21.04.20

Семинар 9

Энергетические пары тензоров

Напомним:

$$\left(\overset{(n)}{T}, \overset{(n)}{C}\right):\ \left(\overset{(I)}{T}, \overset{(I)}{C}\right), \ldots, \left(\overset{(V)}{T}, \overset{(V)}{C}\right)$$

№ (n) пары	Энергетич. тензор напряж. $\overset{(n)}{T}$	Энергетич. тензор деформации $\overset{(n)}{C}$	Энергетич. т-н деформ $\overset{(n)}{C}$
I	$F^T\cdot T^S\cdot F$	$\Lambda=(E-U^{-2})/2$	$-G^{-1}/2$
II	$\frac{1}{2}(F^T\cdot T^S\cdot O+O^T T^S F)$	$E-U^{-1}$	$-U^{-1}$
III	$O^T\cdot T^S\cdot O$	B	$\overset{(III)}{G}$
IV	$\frac{1}{2}(F^{-1}\cdot T^S\cdot O+O^T F^{-1^T})$	$U-E$	U
V	$F^{-1}\cdot T^S\cdot F^{-1^T}$	$C=(U^2-E)/2$	$G/2$

$$\overset{(n)}{T}:\quad \bar{R}_i\otimes\bar{R}_j;\ \bar{R}^i\otimes\bar{R}^j;\ \bar{R}^i\otimes\bar{R}_j;\ \bar{e}_i\otimes\bar{e}_j$$

$$\overset{(I)}{T}:\quad \overset{(I)}{T}=F^T\cdot T\cdot F \qquad T=T^{ij}\,\bar{e}_i\otimes\bar{e}_j;\ T^{ij}=T^{ji}$$

$$F=\frac{\partial x^l}{\partial \overset{\circ}{x}{}^m}\,\bar{e}_l\otimes\bar{e}^m \qquad F^T=\frac{\partial x^s}{\partial \overset{\circ}{x}{}^p}\,\bar{e}^p\otimes\bar{e}_s$$

$$\overset{(I)}{T}=F^T\cdot T\cdot F=\frac{\partial x^s}{\partial \overset{\circ}{x}{}^p}\,\bar{e}^p\otimes\bar{e}_s\cdot T^{ij}\,\bar{e}_i\otimes\bar{e}_j\cdot\frac{\partial x^l}{\partial \overset{\circ}{x}{}^m}\,\bar{e}_l\otimes\bar{e}^m=$$

$$=\frac{\partial x^s}{\partial \overset{\circ}{x}{}^p}\,T^{ij}\,\frac{\partial x^l}{\partial \overset{\circ}{x}{}^m}\,\bar{e}^p\otimes\bar{e}_s\cdot\bar{e}_i\otimes\bar{e}_j\cdot\bar{e}_l\otimes\bar{e}^m=$$

$$\frac{\partial x^s}{\partial \mathring{x}^p} T_{s} \frac{\partial x^l}{\partial \mathring{x}^m} \bar{e}^p \otimes \bar{e}^m$$

$$\overset{(I)}{T} = \frac{\partial x^s}{\partial \mathring{x}^p} \frac{\partial x^l}{\partial \mathring{x}^m} T_{sl}\, \bar{e}^p \otimes \bar{e}^m \qquad (\ell_{\neq}\ \bar{e}_i \otimes \bar{e}_j)$$

$$\overset{(II)}{T} = \frac{1}{2}(F^T \cdot T \cdot O + O^T \cdot T \cdot F) \qquad \bar{e}_i \otimes \bar{e}_j$$

$$T = T^{ij}\, \bar{e}_i \otimes \bar{e}_j \qquad O^{-1} = O^T$$

$$F^T = \frac{\partial x^l}{\partial \mathring{x}^m} \bar{e}^m \otimes \bar{e}_l \qquad O = O^{sp}\, \bar{e}_s \otimes \bar{e}_p$$

$$\overset{(II)}{T} = \frac{1}{2}(F^T \cdot T \cdot O + O^T \cdot T \cdot F) = \frac{1}{2}\Big(\frac{\partial x^l}{\partial \mathring{x}^m} \bar{e}^m \otimes \bar{e}_l \cdot T^{ij} \bar{e}_i \otimes \bar{e}_j \cdot$$

