Quantum Entanglement / Teleportation

Product states $\longrightarrow |A\rangle \otimes |B\rangle$

Entangled states $\rightarrow |A\rangle|B\rangle + |C\rangle|D\rangle$

Bell states

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}\left[|01\rangle - |11\rangle\right]$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left[|01\rangle + |10\rangle\right]$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left[|01\rangle - |10\rangle\right]$$

How to create $|\phi^+\rangle$

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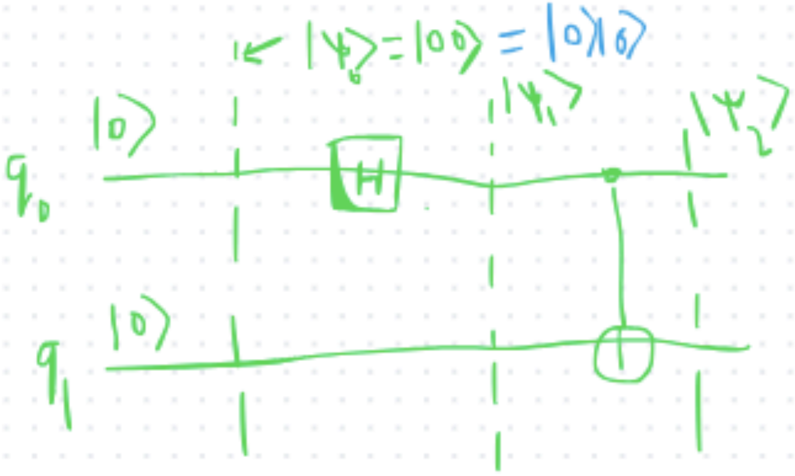


$|\psi_0\rangle = |00\rangle$ (q_0, q_1)

$q_0 \rightarrow$

$$|\psi_0'\rangle = H|0\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left[|00\rangle + |01\rangle\right]$$

