text{Resid SS}/(n-2)}{\text{Total SS}/(n-1)} = 1 - (1 - R^2)\frac{n-1}{n-2} = 0.4678.$$

- A3 is

$$A3 = \text{SE}(\hat{\beta}_2) = \hat{\beta}_2 / T = -0.01336 / -0.092 = 0.1452.$$

- A4 is the p-value $\text{P}(|t_{78}| > 0.092) = 0.9269$.

- A5 is $\hat{\sigma}$ which is the square-root of the mean Total SS in the ANOVA table, $\hat{\sigma} = \sqrt{0.829} = 0.9105$.

- A6 is the degrees of freedom for the Total SS, which is 77.

- A7 is R^2 value given by

$$A7 = 1 - \frac{\text{Resid SS}}{\text{Total SS}} = \frac{63.833}{57.695 + 0.046 + 63.833} = 0.4749$$

- A8 is the F-statistic

$$A8 = \frac{\text{Resid due to } x_1/1}{\text{Resid SS}/(n-p)} = \frac{57.695}{0.829} = 69.596$$