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Семинар 11

1) $T \times \bar{a} = T_{jm} r^j \otimes r^m \times a_i r^i = \frac{1}{\sqrt{g}} \epsilon^{mik} T_{jm} a_i r^j \otimes r_k$

2) Пусть в $\forall$ точке x 3-х мерного пр-ва E^3 $^n\Omega(x)$

$$\Omega^{i_1 \dots i_n} r_{i_1} \otimes r_{i_2} \otimes \dots \otimes r_{i_n}, \quad n \geq 1$$

при переходе $x^i \to x^{i'}$: $\Omega^{i'_1 \dots i'_n} = \bar{P}^{i_1}_{j_1} \dots \bar{P}^{i_n}_{j_n} {}^n\Omega^{j_1 \dots j_n}$

при переходе $x^i \to x^{i'}$:

$$\Omega^{i_1 \dots i_k}{}_{i_{k+1} \dots i_n} = P^{i_1}{}_{j_1} \dots P^{i_k}{}_{j_k} Q^{j_{k+1}}{}_{i_{k+1}} \dots Q^{j_n}{}_{i_n} \Omega^{j_1 \dots j_k}{}_{j_{k+1} \dots j_n}$$

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$$\epsilon = \sqrt{g}\, \epsilon_{ijk} r^i \otimes r^j \otimes r^k = \frac{1}{\sqrt{g}} \epsilon^{ijk} r_i \otimes r_j \otimes r_k$$

$$\epsilon_{ijk} = \epsilon^{ijk} = \begin{cases} 1, & (ijk) \text{ - четная подст.} \\ 0, & (ijk) \text{ - есть повторяющ. инд.} \\ -1, & (ijk) \text{ - нечетн. подст.} \end{cases}$$

Дифференц. форма закона изменения мом. кол-ва движ.

$$\frac{d}{dt}\int_V \rho\left(\bar{x}\times\bar{v}+\frac{1}{\partial s}K_m\right)dV = \frac{d}{dt}\int_{\mathring{V}}\mathring{\rho}\left(\bar{x}\times\bar{v}+\frac{1}{\partial s}K_m\right)d\mathring{V} =$$

$$= \int_V \rho\left(\bar{x}\times\frac{d\bar{v}}{dt}+\frac{1}{\partial s}\frac{dK_m}{dt}\right)dV$$

$$\int_\Sigma \bar{x}\times t_n\, d\Sigma = -\int_\Sigma n\cdot(T\times\bar{x})\,d\Sigma = -\int_V \nabla\cdot(T\times\bar{x})\,dV =$$

$$\ominus\ \Big[\nabla\cdot(T\times\bar{x}) = r^i\frac{\partial}{\partial x^i}(T\times\bar{x}) = r^i\left(\frac{\partial T}{\partial x^i}\times\bar{x}+T\times\frac{\partial\bar{x}}{\partial x^i}\right) =$$

$$= r^i\frac{\partial T}{\partial x^i}\times\bar{x} + r^i T\times r^i = -\bar{x}\times\nabla\cdot T + \epsilon\cdot\cdot T,$$

покажем, что $r^i T\times r_i = \epsilon\cdot\cdot T$

$$T\times r_i = T_{jm}\, r^j\otimes r^m\times\delta_{ip}r^p = \frac{1}{\sqrt{g}}\epsilon^{mik}T_{jm}r_k = \frac{1}{\sqrt{g}}\epsilon^{mik}T_m r_k$$

$$\epsilon\cdot\cdot T = \frac{1}{\sqrt{g}}\epsilon^{ijk}\, r_i\otimes r_j\otimes r_k\cdot\cdot T_{pl}\, r^p\otimes r^l =$$

$$= \frac{1}{\sqrt{g}}\epsilon^{ijk}T_{pl}\,\delta^p_j\delta^l_k\, r_i = \frac{1}{\sqrt{g}}\epsilon^{ijk}T_{jk}r_i$$

$$-\frac{1}{\sqrt{g}}\epsilon^{kji}T_{kj}r_i = \frac{1}{\sqrt{g}}\epsilon^{jki}T_{kj}r_i\Big]\ \ominus$$

$$\ominus\ \int_V\left(\frac{\rho}{\partial s}\frac{dK_m}{dt} - \rho h_m - \nabla\cdot M + \epsilon\cdot\cdot T\right)dV = 0$$

$$\rightarrow \frac{\delta}{\delta t}\frac{dK_m}{dt} = \rho h_m + \nabla\cdot M - \varepsilon\cdot\cdot T$$

