Задача 2 M и $\overset{\circ}{M}$ $\overset{\circ}{M} = \sqrt{\frac{g}{\overset{\circ}{g}}}\, F^{-1} \cdot M$

$\overset{\circ}{M}$ – Лагранж. тензор моментных напряжений в $\overset{\circ}{K}$

M – тензор моментных напряжений в K

$\bar{n} M \, d\Sigma = \overset{\circ}{\bar{n}} \overset{\circ}{M} \, d\overset{\circ}{\Sigma}$

$\bar{n} \, d\Sigma = \sqrt{\frac{g}{\overset{\circ}{g}}}\, \overset{\circ}{\bar{n}} \cdot F^{-1} d\overset{\circ}{\Sigma}$

$$\Rightarrow \quad \overset{\circ}{M} = \sqrt{\frac{g}{\overset{\circ}{g}}}\, F^{-1} M$$

т.к. $\sqrt{\frac{g}{\overset{\circ}{g}}}\, \overset{\circ}{\bar{n}} F^{-1} \cdot M \, d\overset{\circ}{\Sigma} = \overset{\circ}{\bar{n}} \overset{\circ}{M} \, d\Sigma$

Самостоятельная работа

Доказать: $$\frac{d}{dt} \int_{\overset{\circ}{V}} \overset{\circ}{\rho} \left(\bar{x} \times \bar{v} + \frac{1}{\partial s} K_m \right) d\overset{\circ}{V} = \int_{\overset{\circ}{V}} \overset{\circ}{\rho} \left(\bar{x} \times \bar{f} + h_m \right) d\overset{\circ}{V} + \int_{\overset{\circ}{\Sigma}} \overset{\circ}{\bar{n}} \left(-P \times \bar{x} + \overset{\circ}{M} \right) d\overset{\circ}{\Sigma}$$

Преобразуем к виду $\frac{\rho}{\partial s} \cdot \frac{dK_m}{dt} = \overset{\circ}{\rho} h_m + \overset{\circ}{\nabla} \overset{\circ}{M} - E \cdot\cdot (F \cdot P)$

Док-во:

1) $\frac{d}{dt}\int_{\mathring V}\mathring\rho(\bar x\times\bar v+\frac{1}{\partial s}K_m)d\mathring V=\int_{\mathring V}\mathring\rho(\bar x\times\frac{d\bar v}{dt}+\frac{1}{\partial s}\frac{dK_m}{dt})d\mathring V$

2) $\int_{\mathring\Sigma}\mathring n(-P\times\bar x+\mathring M)d\mathring\Sigma=-\int_{\mathring\Sigma}\mathring n(P\times\bar x)d\mathring\Sigma+\int_{\mathring\Sigma}\mathring n\mathring M d\mathring\Sigma=$

$=\int_{\mathring\Sigma}\bar x\times(\mathring n\cdot P)d\mathring\Sigma+\int_{\mathring V}\mathring\nabla\mathring M dV=\int_{\mathring\Sigma}\bar x\times(\mathring n\cdot T)d\mathring\Sigma+$

$+\int_{\mathring V}\mathring\nabla\mathring M d\mathring V=-\int_{\mathring V}\nabla(T\times\bar x)d\mathring V+\int_{\mathring V}\mathring\nabla\mathring M d\mathring V=$

$=\int_{\mathring V}(\bar x\cdot\nabla T-\epsilon\cdot\cdot T+\mathring\nabla\mathring M)d\mathring V$

$\int_{\mathring V}\mathring\rho\,\bar x\times\frac{d\bar v}{dt}+\frac{\mathring\rho}{\partial s}\frac{dK_m}{dt}-\mathring\rho\,\bar x\times f-\mathring\rho h_m-\bar x\times\nabla T+$

$+\epsilon\cdot\cdot T-\mathring\nabla\mathring M)d\mathring V=\int_{\mathring V}(\mathring\rho\,\bar x+\cancelto{0}{(\frac{d\bar v}{dt}-f-\frac{\nabla T}{\mathring\rho})}+$

$+\frac{\mathring\rho}{\partial s}\cdot\frac{dK_m}{dt}-\mathring\rho h_m+\epsilon\cdot\cdot T-\mathring\nabla\mathring M)d\mathring V=$

$=\int_{\mathring V}(\frac{\mathring\rho}{\partial s}\frac{dK_m}{dt}-\mathring\rho h_m+\epsilon\cdot\cdot T-\mathring\nabla\mathring M)d\mathring V=0$

В силу произвольности объема $\mathring V$ имеем:

$\frac{\mathring\rho}{\partial s}\frac{dK_m}{dt}=\mathring\rho h_m-\epsilon\cdot\cdot(F\cdot P)+\mathring\nabla\mathring M$ #