SM-1402: Exercise 3 (The Normal Distribution)

1. For the following normal probabilities, sketch a shaded region under the bell curve that corresponds to the required probabilities. Then "prove'' each statement below, where $a, b > 0$ and $b > a$.
   (a) $\Pr(Z \leq -a) = \Phi(-a) = 1 - \Phi(a)$
   (b) $\Pr(a \leq Z \leq b) = \Phi(b) - \Phi(a)$
   (c) $\Pr(-a \leq Z \leq b) = \Phi(a) + \Phi(b) - 1$
   (d) $\Pr(-b \leq Z \leq -a) = \Phi(a) - \Phi(b)$
   (e) $\Pr(|Z| \leq a) = \Pr(-a \leq Z \leq a) = 2\Phi(a) - 1$
   (f) $\Pr(|Z| \geq a) = \Pr(\{Z < -a\} \cup \{Z > a\}) = 2\Big(1 - \Phi(a)\Big)$

2. Assume X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the following:
   (a) $\Pr(X < 10)$.
   (b) $\Pr(X > 9)$
   (c) $\Pr(6 < X < 14)$
   (d) $\Pr(X < 2)$
   (e) $\Pr(-2 < X < 8)$

3. Let $Z \sim \mathrm{N}(0, 1)$. Find the value of z such that
   (a) $\Pr(Z < z) = 0.5$
   (b) $\Pr(|Z| < z) = 0.90$
   (c) $\Pr(|Z| < z) = 0.95$
   (d) $\Pr(|Z| < z) = 0.99$

4. Suppose the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 squared milliamperes.
   (a) What is the probability that a measurement will exceed 13 milliamperes?
   (b) What is the probability that a current measurement is between 9 and 11 milliamperes?

5. Recall, from Exercise 1 (and 2), the heights of 20 students in centimetres.

147	151	153	158	158	162	165	165	166	168
168	168	171	173	173	177	178	179	184	189

   Using the methods discussed in the lectures, determine whether or not the data is normally distributed.

6. A random sample of 10 measurements are given as follows:

64.1 64.7 64.5 64.6 64.5 64.3 64.6 64.8 64.2 64.3

Assume that the population is normally distributed with $\sigma = 1$, find a 95% confidence interval for the mean.

7. A factory has 1000 workers. Let X_i be the monthly salary of worker i in Brunei dollars. A simple random sample of 100 of these workers yielded the following statistics

$$\sum_i X_i = 99\,969 \qquad \text{and} \qquad \sum_i X_i^2 = 99\,980\,896.$$

   (a) Calculate a 90% confidence interval for the mean weekly salary of all workers in the factory. State any assumptions you make in your calculation.

   (b) How many more workers should be sampled if it is required that the estimate is to be within B$2.50 of the true average (again, with 90% confidence)? Note this means a tolerance of B$2.50–equivalent to a confidence interval width of B$5.00.