Самостоятельная работа

Даны выражения тензоров в требуемых диадн. базисах. Выполнить ф-лы:

$$E = \bar{R}_i \otimes \bar{R}^i$$

$$U = u^{ij} \bar{R}_i \otimes \bar{R}_j$$

$$\bar{R}^k = g^{kn} \bar{R}_n$$

$$U^{-1} = \tilde{u}^{ij} \bar{R}_i \otimes \bar{R}_j$$

$$U^{-2} = \tilde{u}^k{}_m \tilde{u}^m{}_l \bar{R}_k \otimes \bar{R}^l$$

$$U^{-1} \cdot U = E \Rightarrow \tilde{u}^{ij} \bar{R}_i \otimes \bar{R}_j \cdot U^{mn} \bar{R}_m \otimes \bar{R}_n =$$

$$= \tilde{u}^{ij} U^{mn} \bar{R}_i \otimes \bar{R}_j \cdot \bar{R}_m \otimes \bar{R}_n = \tilde{u}^{ij} U^{mn} g_{jm} \bar{R}_i \otimes (g_{ni} \bar{R}^i) =$$

$$= \tilde{u}^i{}_m \cdot U^m{}_i \cdot \bar{R}_i \otimes \bar{R}^i = \bar{R}_i \otimes \bar{R}^i = E \checkmark$$

$$F = \frac{\partial x^i}{\partial \mathring{x}^l} \frac{\partial x^k}{\partial x^l} \bar{R}_i \otimes \bar{R}_k$$

$$F^{-1} = \frac{\partial \mathring{x}^l}{\partial x^i} \frac{\partial x^l}{\partial x^k} \bar{R}_i \otimes \bar{R}^k$$

$$F^{T} = \frac{\partial x^i}{\partial \mathring{x}^l} \frac{\partial x^k}{\partial x^l} \bar{R}_k \otimes \bar{R}_i$$

$$F^{-1T} = \frac{\partial \mathring{x}^l}{\partial x_i} \frac{\partial x^l}{\partial x_k} \bar{R}_k \otimes \bar{R}_i$$

$$\tilde{u}^{ij}\,\tilde{u}^{mn}\,\bar{R}_i\otimes\bar{R}_j\cdot\bar{R}_m\otimes\bar{R}_n=\tilde{u}^i{}_m\,\tilde{u}^{mn}\,\bar{R}_i\otimes\bar{R}_n=$$

$$=\tilde{u}^i{}_m\,\tilde{u}^m{}_l\,\bar{R}_i\otimes\bar{R}^l$$

$$u^2=u^{ij}\bar{R}_i\otimes\bar{R}_j\,u^{mn}\bar{R}_m\otimes\bar{R}_n=u^i{}_m u^{mn}\bar{R}_i\otimes\bar{R}_n$$

$$\overset{(I)}{C}=\frac{1}{2}(E-u^2)=\frac{1}{2}(\bar{R}_i\otimes\bar{R}^i-\tilde{u}^i{}_m\tilde{u}^m{}_l\bar{R}_i\otimes\bar{R}^l)=$$

$$=\frac{1}{2}(\delta^{il}\,\bar{R}_i\otimes\bar{R}^l-\tilde{u}^i{}_m\tilde{u}^m{}_l\,\bar{R}_i\otimes\bar{R}^l)=$$

$$=\frac{1}{2}(\bar{R}_i\otimes\bar{R}^l)(\delta^{il}-\tilde{u}^i{}_m\tilde{u}^m{}_l)=$$

$$=\frac{1}{2}(\bar{R}_i\otimes\bar{R}_p)(g^{lp}\delta^{il}-\tilde{u}^i{}_m\tilde{u}^m{}_l\cdot g^{lp})=\frac{1}{2}(\bar{R}_i\otimes\bar{R}_p)\cdot$$

$$\cdot(g^{ip}-\tilde{u}^i{}_m\tilde{u}^{mp})=\frac{1}{2}(\bar{R}^k\otimes\bar{R}^l)(g_{kl}-\tilde{u}_{km}\tilde{u}^m{}_l)$$

$$\overset{(II)}{C}=E-u^{-1}=\bar{R}_m\otimes\bar{R}^m-\tilde{u}^{mn}\bar{R}_m\otimes\bar{R}_n=g\,\bar{R}_m\otimes\bar{R}_n$$

$$-\tilde{u}^{mn}\bar{R}_m\otimes\bar{R}_n=(\bar{R}_m\otimes\bar{R}_n)(g^{mn}-\tilde{u}^{mn})=$$

$$=(\bar{R}^i\otimes\bar{R}_n)(\delta^n_j-\tilde{u}^n_i)=(\bar{R}^i\otimes\bar{R}^j)(g_{jn}\delta^n_i-\tilde{u}_{ij})=$$

