

Vote Selling in the United States: Introducing Support Vector Machine Methods to Analyzing Conjoint Experimental Data

HÉCTOR BAHAMONDE ^{*1} and
CRISTOBAL QUININAO ^{†2}

¹Assistant Professor, Instituto de Ciencias Sociales, O'Higgins University

²Assistant Professor, Instituto de Ciencias de la Ingeniería O'Higgins University

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Abstract

Democracy has traditionally been theorized as a multidimensional concept (Dahl 1971). Yet, the quantitative study of clientelism—as a democratic failure—has been mostly studied from a unidimensional perspective. We argue that to better understand clientelism, quantitative studies should situate the phenomena within the multidimensionality of democracy to understand which specific subdimension should fail to explain vote selling. By leveraging a conjoint experiment on hypothetical vote selling in the United States—a traditionally considered consolidated democracy—this paper makes both methodological and substantive contributions to the literature. Using machine learning techniques, the paper identifies that among the five conjoint attributes considered in this study, the one related to the co-dependence of the Presidency on Congress is the only one that explains individual levels of vote selling. These findings are particularly relevant to the study of American democracy given the fact that the data were collected during the campaign period where Donald Trump was elected.

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Keywords— conjoint designs; vector support machines; support for democracy; United States.

^{*}hector.bahamonde@uoh.cl; www.HectorBahamonde.com.

[†]cristobal.quininao@uoh.cl; <https://cquininao.wordpress.com>.

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I. TOWARD A MULTIDIMENSIONAL STUDY OF CLIENTELISM

Democracy has been theorized as a multidimensional concept. Specifically referring to *polyarchies*, Dahl (1971, p. 3) explains that full democracies should satisfy a number of dimensions which speak to certain institutional guarantees that create opportunities to (1) formulate political and social preferences, (2) signify those preferences, and (3) have preferences weighted equally when conducting a government.

Yet, clientelism—as a democracy failure—has been studied almost exclusively from a unidimensional perspective. We believe that there exists a methodological and conceptual alignment—one that biases our inferences. On the one hand, qualitative, historical and/or ethnographically-based contributions describe clientelist transactions as complex and multidimensional. By employing qualitative techniques, researchers are able to provide “thick descriptions” (Goertz 1973) of the phenomena at hand (Posada-Carbó 1996; Sabato 2001; Auyero 2000; Szwarcberg 2013; Borges 2019). On the other hand, statistical, survey, and/or experimentally-based work mostly explores singular aspects related to clientelism—typically, the effect of a single variable (or treatment) on the probability of clientelism.¹ For example, using a field experiment in Benin, Wantchekon (2003) stresses the role of “incumbency” on vote buying, while Jensen and Justesen (2014, p. 227) focus on the impact of “poverty” on vote buying. While the quantitative literature on clientelism has advanced on a number of important questions, most studies concentrate their efforts on a single variable which (when possible) is manipulated in an experimental or quasi-experimental design (Corstange 2012; Imai, Bethany Park, and Kenneth Greene 2015; Nichter and Peress 2017; Hicken et al. 2015; Hicken et al. 2018; Michael and Thachil 2018; Bratton 2008; Weitz-Shapiro 2012; González-Ocantos, Kiewiet de Jonge, and Nickerson 2014; Bahamonde 2018; Bahamonde 2020; Oliveros 2016). Since the approach (i.e. unicausal/multicausal) is correlated with the method (quantitative/qualitative), we believe this methodological and conceptual alignment represents an important gap in the literature.

Substantively, we argue that to better understand the motivations behind clientelism and the micro-dynamics that drive it, studies should situate the phenomena within the *multidimensionality of democracy*. In other words, What are the *causes* of clientelism? Which *dimensions* of democracy—as described by Dahl (1971)—should fail to produce clientelism? While qualitative researchers are better equipped to properly answer these questions, there are some quantitative techniques that might provide broader explanations for the causes of clientelism. We do not argue that these quantitative tools might give us the kind of rich explanations ethnographies provide. However, we hope this paper provides multidimensional answers to a multidimensional concept within the quantitative framework. Ultimately, this paper tries to provide a multidimensional explanation for clientelism within the “effects of causes” approach (Pearl 2015). Exploiting a novel conjoint dataset, this paper developed an experimental design which sought to answer *Which of the three democratic dimensions explained by Dahl (1971) should fail to produce clientelism in the United States?*

Since the vote-buying literature mostly considers developing countries and describes vote sellers as

¹Quantitative scholars then usually focus on the “effects of causes” rather than on the “causes of effects” (Pearl 2015).

poor (Weitz-Shapiro 2014, p. 12), uneducated (González-Ocantos, Kiewiet de Jonge, and Nickerson 2014), and undemocratic (Carlin and Moseley 2015), the willingness to sell votes in the United States should be low, making it a difficult case study on vote selling.² And such, this study follows a “least-likely” design presenting the United States as a “crucial case.” As Levy (2008, p. 12) explains, “[i]nferential leverage from a least likely case is enhanced if our theoretical priors for the leading alternative explanation make it a most likely case for that theory.” However, the evidence that this paper presents may be associated with a probable erosion of American democracy (Levitsky and Ziblatt 2018). Foa and Mounk (2016, p. 7) document a deep “crisis of democratic legitimacy [that] extends across a [...] wider set of indicators” in the United States. They find that 26% of millennials declare that it is “unimportant” in a democracy for people to “choose their leaders in free elections” (Foa and Mounk (2016, p. 10) and Foa and Mounk (2017)). Our study aims to contribute to this debate by presenting experimental evidence that links the democracy theory literature with the clientelism literature.

The methodological contribution of this paper is twofolds. First, this paper contributes to the literature by leveraging a conjoint experiment on hypothetical vote selling in the United States, a traditionally considered consolidated democracy. Most quantitative studies have been conducted in developing countries, seriously narrowing the scope of our inferences. In part, this is because the clientelism literature usually focuses on realized behaviors only—that is, actual clientelist transactions. Unfortunately, by ignoring attitudes of *potential* vote sellers, particularly when it comes to the willingness to sell, selection bias seriously threatens causal inferences. Geddes (1990, p. 131) explains the well-known selection issues of studying “only cases that have achieved the outcome of interest.” Thus, and following the lead of González-Ocantos, Kiewiet de Jonge, and Nickerson (2014) and Bahamonde (2020), this paper presents experimental evidence of hypothetical willingness to sell the vote in the United States.

Second, we introduce machine learning techniques, particularly support vector machine analyses (SVM) for analyzing conjoint datasets. SVMs rely on computational algorithms that solve classification problems in a data-driven fashion. One of the main advantages of SVM methods is that their optimal classification properties are robust even when working with multiple dimensions at the same time. Since this paper makes the case for a multidimensional approach to the study of clientelism, we claim the method is appropriate and relevant to the discipline. From a technical standpoint, SVMs separate groups of observations in a hyperplane. A hyperplane is a geometrical space where observations—survey respondents, in this case—are located. As explained later, the three democracy dimensions described by Dahl (1971) were operationalized in five different subdimensions (i.e. conjoint attributes). This paper employed SVM methods to group most-similar observations according to their preferences toward the five conjoint attributes. After the classification problem was solved, standard regression techniques were used to study correlations between the preferences of survey participants toward the five conjoint attributes and their willingness to sell the vote—captured by a question asked during the same study.

We claim this set of techniques might provide new insights about the *specific* democracy

²However see Bahamonde (2020).

dimensions (Dahl 1971) that should fail to make vote selling most likely. In fact, we find that among the five attributes, the only one that ought fail to make vote selling most likely, is the presence of presidential checks and balances. That is, individuals who systematically do not value accountable Presidents, thus preferring a fully autonomous President—one that governs *without* a Congress—are more likely to sell their vote. We believe that this contribution is novel. The authors are not aware of any study in the quantitative literature of clientelism that conceives democracy as a multidimensional concept. In fact, the literature usually assumes that clientelism emerges when “democracy”—as a whole—fails. This paper takes a step further explaining which *specific* democracy attribute ought to fail to make vote selling most likely.

The paper continue as follows. **First**, we explain the logic of conjoint analyses and their main contribution to political science. **Second**, we introduce a novel dataset of U.S. voters representative at the country level. In that section we **analyze** the conjoint dataset exploiting the traditional conjoint approach and explain its shortcomings. **Third**, we introduce the support vector machine approach to analyzing conjoint data, and proceed to explaining its main advantages and show our main results. Finally, in the **Appendix** section we make available **R** (**Listing 1**) and **Python** (**Listing 2**) routines not only to replicate the data analyses in this study, but also to spread these methods among interested researchers, to hopefully move this area of research forward. The **final section** concludes.

