SM-1402: Exercise 5 (Linear relationships)

1. Each year, all American Bar Association (ABA) approved law schools within the United States must make disclosures to the ABA, which includes the GPA and LSAT score of incoming students. The Grade Point Average (GPA) is a number on scale from 0.0 to 4.0 that students earn when studying at undergraduate level, while the Law School Admission Test (LSAT) the test law school applicants take prior to graduate-level law school.

   A sample of 15 GPA & LSAT scores were obtained. Calculate the correlation between GPA and LSAT scores. What can you conclude from this correlation number?

i	1	2	3	4	5	6	7	8
LSAT	576	635	558	578	666	580	555	661
GPA	3.39	3.30	2.81	3.03	3.44	3.07	3.00	3.43

i	9	10	11	12	13	14	15
LSAT	651	605	653	575	545	572	594
GPA	3.36	3.13	3.12	2.74	2.76	2.88	2.96

2. Let $X \sim t_k$. For each value of k and x, find the probability value A such that $\Pr(X < x) = A$.

   (a) $k = 3, x = 4.541$

   (b) $k = 8, x = 1.397$

   (c) $k = 29, x = 3.04$

   (d) $k = 200, x = 1.96$

3. Social security contributions are compulsory payments paid to general government that confer entitlement to receive a (contingent) future social benefit. The table below lists the USA social security costs for 7 years between 1965 to 1992.

Year	1965	1970	1975	1980	1985	1990	1992
$x =$ no. years from 1960	5	10	15	20	25	30	32
$y =$ social security cost ($B)	17.1	29.6	63.6	117.1	186.4	246.5	285.1

   (a) Plot the data y against x.

   (b) Compute $\sum x$, $\sum y$, $\sum x^2$, and $\sum xy$, and therefore fit the data to a simple linear regression model $y = \alpha + \beta x + \epsilon$. Superimpose the fitted regression line onto the plot in (a).

   (c) The correlation coefficient between x and y was calculated to be $r = 0.98$. Test the hypothesis $H_0 : \beta = 0$. What can be concluded on the social security costs from the test?

   (d) Compute the value of the coefficient of determination R^2.

   (e) Plot the residuals against x. Are you happy with the fitted model?