QC101: Assignment from Lecture 0 & 1

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July 21, 2024

1. The wavefunction for the 1D particle-in-a-box problem is given by $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$. Show that (A) $\langle \psi_1 | \psi_1 \rangle$ is normal and (B) $\langle \psi_1 | \psi_2 \rangle$ is orthogonal. [Note that inner product $\langle \psi_m | \psi_n \rangle \equiv \int_{-\infty}^{\infty} dx\, \psi_m^*(x) \psi_n(x)$.]

2. The three Pauli matrices are: $\sigma_1 = \mathbf{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\sigma_2 = \mathbf{Y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, and $\sigma_3 = \mathbf{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

   A. Show $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ are Hermitian and unitary. [$\mathbf{A}$ is Hermtian if $\mathbf{A}^\dagger = \mathbf{A}$ and unitary if $\mathbf{A}^\dagger \mathbf{A} = \mathbf{1}$; $\mathbf{1}$ is the identity matrix. Here † indicates conjugate+transpose operation.]

   B. What are the eigenvalues and eigenvectors of $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$? Can you check if $\mathrm{Tr}\,\sigma_i = \sum_i \lambda_i$ and $\mathrm{Det}\,\sigma_i = \prod_i \lambda_i$ where Tr and Det imply the trace and determinant respectively.

   C. What are the actions of $\mathbf{X} + \mathbf{Z}$ and $\mathbf{XZ}$ on states $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$?

   D. Can you identify $\mathbf{X} + \mathbf{Z}$ and $\mathbf{XZ}$ with some other names?

PS: Please feel free to point out any mistakes or typographical errors.