II. TRADITIONAL CONJOINT ANALYSES

Conjoint designs ask respondents to choose from hypothetical profiles that combine multiple attributes, “enabling researchers to estimate the relative influence of each attribute value on the resulting choice or rating” (Hainmueller, Hopkins, and Yamamoto 2014, p. 2). Typically, survey participants are given a number of “tasks” where they have to make a number of choices between—usually—two set of profiles. It is generally accepted that Luce and Tukey (1964) started the conjoint design (Green and Srinivasan 1978; Franchino and Zucchini 2015).

This methodology has been widely used in marketing research to measure “consumer trade-offs among multi-attributed products and services” (Lenk et al. 1996, p. 174). Typically, researchers in that field would assign arbitrary utilities to investigate “how much difference each attribute could make in the total utility of a product” (Orme 2010, p. 79). Utilities were assigned according to general expectations, for instance, a “respondent generally prefers higher gas mileage to lower gas mileage” (Green and Srinivasan 1978, p. 107). At the time this seemed particularly interesting given the impossibility to truly randomize the set of attributes. Hence, the analyst needed to set the utilities associated with every attribute in advance, usually building a small number of attribute profiles or “combinations” (Lenk et al. 1996, p. 175). Much research was done arguing how ranked attributes or attribute ratings were better than using assigned utilities (Carmone, Green, and Jain 1978, p. 301). For instance, Louviere, Flynn, and Carson (2010, p. 60) criticize the use of arbitrary utilities assigned to every attribute, making traditional conjoint analyses incompatible with economic theory. Since early conjoint methods exploited the “additive measurements” of the utilities associated

with the respective attribute (Luce and Tukey 1964, p. 2), some times that led to non-accounted-for nonlinearities.

A number of topics have been studied, such as preferences for health care (Ryan 2000), vaccine decision making (Seanehia et al. 2017), preferences for energy-saving measures (Poortinga et al. 2003), preferences toward different food packagings (Silayoi and Speece 2007), consumer demand for fair trade (Hainmueller, Hiscox, and Sequeira 2015), evaluations of teaching performance (Kuzmanovic et al. 2013), roommate choice (Shafranek 2019) and renter behavior (Hankinson 2018).

Hainmueller, Hopkins, and Yamamoto (2014) “introduced conjoint analysis to political science as a survey experimental method for causal inference” (Horiuchi, Markovich, and Yamamoto 2020, p. 1), particularly making conjoint designs compatible with the potential outcomes framework of causal inference (Rubin 1974). Since then, a number of important studies have been published, making a very common tool for causal inference in political science (Cuesta, Egami, and Imai 2021). Just to name a few examples in political science, conjoint designs have been used to study attitudes toward immigrants (Hainmueller and Hopkins 2015), preferences toward political candidates (Franchino and Zucchini 2015; Horiuchi, Smith, and Yamamoto 2017; Horiuchi, Smith, and Yamamoto 2020; Mares and Visconti 2020), the role of candidate sex on voter choice (Ono and Burden 2019) and the role of the information environment in partisan voting (Peterson 2017). In part, this is due to the simplicity of the main quantity of interest developed by Hainmueller, Hopkins, and Yamamoto (2014, p. 3)—the *average marginal component effect* (AMCE).³ The quantity equals the counterfactual probability where a specific characteristic would be chosen if the value of that characteristic is absent (Hainmueller, Hopkins, and Yamamoto 2014, p. 11).⁴ Since the AMCE does not rely on arbitrary utility assignments nor does resort to functional form assumptions (Hainmueller, Hopkins, and Yamamoto 2014, p. 3), it has become a very common quantity of interest in political science, specially, because it also avoids “unnecessary statistical assumptions” at the same time that improves “internal validity than the more model-dependent procedures” (Hainmueller, Hopkins, and Yamamoto 2014, pp. 2–3).⁵ Importantly, they show that “when attribute levels are randomized independently from one another, the ordinary least squares (OLS) estimates of the coefficients from the linear regression of the choice indicator on the set of dummy variables for the levels of the attributes provide unbiased and consistent estimates of the AMCEs” (Horiuchi, Smith, and Yamamoto 2017, p. 14).⁶ Others have argued that when attribute levels are randomized, the design “reduces social desirability bias by providing many potential reasons for supporting or opposing a proposed [attribute]” (Hankinson

³Due to space concerns, we are not deriving the AMCE here. The AMCE has been well explained and widely used before. See Equation 5 in Hainmueller, Hopkins, and Yamamoto (2014, p. 11). In addition to that, see Egami and Imai (2019), who have introduced another quantity of interest, the average marginal interaction effect (AMIE).

⁴Importantly, Leeper, Hobolt, and Tilley (2020, p. 6) explain that arbitrary choice of reference category when computing the AMCE might introduce “highly distorted descriptive interpretations of preferences among subgroups of respondents.”

⁵Yet, some necessary assumptions need to be made. For instance, in order to make statistical inferences, the AMCE depends on (clustered) standard errors, which in turn rely on the central limit theorem. See Hainmueller, Hopkins, and Yamamoto (2014, p. 17).

⁶Hainmueller, Hopkins, and Yamamoto (2014, p. 15) show that OLS estimators have “identical” properties to the subclassification estimators, and therefore this “implies that the linear regression estimator is fully nonparametric.”

2018, p. 7),⁷ while others offer guidance regarding the number of attributes by developing a two-stage conjoint design (Bansak et al. 2019).

III. STUDYING CLIENTELISM MULTIDimensionALLY VIA CONJOINT DESIGNS

Our multidimensional approach toward the study of vote selling is novel in the quantitative literature. Quantitative contributions on vote buying, vote selling and clientelism in general, are usually unidimensional. Survey experiments have been widely used to study this phenomena. For instance, Bahamonde (2020), González-Ocantos, Jonge, et al. (2012), González-Ocantos, Kiewiet de Jonge, and Nickerson (2014), and González-Ocantos, Kiewiet de Jonge, and Nickerson (2015) use list experiments to study the effect of selling prices or specific issues related to norms and legitimacy on the probability of vote selling. While these and other studies have advanced a number of important questions in the discipline, unfortunately, they are able to study one aspect at a time, mainly, by manipulating a word, a sentence, a framing or a price. As Hainmueller, Hopkins, and Yamamoto (2014, p. 2) point out, these designs “have an important limitation for analyzing multidimensional decision making.” We fill this gap by introducing a multidimensional conjoint-based approach to studying vote selling in the United States.

Our contribution builds directly on Carlin and Singer (2011), Carlin and Moseley (2015), and Carlin (2018). Using survey data, they build a series of multidimensional indexes to measure—in Dahlian terms—attitudes towards democracy. Particularly, using the Q-Method and cluster analyses, they account for the multifaceted views towards a democracy. Considering their operationalization strategy of Dahl’s conceptualization of democracy (Table 1) but also leveraging the Hainmueller, Hopkins, and Yamamoto (2014) approach to designing conjoint experiments, we implemented a conjoint design (Table 2) aimed to studying the multidimensionality of conditions that make vote selling most likely in the United States. Particularly, we are interested in specifying which of the three democracy dimensions of Dahl (1971) ought to fail to make vote selling most likely in the United States.

Conjoint designs are suitable tools to “determine which components of the manipulation produce the observed effect” (Hainmueller, Hopkins, and Yamamoto 2014, p. 2). Following Dahl (1971), Table 1 specifies three general dimensions that should be satisfied for a country to be considered democratic (first column). Every dimension has a number of requirements (second column). Based on Carlin and Singer (2011), Carlin and Moseley (2015), and Carlin (2018), we operationalized these requirements for the conjoint experiment by devising five attributes (third column): (1) *media can confront the government*, (2) *president cannot rule without congress*, (3) *citizens can vote in the next two elections*, (4) *citizens can run for office for the next two elections* and (5) *citizens can associate with others and form groups*.

Given that conjoint designs are able “to identify the causal effects of various components of a treatment in survey experiments” (Hainmueller, Hopkins, and Yamamoto 2014, p. 2), we claim

⁷However, see Horiuchi, Markovich, and Yamamoto (2020).

