$$\overset{(v)}{T} = \frac{\partial \overset{\circ}{x}^{l}}{\partial x^{m}} \bar{e}_l \otimes \bar{e}^{m} \cdot T^{ij} \bar{e}_i \otimes \bar{e}_j \cdot \frac{\partial \overset{\circ}{x}^{s}}{\partial x^{p}} \bar{e}^{p} \otimes \bar{e}_s =$$

$$= \frac{\partial \overset{\circ}{x}^{l}}{\partial x^{m}} T^{ij} \frac{\partial \overset{\circ}{x}^{s}}{\partial x^{p}} \cdot \bar{e}_l \otimes \bar{e}^{m} \cdot \bar{e}_i \otimes \bar{e}_j \cdot \bar{e}^{p} \otimes \bar{e}_s \Rightarrow$$

$$\Rightarrow \overset{(v)}{T} = \frac{\partial \overset{\circ}{x}^{l}}{\partial x^{m}} \frac{\partial \overset{\circ}{x}^{s}}{\partial x^{p}} T^{mp} \bar{e}_l \otimes \bar{e}_s$$

Самостоятельная работа

① Выразим $\overset{(u)}{T}$ в $\bar{R}_i \otimes \bar{R}_j$

$$F = \bar{R}_i \otimes \overset{\circ}{\bar{R}}{}^{i} \Rightarrow$$

$$F^{T} = \overset{\circ}{\bar{R}}{}^{i} \otimes \bar{R}_i \, ; \quad F^{-1} = \overset{\circ}{\bar{R}}_i \otimes \bar{R}^{i} \, ; \quad F^{-1T} = \bar{R}^{i} \otimes \overset{\circ}{\bar{R}}_i$$

$$\overset{\circ}{\bar{R}}{}^{i} = \frac{\partial x^{i}}{\partial \overset{\circ}{x}^{l}} \bar{e}^{l} \, ; \quad \bar{R}_k = \frac{\partial x^{l}}{\partial x^{k}} \bar{e}_l \Rightarrow \overset{\circ}{\bar{R}}{}^{i} = \frac{\partial x^{i}}{\partial \overset{\circ}{x}^{l}} \cdot \frac{\partial x^{k}}{\partial x^{l}} \bar{R}_k$$

$$\overset{\circ}{\bar{R}}_i = \frac{\partial \overset{\circ}{x}^{l}}{\partial x} \bar{e}_l \, ; \quad \bar{e}_l = \frac{\partial x^{k}}{\partial x^{l}} \bar{R}_k \Rightarrow \overset{\circ}{\bar{R}}_i = \frac{\partial \overset{\circ}{x}^{l}}{\partial x^{i}} \cdot \frac{\partial x^{k}}{\partial x^{l}} \bar{R}_k$$

$$F = \frac{\partial x^{i}}{\partial \overset{\circ}{x}^{l}} \cdot \frac{\partial x^{k}}{\partial x^{l}} \bar{R}_i \otimes \bar{R}_k$$

$$F^{-1} = \frac{\partial \overset{\circ}{x}^{l}}{\partial x^{i}} \frac{\partial x^{l}}{\partial x^{k}} \bar{R}_i \otimes \bar{R}_k$$

$$F^{T} = \frac{\partial x^{i}}{\partial \overset{\circ}{x}^{l}} \frac{\partial x^{k}}{\partial x^{l}} \bar{R}_k \otimes \bar{R}_i$$

$$F^{-1T} = \frac{\partial \overset{\circ}{x}^{l}}{\partial x^{0}} \frac{\partial x^{l}}{\partial x^{k}} \bar{R}_k \otimes \bar{R}_0$$

$$T = T^{mn}\bar{R}_m \otimes \bar{R}_n$$
$$O = O^{sp}\bar{R}_s \otimes \bar{R}_p$$
$$O^T = O^{sp}\bar{R}_p \otimes \bar{R}_s$$

