## Objectives

For this homework, you will review the classification of points (ordinary, regular singular, irregular singular) for specific differential equations, use Fuchs' theory to anticipate the properties of expansions of solutions about specific points, and compute those expansions.

## Instructions

Use Mathematica to solve the following problems. Use the template introduced for the previous homework. Write up discussions of your results.

## Problems

1. (Bender & Orszag, Problem 3.4) Classify the points at 0 and $\infty$ of the following differential equations.

   (a) $x^7 \dfrac{\mathrm{d}^4 y}{\mathrm{d}x^4} = \dfrac{\mathrm{d}y}{\mathrm{d}x}$.

   (b) $x^3 \dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3} = y$.

   (c) $\dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3} = x^3 y$.

   (d) $x^2 \dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = e^{1/x} y$.

   (e) $(\tan x) \dfrac{\mathrm{d}y}{\mathrm{d}x} = y$.

   (f) $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = (\log x) y$.

   One way to use Mathematica to help understand whether a function is analytic or not is to use the `Series` command and interpret the results.

2. (Bender & Orszag, Problem 3.5) For the initial-value problem,

$$(x-1)(x-2)y'' + (4x-6)y' + 2y = 0, \quad y(0) = 2, \quad y'(0) = 1.$$

   where might one expect the series solution about $x = 0$ to converge? Compute the Taylor expansion about $x = 0$ of the solution and determine the radius of convergence.

3. (Bender & Orszag, Problem 3.24) For the following differential equations, where would you expect the series solution about $x = 0$ to converge? Find series expansions of all the solutions to the following differential equations about $x = 0$. Try to sum in closed form any infinite series that appear.

   (a) $xy'' + y = 0$

   (b) $y'' + (e^x - 1)y = 0$

   (c) $(\sin x)y'' - 2(\cos x)y' - (\sin x)y = 0$