Dahl's Polyarchy Dimension	Dahl's Requirements for a Democracy	Experimental Operationalization for Conjoint Design
Formulate preferences	Freedom of expression	Media can confront the government
	Alternative sources of information	Media can confront the government
	Right of political leaders to compete for support	President cannot rule without Congress
	Right to vote	Citizens can vote in the next two elections
	Freedom to form and join organizations	Citizens can associate with others and form groups
Signify preferences	Freedom of expression	Media can confront the government
	Alternative sources of information	Media can confront the Government
	Right of political leaders to compete for support	President cannot rule without Congress
	Right to vote	Citizens can vote in the next two elections
	Free and fair elections	Citizens can vote in the next two elections
	Eligibility for public office	Citizens can run for office for the next two elections
	Freedom to form and join organizations	Citizens can associate with others and form groups
Preferences are weighted equally in conduct of government	Freedom of expression	Media can confront the government
	Alternative sources of information	Media can confront the Government
	Right of political leaders to compete for support/votes	President cannot rule without Congress
	Right to vote	Citizens can vote in the next two elections
	Free and fair elections	Citizens can vote in the next two elections
	Institutions for making government policies depend on votes and other expressions of preference	Citizens can vote in the next two elections
	Eligibility for public office	Citizens can run for office for the next two elections
	Freedom to form and join organizations	Citizens can associate with others and form groups

Table 1: Dimensions of Democracy (Dahl 1971) and Their Corresponding Experimental Operationalizations.

Note: Dahl (1971) specifies three general dimensions that should be satisfied for a country to be considered democratic (first column). Every dimension has a number of requirements (second column). Based on Carlin and Singer (2011), Carlin and Moseley (2015), and Carlin (2018), we operationalized these requirements for the conjoint experiment by devising five attributes (third column). As Table 2 shows, all participants were asked to choose between hypothetical candidates that either supported or rejected each of these five attributes.

that this is an appropriate tool to shed some light on the multi-causal study of clientelism. To study which democratic dimension(s) should fail to produce clientelism, we presented subjects (as in Table 2) two hypothetical candidates that supported (or not) every policy (attribute)—as operationalized in Table 1. We recognize that the resulting candidate profiles are highly unlikely. Unlikely profiles (such as doctors with no education) have been a big concern in the conjoint literature. So far the suggestion has been to delete them before hand by restricting randomization of certain unlikely profiles (Hainmueller, Hopkins, and Yamamoto 2014) or by marginalizing “factors over the target population distribution” via the population AMCE (Cuesta, Egami, and Imai 2021, p. 12). While acknowledging the advantages of both approaches, our goal is identifying a set of

<p>In the next section you will see 10 different candidates presented in pairs. Each candidate supports different policies. Some candidates might or might not share some similarities/differences. You might not like any of them, but we want to know which candidate represents the lesser of the two evils for you. You might want to focus your attention on the issues that you care about the most.</p>	
Candidate 1	Candidate 2
Media CAN confront the government	Media CANNOT confront the government
President CANNOT rule without Congress	President CAN rule without Congress
Citizens CANNOT vote in the next two elections	Citizens CANNOT vote in the next two elections
Citizens CAN run for office for the next two elections	Citizens CAN run for office for the next two elections
Citizens CAN associate with others and form groups	Citizens CANNOT associate with others and form groups
Which of these candidates represents the lesser of the two evils for you?	
Candidate 1 <input type="checkbox"/>	Candidate 2 <input type="checkbox"/>

Table 2: A Multidimensional Approach to Studying Clientelism: A Conjoint Design (example).

Note: Participants were asked to choose between two hypothetical candidates (*Candidate 1* and *Candidate 2*). Every entry was filled at random according to the five different attributes explained in [Table 1](#). In practice, every subject chose between two unique hypothetical candidates. Note that in order to highlight the differences between the two candidates, the can and cannot were capitalized. The idea was to minimize experimental fatigue.

democratic attributes that, when absent, make clientelism more likely. In fact, external validity seems to be the trade-off when building a case study according to the *least*-likely case design (Levy 2008). In addition, one of the methodological contributions of this paper is to overcome selection bias by studying *hypothetical* behaviors, specially the ones where the outcome of interest has not been produced (Geddes 1990). And finally, there are several survey experiments that have fielded hypothetical questions, mostly putting respondents in experimental conditions that do not necessarily mimic reality. For instance, Bahamonde (2020) finds that a big portion of U.S. voters would be willing to sell their vote to an hypothetical candidate in exchange for money, while Ballard-Rosa, Martin, and Scheve (2017) examine a number of tax proposals “that are infeasible in the real world politics” (Cuesta, Egami, and Imai 2021, p. 5).

[Table 2](#) shows one possible realization of the experiment. It is important to note that every attribute was randomly assigned, and consequently, every participant in practice chose between

two unique hypothetical candidates. Also, in order to minimize experimental fatigue, the CAN and CANNOT were capitalized.

The study considered a **direct question** about the intention to sell the vote. As a whole, the conjoint experiment was framed as a study about crime in the United States, not as a study about clientelism. Participants were asked to read **an excerpt** mentioning a number of crimes. All were formatted as news pieces. The idea was to explain “vote selling” to “newsreaders.” To further prevent bias, the direct question stated that there was the hypothetical possibility of doing one of the illegal things mentioned in the excerpt. And that this possibility would be randomly assigned. However, all participants were directly asked whether they would be interested in selling their vote. Following Bahamonde (2020), to capture the willingness to sell without the potential costs, participants were asked whether they would be willing to accept the offer, assuming they would not go to jail. After answering the conjoint portion of the study, participants were asked to answer a battery of socio-demographic and political questions.

Ultimately, our design will allow a series of hypotheses tests between every of the five democracy attributes and the vote selling question. Typical conjoint analyses offer descriptive associations between hypothetical attributes. While these analyses have advanced a number of important research avenues, they do not permit statistical associations between the selected profiles and the respondents attitudes or preferences. By introducing support vector machine techniques to analyzing conjoint experiments, we are able to do so. As we explain later, this approach improves our causal inferences by permitting statistical correlations between the selected conjoint profiles (democracy) and the respondents attitudes (vote selling). We also are able to control for other observables (the socio-demographic battery). Before presenting the machine learning approach, we first present our novel dataset and analyzes it first by using the traditional AMCE-based conjoint approach.

I. Classic Conjoint Data Approach

Collected in 2016, the data (N=1,108) are representative at the national level.⁸ Figure 1 shows the geographical distribution of survey respondents grouped by party identification. Following Hainmueller, Hopkins, and Yamamoto (2014), we computed the AMCE via the OLS estimator using clustered standard errors. In this section we present a classic conjoint analysis. Particularly, we show the hypothetical candidates’ attributes that were selected by survey respondents.

⁸1,108 respondents, everyone answering 5 tasks with 2 candidates each. *Research Now SSI* collected the data between March 2 and March 6 2016. Survey respondents belong to the online panel owned and administered by SSI. Notice of IRB exemption Protocol #E16-292 is kept on file at the Office of Research and Regulatory Affairs of University.

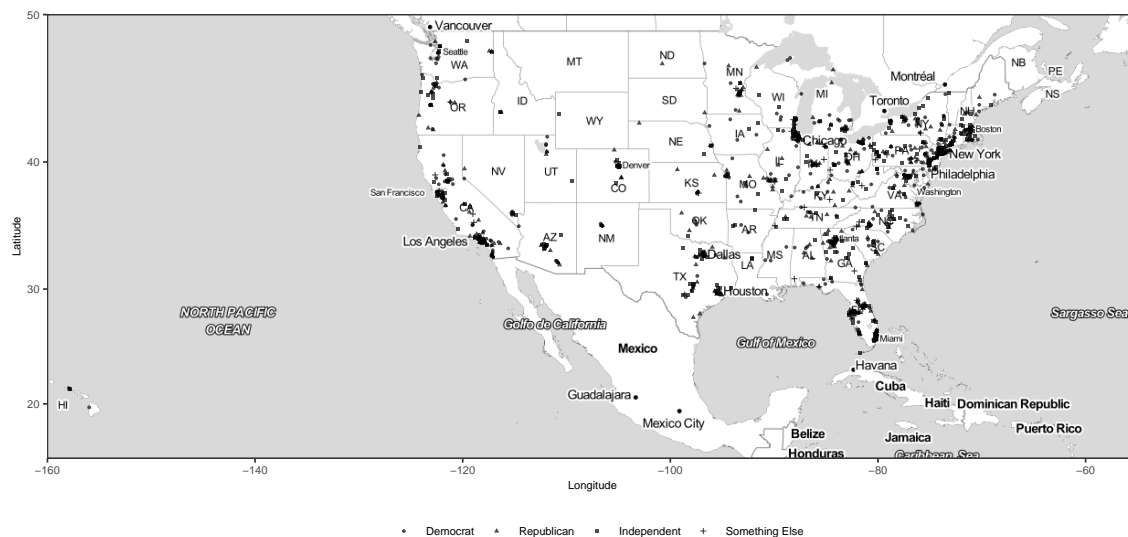


Figure 1: Geographical Distribution of Survey Respondents by Party Identification.

