

Formalización de las matemáticas con Lean.  
Un caso de estudio: Geometría euclídea plana.

Facultad de Ciencias Matemáticas.  
Trabajo dirigido por Jorge Carmona Ruber.

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Septiembre de 2023

## Proposición 1

```
lemma line_has_external_point
  {Point Line : Type*} [ig : incidence_geometry Point Line] :
   $\forall l : \text{Line}, \exists A : \text{Point}, \neg A \sim l :=$ 
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begin
  rcases ig.I3 with ⟨A, B, C, ⟨_, h1⟩⟩,

end

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l : Line
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  have hA1 :  $A \sim l$ , { tauto },
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  specialize h1 l hA1 hB1,
  exact h1 hC1
end
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