SM-2401 Geometry Class Test 1
2020/21 Semester 1
17 September 2020
Time allowed: 60 minutes

Instructions:

- This is an **open-book**, **take-home** test. You are allowed 60 minutes to answer the questions, and another 30 minutes to upload your solutions to Canvas. Late solutions will be penalised.
- There are three (3) questions totalling 30 marks and one (1) bonus question for 1 mark. The total attainable marks is 30 only.
- Answer **ALL** questions on a separate answer sheet.
- Ensure that you have written your name and student number on your answer sheets that you are submitting.
- The use of calculators is allowed.

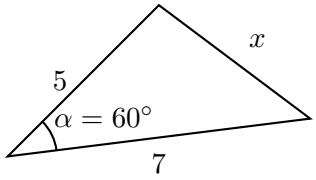
Question:	1	2	3	4	Total
Marks:	10	10	10	1	30

1. (10 marks) Mark each of the following statements as either TRUE or FALSE.
   (a) A line in plane geometry has width but no length.
   ☐ TRUE √ **FALSE**
   (b) Two angles whose measures add together to give 180 are said to be complementary angles.
   ☐ TRUE √ **FALSE**
   (c) Two or more angles are said to be congruent if they have the same angle measure.
   √ **TRUE** ☐ FALSE
   (d) A triangle is scalene if all three of its sides have different lengths.
   √ **TRUE** ☐ FALSE
   (e) If all the medians of a triangle are also altitudes, the triangle is equilateral.
   √ **TRUE** ☐ FALSE
   (f) An obtuse triangle can never be isosceles.
   ☐ TRUE √ **FALSE**
   (g) A triangle $\triangle ABC$ has side lengths $|\overline{AB}| = 2$, $|\overline{BC}| = 2$, and $|\overline{AC}| = 3$. The triangle is
      i. isoceles √ **TRUE** ☐ FALSE
      ii. a right triangle ☐ TRUE √ **FALSE**
   (h) If a sector of a circle has arc length 4 and central angle 60°, then the area of the circle is $144/\pi$.
   √ **TRUE** ☐ FALSE
   (i) If a circle is inscribed inside a triangle $\triangle ABC$, and a second circle is circumscribed around $\triangle ABC$, then the two circles are always concentric.
   ☐ TRUE √ **FALSE**

2. (a) (2 marks) The angles of a triangle have measures $2x + 10$, $3x - 15$, and $4x - 40$ for some number x. Show that the triangle is equilateral. *Hint: All angles in an equilateral triangle are congruent to each other.*

Solution: The sum of the angles is 180, so $(2x+10)+(3x-15)+(4x-40) = 180$, yielding $x = 25$. All 3 angles of the triangle can be shown to have measure 60, thus making it an equilateral triangle.

(b) (2 marks) Use the Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos\alpha$ to calculate the value of x for the following triangle:



Solution:

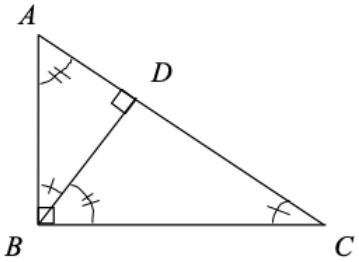
$$\begin{aligned} x^2 &= 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cos 60 \\ &= 25 + 49 - 70/2 = 39 \end{aligned}$$

and therefore $x = \sqrt{39} = 6.24$.

(c) (2 marks) Show that a triangle is with the side lengths side lengths 5, 12 and 13 is a right triangle.

Solution: Since $5^2 + 12^2 = 13^2$ the side of length 13 is the hypothenuse of a right triangle according to Pythagoras' Theorem.

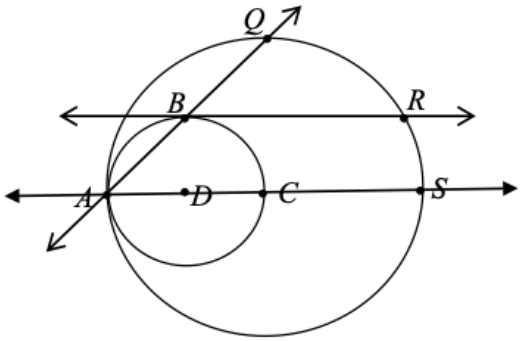
(d) In the diagram below, the angle $\angle ABC$ and $\angle ADB$ are right angles.



i. (2 marks) If $m\angle DBC = 40$, what is $m\angle BAD$? **Ans:** 40.

ii. (2 marks) If $|\overline{AB}| = 3$ and $|\overline{AC}| = 5$, what is $|\overline{BC}|$? **Ans:** 4.