Note: The data ($N=1,108$) were collected by Research Now SSI between March 2 and March 6 2016 and are representative at the national level. Survey respondents belong to the online panel owned and administered by SSI.

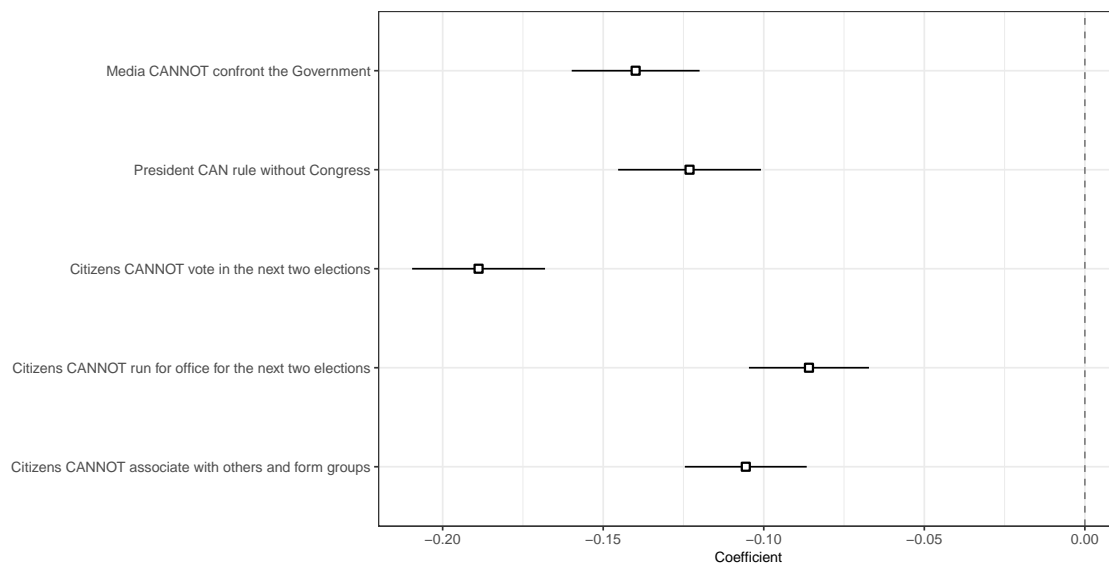


Figure 2: Classic AMCE Analysis: Candidate Selection and Dahl's Democratic Dimensions.

Note: Following Hainmueller, Hopkins, and Yamamoto (2014), the figure shows the corresponding AMCEs for every of the attributes explained in Table 1. All attributes are based on Dahl (1971). All reference categories were omitted—all of them are at the 0 vertical line and represent the opposite of the attribute shown in the plot. For substantive reasons, all categories displayed in the figure represent the non-democratic side of the attributes. The figure strongly suggests that respondents systematically preferred hypothetical candidates who supported democratic policies.

Figure 2 suggests that respondents systematically preferred hypothetical candidates who supported democratic policies. Authoritarian candidates that can rule without Congress, or political systems in which there is controlled mass media, or where citizens are not allowed to vote, run for office or associate with others, are systematically rejected by the nationally represented pool of respondents. These analyses are not surprising as they conform with our theoretical priors, i.e. the United States has (traditionally) been considered a strong democracy.

While classic conjoint analyses provide consistent causal estimates, they unfortunately overlook respondent’s preferences. As Figure 2 suggests, analysts can *describe* aggregate behaviors but cannot establish relationships between the respondent’s preferences and her attribute choices. In other words, analysts are not able to tell a story of *why* respondents choose what they chose. By implementing support vector machine methods we are able to introduce additional covariates—such as respondent socio-economic and political preferences—and explore the reasons that make vote selling more likely in the United States. Why we still believe description is “good science” (Gerring 2012), the descriptive nature of the classic conjoint design (for instance, via the AMCE) might (wrongly) suggest that democratic values scored high in the United States. After all, Figure 2 strongly indicates that non-democratic candidate attributes are systematically rejected (i.e. all coefficients are negative). And as a consequence, that might imply that the intention to sell the vote should be low. The ability to introduce covariates, such as the intention to sell the vote, might shed some light about how healthy or broken democratic values were at the time were the data were collected (which coincides with the campaign period that gave Donald Trump the U.S. presidency). We claim this is an exceptional opportunity to study democracy, and particularly, the democratic attitudes that, when broken, explain vote selling. Next section introduces support vector machine methods that allow analyzing conjoint data considering subject preferences and/or attitudes.

IV. INTRODUCING SUPPORT VECTOR MACHINES METHODS TO ANALYZING CONJOINT DATA

Support vector machine methods are a subclass of machine learning techniques, which in turn are a branch of the artificial intelligence field. The main goal of these methods is to use computational algorithms to learn from data (Miyamoto et al. 2015). For instance, in agricultural sciences machine learning methods are used to identify between crops and grass using satellite images as data. There are mainly three branches of machine learning techniques: supervised learning, unsupervised learning and reinforcement learning. Based on a subset of the complete dataset (the “training” dataset), the goal of supervised learning is to identify a mathematical function able to map a vector (“input”) to another vector (“output”) based on input-output data pairs which are used as examples.⁹ For instance, the analyst can define how crops and grass look like (output). Once the mathematical function is defined, supervised learning techniques should be able to automatically recognize newly

⁹A further demonstration is in the Appendix.

decia set,
ok?

decia find,
ok?

presented images of crops and grass (input) and classify them (map) into their respective categories (output)—i.e. “grass” or “crops.” Unsupervised learning is similar to supervised learning, but the training dataset have only the input component. Thus, the computational algorithms in this case should be able to find patterns in the dataset without having prior information. Finally, reinforcement learning was inspired by behavioral psychology, and its main goal is to determine what actions an algorithm should perform in order to maximize some notion of “accumulated reward.”

Support vector machines (SVMs) are a set of techniques of supervised learning which might be extended to classification applications and/or standard regression techniques (Maimon and Rokach 2005, Ch. 12). Supervised learning techniques have been used in political science before (D’Orazio et al. 2014; Kevin Greene, Baekkwon Park, and Colaresi 2019; Cantú and Saiegh 2011). In this paper we are interested in applying SVM methods to—first—solve a classification problem, and then in a second stage, apply OLS methods. The first stage is a data-driven process which consists of defining a full set of democratic attitudes, and then finding a classification rule (i.e. a mathematical function) that takes survey participants and classify them along the constructed democratic attitudes spectrum. In particular, we are interested in learning about the political preferences of survey participants by using the information participants provided when making choices between the two hypothetical candidates and the policies those candidates endorsed (as exemplified in Table 2). Since the focus is to account for the multi-dimensionality of democracy, we perform these analyses for all five democracy attributes. In the second stage we use standard regression methods to study correlations between the five democracy attributes (obtained in the first stage) and respondent’s socio-political attitudes, particularly, the willingness to sell the vote. In sum, we exploit the conjoint dataset to first classify all participants within a common space for all five attributes. Then we fit five separate OLS regression models between those five spaces and the willingness to sell the vote (controlling for relevant covariates).

Stating the problem Consider the following classification task: estimate a function $f : \mathbb{R}^J \rightarrow \{-1, 1\}$ using a system of input-output training data pairs. The f function should be able to map every participant attitudes toward the five democracy attributes and classify those attitudes within a common numeric space.

As exemplified in Table 2, all hypothetical political candidates took stances in favor (or against) the five democracy policies. For an input $\mathbf{x} \in \{0, 1\}^5$, which represents the range of possible political stances of a candidate, the function f maps survey participants with all democracy attributes, linking both in what could be described as the “decision” made by the survey participant. For instance, hypothetical candidates 1 and 2 in Table 2 are represented by vectors $(1, 1, 0, 1, 1)$ and $(0, 0, 0, 1, 0)$, respectively. Then, the function f connects the respondent’s attribute choices (embodied by hypothetical candidates 1 or 2) with the respondent herself (i.e. her “decision”).

Constructing the vector weight \mathbf{w}^i One way to motivate our approach is via the latent variable and the standard maximization utility approaches. For simplicity, we assume that participant preferences can be modeled by an additive linear utility function, and that the decision function

f takes a utility sign $\{+, -\}$ depending on the attribute chosen—more details are provided below. Note that coding sign is completely irrelevant to the substantive policies endorsed by the candidates. If the data can be partitioned by an hyperplane (which is the case in most randomized conjoint designs), such functions can be defined for every participant i .

