Binary probit regression with I-priors

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http://phd3.haziqj.ml

Outline

Priors on regression functions

- For $i = 1, \dots, n$, consider the regression model

$$y_i = \alpha + f(x_i) + \epsilon_i$$
$$(\epsilon_1, \dots, \epsilon_n) \sim \mathrm{N}(0, \psi^{-1}\mathbf{I}_n)$$

where $f \in \mathcal{F}$, $y_i \in \mathbb{R}$, and $x_i = (x_{i1}, \dots, x_{ip}) \in \mathcal{X}$.

The I-prior

- Let $\mathcal{F}$ be a reproducing kernel Hilbert space (RKHS) with reproducing kernel $h_\lambda : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$. An I-prior on f is

$$\big(f(x_1), \dots, f(x_n)\big)^\top \sim \mathrm{N}\left(\mathbf{f}_0, \mathcal{I}(f)\right)$$

with $\mathbf{f}_0$ a prior mean, and $\mathcal{I}$ the Fisher information for f, given by

$$\mathcal{I}\big(f(x_i), f(x_j)\big) = \sum\sum_{x,x' \in \mathcal{X}\times\mathcal{X}} \psi h_\lambda(x_i, x) h_\lambda(x_j, x').$$

- The I-prior regression model for $i = 1, \dots, n$ becomes

$$y_i = \alpha + \sum_{k=1}^n h_\lambda(x_i, x_k) w_k + \epsilon_i$$
$$(w_1, \dots, w_n) \sim \mathrm{N}(0, \psi \mathbf{I}_n)$$
$$(\epsilon_1, \dots, \epsilon_n) \sim \mathrm{N}(0, \psi^{-1} \mathbf{I}_n)$$

W. Bergsma (2017). "Regression with I-priors". *Manuscript in preparation*

RKHS sample paths

Relation to Gaussian process regression

Roadmap

End

Thank you!