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On the Robustness of Global Feature Effect Explanations

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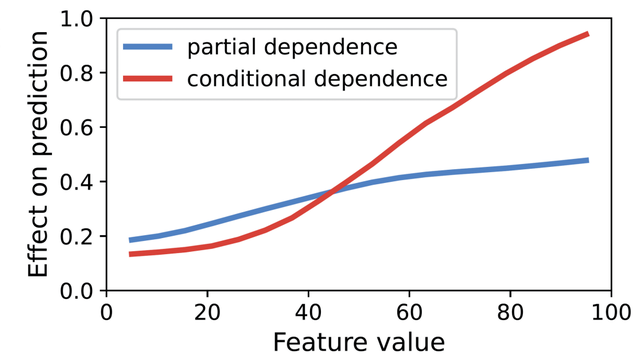
TL;DR: Theoretical bounds for the robustness of feature effect explanations to data and model perturbations.

0. Feature effect explanations (partial dependence, ALE)

$$\text{PD}_s(\mathbf{x}_s; f, p_{\mathbf{X}}) \coloneqq \mathbb{E}_{\mathbf{X}_{\bar{s}} \sim p_{\mathbf{X}_{\bar{s}}}} \left[f(\mathbf{x}_s, \mathbf{X}_{\bar{s}}) \right] \coloneqq \int f(\mathbf{x}_s, \mathbf{x}_{\bar{s}}) p_{\mathbf{X}_{\bar{s}}}(\mathbf{x}_{\bar{s}}) d\mathbf{x}_{\bar{s}}$$

$$\text{CD}_s(\mathbf{x}_s; f, p_{\mathbf{X}}) \coloneqq \mathbb{E}_{\mathbf{X}_{\bar{s}} \sim p_{\mathbf{X}_{\bar{s}} | \mathbf{X}_s = \mathbf{x}_s}} \left[f(\mathbf{x}_s, \mathbf{X}_{\bar{s}}) \right] \coloneqq \int f(\mathbf{x}_s, \mathbf{x}_{\bar{s}}) p_{\mathbf{X}_{\bar{s}} | \mathbf{X}_s = \mathbf{x}_s}(\mathbf{x}_{\bar{s}} | \mathbf{x}_s) d\mathbf{x}_s$$

Example:



1. Robustness to data perturbations

Assumption 1. *We assume that the model f has bounded predictions, i.e., there exists a constant B such that $|f(\mathbf{x})| \leq B$ for all $\mathbf{x} \in \mathbb{R}^p$.*

Theorem 2. *The robustness of partial dependence and conditional dependence to **data perturbations** is given by the following formulas*

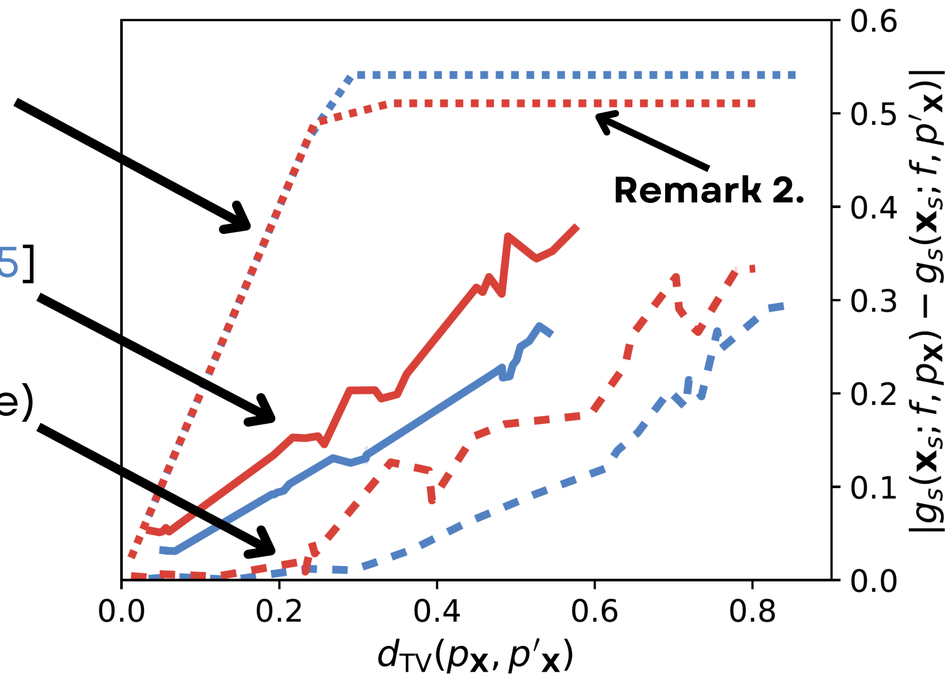
$$\left| \text{PD}_s(\mathbf{x}_s; f, p_{\mathbf{X}}) - \text{PD}_s(\mathbf{x}_s; f, p'_{\mathbf{X}}) \right| \leq 2B \cdot d_{\text{TV}}\left(p_{\mathbf{X}_{\bar{s}}}, p'_{\mathbf{X}_{\bar{s}}}\right),$$

$$\left| \text{CD}_s(\mathbf{x}_s; f, p_{\mathbf{X}}) - \text{CD}_s(\mathbf{x}_s; f, p'_{\mathbf{X}}) \right| \leq 2B \cdot d_{\text{TV}}\left(p_{\mathbf{X}_{\bar{s}} | \mathbf{X}_s = \mathbf{x}_s}, p'_{\mathbf{X}_{\bar{s}} | \mathbf{X}_s = \mathbf{x}_s}\right),$$

where the total variation distance d_{TV} is defined via the l_1 functional distance.

