July 20, 2024

Lecture 1: Quantum Gates

Last session

Qubits → $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Ortho ⊥ props

Classical → OR, AND, NOT, XOR

Quantum

Pauli matrices?

$$\sigma_1 \to X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \to \sigma_1 \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \to \sigma_2 \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \to \sigma_3$$

All → 2×2 matrix

$\mathrm{Tr}\,\sigma_i \to \sum$ diag elmts $= 0 \quad \forall i$

X on $|0\rangle, |1\rangle$

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

NOT

$$X|\alpha\rangle = |\bar{\alpha}\rangle \qquad |\bar{0}\rangle = |1\rangle \quad |\bar{1}\rangle = |0\rangle$$

X acts as NOT operator

PHASE

Z

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \to |0\rangle$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle = e^{i\pi}|1\rangle$$

phase flip

phase change only qubit in $|1\rangle$

$$\boxed{Z|\alpha\rangle = (-1)^{\alpha}|\alpha\rangle}$$

$\alpha = 0$, $Z|0\rangle = |0\rangle$

$= 1$, $Z|1\rangle = (-1)^{1}|1\rangle = -|1\rangle$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$\tilde{H}|0\rangle = \begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} = |0\rangle + |1\rangle$$

$$\tilde{H}|1\rangle = \begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}1\\ -1\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix} - \begin{bmatrix}0\\ 1\end{bmatrix}$$

$$= |0\rangle - |1\rangle$$

MIXER / SUPERPOSER

$$\tilde{H}|\alpha\rangle = (-1)^{\alpha}|\alpha\rangle + |\bar{\alpha}\rangle$$

$\alpha = 0$: $|0\rangle + |1\rangle$

$= 1$: $-|1\rangle + |0\rangle$

$$\boxed{H|\alpha\rangle = \frac{1}{\sqrt{2}}\left[(-1)^{\alpha}|\alpha\rangle + |\bar{\alpha}\rangle\right]} \rightarrow |\psi\rangle$$

Hadamard gate

$$\langle\psi|\psi\rangle = \frac{1}{2} + \frac{1}{2} = 1$$

$|1\rangle$ say

$$H|\psi\rangle = |\psi'\rangle = \left[|0\rangle - |1\rangle\right]\frac{1}{\sqrt{2}}$$

$$\langle\psi'|\psi'\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\underbrace{\langle 0|0\rangle}_{1} + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\underbrace{\langle 1|1\rangle}_{1} = 1$$