Two Qubit systems & gates

$|0\rangle \rightarrow$ gnd state

$|1\rangle \rightarrow$ excited state

$|\alpha\rangle|\beta\rangle \rightarrow |\alpha\rangle \otimes |\beta\rangle$

$|\alpha\rangle$ → sys 1, $|\beta\rangle$ → sys 2

where sys: ↑ ↓

$$|0\rangle|0\rangle \equiv |00\rangle$$
$$|0\rangle|1\rangle \equiv |01\rangle$$
$$|1\rangle|0\rangle \equiv |10\rangle$$
$$|1\rangle|1\rangle \equiv |11\rangle$$

* gen. state: $|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

Case 1: $|\psi\rangle = \prod_i |\phi_i\rangle$ eg $= |\phi_1\rangle \otimes |\phi_2\rangle$

Eg. $|\psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$ ← 2-qubit

$|\phi_1\rangle = ?$ $|\phi_2\rangle = ?$

← separable / product states

$|\phi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = |\phi_2\rangle$ ← single qubit

Case 2

$$|\psi\rangle \neq \prod_i |\phi_i\rangle$$

Eg. $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ ← entangled states

Matrix reprstn

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\alpha\rangle = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \beta = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$|\alpha\rangle \otimes |\beta\rangle = \begin{bmatrix} a_1 |\beta\rangle \\ a_2 |\beta\rangle \end{bmatrix}$$

$$= \begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{bmatrix} \leftarrow 4\times 1 \text{ matrx}$$

Ex Is $|\alpha\beta\rangle = |\beta\alpha\rangle$?

$$|01\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ 0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

2-qubit gates

CNOT / CX

controlled

q_1 ← control bit

q_2 ← target

Qiskit convention for multi-qubit state $|q_n q_{n-1} \dots q_2 q_1\rangle$

$CX(q_1, q_2)$: Operate NOT on q_2 when $q_1 = |1\rangle$

Eg $|\psi\rangle = |q_2 q_1\rangle = |10\rangle$

$CX(q_1,q_2)\,|\psi\rangle = |10\rangle$

$CX(1,2)$

$CX(0,1)$ ← when code in Python

$|\psi\rangle = |\overset{1st}{1}\overset{0th}{1}\rangle$

$CX(0,1) = |01\rangle$

$\boxed{\text{Ex} \quad \text{Find } CX(0,1)\,|\alpha\beta\rangle \;\; \forall\, \alpha, \beta}$

Ex. Find $CX(0,1)\,|\alpha\beta\rangle \;\forall\, \alpha, \beta$

I/p	O/p
$\|00\rangle$	—
$\|01\rangle$	
$\vdots$	

Find $CX(1,0)$ ~ — —

I	O