Survey participants evaluate between two hypothetical candidates and choose one. They do so for all five tasks. To clarify, for a survey participant i , we have the following conjoint data,

$$(\mathbf{x}_1^{1,i}, \mathbf{x}_1^{2,i}; y_1^i), (\mathbf{x}_2^{1,i}, \mathbf{x}_2^{2,i}; y_2^i), \dots (\mathbf{x}_5^{1,i}, \mathbf{x}_5^{2,i}; y_5^i) \quad (1)$$

where $\mathbf{x}_k^{1,i}$ represents the attributes of candidate 1 shown to citizen i during task k . Similarly, $\mathbf{x}_k^{2,i}$ represents the attributes of candidate 2 shown to citizen i during task k . The corresponding y_k^i is the selected candidate. We coded $y_k^i = 1$ when survey participant i selected candidate 1, and $y_k^i = -1$ when survey participant i selected candidate 2. Since we are trying to characterize a linear function for every i , survey participants can be mapped by a vector weight $\mathbf{w}^i \in \mathbb{R}^5$ and an intercept b^i . Indeed, for a candidate \mathbf{x} , the function,

$$u_i(\mathbf{x}) = \mathbf{w}^i \cdot \mathbf{x} + b, \quad (2)$$

models the utility function of every survey participant i . For consistency, we identify weights $\mathbf{w}^i \in \mathbb{R}^5$ and intercept $b \in \mathbb{R}$ such that,

$$\mathbf{w}^i \cdot (\mathbf{x}_k^{1,i} - \mathbf{x}_k^{2,i}) > 0 \quad \Leftrightarrow \quad y_k^i = 1, \quad (3)$$

and

$$\mathbf{w}^i \cdot (\mathbf{x}_k^{1,i} - \mathbf{x}_k^{2,i}) < 0 \quad \Leftrightarrow \quad y_k^i = -1, \quad (4)$$

implying that whenever two candidates $\mathbf{x}_1^{1,i}$ and $\mathbf{x}_1^{2,i}$ are presented, the survey respondent will choose the one that provides him with a larger utility.

Optimization strategy Note that, within this framework, it is sufficient to consider the differences between the democracy attributes among the two hypothetical candidates. In fact, the selected hypothetical candidate and their corresponding policy stands are observed quantities. **This is slightly different from the question: for a candidate \mathbf{x} should you select her?** Therefore, and from a theoretical standpoint, unlike u_i which cannot be directly constructed, \mathbf{w}^i is the only observed quantity of interest which is accessed within the space of differences between candidates. We define then the centered coordinates $\mathbf{z}_k^i \in \{-1, 0, 1\}$ as,

$$\mathbf{z}_k^i = \mathbf{x}_k^{1,i} - \mathbf{x}_k^{2,i}, \quad (5)$$

hence, from now on, the intercept b will be ignored because a function of the type $f_i(\mathbf{z}_k) =$

$\text{sign}(\mathbf{w}_i \cdot \mathbf{z}_k^i)$ is mathematically sufficient. Indeed, equations Equation 3 and Equation 4 become,

decia “-”

$$y_k^i (\mathbf{w}^i \cdot \mathbf{z}_k^i) > 0. \quad (6)$$

It should be clear by now that the challenge is to identify weights \mathbf{w}^i for every survey participant i . Under the data separability assumption, it has been shown by Vapnik and Chervonenkis (1991) that it suffices to focus on the margin, defined as the minimal distance of a sample to the decision surface. For notation simplicity, the dependence of y^i and \mathbf{z}_k^i on i will be dropped. By rescaling \mathbf{w}_i we know that the closest points to the hyperplane must satisfy,

$$|\mathbf{w}^i \cdot \mathbf{z}_k| = 1, \quad (7)$$

and if two observations \mathbf{z}_k and \mathbf{z}_m belong to different classes (i.e. the selected candidate), then the margin is defined as the distance of these two points to the hyperplane such that,

$$\frac{\mathbf{w}^i}{\|\mathbf{w}^i\|} \cdot (\mathbf{z}_k - \mathbf{z}_m) = \frac{2}{\|\mathbf{w}^i\|}. \quad (8)$$

It should be clear by now that for each survey participant i , the optimal hyperplane is the solution to the following optimization problem,

$$\begin{aligned} \min_{\mathbf{w}^i} \quad & \frac{1}{2} \|\mathbf{w}^i\|^2 \\ \text{subject to} \quad & y_k (\mathbf{w}^i \cdot \mathbf{z}_k) \geq 1, \quad k = 1, 2, 3, 4, 5. \end{aligned} \quad (9)$$

Regarding the actual dataset analyzed in this study, it is unknown if these data can *a priori* be separated by an hyperplane. To allow for bad classification issues, Cortes and Vapnik (1995) introduced the concept of “slack variables” ξ_i that relax the optimization problem restrictions (Equation 9):

$$y_k (\mathbf{w}^i \cdot \mathbf{z}_k) \geq 1 - \xi_k, \quad \xi_k \geq 0, \quad k = 1, 2, 3, 4, 5. \quad (10)$$

All in all, this allows controlling for both the classification strength of $\|\mathbf{w}^i\|$ (or the capacity of the algorithm to correctly classify the data), and the sum of the slack variables $\sum_{k=1}^5 \xi_k$ which account for possible errors in classification. Since we allow the learning algorithm some degree of deviations during the classification process, we need to account for this error in the optimization problem. By doing so, we will find a new \mathbf{w} that might not be the unique solution to Equation 9—that in the non-linearly separable case does not exist—but still is good enough in the sense that the number of training data-pairs misclassified is small. By doing so the following trade-off problem is encountered: finding a \mathbf{w} with small norm that classifies properly a large proportion of the data. A widely used solution to that trade-off is the *C*-SVM or “soft margin classifying,” which is based in the minimization of the following objective function,

$$\min_{\mathbf{w}^i, \xi} \frac{1}{2} \|\mathbf{w}^i\|^2 + C \sum_{k=1}^5 \xi_k, \quad (11)$$

where the regularization constant $C > 0$ determines the trade-off between the empirical error and the complexity term.

We solved the optimization problem with the `Python` library “*sklearn*.” We also present in the Appendix a *R* routine that solves the problem directly using the steepest descent approach. Due to computational complexity and possible convergence issues, this routine is not recommendable when the number of training-data pairs is large. However, since the optimization problem is convex, algorithms converge to a unique solution but the convergence time might be long.

I. Analyzing The Conjoint Data via Support Vector Machines

The SVM approach has been used to solve a classification problem. Via a data-driven process described in the previous section, five \mathbf{w}_i variables were constructed, one per democracy attribute (as shown in third column of [Table 1](#)). These five vectors conform five different dependent variables. In this section we estimate five OLS multivariate lineal models (see [Table A2](#)), where the covariate of interest is the declared willingness to sell the vote (“**sell vote**”). Distributions of the five dependent variables are plotted in [Figure 3](#). Given that they represent different attributes of the same concept—i.e., “support for democracy”—it is not a surprise that these distributions look very similar. From a substantive standpoint, the interest is on finding statistically significant predictors (if any) across all/some (or none of) the five democracy attributes. Following the literature on clientelism,¹⁰ the following covariates were included: **woman**, **party id.**, **ideology**, **education**, **political knowledge**, **registered to vote**, **trust in Federal Gov.**, **income** and **sell vote** (which captures the willingness to the sell the vote).¹¹

¹⁰See particularly Auyero (2000), Kitschelt (2000), Brusco, Nazareno, and Stokes (2004), Calvo and Murillo (2004), Stokes (2005), Nazareno, Brusco, and Stokes (2008), Nichter (2008), González-Ocantos, Jonge, et al. (2012), Weitz-Shapiro (2012), Szwarcberg (2013), Vicente (2014), González-Ocantos, Kiewiet de Jonge, and Nickerson (2014), Rueda (2015), Zarazaga (2014), González-Ocantos, Kiewiet de Jonge, and Nickerson (2015), Schaffer and Baker (2015), Holland and Palmer-Rubin (2015), Zarazaga (2016), Rueda (2017), Oliveros (2016), Bahamonde (2018), Bahamonde (2020), and Murillo, Oliveros, and Zarazaga (2021).

¹¹[Table A1](#) shows summary statistics.

