Objectives

For this homework, you will work on some basic Mathematica commands that will help you throughout the semester on future homeworks.

Instructions

Find the Mathematica notebook entitled, `Math 223 - HW01 - Name.nb`.

- Rename the file and replace "Name" with your name.
- Open the Mathematica notebook. Replace "<Insert Name>" with your name and replace "<Insert Date>" with the due date for the assignment.
- Complete the following problems in the notebook.

Problems

1. Use the `Series` command to compute the Taylor series of $(1+x)^{1/x}$ about $x = 0$. Explore different parameter values to make sure you understand how to use this command. Use the `Plot` command to plot a comparison of this function with the polynomial of degree 4 given by the partial sum of the Taylor series over the interval $[-1, 4]$.
2. Use the `Integrate` command to compute $\int_0^{3/2} \sin(x^2)\mathrm{d}x$. Then use `Series` within `Integrate` to integrate the polynomial of degree 6 given by the partial sum of the Taylor series of $\sin(x^2)$ about $x = 0$. Use the `N` command to compute the absolute error made by this approximation.
3. Use the `DSolve` command to solve the following two-point boundary value problem:

$$\begin{aligned} \epsilon y'' + (1+\epsilon)y' + y &= 0, \quad 0 < x < 1, \\ y(0) &= 0, \\ y(1) &= e^{-1}. \end{aligned}$$

   Read the documentation to learn how to plot the solution for different values of $0 < \epsilon \leq 1$. Comment on how the solution changes with ϵ.