

Objectives

Practice applying the method of multiple scales to study weakly nonlinear oscillators.

Instructions

Use Mathematica to solve the following problems. Use the template introduced for the previous homework. Write up discussions of your results.

Problems

1. For the van der Pol oscillator,

$$y'' + y + \epsilon(y^2 - 1)y' = 0,$$

apply the method of multiple scales and show that it has a stable limit cycle that is nearly circular with radius $r = 2 + O(\epsilon)$.

2. Use the method of multiple scales to study

$$\frac{d^2 y}{dt^2} + y + \epsilon y^2 \frac{dy}{dt} = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

Describe the long-time behavior of the solution based on your results.