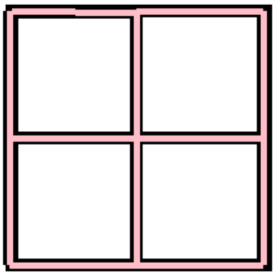
SM4202 Exercise 2

1. Recall the *Gambler's ruin* example from the lectures. Suppose you play for the first four tosses only.
   (a) What is the probability that you win at least three times?
   (b) What are your expected winnings? *Hint: Count losses as negative winnings.*
   (c) Suppose you play for the first four tosses only, but in addition withdraw after your first loss. What are your expected winnings from this strategy?
   (d) Suppose you play for the first four tosses only, but in addition withdraw after your first win. What are your expected winnings from this strategy?

2. Consider a square 3×3 lattice of pipes. Suppose each of the 12 edges independently is open with probability p and closed with probability $1-p$. Compute the probability that there is no open path from the centre to the boundary.



3. (a) Consider a model which puts four points down in a unit disk, independently and uniformly at random. What is the probability that in the centred disk of radius $1/2$ there are no points?
   (b) How does the answer change if now the model is that a Poisson (mean 1) number of points are put down in a unit disk, independently and uniformly at random?

4. Recall the *Brownian motion* example from lectures. Model the motion X as follows: for $t \geq 0$, $s \geq 0$, we suppose $X_{t+s} - X_t$ has a normal distribution of mean zero and variance s, independent of behaviour of X previous to time t.
   (a) If $X_0 = 1$ then what is the chance that X_1 is positive?
   (b) If $X_0 = 1$ then what is the chance that both $X_{1/2}$ and X_1 are positive?

5. Recall the *Changing words* example from lectures.
   (a) What is the state-space if the words have just one letter (and no emojis/'txting' are allowed!)?
   (b) Compute the probability of moving from one state to a different state in just one step, if the move is at all possible.
   (c) Exhibit a sequence of valid moves from `fail` to `pass`.

6. Consider two tireless and reasonably equally matched tennis players, A and B. Sketch a diagrammatic description of the state-space for a random game played between A and B.