1) $\overset{(I)}{T} = F^T \cdot T \cdot F$

$$\overset{(I)}{T} = \frac{\partial x^i}{\partial \overset{\circ}{x}^l}\frac{\partial x^k}{\partial \overset{\circ}{x}^l}\bar{R}_k \otimes \bar{R}_i \cdot T^{mn}\bar{R}_m \otimes \bar{R}_n \cdot \frac{\partial x^p}{\partial \overset{\circ}{x}^s}\frac{\partial x^t}{\partial x^s}\bar{R}_p \otimes \bar{R}_t =$$

$$= \frac{\partial x^i}{\partial \overset{\circ}{x}^l}\cdot\frac{\partial x^k}{\partial x^l} T^{mn}\cdot\frac{\partial x^p}{\partial \overset{\circ}{x}^s}\frac{\partial x^t}{\partial x^s}\bar{R}_k \otimes \bar{R}_i \cdot R_m \otimes \bar{R}_n \cdot \bar{R}_p \otimes \bar{R}_t =$$

$$= \frac{\partial x^i}{\partial \overset{\circ}{x}^l}\frac{\partial x^k}{\partial x^l}\frac{\partial x^p}{\partial \overset{\circ}{x}^s}\frac{\partial x^t}{\partial x^s} T_{ip}\bar{R}_k \otimes \bar{R}_t$$

$$\overset{(I)}{T} = \frac{\partial x^i}{\partial \overset{\circ}{x}^l}\frac{\partial x^k}{\partial x^l}\frac{\partial x^p}{\partial \overset{\circ}{x}^s}\frac{\partial x^t}{\partial x^s} T_{ip}\bar{R}_k \otimes \bar{R}_t$$

2) $\overset{(II)}{T} = \frac{1}{2}(F^T \cdot T \cdot O + O^T \cdot T \cdot F)$

$$\overset{(II)}{T} = \frac{1}{2}\Big(\frac{\partial x^i}{\partial \overset{\circ}{x}^l}\frac{\partial x^k}{\partial x^l}\cancel{\frac{\partial x^p}{\partial \overset{\circ}{x}^s}}\bar{R}_k \otimes \bar{R}_i \cdot T^{mn}\bar{R}_m \otimes \bar{R}_n \cdot O^{sp}\bar{R}_s \otimes \bar{R}_p +$$
$$+ O^{sp}\bar{R}_p \otimes \bar{R}_s \cdot T^{mn}\bar{R}_m \otimes \bar{R}_n \cdot \frac{\partial x^i}{\partial \overset{\circ}{x}^l}\frac{\partial x^k}{\partial x^l}\bar{R}_i \otimes \bar{R}_k\Big) =$$

$$= \frac{1}{2}\frac{\partial x^i}{\partial \overset{\circ}{x}^l}\frac{\partial x^k}{\partial x^l} O^{sp}\big(T^{mn}\bar{R}_k \otimes \bar{R}_i \cdot \bar{R}_m \otimes \bar{R}_n \cdot \bar{R}_s \otimes \bar{R}_p +$$

$$+ T^{mn}\bar{R}_p \otimes \bar{R}_s \cdot \bar{R}_m \otimes \bar{R}_n \cdot \bar{R}_i \otimes \bar{R}_k\big) =$$

$$= \frac{1}{2}\frac{\partial x^i}{\partial \overset{\circ}{x}^l}\frac{\partial x^k}{\partial x^l} T_{is} O^{sp}(\bar{R}_k \otimes \bar{R}_p + \bar{R}_p \otimes \bar{R}_k)$$

$$\overset{(II)}{T} = \frac{1}{2}\frac{\partial x^i}{\partial \overset{\circ}{x}^l}\frac{\partial x^k}{\partial \overset{\circ}{x}^l} T_{is}\, O^{sp}(\bar{R}_k \otimes \bar{R}_p + \bar{R}_p \otimes \bar{R}_k)$$

3) $\overset{(III)}{T} = O^T \cdot T \cdot O$

$$\overset{(III)}{T} = O^{sp}\, \bar{R}_p \otimes \bar{R}_s \cdot T^{mn} \bar{R}_m \otimes \bar{R}_n \cdot O^{ij} \bar{R}_i \otimes \bar{R}_j =$$

$$= O^{sp}\, T^{mn}\, O^{ij}\, \bar{R}_p \otimes \bar{R}_s \cdot \bar{R}_m \otimes \bar{R}_n \cdot R_i \otimes \bar{R}_j =$$

$$= O^{sp}\, T_{si}\, O^{ij}\, \bar{R}_p \otimes \bar{R}_j$$

4) $\overset{(IV)}{T} = \frac{1}{2}(F^{-1} \cdot T \cdot O + O^T \cdot T \cdot F^{-1T})$

