28.04.20

Семинар 10

Продолжаем

$\overset{I}{\bar{e}}, \overset{II}{\bar{C}}, \overset{IV}{\bar{C}}, \overset{V}{\bar{C}}, \quad \overset{(I)}{G}, \overset{(II)}{G}, \overset{IV}{G}, \overset{(V)}{G}$

U - ?

Теорема о полярном разложении

$F = O \cdot U$ или $F = V \cdot O$

U, V - симметр. $(U = U^{ij}\bar{e}_i \otimes \bar{e}_j, \; U^{ij} = U^{ji})$

- положит. опр. $(\forall \bar{a} \quad \bar{a} \cdot U \cdot \bar{a} > 0)$

O - ортогон. тензор

$\overset{(I)}{C} = \Lambda = \frac{1}{2}(E - U^{-2})$

$\bar{a} = a^l \bar{e}_l, \quad \bar{a} = a^n \bar{e}_n$, т.е. $\bar{a} \cdot U \cdot \bar{a} = a^l \bar{e}_l U^{ij} \bar{e}_i \otimes \bar{e}_j a^n \bar{e}_n =$

$= a^l \cdot U^{ij} a^n \bar{e}_l \bar{e}_i \otimes \bar{e}_j \bar{e}_n = a^l U^{ij} a^n \delta_{li} \otimes \delta_{jn} =$

$= a_i U^{ij} a_j = U^{ij} a_i a_j > 0$

U^{-2} - ? $\rightarrow U^{-1}$ - ?

$U^{-1} \cdot U = E = \bar{e}_k \otimes \bar{e}^k = \bar{e}_k \otimes \bar{e}_k$

$U^{-1} = \tilde{U}^{mn} \bar{e}_m \otimes \bar{e}_n$

$\tilde{U}^{mn} \bar{e}_m \otimes \bar{e}_n \cdot U^{ij} \bar{e}_i \otimes \bar{e}_j = \tilde{U}^{mn} \cdot U^{ij} \bar{e}_m \otimes \bar{e}_n \cdot \bar{e}_i \otimes \bar{e}_j =$

$= \tilde{u}^{nm} u^{ij} \delta_{ni} \bar{e}_m \otimes \bar{e}_j = \tilde{u}_i^{\ m} u^{ij} \bar{e}_m \otimes \bar{e}_j = E$

$\bar{e}_j = \delta_{jm} e^m$, тогда $\tilde{u}^m_{\ i} u^{ij} \delta_{jm} \bar{e}_m \otimes \bar{e}^m \Rightarrow \tilde{u}_{ji} u^{ij} = 1$

$u^{-1} = \tilde{u}^{mn} \bar{e}_m \otimes \bar{e}_n$

$u^{-2} = u^{-1} \cdot u^{-1} = \tilde{u}^{mn} \bar{e}_m \otimes \bar{e}_n \cdot \tilde{u}^{kl} \bar{e}_k \otimes \bar{e}_l = \tilde{u}^{mn} \tilde{u}^{kl} \cdot$

$\cdot \bar{e}_m \otimes \bar{e}_n \, \bar{e}_k \otimes \bar{e}_l = \tilde{u}^m_{\ k} \tilde{u}^{kl} \bar{e}_m \otimes \bar{e}_l = \tilde{u}^m_{\ k} \tilde{u}^k_{\ l} \delta_{ep} \bar{e}_m \otimes \bar{e}^p$

$\overset{(I)}{\bar{C}} = \frac{1}{2}(E - u^{-2}) = (\bar{e}_m \otimes \bar{e}^m - \tilde{u}^m_{\ k} \tilde{u}^{kl} \delta_{ep} \bar{e}_m \otimes \bar{e}^p) \cdot \frac{1}{2} =$

$= (\delta^m_{\ p} \bar{e}_m \otimes \bar{e}^p - \tilde{u}^m_{\ k} \tilde{u}^{kl} \delta_{ep} \bar{e}_m \otimes \bar{e}_p) \cdot \frac{1}{2} =$

$= \frac{1}{2}(\delta^m_{\ p} - \tilde{u}^m_{\ k} \tilde{u}^{kl} \delta_{ep}) \cdot \bar{e}_m \otimes \bar{e}_p$

$\overset{(II)}{\bar{C}} = E \cdot u^{-1} = \bar{e}_k \otimes \bar{e}^k - \tilde{u}^{kl} \bar{e}_k \otimes \bar{e}_l = (\delta^{lk} - \tilde{u}^{kl}) \bar{e}_k \otimes \bar{e}_l$

$\overset{(IV)}{C} = u - E = u^{kl} \bar{e}_k \otimes \bar{e}_l - \bar{e}_k \otimes \bar{e}^k = (u^{kl} - \delta^{kl}) \bar{e}_k \otimes \bar{e}_l$

$\overset{(V)}{C} = \frac{1}{2}(u^2 - E)$

$u^2 = u \cdot u = u^{kl} \bar{e}_k \otimes \bar{e}_l \cdot u^{mn} \bar{e}_m \otimes \bar{e}_n = u^{kl} u^{mn} \bar{e}_k \otimes \bar{e}_l \cdot$

$\cdot \bar{e}_m \otimes \bar{e}_n = u^{kl} u^{mn} \delta_{em} \bar{e}_k \otimes \bar{e}_n = u^{kl} u_l^{\ n} \bar{e}_k \otimes \bar{e}_n$

