Objectives

Practice computing the asymptotic behavior of Fourier integrals.

Instructions

Use Mathematica to solve the following problems. Use the template introduced for the previous homework. Write up discussions of your results.

Problems

1. Find the asymptotic behavior of
$$\int_0^2 (\sin t + t) e^{\mathrm{i}xt} \mathrm{d}t,$$
as $x \to +\infty$ up to terms involving $\frac{1}{x^2}$.

2. Find the leading behavior of
$$\int_0^1 \sqrt{t(1-t)}(t+a)^{-x} \mathrm{d}t, \quad x \to +\infty, \quad a > 0.$$

3. Using the method of stationary phase to find the leading behaviors of the following integrals as $x \to +\infty$.
   (a) $\displaystyle\int_0^1 \tan(t) e^{\mathrm{i}xt^4} \mathrm{d}t,$
   (b) $\displaystyle\int_{1/2}^2 (1+t) e^{\mathrm{i}x(t^3/3-t)} \mathrm{d}t.$
   (c) $\displaystyle\int_0^1 \cos(xt^4) \tan t \mathrm{d}t$