

Lab 3: Water Rocket Final Report

Rocket Drawing and Final Design Analysis

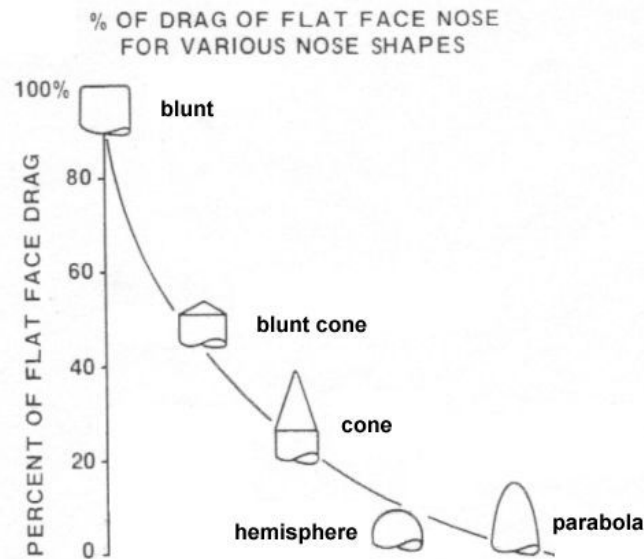
For the construction of the rocket, we were somewhat limited by the materials we had available. We used Pepsi brand 2L bottles, polystyrene modeling foam, glue, and tape as the construction materials.

The body of the rocket was made of a whole 2 liter bottle. The nozzle was trimmed shorter to fit on to the launch pad, and several layers of scotch tape were added to the interior of the nozzle to provide a better seal with the launch tube. The nose cone was made of another 2 liter bottle with the bottom and neck/opening cut off. Using a second bottle as the nose cone ensured a perfect fit over the existing body of the rocket. We glued rounded modeling foam into the top of the nose cone to close the hole created by the trimming of the nozzle/opening and to make the nose cone as close to a parabolic shape as possible. We were aiming for a parabolic nose cone shape because parabolic nose cones (for subsonic flight) provide the lowest flat face drag of any nose cone shape (as seen in Figure 1 below). Strategically adding modeling foam, we built a payload compartment of sorts to ensure our bagel wouldn't move through the duration of the flight or between flights. This also added the needed weight in order to move the center of gravity such that the rocket's flight would be stable. Our calculations for the center of gravity are detailed later in this lab report. After determining that the center of gravity for our proposed rocket design would allow the performance we desired, we continued with the design process.

The fins were designed in OpenRocket and cut out of balsa wood. We made three sets of fins because we (correctly) assumed the fins would break after each launch. We designed our fins such that the static margin was negative even with the entire mass of water in the rocket so it would be stable from the very moment it was released from the launch pad. We now understand that this caused our fins to be oversized, as the entirety of the water is expelled within the first 50 milliseconds. Such large fins were not needed for the overwhelming majority of the rocket's flight. Despite being too large, our fins suited our needs and we determined our design was stable, and so continued to refine the design

of our rocket. Our methods for determining the center of pressure and stability margin are described in more detail later in this lab report.