$$\cdot O^{sp} \bar{e}_s \otimes \bar{e}_p + O^{sp} \bar{e}_p \otimes \bar{e}_s \cdot T^{ij} \bar{e}_i \otimes \bar{e}_j \cdot \frac{\partial x^l}{\partial \mathring{x}^m} \bar{e}_l \otimes \bar{e}^m\Big) =$$

$$= \frac{1}{2}\Big(\frac{\partial x^l}{\partial \mathring{x}^m} T^{ij} O^{sp} \cdot \bar{e}^m \otimes \bar{e}_l \cdot \bar{e}_i \otimes \bar{e}_j \cdot \bar{e}_s \otimes \bar{e}_p +$$

$$+ O^{sp} T^{ij} \frac{\partial x^l}{\partial \mathring{x}^m} \bar{e}_p \otimes \bar{e}_s \otimes \bar{e}_i \otimes \bar{e}_j \cdot \bar{e}_l \otimes \bar{e}^m\Big) =$$

$$= \frac{1}{2}\Big(\frac{\partial x^l}{\partial \mathring{x}^m} T_{ls} O^{sp}\, \bar{e}^m \otimes \bar{e}_p + O^{sp} T_{sl} \frac{\partial x^l}{\partial \mathring{x}^m} \bar{e}_p \otimes \bar{e}^m\Big) =$$

$$= \frac{1}{2} \frac{\partial x^l}{\partial \mathring{x}^m} T_{ls} O^{sp} \cdot (\bar{e}_m \otimes \bar{e}^p + \bar{e}^p \otimes \bar{e}_m)$$

$$\overset{(II)}{T} = \frac{1}{2} \frac{\partial x^l}{\partial \mathring{x}^m} T_{ls} \cdot O^{sp} (\bar{e}_m \otimes \bar{e}^p + \bar{e}^p \otimes \bar{e}_m) \qquad (\text{в } \bar{e}_i \otimes \bar{e}_j)$$

$$\overset{(III)}{T} = O^T \cdot T \cdot O$$

$$T^{ij}\,\bar{e}_i \otimes \bar{e}_j \;;\quad O = O^{sp}\,\bar{e}_s \otimes \bar{e}_p \;;\quad O^T = O^{mn}\,\bar{e}_n \otimes \bar{e}_m$$

$$\overset{(III)}{T} = O^T \cdot T \cdot O = O^{mn}\,\bar{e}_n \otimes \bar{e}_m \cdot T^{ij}\,\bar{e}_i \otimes \bar{e}_j \cdot O^{sp}\,\bar{e}_s \otimes \bar{e}_p =$$

$$= O^{mn}\,T^{ij}\,O^{sp} \cdot \bar{e}_n \otimes \bar{e}_m \cdot \bar{e}_i \otimes \bar{e}_j \cdot \bar{e}_s \otimes \bar{e}_p =$$

$$= O^{mn} \cdot T_{ms}\,O^{sp}\,\bar{e}_n \otimes \bar{e}_p$$

$$\overset{III}{T} = O^{mn}\,T_{ms}\,O^{sp}\,\bar{e}_n \otimes \bar{e}_p \quad (\text{в } \bar{e}_i \otimes \bar{e}_j)$$

$$F^{-1} = \frac{\partial \overset{\circ}{x}^l}{\partial x^m}\,\bar{e}_l \otimes \bar{e}^m \;;\quad F^{-1T} = \frac{\partial \overset{\circ}{x}^l}{\partial x^m}\,\bar{e}^m \otimes \bar{e}_l \;;\quad T^{ij}\,\bar{e}_i \otimes \bar{e}_j$$

$$O = O^{sp}\,\bar{e}_s \otimes \bar{e}_p \;;\quad O^T = O^{sp}\,\bar{e}_p \otimes \bar{e}_s$$

$$\overset{IV}{T} = \frac{1}{2}\Big(\frac{\partial \overset{\circ}{x}^l}{\partial x^m}\,\bar{e}_l \otimes \bar{e}^m\,T^{ij}\,\bar{e}_i \otimes \bar{e}_j \cdot O^{sp}\,\bar{e}_s \otimes \bar{e}_p +$$

$$+ O^{sp}\,\bar{e}_p \otimes \bar{e}_s\,T^{ij}\,\bar{e}_i \otimes \bar{e}_j \cdot \frac{\partial \overset{\circ}{x}^l}{\partial x^m}\,\bar{e}^m \otimes \bar{e}_l\Big) =$$