$$=(\bar{R}^i\otimes\bar{R}^j)(g_{ji}-\tilde{u}_{ij})$$

$$\overset{(V)}{C}=\frac{1}{2}(u^2-E)=\frac{1}{2}(u^i{}_m u^{mn}\bar{R}_i\otimes\bar{R}_n-\bar{R}_i\otimes\bar{R}^i)=$$

$$=\frac{1}{2}(\bar{R}_i\otimes\bar{R}_n)(u^i{}_m u^{mn}-g^{in})=\frac{1}{2}(\bar{R}_i\otimes\bar{R}^p)(u^i{}_m u^m{}_p$$

$$=\frac{1}{2}(\bar{R}^t\otimes\bar{R}^p)(u_{tm}u^m{}_p-g_{tp})$$

$$\overset{(I)}{G} = -\frac{1}{2} G^{-1} = -\frac{1}{2}(F^{-1} \cdot F^{-T}) = -\frac{1}{2} \frac{\partial \overset{\circ}{x}^l}{\partial x^i} \frac{\partial x^l}{\partial x^k} \bar{R}_i \otimes \bar{R}_k \cdot$$

$$\cdot \frac{\partial \overset{\circ}{x}^n}{\partial x^j} \frac{\partial x^n}{\partial x^p} \bar{R}_j \otimes \bar{R}_p = -\frac{1}{2} \frac{\partial \overset{\circ}{x}^l}{\partial x^i} \frac{\partial x^l}{\partial x^k} \frac{\partial \overset{\circ}{x}^n}{\partial x^j} \frac{\partial x^n}{\partial x^p} \bar{R}_i \otimes \bar{R}_k \cdot$$

$$\cdot \bar{R}_j \otimes \bar{R}_p = -\frac{1}{2} \frac{\partial \overset{\circ}{x}^l}{\partial x^i} \frac{\partial x^l}{\partial x^k} \frac{\partial \overset{\circ}{x}^n}{\partial x^j} \frac{\partial x^n}{\partial x^p} g_{kj} \bar{R}_i \otimes \bar{R}_p =$$

$$= -\frac{1}{2} \frac{\partial \overset{\circ}{x}^l}{\partial x^i} \frac{\partial x^l}{\partial x^k} \frac{\partial \overset{\circ}{x}^n}{\partial x^j} \frac{\partial x^n}{\partial x^p} g_{kj}\, g_{pt} \bar{R}_i \otimes \bar{R}^t =$$

$$= -\frac{1}{2} \frac{\partial \overset{\circ}{x}^l}{\partial x^i} \frac{\partial x^l}{\partial x^k} \frac{\partial \overset{\circ}{x}^n}{\partial x^j} \frac{\partial x^n}{\partial x^p} g_{kj}\, g_{pt}\, g_{iq} \bar{R}^q \otimes \bar{R}^t$$

$$\overset{(II)}{G} = -U^{-1} = -\tilde{U}^{ij} \bar{R}_i \otimes \bar{R}_j = -\tilde{U}^i_m \bar{R}_i \otimes \bar{R}^m = -\tilde{U}_{km} \bar{R}^k \otimes \bar{R}^m$$

$$\overset{(III)}{G} = U = U^{ij} \bar{R}_i \otimes \bar{R}_j = U^i_p \bar{R}_i \otimes \bar{R}^p = U_{tp} \bar{R}^t \otimes \bar{R}^p$$

$$\overset{(IV)}{G} = \frac{1}{2} G = \frac{1}{2} F^T \cdot F = \frac{1}{2} \frac{\partial x^i}{\partial \overset{\circ}{x}^l} \frac{\partial x^k}{\partial x^l} \frac{\partial x^j}{\partial \overset{\circ}{x}^m} \frac{\partial x^n}{\partial x^m} g_{ij} \bar{R}_k \otimes \bar{R}_n =$$

$$= \frac{1}{2} \frac{\partial x^i}{\partial \overset{\circ}{x}^l} \frac{\partial x^k}{\partial x^l} \frac{\partial x^j}{\partial \overset{\circ}{x}^m} \frac{\partial x^n}{\partial x^m} g_{ij}\, g_{ns} \bar{R}_k \otimes \bar{R}^s =$$

$$= \frac{1}{2} \frac{\partial x^i}{\partial \overset{\circ}{x}^l} \frac{\partial x^k}{\partial x^l} \frac{\partial x^j}{\partial \overset{\circ}{x}^m} \frac{\partial x^n}{\partial x^m} g_{ij}\, g_{ns}\, g_{kt} \bar{R}^t \otimes \bar{R}^s$$