$$\overset{(IV)}{T} = \frac{1}{2}\Big(\frac{\partial \overset{\circ}{x}^l}{\partial x^i} \cdot \frac{\partial x^l}{\partial x^k} \bar{R}_i \otimes \bar{R}_k \cdot T^{mn} \bar{R}_m \otimes \bar{R}_n \cdot O^{sp} \bar{R}_s \otimes \bar{R}_p +$$

$$+ O^{sp} \bar{R}_p \otimes \bar{R}_s \cdot T^{mn} \bar{R}_m \otimes \bar{R}_n \cdot \frac{\partial \overset{\circ}{x}^l}{\partial x^i}\frac{\partial x^l}{\partial x^k} \bar{R}_k \otimes \bar{R}_i\Big) =$$

$$= \frac{1}{2}\frac{\partial \overset{\circ}{x}^l}{\partial x^i}\frac{\partial x^l}{\partial x^k} O^{sp} \cdot \big(T^{mn} \bar{R}_i \otimes \bar{R}_k \cdot \bar{R}_m \otimes \bar{R}_n \cdot \bar{R}_s \otimes \bar{R}_p +$$

$$+ T^{mn} \bar{R}_p \otimes \bar{R}_s \cdot \bar{R}_m \otimes \bar{R}_n \cdot \bar{R}_k \otimes \bar{R}_i\big) =$$

$$= \frac{1}{2}\frac{\partial \overset{\circ}{x}^l}{\partial x^i}\frac{\partial x^l}{\partial x^k} O^{sp} T_{ks} (\bar{R}_i \otimes \bar{R}_p + \bar{R}_p \otimes \bar{R}_i)$$

$$\overset{(IV)}{T} = \frac{1}{2}\frac{\partial \overset{\circ}{x}^l}{\partial x^i}\frac{\partial x^l}{\partial x^k} O^{sp} T_{ks} (\bar{R}_i \otimes \bar{R}_p + \bar{R}_p \otimes \bar{R}_i)$$

5) $\overset{(V)}{T} = F^{-1} T \cdot F^{-1T}$

$$\overset{(V)}{T} = \frac{\partial \overset{\circ}{x}^l}{\partial x^i}\frac{\partial x^l}{\partial x^k}\frac{\partial \overset{\circ}{x}^t}{\partial x^j}\frac{\partial x^t}{\partial x^p} T^{mn} \bar{R}_i \otimes \bar{R}_k \cdot \bar{R}_m \otimes \bar{R}_n \cdot \bar{R}_p \otimes \bar{R}_j =$$

$$= \frac{\partial \tilde{x}^l}{\partial x^i}\frac{\partial x^l}{\partial x^k}\frac{\partial \tilde{x}^t}{\partial x^j}\frac{\partial x^t}{\partial x^p} T_{kp}\, \bar{R}_i \otimes \bar{R}_j$$

$$\overset{(v)}{T} = \frac{\partial \tilde{x}^l}{\partial x^i}\frac{\partial x^l}{\partial x^k}\frac{\partial \tilde{x}^t}{\partial x^j}\frac{\partial x^t}{\partial x^p} T_{kp}\, \bar{R}_i \otimes \bar{R}_j$$

(2) $O^{-1} = O^T$; O – ортогон. ; O^{sp} – ?

$O = O^{sp}\,\bar{e}_s \otimes \bar{e}_p$;

$O^T = O^{sp}\,\bar{e}_p \otimes \bar{e}_s$

$$E = O \cdot O^{-1} = O \cdot O^T = O^{sp}\,\bar{e}_s \otimes \bar{e}_p \cdot O^{mn}\,\bar{e}_n \otimes \bar{e}_m =$$

$$= O^{sp} O^{mn}\,\bar{e}_s \otimes \bar{e}_p \cdot \bar{e}_n \otimes \bar{e}_m = O^{sp} O^{m}{}_{p}\,\bar{e}_s \otimes \bar{e}_m = \delta_{sm}\,\bar{e}_s \otimes \bar{e}_m \Rightarrow$$

$$\Rightarrow O^{sp} O^{m}{}_{p} = \delta_{sm} = O^{sp} O_{mp} \quad \Rightarrow$$

$\Rightarrow$ на 9 компонент тензора O наложено 6 связей $\Rightarrow$ произвольный ортогон. тензор имеет не более 3 независим. комп.