<u>Задача 1</u> Показать, что для СС

$$E\cdot\cdot(F\cdot P) = 0 \Leftrightarrow F\cdot P$$

$$h_m = 0,\ K_m = 0,\ M = 0$$

$$E\cdot\cdot T = -\sqrt{g}\sum_{\substack{\alpha=1 \\ \alpha\neq\beta\neq\gamma\neq\alpha}}^{3}\left(T^{\alpha\beta} - T^{\beta\alpha}\right) r^{\gamma} = 0$$

$T = T^{\gamma}$ $\quad$ T опр. на площ. $d\Sigma$

$$n\, d\Sigma = \sqrt{\frac{g}{\mathring{g}}}\,\mathring{n}\cdot F^{-1} d\mathring{\Sigma}$$

$$t_n\, d\Sigma = n\cdot T\, d\Sigma = \sqrt{\frac{g}{\mathring{g}}}\,\mathring{n}\cdot F^{-1}\cdot T\, d\mathring{\Sigma} = \mathring{n}\cdot P\, d\mathring{\Sigma} = \mathring{t}_n\, d\mathring{\Sigma}$$

$$P = \sqrt{\frac{g}{\mathring{g}}}\,F^{-1}\cdot T$$

$$E\cdot\cdot T = 0 \qquad T = \sqrt{\frac{\mathring{g}}{g}}\,F\cdot P$$

$$E\cdot\cdot(F\cdot P) = 0 \iff (F\cdot P)^T = F\cdot P$$

$$P^T\cdot F^T = F\cdot P$$

Задача 2 M и $\overset{\circ}{M}$ $\overset{\circ}{M} = \sqrt{\frac{g}{\overset{\circ}{g}}}\, F^{-1}\cdot M$

$\overset{\circ}{M}$ – Лагранж. тензор моментных напряжений в K

M – тензор моментных напряжений в K

$$\bar n M d\Sigma = \overset{\circ}{\bar n} \overset{\circ}{M} d\overset{\circ}{\Sigma}$$

$$\bar n\, d\Sigma = \sqrt{\frac{g}{\overset{\circ}{g}}}\, \overset{\circ}{\bar n}\cdot F^{-1} d\overset{\circ}{\Sigma}$$

$$\Rightarrow \quad \overset{\circ}{M} = \sqrt{\frac{g}{\overset{\circ}{g}}}\, F^{-1} M$$

т.к. $\sqrt{\frac{g}{\overset{\circ}{g}}}\, \overset{\circ}{\bar n} F^{-1}\cdot M d\overset{\circ}{\Sigma} = \overset{\circ}{\bar n} \overset{\circ}{M} d\Sigma$

Самостоятельная работа

Доказать: $$\frac{d}{dt}\int_{\overset{\circ}{V}} \overset{\circ}{\rho}\left(\bar x\times\bar v + \frac{1}{\partial s}K_m\right) d\overset{\circ}{V} = \int_{\overset{\circ}{V}} \overset{\circ}{\rho}(\bar x\times \bar f + h_m)\, d\overset{\circ}{V} + \int_{\overset{\circ}{\Sigma}} \overset{\circ}{\bar n}(-P\times \bar x + \overset{\circ}{M})\, d\overset{\circ}{\Sigma}$$

Преобразуем к виду $\frac{\rho}{\partial s}\frac{dK_m}{dt} = \overset{\circ}{\rho} h_m + \overset{\circ}{\nabla}\overset{\circ}{M} - E\cdot\cdot(F\cdot P)$