Check out the paper:

2. Perturb the dataset



3. Robustness to model perturbations

Lemma 2. *The robustness of partial dependence and conditional dependence to **model perturbations** is given by the following formulas*

$$\left| \text{PD}_s(\mathbf{x}_s; f, p_{\mathbf{X}}) - \text{PD}_s(\mathbf{x}_s; f', p_{\mathbf{X}}) \right| \leq \|f - f'\|_{\infty},$$

$$\left| \text{CD}_s(\mathbf{x}_s; f, p_{\mathbf{X}}) - \text{CD}_s(\mathbf{x}_s; f', p_{\mathbf{X}}) \right| \leq \|f - f'\|_{\infty, \mathcal{X}},$$

where $\|f\|_{\infty} \coloneqq \sup_{\mathbf{x} \in \mathbb{R}^p} |f(\mathbf{x})|$ denotes an infinity norm for a function and $\|f\|_{\infty, \mathcal{X}} \coloneqq \sup_{\mathbf{x} \in \mathcal{X}} |f(\mathbf{x})|$ is the same norm taken over the domain $\mathcal{X} \subseteq \mathbb{R}^p$.

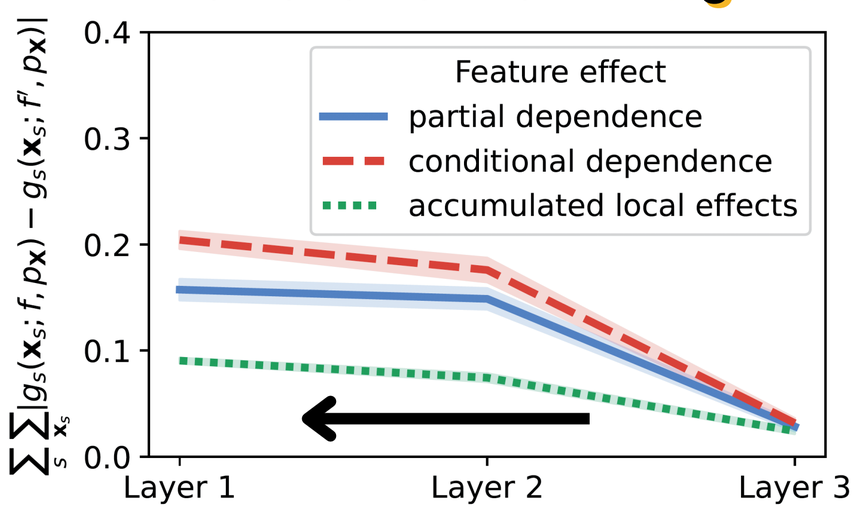
Proof. Follows directly from [26, lemmas 5 & 6].

Theorem 5. *The robustness of accumulated local effects to **model perturbations** is given by the following formula*

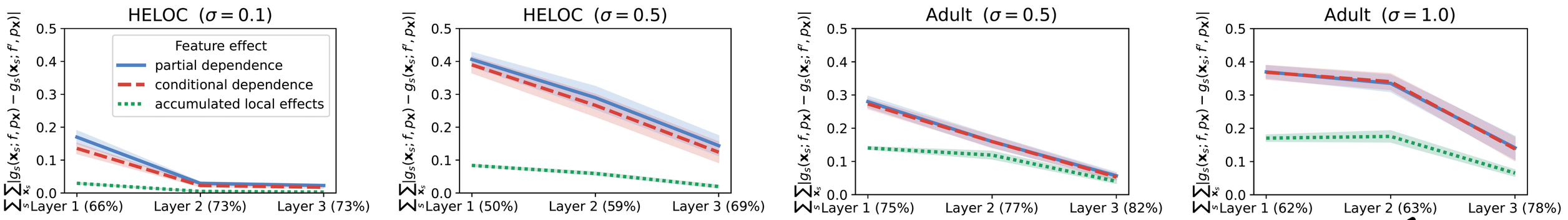
$$\left| \text{ALE}_s(\mathbf{x}_s; f, p_{\mathbf{X}}) - \text{ALE}_s(\mathbf{x}_s; f', p_{\mathbf{X}}) \right| \leq (\mathbf{x}_s - \mathbf{x}_{\min,s}) \cdot \|h - h'\|_{\infty, \mathcal{X}},$$

where $h \coloneqq \frac{\partial f}{\partial \mathbf{x}_s}$ and $h' \coloneqq \frac{\partial f'}{\partial \mathbf{x}_s}$ denote partial derivatives of f and f' respectively.

4. Perturb network weights



(Differential) **ALE does not pass** the model randomization test.



Accuracy drops, but the explanation remains similar.

References

[5] Baniecki et al. *Fooling Partial Dependence via Data Poisoning*. ECML PKDD 2022
[26] Lin et al. *On the Robustness of Removal-Based Feature Attributions*. NeurIPS 2023