Haley Bates-Tarasewicz
Nonlinear Dynamics: The Natural Environment
12.009 PSET 1

Part 1

a)

The functional form is a decaying exponential, which can be seen below in Figures 1 and 2. Exponential decay takes the form $y = N_0x^{-\alpha}$, where $N_0 = y(0)$ and α is the decay constant, both of which are influenced by probability.

For a large probability N_0 will be large. At high probabilities (p = 5%), N_0 was ~2250, while at low probabilities (p = 0.5%), N_0 was only ~130.

Contrarily, for a large probability, α will be small. This can be seen by how quickly the trendline for the histogram approaches zero. At high probabilities where p = 5%, the graph began leveling off around 100, but at low probabilities where p = 0.5%, it leveled out much slower, beginning around 800.

Physically, this means (intuitively) that at higher probabilities, ^{14}C is counted more frequently and there is less time on average between ^{14}C atoms.

Figure 1

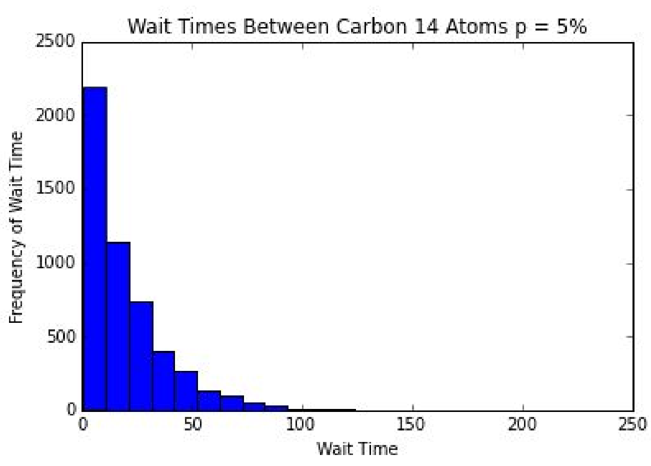
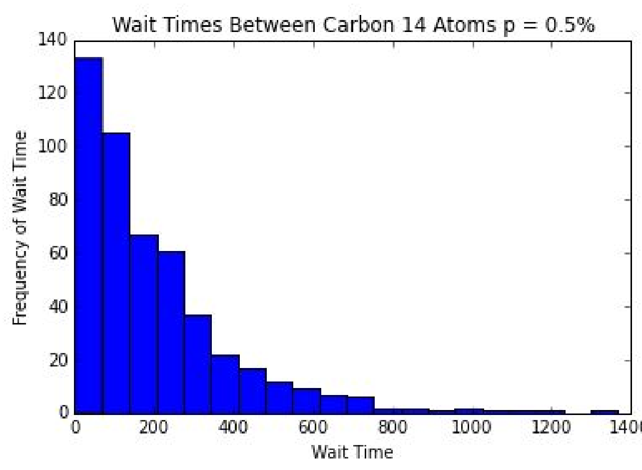


Figure 2



b)

The accuracy of measured isotopic ratio gets better with an increased sample size.

The two plots below (Figure 3 and 4) of the Variance of ^{14}C vs Sample size have the functional form $y = x^{a} * e^{b}$ (the variance is linear on a log/log plot).

The probability affects the graph by changing b. A higher probability provides a larger scaling factor. As can be seen from the below plots, when the probability was high (p = 5%), the largest point was around 10^{-4}. With a lower probability (p = 0.5%), the highest point was less than 10^{-5}.

Figure 3

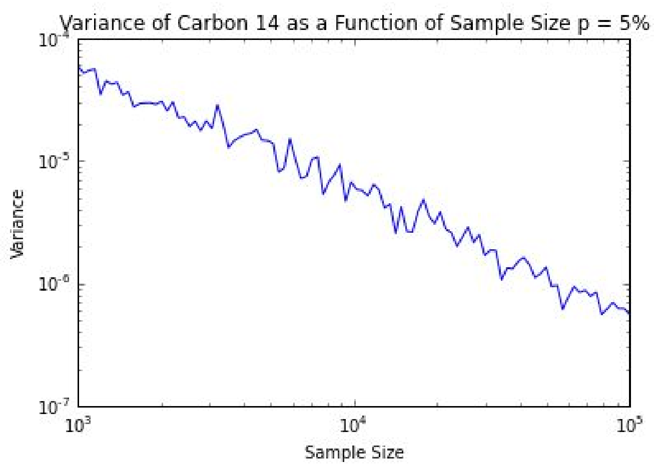
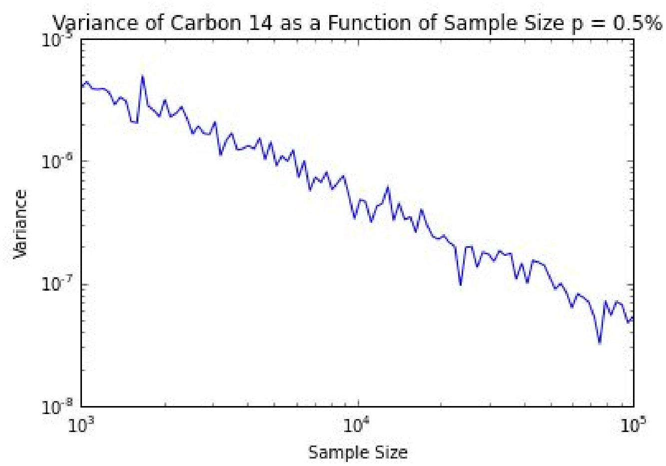


Figure 4



c)

This suggests that older samples will be harder to date. As ^{14}C decays, the ratio between ^{14}C and ^{12}C will become smaller and smaller, and the probability of a ^{14}C atom appearing will be smaller. This means that it will be harder to get a large enough sample size to have low variance, so more material will be needed to obtain precise measurements.

