

SM-4335 Advanced Probability Class Test 1

2021/22 Semester I

17 September 2022

Time allowed: 60 minutes

Instructions:

- There are three (3) questions totalling 30 marks.
- Answer **ALL** questions on a separate answer sheet.
- Ensure that you have written your name and student number on your answer sheets that you are submitting.
- The use of calculators is allowed.

Question:	1	2	3	Total
Marks:	5	5	5	15

1. (5 marks) Mark each of the following statements as either TRUE or FALSE. You do not need to provide a reason for your answer.

- (a) Two sets A and B have the same cardinality if there exists a bijection (a.k.a. one-to-one correspondence) between them.

☐ TRUE ☐ FALSE

- (b) Let A be any set. The empty set is always a subset of A .

☐ TRUE ☐ FALSE

- (c) Let A be any set. Then $|A| < |\mathcal{P}(A)|$.

☐ TRUE ☐ FALSE

- (d) Every algebra is also a σ -algebra.

☐ TRUE ☐ FALSE

- (e) Let A_1, A_2, \dots be sets. Suppose that the limit infimum of these sets exists. Then $x \in \liminf_{n \rightarrow \infty} A_n$ implies that x belongs to infinitely many A_k .

☐ TRUE ☐ FALSE

2. (a) (3 marks) Let \mathcal{F} be a collection of subsets of a set Ω . Provide the three necessary conditions for \mathcal{F} to be an *algebra*.

- (b) (2 marks) Let $\Omega = \{1, 2, 3\}$. Construct the *smallest algebra* from the collection of subsets of Ω , $\mathcal{C} = \{\{1\}, \{2\}\}$.

3. (5 marks) For $n \in \mathbb{N}$, let

$$A_n = \left\{ 1, \frac{n+1}{n}, \frac{n+2}{n}, \dots, \frac{2n-1}{n}, 2 \right\}.$$

- (a) Find the limit inferior and limit superior of the set A_n .

- (b) Does the sequence A_n have a limit? Why, or why not?