$$= \frac{1}{2}\,\frac{\partial \overset{\circ}{x}^l}{\partial x^m} \cdot O^{sp} \cdot T^m{}_s\,(\bar{e}_l \otimes \bar{e}_p + \bar{e}_p \otimes \bar{e}_l)$$

$$\overset{IV}{T} = \frac{1}{2}\,\frac{\partial \overset{\circ}{x}^l}{\partial x^m}\,T^m{}_s\,(\bar{e}_l \otimes \bar{e}_p + \bar{e}_p \otimes \bar{e}_l)$$

$$\overset{V}{T} = F^{-1} \cdot T \cdot F^{-1T} \qquad\qquad \bar{e}_i \otimes \bar{e}_j$$

$$F^{-1} = \frac{\partial \overset{\circ}{x}^l}{\partial x^m}\,\bar{e}_l \otimes \bar{e}^m \;;\quad F^{-1} = \frac{\partial \overset{\circ}{x}^l}{\partial x^m}\,\bar{e}^m \otimes \bar{e}_l \;;\quad T = T^{ij}\,\bar{e}_i \otimes$$

$$\overset{(v)}{T} = \frac{\partial \overset{\circ}{x}^l}{\partial x^m} \bar{e}_l \otimes \bar{e}^m \cdot T^{ij} \bar{e}_i \otimes \bar{e}_j \cdot \frac{\partial \overset{\circ}{x}^s}{\partial x^p} \bar{e}^p \otimes \bar{e}_s =$$

$$= \frac{\partial \overset{\circ}{x}^l}{\partial x^m} T^{ij} \frac{\partial \overset{\circ}{x}^s}{\partial x^p} \cdot \bar{e}_l \otimes \bar{e}^m \cdot \bar{e}_i \otimes \bar{e}_j \cdot \bar{e}^p \otimes \bar{e}_s \Rightarrow$$

$$\Rightarrow \overset{(v)}{T} = \frac{\partial \overset{\circ}{x}^l}{\partial x^m} \frac{\partial \overset{\circ}{x}^s}{\partial x^p} T^{mp} \bar{e}_l \otimes \bar{e}_s$$

Самостоятельная работа

1. Выразим $\overset{(u)}{T}$ в $\bar{R}_i \otimes \bar{R}_j$

$$F = \bar{R}_i \otimes \overset{\circ}{\bar{R}}{}^i$$

$$F^T = \overset{\circ}{\bar{R}}{}^i \otimes \bar{R}_i; \quad F^{-1} = \overset{\circ}{\bar{R}}_i \otimes \bar{R}^i; \quad F^{-1T} = \bar{R}^i \otimes \overset{\circ}{\bar{R}}_i$$

$$\overset{\circ}{\bar{R}}{}^i = \frac{\partial x^i}{\partial \overset{\circ}{x}^l} \bar{e}^l; \quad \bar{R}_k = \frac{\partial x^l}{\partial x^k} \bar{e}_l \Rightarrow \overset{\circ}{\bar{R}}{}^i = \frac{\partial x^i}{\partial \overset{\circ}{x}^l} \cdot \frac{\partial x^k}{\partial x^l} \bar{R}_k$$

$$\overset{\circ}{\bar{R}}_i = \frac{\partial \overset{\circ}{x}^l}{\partial x} \bar{e}_l; \quad \bar{e}_l = \frac{\partial x^k}{\partial x^l} \bar{R}_k \Rightarrow \overset{\circ}{\bar{R}}_i = \frac{\partial \overset{\circ}{x}^l}{\partial x^i} \cdot \frac{\partial x^k}{\partial x^l} \bar{R}_k$$

$$F = \frac{\partial x^i}{\partial \overset{\circ}{x}^l} \cdot \frac{\partial x^k}{\partial x^l} \bar{R}_i \otimes \bar{R}_k$$

$$F^{-1} = \frac{\partial \overset{\circ}{x}^l}{\partial x^i} \frac{\partial x^l}{\partial x^k} \bar{R}_i \otimes \bar{R}_k$$

$$F^T = \frac{\partial x^i}{\partial \overset{\circ}{x}^l} \frac{\partial x^k}{\partial x^l} \bar{R}_k \otimes \bar{R}_i$$

$$F^{-1T} = \frac{\partial \overset{\circ}{x}^l}{\partial x^i} \frac{\partial x^l}{\partial x^k} \bar{R}_k \otimes \bar{R}_i$$