Math 223: Homework 1

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Problem 1

Use the Series command to compute the Taylor series of (1+x)^(1+x) about $x = 0$. Explore different parameter values to make sure you understand how to use this command. Use the Plot command to plot a comparison of this function with the polynomial of degree 4 given by the partial sum of the Taylor series over the interval [-1,4]

In[ ]:=
```
Series[(1 + x)^(1/x), {x, 0, 8}]
```

Out[ ]=

$$e - \frac{e x}{2} + \frac{11 e x^2}{24} - \frac{7 e x^3}{16} + \frac{2447 e x^4}{5760} - \frac{959 e x^5}{2304} + \frac{238\,043 e x^6}{580\,608} - \frac{67\,223 e x^7}{165\,888} + \frac{559\,440\,199 e x^8}{1\,393\,459\,200} + O[x]^9$$

In[ ]:=
```
Series[(1 + x)^(1/x), {x, 0, 10}]
```

Out[ ]=

$$e - \frac{e x}{2} + \frac{11 e x^2}{24} - \frac{7 e x^3}{16} + \frac{2447 e x^4}{5760} - \frac{959 e x^5}{2304} + \frac{238\,043 e x^6}{580\,608} - \frac{67\,223 e x^7}{165\,888} + \frac{559\,440\,199 e x^8}{1\,393\,459\,200} - \frac{123\,377\,159 e x^9}{309\,657\,600} + \frac{29\,128\,857\,391 e x^{10}}{73\,574\,645\,760} + O[x]^{11}$$

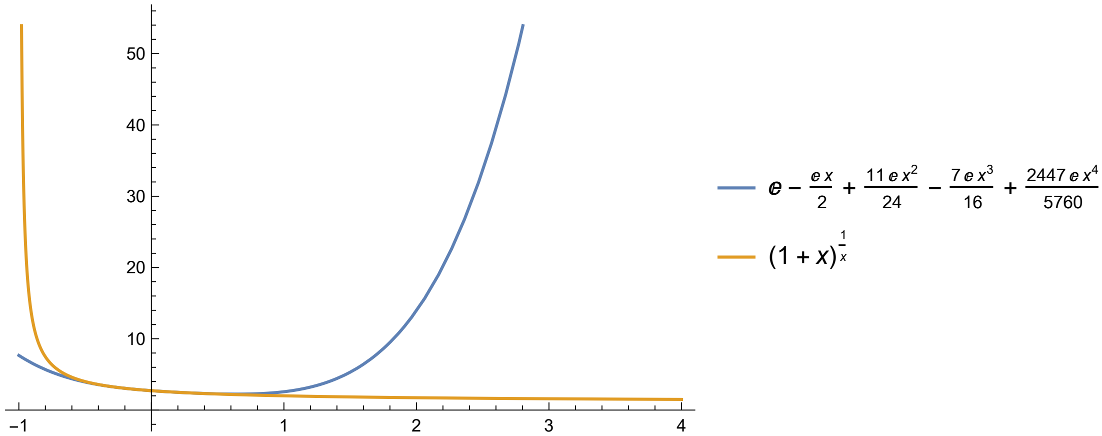
In[ ]:=
```
Series[(1 + x)^(1/x), {x, 0, 12}]
```

Out[ ]=

$$e - \frac{e x}{2} + \frac{11 e x^2}{24} - \frac{7 e x^3}{16} + \frac{2447 e x^4}{5760} - \frac{959 e x^5}{2304} + \frac{238\,043 e x^6}{580\,608} - \frac{67\,223 e x^7}{165\,888} + \frac{559\,440\,199 e x^8}{1\,393\,459\,200} - \frac{123\,377\,159 e x^9}{309\,657\,600} + \frac{29\,128\,857\,391 e x^{10}}{73\,574\,645\,760} - \frac{5\,267\,725\,147 e x^{11}}{13\,377\,208\,320} + \frac{9\,447\,595\,434\,410\,813 e x^{12}}{24\,103\,053\,950\,976\,000} + O[x]^{13}$$

In[ ]:=
```
Plot[{Evaluate[Normal[Series[(1 + x)^(1 / x), {x, 0, 4}]]], (1 + x)^(1 / x)},
  {x, -1, 4}, PlotLegends → "Expressions"]
```

Out[ ]=



Problem 2

Use the Integrate command to compute $\int_0^{3/2} sin(x^2)\, dx$. Then use Series within Integrate to integrate the polynomial of degree 6 given by the partial sum of the Taylor series of $sin(x^2)$ about x =0. Use the N command to compute the absolute error made by this approximation.