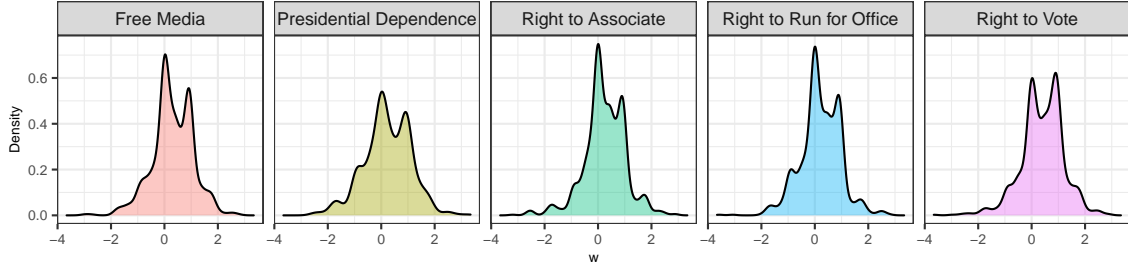


Figure 3: Support Vector Machine Analyses and the Constructed w_i : Five Democracy Attributes.

Note: Leveraging a data-driven process (particularly, support vector machines) and following Dahl (1971), five individual-level scores of support-for-democracy w_i were constructed. Each w_i distribution represents a specific democracy dimension which will be used as a dependent variable to study possible correlations with the willingness to sell the vote. Five different multivariate OLS models were estimated having these distributions as dependent variables—they are shown in Table A2 in the Appendix section and summarized in Figure 4.

Figure 4 shows the point estimates and their estimated uncertainty for all five models. Most importantly, the figure shows that the willingness to sell the vote is negatively correlated with the democracy attribute that speaks to the idea that the President of the United States should not govern without a Congress (“Presidential Dependence”). We believe that individuals who are willing to sell their freedom to vote for money, value strong leaderships that can govern in an unaccounted-for way, that is, without the participation of the American Congress. The other dimensions are not correlated with the willingness to sell.

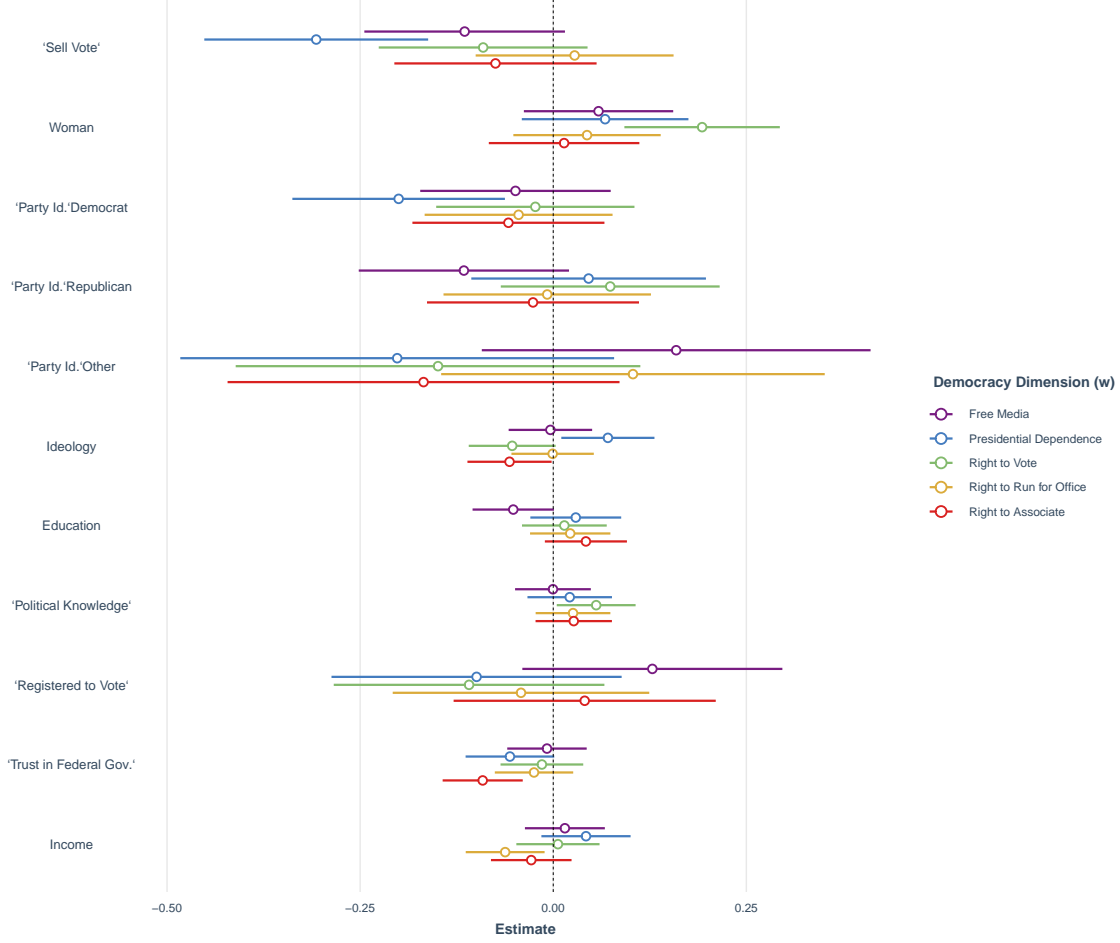


Figure 4: Multivariate Analysis: A Multidimensional Approach to Study Vote Selling and Support for Democracy.

Note: The figure shows correlations between the willingness to sell the vote (*Sell Vote*) across all five democracy dimensions (w_i). All dimensions were constructed following Dahl (1971), operationalized in Table 1 and collected in a conjoint experiment in the United States during 2016 (see Table 2). In this figure, all estimates are OLS (see regression table in Table A2). From a substantive standpoint, the figure shows that from the five democracy attributes, the only one that ought to fail is the belief that the President of the United States may not rule without a Congress (“Presidential Dependence”). In other words, vote sellers systematically believe that the President can govern on his/her own. Importantly, the other democracy attributes are not relevant to explain vote selling.

We believe this finding is relevant. The clientelism literature usually frames the act of selling (or buying) votes as “a democracy” failure. Unfortunately, as we have argued, that explanation, while important, it is too general. This paper explains why it is important to approach democracy from a multidimensional perspective—using the framework provided by Dahl (1971). The immediate

consequence is to identify the particular democracy dimension associated with clientelism, particularly, vote selling. While this paper identifies such dimension, it leaves for future research theorizing about why that particular dimension is associated with vote selling. However, we encourage researchers, particularly political and social psychologists, to investigate the relationship between authoritarian personalities with strong traits of obedience to authority (who would likely let a president govern without a congress) and vote selling.

V. CONCLUSION

Exploiting a novel conjoint dataset representative at the national level and machine learning methods, this paper finds that U.S. voters who score low on the belief that the President should govern with a Congress are more likely to sell the vote. Following the multidimensional approach to study democracy proposed by Dahl (1971), the other dimensions are not relevant to explain vote selling. We believe these findings are relevant to the clientelism literature since most explanations of vote selling see the phenomena as a general democratic failure.

In addition to that, these findings go in line with recent studies pointing out to a possible erosion of American democracy—see Bahamonde (2020), but particularly Levitsky and Ziblatt (2018). The data analyzed in this paper were collected while the political campaign that led Donald Trump to the U.S. Presidency took place. From that perspective, survey respondents not only participated in this study, but also (some of them) voted for Donald Trump. This is particularly important given the events occurred during January 2021 where Trump incited the storming of the U.S. Capitol. Following the lead of Levitsky and Ziblatt (2018), we encourage comparative politics scholars to incorporate the United States (as case) to the pool of cases usually studied while studying clientelism, which are mostly developing countries.

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VI. APPENDIX

I. Experimental Manipulations and Vignettes

Distractor Paragraph. The next paragraph was used to distract subjects from the main purpose of the study, and also to define vote selling. After reading the excerpt, participants were told about the hypothetical possibility of doing one of the illegal things mentioned in the excerpt. And that this possibility would be randomly assigned. However, all participants were directly asked whether they would be interested in selling their vote (as seen on [Direct Question](#)).

Washington, D.C.— A department store downtown had a robbery incident last week, reporting several missing iPods from their inventory. Authorities also inform that a group of local residents are trying to ``sell'' their votes to political candidates ahead of a local election for city council. Residents approached some of the candidates running for office and offered to vote for that candidate in return for monetary compensation. In a different subject matter, the local police station released a report on driving habits and behaviors in the Capitol district last week. Finally, cyber-crime has become an increasingly serious issue in the area in the past few year.

Direct Question. All subjects read the next paragraph, and then *all* answered the direct question:

Now you will be entered into a random lottery for the opportunity to do ONE of the illegal things you just read before. This means that you might be randomly offered to hypothetically do ANY of the activities mentioned before.

