Objectives

Practice using the method of dominant balance to determine the leading behaviors of solutions of nonhomogeneous differential equations, and computing the asymptotic behavior of integrals.

Instructions

Use Mathematica to solve the following problems. Use the template introduced for the previous homework. Write up discussions of your results.

Problems

1. (Bender & Orszag, Problem 3.48) Find the first three terms in the local behavior as $x \to 0^+$ of a particular solution $y(x)$ to
$$y' + xy = x^{-3}.$$

2. (Bender & Orszag, Problem 6.7(h)) Find the leading behavior of $\displaystyle\int_0^1 \frac{e^{-xt}}{(1+t^2)}\mathrm{d}t$ as $x \to 0^+$.

3. Find the asymptotic series for $\displaystyle\int_0^x t^{-1/4}e^{-t}\mathrm{d}t, \quad x \to 0^+$. (Hint: Substitute $s = t^{1/4}$)