Objectives

Practice computing regular perturbation expansions for algebraic and differential equations.

Instructions

Use Mathematica to solve the following problems. Use the template introduced for the previous homework. Write up discussions of your results.

Problems

1. (Bender & Orszag, Problem 7.1 a) Use second-order perturbation theory to find approximations to the roots of $x^2 + x + 6\epsilon = 0$.

2. (Bender & Orszag, Problem 7.10) Compute all of the coefficients in the perturbation series solution to the initial-value problem

$$y' = y + \epsilon x y, \qquad y(0) = 1.$$

   Show that the series converges for all values of ϵ. Also, compute the perturbation series indirectly by expanding the explicit exact solution in powers of ϵ.

3. (Bender & Orszag, Problem 7.19) Show that the solution to the initial-value problem

$$y'' + y + \epsilon y = 0, \quad y(0) = 1, \quad y'(0) = 0,$$

   remains bounded for all real x. Obtain a first-order perturbative approximation to $y(x)$ and show that it is unbounded as $x \to \infty$. Conclude that the first-order approximation is valid only for $|x| < O(\epsilon^{-1})$.