3. (a) In the diagram below, the point C is the centre of the large circle and $\overline{AS}$ is a diameter of this circle. The centre D of the small circle lies on $\overleftrightarrow{AS}$. It passes through A and C and its radius is 1. The line $\overleftrightarrow{BR}$ is parallel to $\overleftrightarrow{AS}$ and a tangent to the small circle. The point of tangency is B.



i. (2 marks) What is $m\angle BAD$?

Solution:
$\triangle BAD$ is a right triangle with $|\overline{AD}| = |\overline{BD}|$, and therefore $m\angle = 45$.

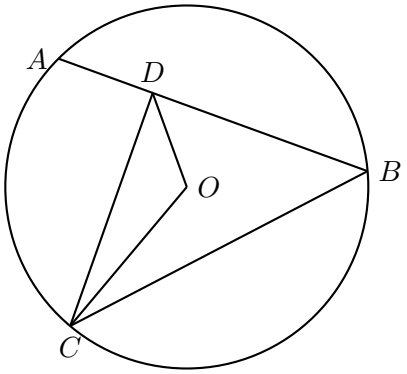
ii. (2 marks) What is the length of the line segment $\overline{AQ}$?

Solution: The triangle $\triangle ACQ$ is a right triangle, whose base is of length 2 and $\angle CAQ = 45$. Thus, $\cos 45 = 2/|\overline{AQ}|$ which implies $|\overline{AQ}| = 2\sqrt{2}$.

iii. (3 marks) What is the measure of the arc $\widehat{QR}$?

Solution: The measure of $\widehat{QS}$ is $m\widehat{QS} = 2\angle BAD = 2 \times 45 = 90$.
Next, construct a perpendicular line segment running from point R to a point on the line $\overleftrightarrow{AS}$ and call this P, say. The the triangle formed by joining points C, R and P form a right triangle, whose hypotenuse $\overline{CR}$ has length 2 (radius of $\odot C$), height $\overline{RP}$ of length 1 (radius of $\odot D$). Thus, the angle $\angle RCS =: \theta$ is obtained using the definition of sine, i.e. $\sin\theta = 1/2$ which implies $m\angle RCS = 30$.
Note that the arc $\widehat{RS}$ has measure equal to the measure of $\angle RCS$, i.e. 30.
Finally, $m\widehat{QR} = m\widehat{QS} - m\widehat{RS} = 90 - 30 = 60$.

(b) (3 marks) Consider the sketch below.



The points A, B and C lie on the circle with centre O. D is a point that lies on the chord $\overline{AB}$. Given that $m\angle ABC = 30$, and that $m\angle BCO = m\angle OCD = 20$, what is the measure of $\angle ODC$?

Solution: Let r be the radius of the cricle $\odot O$. Note that $\triangle OCB$ is isoceles, so $m\angle OCB = 140$, and using the sine rule, we obtain

$$\frac{OB}{\sin 20} = \frac{BC}{\sin 140} \Rightarrow |\overline{BC}| = \frac{r \sin 140}{\sin 20}.$$

Whereas from $\triangle BCD$, we have that $m\angle CDB = 180 - 30 - 2 \cdot 20 = 110$, and using the sine rule we get

$$\begin{aligned} \frac{CD}{\sin 30} = \frac{BC}{\sin 110} \Rightarrow |\overline{CD}| &= \frac{\sin 30}{\sin 110} \times |\overline{BC}| \\ &= \frac{\sin 30}{\sin 110} \times \frac{r \sin 140}{\sin 20} \\ &= r \frac{\sin(2 \cdot 20)}{2 \sin 70 \sin 20} \\ &= r \frac{\cos 20}{\sin 70} = r. \end{aligned}$$

Finally, since $|\overline{OC}| = |\overline{CD}| = r$, we have that $\triangle OCD$ is isoceles, $\angle ODC = (180 - 20)/2 = 80$.

Bonus Question

4. (1 mark) Find the statement which contradicts the following statement:

"If I work hard, then I will become rich"

A. If I work hard, then I will not become rich.

B. If I do not work hard, then I will become rich.

C. I work hard and I do not became rich.

D. I do not work hard and I become rich.

E. I do not work hard and I do not become rich.

F. I do not work hard or I become rich.

G. I work hard or I do not become rich.

H. I do not work hard or I do not become rich.

End of Paper