Problem 1

Plots made for p = 0.4, 0.5, 0.55, 0.9, 0.6, and 0.8

As p approaches p_c , the largest cluster encompasses more and more of the grid, until after p_c when it encompasses most of nearly all of the grid. At p_c, one cluster is about large enough to extend from one end of the grid to the other.

Problem 2

a)
For p near p_c, the probability that a point will be part of the largest cluster q_1 rises dramatically and asymptotically goes to 1. The probability that a point will be part of the second largest cluster q_2 grows like q_1 , until it reaches p_c and then falls asymptotically going to zero.

This is because after p_c is reached, the largest cluster begins to very quickly cover the entire grid. What was the second largest cluster is now absorbed into the largest cluster. In this way, the largest cluster dominates the grid after p_c, and the size discrepancy between the size of the largest and second largest becomes much greater. Now, the second largest cluster is the largest little lump on the edge that managed to not become part of the largest cluster.

b)
P_c in 2D ~ 0.589
P_c in 3D ~ 0.310

c)
The error in 2D is approximately +/- 0.02. This error is mainly due to error in identifying the peak, my own error. I chose an L value large enough that the position of the peak position varied a very small amount when changing L, a negligible amount when compared to my uncertainty in selecting the peak. That is, my L value was large enough that the error in which p_c was shown by the curve was small.

The error in3D is approximately 0.05. This is slightly larger than the error in 2D, simply because my code crashed when trying to used as large an L as in the 2D situation, so the certainty in the peak lining up with the actual value for p_c is lower.

Problem 3

a)

The fractal dimensions in 2D were calculated to be 1.82 from the largest clusters data, and 2.02 from the second largest, which are both close when averaged to the predicted 91/48 = 1.859 value found in class, and converge closer as L goes to infinity.

Fractal dimensions in 3D were calculated to be 2.48 from the largest clusters data, and 2.62 from the second largest, which are also both close to the 2.5 predicted value when averaged, and will converge with L goes to infinity.

b)

d = 2

For $p < p_c$, chose 0.3 and 0.4

For $p > p_c$, chose 0.7 and 0.8

For p close to p_c , chose 0.57

(All plots attached except second largest clusters for p = 0.8. I couldn't graph this, as some of the values for cluster sizes were so small they became infinite when trying to log(M) = -inf)

As p becomes very close to p_c, the theoretical functional form doesn't fit as well. For example, in the p = 0.57 plot, the data is more of a power law than a log.

c)

Relating to the previous problem, the reasons the functional forms don't seem to fit is the same reason there is no discontinuity at $p = p_c$. It's because the theoretical forms are true in the ranges specified exactly for L at infinity. Because in our analysis, L is much much less than infinity, there is "fuzz" in the functional form around where the discontinuity would occur. As L becomes larger, this fuzz is less pronounced and the discontinuity will become more evident.