Part 2

a)

Based on the mechanism, I would expect the wait time distribution between eruptions to look like a normal distribution (perhaps multi-modal), based on the Central Limit Theorem. This would also make sense physically as geysers can have underground reservoirs of steam that bubble up periodically and accelerate surface boiling.

b)

I would expect random variability to express itself by the distribution not being a perfect normal distribution. The Central Limit Theorem states that as random independent variables are added, their sum tends towards a normal distribution, so I would not expect

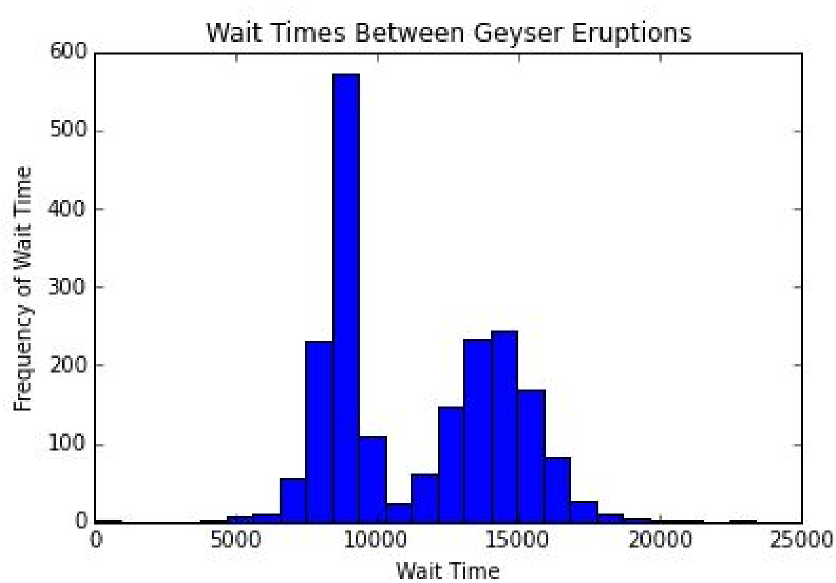
c)

This distribution appears to be a bimodal normal distribution (Figure 5). This is in fact a normal distribution, and is consistent with the proposed theory.

If the geyser had one underground chamber heating water, the result would be a single normal distribution as the reservoir emptied and refilled with a nearly constant rate.

With a bimodal distribution, it hints at two underground chambers fueling the geyser, and the two curves of the distribution correspond to each reservoir emptying and refilling.

Figure 5



d)

The i + 1 indexing shown in Figure 6 shows two tightly clustered groups of data points. The centers of these clusters are at approximately (9000, 15000) and (15000, 9000) respectively. This implies that the average time between eruptions is either 9000, or 15000. These values also match the two peaks of the bimodal distribution seen in Figure 5. This plot reaffirms the bimodal behavior of the geyser, and implies that the timing is periodic between 9000 and 15000.

The i + 2 indexing shown in Figure 7 shows two groups of data points more loosely clustered than those seen in Figure 6. The centers of the clusters are at (9000, 9000) and (15000, 15000) respectively, which mirror the two peaks of the distribution in Figure 5. Because this plot is offset by index + 2 and the centers of the clustered data points have equal x and y values, it reaffirms the hypothesis that the timing of the geyser eruptions is periodic between the two modes.

Figure 6

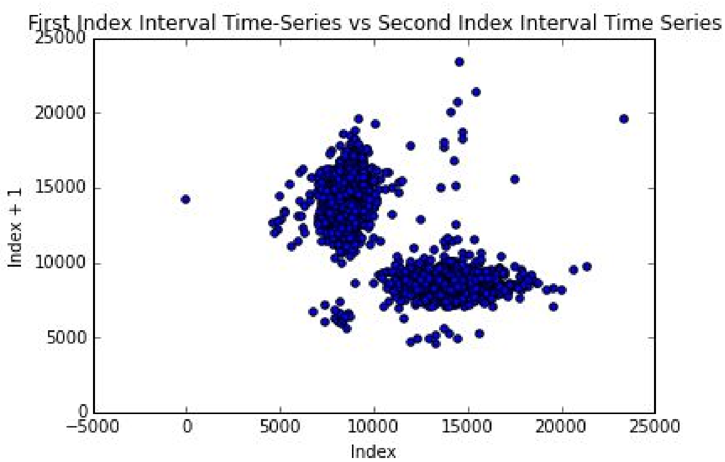


Figure 7