In[ ]:=
```
int = Integrate[Sin[x^2], {x, 0, 3 / 2}]
```

Out[ ]=

$$\sqrt{\frac{\pi}{2}}\ \text{FresnelS}\left[\frac{3}{\sqrt{2\pi}}\right]$$

In[ ]:=
```
ps = Integrate[Series[Sin[x^2], {x, 0, 6}], {x, 0, 3 / 2} ]
```

Out[ ]=

$$\frac{1287}{1792}$$

In[ ]:=
```
Abs[N[ps - int]]
```

Out[ ]=

0.0600458

Problem 3

Use the DSolve command to solve the following two-point boundary value problem (in the Homework 1 pdf). Read the documentation to learn how to plot the solution for different values of $0 < \epsilon \leq 1$. Comment on how the solution changes with ϵ.

In[ ]:=
```
DSolve[{ϵ * y''[x] + (1 + ϵ) * y'[x] + y[x] == 0, y[0] == 0, y[1] == E^(-1)}, y[x], {x, 0, 1}]
```

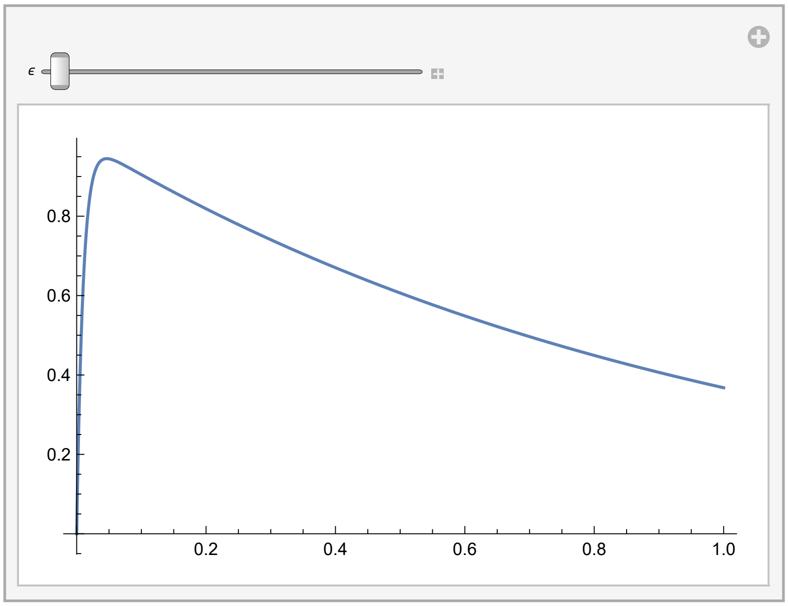
Out[ ]=

$$\left\{\left\{y[x] \to \frac{e^{-\frac{-1+x+x\epsilon}{\epsilon}}\left(-e^{x}+e^{x/\epsilon}\right)}{-e+e^{\frac{1}{\epsilon}}}\right\}\right\}$$

In[ ]:=

$$\text{Manipulate}\left[\text{Plot}\left[\frac{e^{-\frac{-1+x+x\epsilon}{\epsilon}}\left(-e^{x}+e^{x/\epsilon}\right)}{-e+e^{\frac{1}{\epsilon}}}, \{x, 0, 1\}\right], \{\epsilon, 0.01, 0.99\}\right]$$

Out[ ]=



Using the Manipulate command, we can plot the result with differing values for ϵ, specified by the scroll bar. Using said scroll bar, we see that as $\epsilon \to 0$, the solution to the BVP is unreasonable around 0 due to the solution of the BVP when $\epsilon = 0$, being $y(x) = C\,e^{-x}$. Because this is the solution, we see that the given the boundary conditions of $y(0) = 0$, $y(1) = e^{-1}$ will never hold.