QC101- Lecture 0 : Quantumness and qubits

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What makes Q special?

- Superposition of states → Tot prob = 1
- Probability and measurement collapses the superposition state
- properties (observable) can be written in terms of Hermitian matrices (operators)

Classical bits: 0 & 1 → follows Boolean algebra

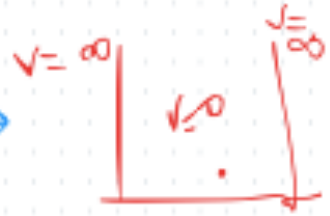
Quantum states rep'd by ket (column matrices)

$$\underbrace{|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}}_{\text{qubits}}$$

Any two level state

e.g. Ground state, 1st excited state

particle in a box →



$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad n_{min} = 1$$

$$H|\psi_n\rangle = E_n|\psi_n\rangle$$

$$E_0 = E_{min} = \frac{\pi^2\hbar^2}{2mL^2} \rightarrow |\psi_0\rangle = |0\rangle$$

$$E_1 = \frac{4\pi^2\hbar^2}{2mL^2} \rightarrow |\psi_1\rangle = |1\rangle$$

A Hermitian matrix's eigenvalues are $\boxed{\text{REAL}}$

It's eigen vectors are $\boxed{\text{ORTHOGONAL}}$

Inner Product

$$\text{bra} \rightarrow \langle \psi_n | \psi_m \rangle \leftarrow \text{ket} = K \delta_{nm} \quad K \rightarrow \text{const}$$

When $K=1 \rightarrow$ ORTHONORMAL

2-state system

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

$|\psi\rangle \rightarrow$ prob. density

$$\text{Prob.} = \langle \psi | \psi \rangle = \|\psi\| = |\psi|^2$$

$$a|0\rangle \xrightarrow{\text{complex conjugate}} a^*\langle 0|$$

$$a\begin{bmatrix}1\\0\end{bmatrix} \longrightarrow a^*\begin{bmatrix}1 & 0\end{bmatrix}^* = \left(a\begin{bmatrix}1\\0\end{bmatrix}\right)^\dagger$$

$$a^*\begin{bmatrix}1 & 0\end{bmatrix}$$

If A is Hermitian then $A = A^\dagger$

$$\langle \psi|\psi\rangle = \left(a^*\langle 0| + b^*\langle 1|\right)\left(a|0\rangle + b|1\rangle\right)$$

$$= |a|^2 \langle 0|0\rangle + |b|^2 \langle 1|1\rangle = a^2 + b^2$$

[orthonormality $\Rightarrow$ $\langle 0|1\rangle = \langle 1|0\rangle = 0$
$\langle 0|0\rangle = \langle 1|1\rangle = 1$]

Norm or prob. con sm. $\Rightarrow$

$$\boxed{\langle \psi|\psi\rangle = a^2 + b^2 = 1}$$

$$\langle 0|1\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= 0 + 0 = 0$$

Ex
show
$\langle 1|0\rangle = 0$

$1 \times ~~2~~ \times 1$

$$\langle 1|1\rangle = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= 0 + 1 = 1$$

Ex
show
$\langle 0|0\rangle = 1$

outer product

$$|1\rangle\langle 1| = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix}$$

2×1 1×2

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Quantum Principles:

1. A state is a superposition of all of its eigenstates
2. E.states are eigenvectors of the quantum property / observable.
3. Observables are represented by Hermitian matrices.
4. Norm of a state ($\langle\psi|\psi\rangle$) must be 1.

Qubit in 2-level quantum state

→ Follow vector (column matrix) algebra

$|0\rangle \rightarrow \hat{x}$

$|1\rangle \rightarrow \hat{y}$

$\vec{\psi} = a\hat{x} + b\hat{y}$

$\langle\psi|\psi\rangle = \vec{\psi}^* \cdot \vec{\psi} = a^*a\, \hat{x}\cdot\hat{x} + b^*b\, \hat{y}\cdot\hat{y}$ $[\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{x} = 0]$

$= |a|^2 + |b|^2 = 1$

my audience → Sleep