SM-1402 Exercise 3

1. A random variable is defined as follows:

x	-2	-1	0	1	2
$f(x)$	1/8	2/8	2/8	2/8	1/8

   Determine the following probabilities

   (a) $\mathrm{P}(X \leq 2)$

   (b) $\mathrm{P}(X > -2)$

   (c) $\mathrm{P}(|X| \leq 1)$

   (d) $\mathrm{P}(\{X \leq -1\} \cup \{X = 2\})$

2. A random variable X has the following probability mass function:

$$f(x) = \frac{2x+1}{25}, \quad x \in \{0, 1, 2, 3, 4\}.$$

   Find

   (a) $\mathrm{P}(X = 4)$

   (b) $\mathrm{P}(X \leq 1)$

   (c) $\mathrm{P}(2 \leq X < 4)$

   (d) $\mathrm{P}(X > -10)$

3. In a semiconductor manufacturing process, three wafers from a lot are tested. Each wafer is classified as pass or fail. Assume that the probability that a wafer passes the test is 0.8 and that wafers are independent. Determine the mass function of the number of wafers from a lot that pass test.

4. Suppose that a days production of 850 manufactured parts contains 50 parts that do not conform to customer requirements. Two parts are selected at random, without replacement, from the batch. Let the random variable X equal the number of nonconforming parts in the sample. Find the cumulative distribution function of X.

5. Given that a discrete r.v. X has cdf

$$F(x) = \begin{cases} 0 & x < -10 \\ 0.25 & -10 \leq x < 30 \\ 0.75 & 30 \leq x < 50 \\ 1 & 50 \leq x \end{cases}$$

   Find

   (a) $\mathrm{P}(X \leq 50)$

   (b) $\mathrm{P}(X \leq 40)$

(c) $P(40 \le X \le 60)$

(d) $P(X < 0)$

(e) $P(0 \le X < 10)$

(f) $P(-10 \le X < 10)$

6. A r.v. X has expectation $E(X) = 3$ and variance $\mathrm{Var}(X) = 2.5$. Let $Y = 2X$, $Z = 4X + 2$ and $W = 3X - 1$. Find the means and variances of the three r.v. Y, Z and W.

7. Calculate the expectation and variance of X, when X has the following distribution:

x	0	1	2	3
$P(X = x)$	0.1	0.2	0.5	0.2

8. Let X be a binomial random variable with $p = 0.1$ and $n = 10$. Calculate the following probabilities.

   (a) $P(X \le 2)$

   (b) $P(X > 8)$

   (c) $P(X = 4)$

   (d) $P(5 \le X \le 7)$

9. The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.

   (a) What is the probability that for exactly three calls the lines are occupied?

   (b) What is the probability that for at least one call the lines are not occupied?

   (c) What is the expected number of calls in which the lines are all occupied?

10. Because not all airline passengers show up for their reserved seat, an airline sells 125 tickets for a flight that holds only 120 passengers. The probability that a passenger does not show up is 0.10, and the passengers behave independently.

    (a) What is the probability that every passenger who shows up can take the flight?

    (b) What is the probability that the flight departs with empty seats?

11. Suppose that the number of flaws in a thin copper wire follows a Poisson distribution with a mean of 2.3 flaws per millimeters.

    (a) Determine the probability of exactly 2 flaws in 1 millimeter of wire.

    (b) Determine the probability of 10 flaws in 5 millimeters of wire.

    (c) Determine the probability of at least 1 flaw in 2 millimeters of wire.

12. Astronomers treat the number of stars in a given volume of space as a Poisson random variable. The density in the Milky Way Galaxy in the vicinity of our solar system is one star per 16 cubic light years.

(a) What is the probability of two or more stars in 16 cubic light years?

(b) How many cubic light years of space must be studied so that the probability of one or more stars exceeds 0.95?

13. Traffic flow is traditionally modelled as a Poisson distribution. A traffic engineer monitors the traffic flowing through an intersection with an average of 6 cars per minute.

    (a) What is the probability of no cars through the intersection within 30 seconds?

    (b) What is the probability of three or more cars through the intersection within 30 seconds?

    (c) Calculate the minimum number of cars through the intersection so that the probability of this number or fewer cars in 30 seconds is at least 90%.

    (d) If the variance of the number of cars through the intersection per minute is 20, is the Poisson distribution appropriate?