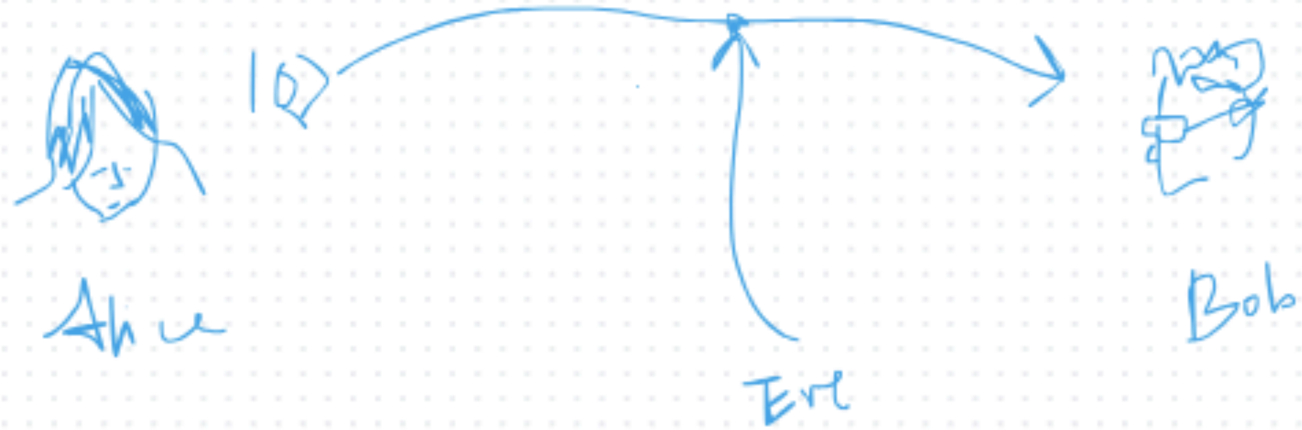
Is product state

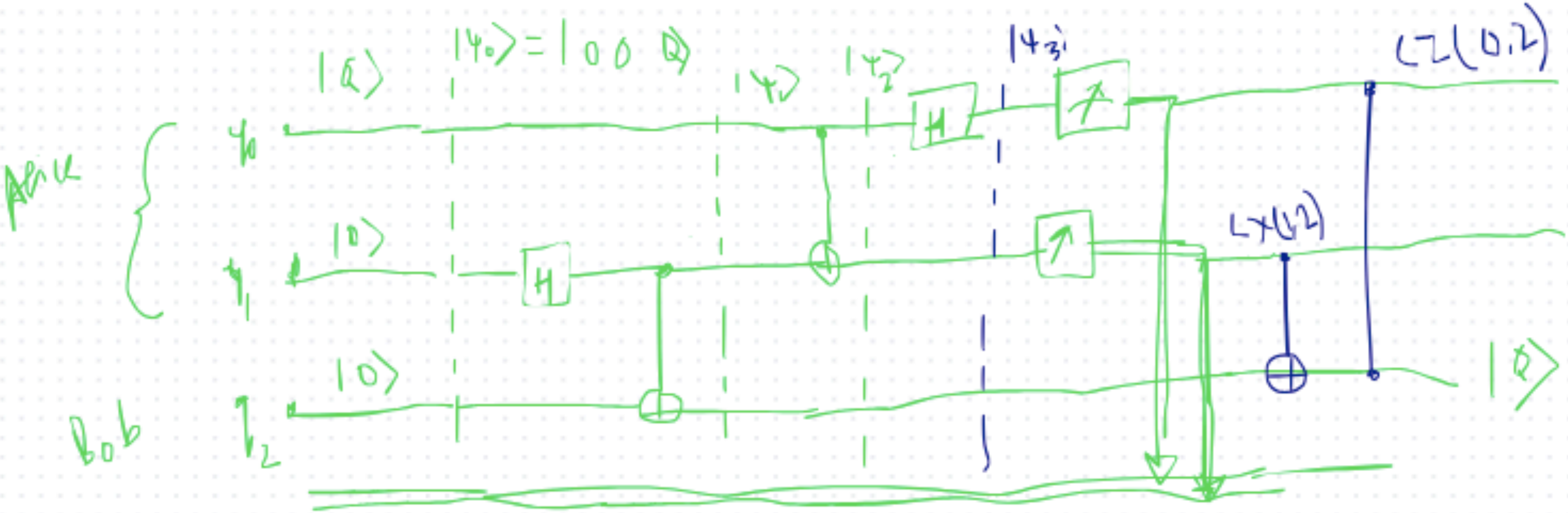
$$|\psi_2\rangle = CX(0,1)\,|\psi_1\rangle$$

$$= \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right] = |\phi^+\rangle$$

Ex: Show how to create other 3 Bell states

Quantum Teleportn





$$|\psi_1\rangle = |Q\rangle|\phi^+\rangle = \frac{1}{\sqrt{2}}\left[|00Q\rangle + |11Q\rangle\right]$$

$|Q\rangle = \alpha|0\rangle + b|1\rangle$

where $|a|^2 + |b|^2 = 1$

$$|\psi_2\rangle = CX(0,1)|\psi_1\rangle = \overset{CX(0,1)}{\left[|00\rangle(a|0\rangle + b|1\rangle) + |11\rangle(a|0\rangle + b|1\rangle)\right]}$$

$$= \frac{1}{\sqrt{2}}\left[a|000\rangle + b|011\rangle + a|110\rangle + b|101\rangle\right]$$

$$|\psi_3\rangle = \overset{H(0)}{H_0}|\psi_2\rangle = \frac{1}{2}\left\{a|00\rangle(|0\rangle+|1\rangle) + b|01\rangle(|0\rangle-|1\rangle)\right.$$

$$\left. + a|11\rangle(|0\rangle+|1\rangle) + b|10\rangle(|0\rangle-|1\rangle)\right\}$$

$|\tilde{Q}\rangle|??\rangle$

$\downarrow$

$a|x\rangle + b|y\rangle$

$x, y \leftarrow 0, 1$

$$= \frac{1}{2}\left[\overbrace{(a|0\rangle + b|1\rangle)}^{Q}|00\rangle + (a|0\rangle - b|1\rangle)|01\rangle + (a|1\rangle + b|0\rangle)|10\rangle + (a|1\rangle - b|0\rangle)|11\rangle\right]$$

Alice

Bob

Bob

? XZ or ZX ? on $|\psi_3\rangle$

$XZ \to X(a|1\rangle - b|0\rangle)|1\rangle$

$= (a|0\rangle - b|1\rangle)|1\rangle$ ✗

$ZX \to Z(a|0\rangle - b|1\rangle)|1\rangle$

[redacted] $a|0\rangle + b|1\rangle)|1\rangle$ ✓

Alice's lab's possible outcome $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

Bob's table

Alice's outcome	Bob's gates
00	I ← identity
01	[redacted] Z [redacted]
10	[redacted] $\cancel{X}$ [redacted]
11	[redacted] Z [redacted] X [redacted]

[redacted]

[redacted] Z^{α} [redacted] X^{β} $\alpha, \beta \in 0, 1$

Alice's original $|\phi\rangle$ gets destroyed

NO CLONING

No cloning theorem

→ can't create same q state without destroying it.