If you cannot join our Zoom session and share your camera for any reasons,
you must let the instructor know. If you haven't done so, you are not allowed to start until you send an email to ckim103@ucmerced.edu.

If you have any internet connection issue and are disconnected during the exam,
please continue to work on the exam and finish it. After uploading your exam to CatCourses, you will need to inform the instructor via email (ckim103@ucmerced.edu) that you have not acted outside the university academic integrity guidelines.

Exam timeline

- 11:55am – 12:05pm (10 minutes): Download the exam (this PDF file).
  - Read the instructions carefully.
  - Copy and sign the honor code.
  - You can read the exam but are NOT allowed to start it yet.
- 12:05pm – 1:05pm (60 minutes): Work on the exam.
- 1:05pm – 1:20pm (15 minutes): Upload your finished exam to CatCourses.
  - If you finish early, you can proceed to upload it before the time.

Read the following instructions carefully

- This exam lasts 60 minutes.
- The total exam score is 40 points = 2 pts $\times$ 5 problems (Part I) + 5 pts $\times$ 6 problems (Part II). There are ***2 extra points*** for tasks in framed boxes. Your score will be converted using the formula min(your score, 30 pts).
- You must write your solution on paper (i.e. not on an electronic device) and upload a PDF copy of your exam to CatCourse.

[extra 1 pt] On the first page, copy the following Academic Integrity Statement and **sign** it:

By completing this exam, I acknowledge and confirm that I will not give or receive any unauthorized assistance on this exam. I will conduct myself within the university academic integrity guidelines.

Part I

(5 problems $\times$ 2 pts)

Determine whether each statement is true or false. Justify your answer.

1. The second-order central difference approximation for $f''(x)$ is given as

$$\frac{f(x+\Delta x)-2f(x)+f(x-\Delta x)}{2\Delta x}. \tag{1}$$

2. If the partial sum $S_N(x)$ of the Fourier series of function $f(x)$ converges uniformly to $f(x)$, $f(x)$ is continuous everywhere.

3. For the least-squares solution $\tilde{\mathbf{x}}$ to $A\mathbf{x}=\mathbf{b}$, its residual error $\mathbf{e}=A\tilde{\mathbf{x}}-\mathbf{b}$ belongs to the null space of A^T.

4. Since the local truncation error for the explicit trapezoid scheme is $O(\Delta t^2)$, the scheme is a second-order method.

5. If the integration function $f(x)$ is not infinitely differentiable, a Newton-Cotes quadrature for $\int_a^b f(x)dx$ may exhibit lower-order convergence.

Part II

(6 problems $\times$ 5 pts)

1. [5 pts] Assume that we are given a data set $\{(x_1,y_1),(x_2,y_2),\dots,(x_m,y_m)\}$ ($m \geq 3$) and three functions $f_1(x)$, $f_2(x)$, $f_3(x)$. We consider the following linear combination of the three functions,

$$f(x)=a_1f_1(x)+a_2f_2(x)+a_3f_3(x), \tag{2}$$

and want to determine the values of a_1, a_2, a_3 so that $f(x)$ gives the *best* fit to the data points.

   (a) Construct a linear system that determines the optimal values of a_1, a_2, a_3.

   (b) Briefly explain in which sense this choice gives the *best* fit.

   (c) State when the optimal values are uniquely determined.

2. [5 pts] Consider the initial value problem: $u'(t)=F(t,u(t)),\ u(t_0)=u_0$.

   (a) Write down the backward Euler scheme.

   (b) Define the local truncation error τ_{k+1}.

   (c) Show that

$$\tau_{k+1}=-\frac{\Delta t^2}{2}(F_t+FF_u)|_{t=t_k,u=u(t_k)}+O(\Delta t^3). \tag{3}$$

3. [5 pts] Consider the initial value problem: $u'(t) = F(t, u(t)),\ u(t_0) = u_0$.
   (a) Write down the explicit midpoint scheme.
   (b) Write a MATLAB function for the scheme
   ```
   function [tvals,uvals] = explicit_midpoint(F,t0,T,u0,n)
   ```
   (c) Briefly explain how you would validate your implementation.

4. [5 pts] Consider the following Gauss-Legendre quadrature:

$$\int_{-1}^{1} f(x) \approx \sum_{i=1}^{3} w_i f(x_i) \quad \text{where } x_1 = -\sqrt{\frac{3}{5}},\ x_2 = 0,\ x_3 = \sqrt{\frac{3}{5}},\ w_1 = \frac{5}{9},\ w_2 = \frac{8}{9},\ w_3 = \frac{5}{9}. \tag{4}$$

   (a) Using the quadrature, compute $\int_{-1}^{1} x^2 dx$.
   (b) Using the quadrature (and integration by substitution), compute $\int_{-2}^{1} x^2 dx$.
   (c) Are these values exact? Briefly explain why.
   (d) Briefly explain how the quadrature points x_1, x_2, and x_3 are obtained.

5. [5 pts]
   (a) Using the method of undetermined coefficients, determine the values of A and B:

$$f''(x) \approx \frac{2f(x) - 5f(x - \Delta x) + Af(x - 2\Delta x) + Bf(x - 3\Delta x)}{\Delta x^2}. \tag{5}$$

   (b) What is the order of this approximation?

6. [5 pts] Using the idea of Richardson extrapolation, obtain Simpson's rule from trapezoid rule.

[extra 1 pt] Write down the time that you finish your exam.

- Proceed to scan (or take clear pictures of) your exam and upload it to CatCourses.
- Include your cheat sheet.
- If, for any reason, you failed to share your camera during the full period of the exam (e.g. disconnection), you must inform the instructor via email that you have not acted outside the university academic integrity guidelines.