$\overset{(V)}{\bar{C}} = (u^{kl} u_l^{\ n} \bar{e}_k \otimes \bar{e}_n - \bar{e}_k \otimes \bar{e}^k) \cdot \frac{1}{2} = \frac{1}{2}(u^{kl} u_l^{\ n} - \delta^n_{\ k})$

$\cdot \bar{e}_k \otimes \bar{e}_n$

Мера деформации

1) Правая мера деформации Коши – Грина G

$$G = g_{ij} \overset{\circ}{\vec{R}}{}^i \otimes \overset{\circ}{\vec{R}}{}^j = F^T \cdot F = E + 2C$$

$$C = \frac{1}{2}(g_{ij} - \overset{\circ}{g}_{ij}) \overset{\circ}{\vec{R}}{}^i \otimes \overset{\circ}{\vec{R}}{}^j = \varepsilon_{ij} \overset{\circ}{\vec{R}}{}^i \otimes \overset{\circ}{\vec{R}}{}^j$$ – прав. тенз. деформ. К – Г (Гр. – Лагр.)

$$F = \frac{\partial x^m}{\partial \overset{\circ}{x}{}^i} \bar{e}_m \otimes \bar{e}^i, \quad F^T = \frac{\partial x^m}{\partial \overset{\circ}{x}{}^i} \bar{e}^i \otimes \bar{e}_m$$

$$F^{-1} = \frac{\partial \overset{\circ}{x}{}^m}{\partial x^i} \bar{e}_m \otimes \bar{e}^i \quad ; \quad F^{-1T} = \frac{\partial \overset{\circ}{x}{}^m}{\partial x^i} \bar{e}^i \otimes \bar{e}_m$$

2) Левая мера деформ. Альманзи

$$g = \overset{\circ}{g}_{ij} \vec{R}^i \otimes \vec{R}^j = (F^{-1})^T \cdot F^{-1} = E - 2A$$

$$A = \frac{1}{2}(g_{ij} - \overset{\circ}{g}_{ij}) \vec{R}^i \otimes \vec{R}^j = \varepsilon_{ij} \vec{R}^i \otimes \vec{R}^j$$ – лев. тензор деформации Альманзи

$$G^{-1} = F^{-1} \cdot F^{-1T} = E - 2\Lambda$$ – правая мера деформации Альманзи

$$\Lambda = \frac{1}{2}(\overset{\circ}{g}{}^{ij} - g^{ij}) \overset{\circ}{\vec{R}}_i \otimes \overset{\circ}{\vec{R}}_j = \varepsilon^{ij} \overset{\circ}{\vec{R}}_i \otimes \overset{\circ}{\vec{R}}_j$$ – прав. тензор деформ. Альманзи

$$G^{-1} = g^{ij} \overset{\circ}{\vec{R}}_i \otimes \overset{\circ}{\vec{R}}_j$$

$\overset{\circ}{g}{}^{-1} = \overset{\circ}{g}{}^{ij}\, \bar{R}_i \otimes \bar{R}_j = F \cdot F^{T} = E + 2J$ — левая мера деформ. Коши-Грина

$J = \frac{1}{2}\left(\overset{\circ}{g}{}^{ij} - g^{ij}\right) \bar{R}_i \otimes \bar{R}_j = \varepsilon^{ij} \bar{R}_i \otimes \bar{R}_j$ — лев. тенз. деформ. Коши-Грина

$\overset{(I)}{G} = -\frac{1}{2} G^{-1} = \left(F^{-1} \cdot F^{-1T}\right)\left(-\frac{1}{2}\right) = -\frac{1}{2} \frac{\partial \overset{\circ}{x}{}^{m}}{\partial x^{i}}\, \bar{e}_m \otimes \bar{e}^{i} \cdot$

$\cdot \frac{\partial \overset{\circ}{x}{}^{n}}{\partial x^{k}}\, \bar{e}^{k} \otimes \bar{e}_n = -\frac{1}{2} \frac{\partial \overset{\circ}{x}{}^{m}}{\partial x^{i}} \cdot \frac{\partial \overset{\circ}{x}{}^{n}}{\partial x^{k}}\, \bar{e}_m \otimes \bar{e}^{i} \cdot \bar{e}^{k} \otimes \bar{e}_n =$

$= -\frac{1}{2} \frac{\partial \overset{\circ}{x}{}^{m}}{\partial x^{i}} \cdot \frac{\partial \overset{\circ}{x}{}^{n}}{\partial x^{k}}\, \delta^{ik}\, \bar{e}_m \otimes \bar{e}_n$

$\overset{(II)}{G} = \frac{1}{2} G = \frac{1}{2} F^{T} \cdot F = \frac{1}{2} \cdot \frac{\partial x^{n}}{\partial \overset{\circ}{x}{}^{i}}\, \bar{e}^{i} \otimes \bar{e}_m \cdot \frac{\partial x^{n}}{\partial \overset{\circ}{x}{}^{m}}\, \bar{e}_n \otimes \bar{e}^{k} =$

$= \frac{1}{2} \frac{\partial x^{m}}{\partial \overset{\circ}{x}{}^{i}} \frac{\partial x^{n}}{\partial \overset{\circ}{x}{}^{k}}\, \delta_{mn}\, \bar{e}^{i} \otimes \bar{e}^{k}$