Objectives

Practice computing the asymptotic behavior of Laplace integrals.

Instructions

Use Mathematica to solve the following problems. Use the template introduced for the previous homework. Write up discussions of your results.

Problems

1. Find the first two terms of the asymptotic expansion of

$$\int_5^9 \frac{e^{-xt}}{t}\mathrm{d}t, \quad x \to +\infty.$$

2. Use Watson's lemma to determine the asymptotic expansion of

$$I(x) = \int_0^{\pi} e^{-xt}t^{-1/3}\cos t\mathrm{d}t, \quad x \to +\infty.$$

3. Use Laplace's method to determine the leading behavior of

$$I(x) = \int_{-1/2}^{1/2} e^{-x\sin^4 t}\mathrm{d}t, \quad x \to +\infty.$$

4. The L_p norm of a function g is given by $\|g\|_p = (I(p))^{1/p}$ where

$$I(p) = \int_a^b |g(t)|^p\mathrm{d}t.$$

   Assuming that $|g(t)| \in C^4$ and that it attains its unique maximum on $t = c$ inside $[a, b]$ with $g(c) \neq 0$, use Laplace's method to show that the L_p norm converges to the "maximum" norm as $p \to \infty$.

5. Show that

$$\int_0^{\infty} \log\left(\frac{u}{1-e^{-u}}\right)\frac{e^{-ku}}{u}\mathrm{d}u \sim \frac{1}{2k}, \quad k \to \infty.$$