After a random assignment, you have been selected for the opportunity to hypothetically sell your vote. This means that you will have the hypothetical opportunity to accept money from a candidate for your vote. Would you be willing to accept the offer, assuming you would not go to jail? By selecting ``Yes,'' you could earn up to \$ 1,000.

II. Summary Statistics

Table A1

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Right to Run for Office	1,005	0.212	0.745	−3.575	−0.133	0.759	2.801
Right to Associate	1,005	0.247	0.768	−3.194	−0.064	0.793	2.890
Free Media	1,005	0.308	0.759	−3.018	−0.009	0.905	2.795
Presidential Dependence	1,005	0.228	0.870	−2.609	−0.302	0.926	3.344
Right to Vote	1,005	0.399	0.795	−3.664	0.000	0.944	2.863
Woman	1,005	1.567	0.496	1	1	2	2
Ideology	1,005	2.940	1.107	1	2	4	5
Registered to Vote	1,005	1.093	0.290	1	1	1	2
Trust in Federal Gov.	1,005	2.612	1.125	1	2	3	5
Income	1,005	7.021	3.805	1	4	10	14
Education	1,005	4.811	1.707	1	4	6	7
Political Knowledge	1,005	0.663	0.372	0.000	0.402	0.871	2.432
Sell Vote	1,005	0.161	0.368	0	0	0	1

III. Algorithms

```
1 # Libraries
2 library(pacman)
3 library(e1071)
4 library(xlsx)
5 library(openxlsx)
6 library(readxl)
7
8 # Read merged Conjoint Data
9 load("conjoint.RData")
10
11 # numeric variables filter
12 str2int <- function(x) as.numeric(!grepl("CANNOT",x,fixed=TRUE))
13 d_mod <- data.frame(lapply(d[5:9], str2int),d[10])
14
15 # supervector with individual preferences type='C-classification '
16 W <- data.frame(NULL)
17 Training <- data.frame(NULL)
18 for (k in seq(1,max(d$idnum))){
19   cand1 <- subset(d,idnum==k & candidate==1,c("at.run","at.asso","at.press","at.
      presaut","at.vote"))
20   cand2 <- subset(d,idnum==k & candidate==2,c("at.run","at.asso","at.press","at.
      presaut","at.vote"))
21   cand1 <- data.frame(lapply(cand1, str2int))
22   cand2 <- data.frame(lapply(cand2, str2int))
23   X_train <- cand1-cand2
24   sel1 <- subset(d,idnum==k & candidate==1,"selected")
25   sel2 <- subset(d,idnum==k & candidate==2,"selected")
```

```

26 y_train <- sel1-sel2
27
28 x <- as.matrix(X_train)
29 y <- as.matrix(y_train)
30 w_answer <- getsvm(x)
31 w_answer <- c(k,w_answer)
32 W <- rbind(W,w_answer)
33
34 training_data <- data.frame(X_train,y_train)
35 Training <- rbind(Training,training_data)
36
37 print(k)
38 }
39 names(W) <- c("k","w1","w2","w3","w4","w5")
40 A <- cbind(W,Training)
41
42 wb <- createWorkbook("weights_scratch.xlsx")
43 addWorksheet(wb,"Pesos")
44 writeData(wb,sheet="Pesos",W,startCol=1,startRow=1,rowNames = FALSE)
45 saveWorkbook(wb, file = "weights_scratch.xlsx", overwrite = TRUE)
46
47 wb <- createWorkbook("weights_and_data.xlsx")
48 addWorksheet(wb,"Weights n Data")
49 writeData(wb,sheet="Weights n Data",A,startCol=1,startRow=1,rowNames = FALSE)
50 saveWorkbook(wb, file = "weights_and_data.xlsx", overwrite = TRUE)
51
52 # write.csv(W,".csv", row.names = TRUE)
53
54 svm_gradient <- function(x, eta=0.001,R=10000){

```

```

55 X <- x
56 n <- nrow(X) # number of sample
57 p <- ncol(X) # number of feature+1 (bias)
58 w_initial <- rep(0,p)
59 W <- matrix(w_initial ,nrow = R+1,ncol = p,byrow = T) # matrix put initial guess
    and the procedure to do gradient descent
60 for(i in 1:R){
61   for(j in 1:p)
62   {
63     W[i+1,j] <- W[i,j] + eta * sum(((y*(X[i,]%W[i,]) < 1) * 1 - y * X[i,j] )
64   }
65 }
66 return(W)
67 }
68
69 getsvm <- function(x){
70   w_answer <- svm_gradient(x)[nrow(svm_gradient(x)),]
71   return(w_answer )
72 }

```

svm_script.R

```

1 # svm.py
2 import numpy as np # for handling multi-dimensional array operation
3 import pandas as pd # for reading data from csv
4 from sklearn.svm import LinearSVC # for classification problem
5 from sklearn.pipeline import make_pipeline # create pipeline
6 from sklearn.preprocessing import StandardScaler # scaling data
7
8 # but following work good enough

```

```

9 reg_strength = 10000 # regularization strength
10 learning_rate = 0.000001
11 #init()
12 data = pd.read_csv('./dataMergedConjoint.csv')
13 # SVM only accepts numerical values.
14 # Therefore, we will transform the categories into
15 # values 1 and 0.
16
17 # at.run
18 cannot_map = {'Citizens CANNOT run for office for the next two elections':0, '
    Citizens CAN run for office for the next two elections':1}
19 data['at.run'] = data['at.run'].map(cannot_map)
20 # at.asso
21 cannot_map = {'Citizens CANNOT associate with others and form groups':0, '
    Citizens CAN associate with others and form groups':1}
22 data['at.asso'] = data['at.asso'].map(cannot_map)
23 # at.press
24 cannot_map = {'Media CANNOT confront the Government':0, 'Media CAN confront the
    Government':1}
25 data['at.press'] = data['at.press'].map(cannot_map)
26 # at.presaut
27 cannot_map = {'President CANNOT rule without Congress':1, 'President CAN rule
    without Congress':0}
28 data['at.presaut'] = data['at.presaut'].map(cannot_map)
29 # at.vote
30 cannot_map = {'Citizens CANNOT vote in the next two elections':0, 'Citizens CAN
    vote in the next two elections':1}
31 data['at.vote'] = data['at.vote'].map(cannot_map)
32

```

```

33 # drop last column (extra column added by pd)
34 # and unnecessary first column (id)
35 # data.drop(data.columns[[-1 0]], axis=1, inplace=True)
36 # put features & outputs in different DataFrames for convenience
37 Y = data.loc[:, 'selected'] # all rows of 'diagnosis'
38 X_c1 = data.iloc[range(0,11080,2),[5,6,7,8,9]] # all feature rows candidate 1
39 X_c2 = data.iloc[range(1,11080,2),[5,6,7,8,9]] # all feature rows candidate 1
40 X = X_c1.values-X_c2.values
41 X = pd.DataFrame(X)
42 Y_c1 = Y.iloc[range(0,11080,2)] # all feature rows candidate 1
43 Y_c2 = Y.iloc[range(1,11080,2)] # all feature rows candidate 1
44 Y = Y_c1.values-Y_c2.values
45 Y = pd.DataFrame(Y)
46 W = pd.DataFrame(data=None, columns=['k', 'w.at.run', 'w.at.asso', 'w.at.press', 'w.at.
    presaut', 'w.at.vote', 'selected', 'at.run', 'at.asso', 'at.press', 'at.presaut', 'at.
    vote'])
47
48 print("training started...")
49 for i in list(range(int(len(Y)/5))):
50     print(i)
51     X_train = X.iloc[5*i:5*(i+1),:]
52     #X_train = [X_train.iloc[0,:],X_train.iloc[1,:],X_train.iloc[2,:],X_train.iloc
        [3,:],X_train.iloc[4,:]]
53     y_train = Y.iloc[5*i:5*(i+1)]
54     if (-1 in np.array(y_train)) and (1 in np.array(y_train)):
55         #clf = make_pipeline(StandardScaler(),LinearSVC(random_state=0, tol=1e-5,
            fit_intercept=False))
56         clf = LinearSVC(random_state=0, tol=1e-5, fit_intercept=False, C = 10, max_
            iter = 2000)

```

```

57     clf.fit(X_train, y_train.values.ravel())
58     #print(clf.decision_function(np.eye(5)))
59     w=list(clf.decision_function(np.eye(5)))
60
61     w = [i+1]+w
62     w = pd.DataFrame({'k': [w[0],w[0],w[0],w[0],w[0]],
63                        'w.at.run': [w[1],w[1],w[1],w[1],w[1]],
64                        'w.at.asso': [w[2],w[2],w[2],w[2],w[2]],
65                        'w.at.press': [w[3],w[3],w[3],w[3],w[3]],
66                        'w.at.presaut': [w[4],w[4],w[4],w[4],w[4]],
67                        'w.at.vote': [w[5],w[5],w[5],w[5],w[5]]})
68     #aux=pd.DataFrame(np.ones((5,1))*w)
69     w['selected']=y_train.values
70     w['at.run']=X_train[0].values
71     w['at.asso']=X_train[1].values
72     w['at.press']=X_train[2].values
73     w['at.presaut']=X_train[3].values
74     w['at.vote']=X_train[4].values
75     W = pd.concat([W,w])
76
77 pd.DataFrame(W).to_excel(r'./File Name.xlsx', index = False)

```

svm_script.py

IV. Regression Table: OLS Analyses using the SVM Approach to Analyzing Conjoint Data

	Free Media	Presidential Dependence	Right to Vote	Right to Run for Office	Right to Associate
'Sell Vote'	-0.11 (0.07)	-0.31*** (0.07)	-0.09 (0.07)	0.03 (0.07)	-0.07 (0.07)
Woman	0.06 (0.05)	0.07 (0.05)	0.19*** (0.05)	0.04 (0.05)	0.01 (0.05)
'Party Id.'Democrat	-0.05 (0.06)	-0.20** (0.07)	-0.02 (0.07)	-0.04 (0.06)	-0.06 (0.06)
'Party Id.'Republican	-0.12 (0.07)	0.05 (0.08)	0.07 (0.07)	-0.01 (0.07)	-0.03 (0.07)
'Party Id.'Other	0.16 (0.13)	-0.20 (0.14)	-0.15 (0.13)	0.10 (0.13)	-0.17 (0.13)
Ideology	-0.00 (0.02)	0.06* (0.03)	-0.05 (0.03)	-0.00 (0.02)	-0.05* (0.03)
Education	-0.03 (0.02)	0.02 (0.02)	0.01 (0.02)	0.01 (0.02)	0.02 (0.02)
'Political Knowledge'	-0.00 (0.07)	0.06 (0.07)	0.15* (0.07)	0.07 (0.07)	0.07 (0.07)
'Registered to Vote'	0.13 (0.09)	-0.10 (0.10)	-0.11 (0.09)	-0.04 (0.08)	0.04 (0.09)
'Trust in Federal Gov.'	-0.01 (0.02)	-0.05 (0.03)	-0.01 (0.02)	-0.02 (0.02)	-0.08*** (0.02)
Income	0.00 (0.01)	0.01 (0.01)	0.00 (0.01)	-0.02* (0.01)	-0.01 (0.01)
R ²	0.02	0.07	0.03	0.01	0.03
Adj. R ²	0.01	0.06	0.02	0.00	0.02
Num. obs.	1005	1005	1005	1005	1005

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$. Every column represents each of Dahl (1971) democracy dimensions. All models OLS. Intercept omitted.

Table A2: *Statistical models*

V. Data-pairs

Consider the following set of training data-pairs $\{(\mathbf{z}_k, y_k)\}_{k=1}^n$ with $\mathbf{z}_k \in \mathbb{R}^2$ and $y_k \in \{-1, 1\}$. We look for an hyperplane going through the origin such that,

$$\mathbf{w} \cdot \mathbf{z}_k > 0 \quad \Leftrightarrow \quad y_k = 1 \quad \text{and} \quad \mathbf{w} \cdot \mathbf{z}_k < 0 \quad \Leftrightarrow \quad y_k = -1, \quad (12)$$

in the case of $z_k \in \mathbb{R}^2$ we have that such hyperplane is simply a straight line with a 0 intercept,

$$L(\mathbf{z}) := L((z_1, z_2)) = w_1 z_1 + w_2 z_2. \quad (13)$$

Now, if $z_k = (z_{k,1}, z_{k,2})$, then we look for \mathbf{w} such that,

$$y_k (w_1 z_{k,1} + w_2 z_{k,2}) > 0, \quad \forall k = 1, \dots, n. \quad (14)$$

Since $n \in \mathbb{N}$ is fixed, there must be some $\delta = \delta(\mathbf{w})$ such that,

$$\delta = \min_{k=1, \dots, n} y_k (w_1 z_{k,1} + w_2 z_{k,2}) > 0, \quad (15)$$

therefore, by redefining $\mathbf{w} = \mathbf{w}/\delta$, equation Equation 14 can be rewritten as,

$$y_k (w_1 z_{k,1} + w_2 z_{k,2}) > 1 \quad (16)$$

moreover, the points \mathbf{z}_k closest to $L(\mathbf{z})$ can be defined such that,

$$y_k (w_1 z_{k,1} + w_2 z_{k,2}) = 1. \quad (17)$$

Recall that the distance between a point $\mathbf{z} = (z_1, z_2) \in \mathbb{R}^2$ to the line L is given by,

$$\text{distance}(\mathbf{z}, L) = \frac{|w_1 z_1 + w_2 z_2|}{\sqrt{w_1^2 + w_2^2}}. \quad (18)$$

Let $(z_{k,1}, z_{k,2})$ a training data-pair belonging to the class labeled +1, and $(z_{m,1}, z_{m,2})$ a training data-pair belonging to the class labeled -1. Assume furthermore that $(z_{k,1}, z_{k,2})$ is one of the points in the 1 class that is closest to the optimal hyperplane. Similarly, assume that $(z_{m,1}, z_{m,2})$ is one of the points in the -1 class that is closest to the optimal hyperplane. The margin is defined as the sum of the distances between $(z_{k,1}, z_{k,2})$ and $(z_{m,1}, z_{m,2})$ to the optimal hyperplane. That is,

$$\text{margin} = \frac{|w_1 z_{k,1} + w_2 z_{k,2}|}{\sqrt{w_1^2 + w_2^2}} + \frac{|w_1 z_{m,1} + w_2 z_{m,2}|}{\sqrt{w_1^2 + w_2^2}} \quad (19)$$

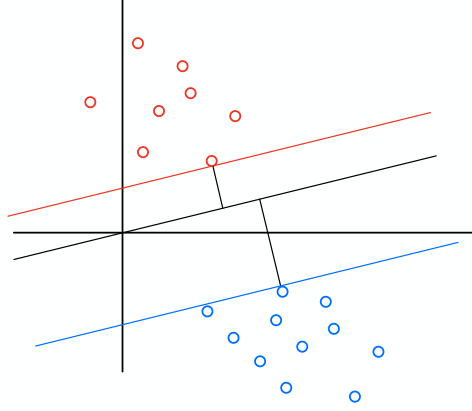


Figure A1: A two dimensional example of an optimal hyperplane. Red dots correspond to the training data-points labeled -1, blue dots correspond to the training data-points labeled as +1. The margin is defined as the sum of the distance between points in different classes to the hyperplane.

since $y_k = 1$ then we have that,

$$y_k (w_1 z_{k,1} + w_2 z_{k,2}) = w_1 z_{k,1} + w_2 z_{k,2} = 1 > 0 \quad (20)$$

and

$$y_m (w_1 z_{m,1} + w_2 z_{m,2}) = -(w_1 z_{m,1} + w_2 z_{m,2}) = 1 > 0 \quad (21)$$

therefore, we can rewrite the margin as,

$$\text{margin} = \frac{w_1 z_{k,1} + w_2 z_{k,2}}{\sqrt{w_1^2 + w_2^2}} - \frac{w_1 z_{m,1} + w_2 z_{m,2}}{\sqrt{w_1^2 + w_2^2}} = \frac{(w_1, w_2) \cdot (z_{k,1} - z_{m,1}, z_{k,2} - z_{m,2})}{\sqrt{w_1^2 + w_2^2}}, \quad (22)$$

or simply by $\frac{\mathbf{w}}{\|\mathbf{w}\|} \cdot (\mathbf{z}_k - \mathbf{z}_m) = \frac{2}{\|\mathbf{w}\|}$. To find the optimal hyperplane is, in this example, to find the values of w_1 and w_2 such that the margin is the largest possible. This is achieved with the vector \mathbf{w} with smallest norm, and we find out the optimization problem stated in the main text,

$$\begin{aligned} & \min_{\mathbf{w}} \frac{1}{2} \|\mathbf{w}^i\|^2 \\ & \text{subject to } y_k (\mathbf{w} \cdot \mathbf{z}_k) \geq 1, \quad k = 1, \dots, n. \end{aligned} \quad (23)$$