Figure 1: Flat face drag of nose cone shapes¹

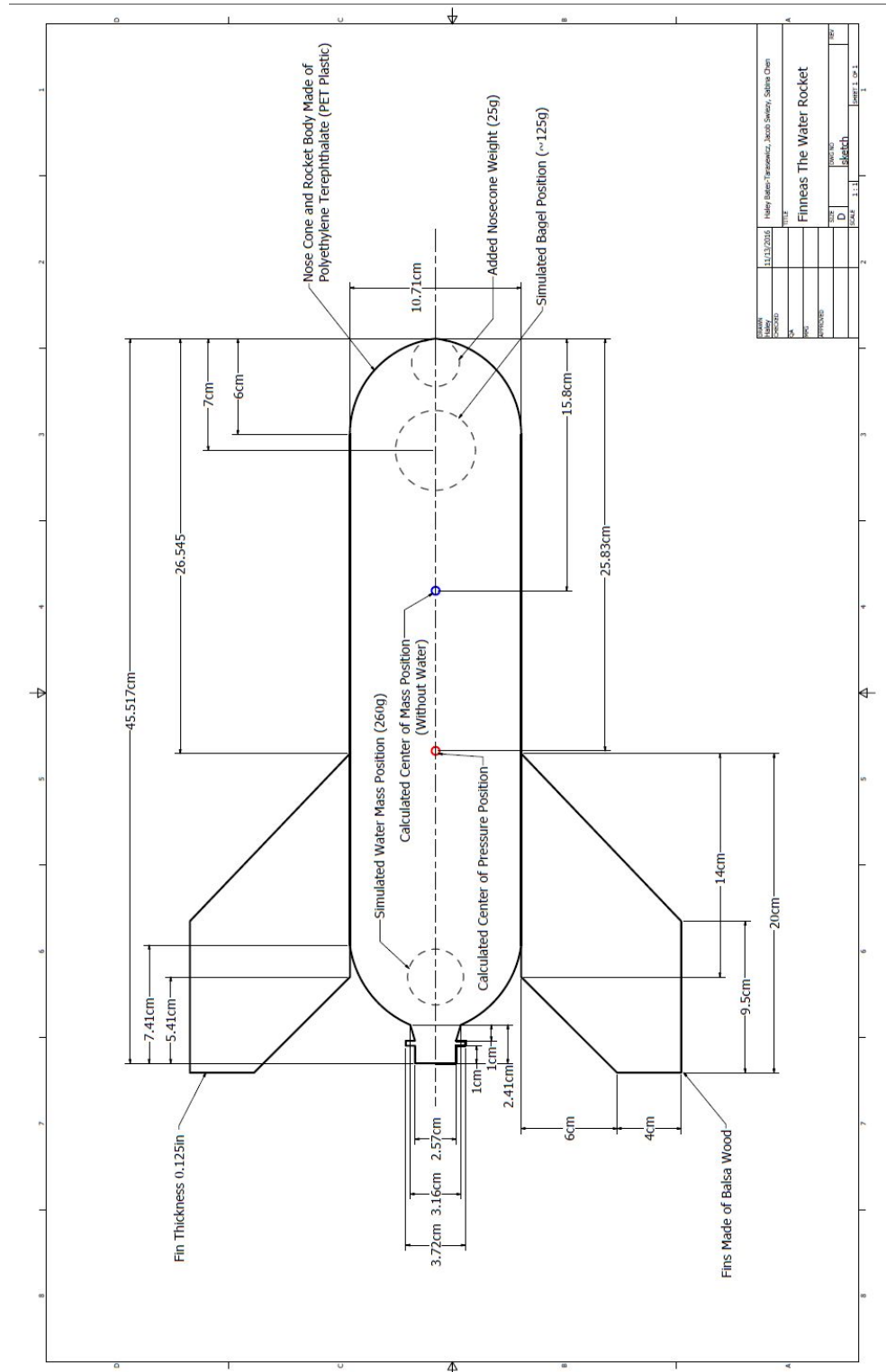


While designing, we kept mass and coefficient of drag in mind. We tested our rocket in the wind tunnel and found our coefficient of drag to be 0.38. This was a little higher than what we were hoping to achieve and was probably caused by the oversized fins. However, this coefficient of drag was within the range we prepared for in Part 1 of this lab, and we continued with the finalization of our rocket design. From the rocket mass and coefficient of drag, we were able to determine the water mass needed to achieve a 50m flight distance. Our methods for determining the performance of the rocket are outlined later in this lab report.

A full schematic of our rocket can be seen in Figure 2 below.

¹ <http://www.aerospaceweb.org/question/aerodynamics/q0151.shtml>

Figure 2: Finneus the Water Rocket Working Drawing



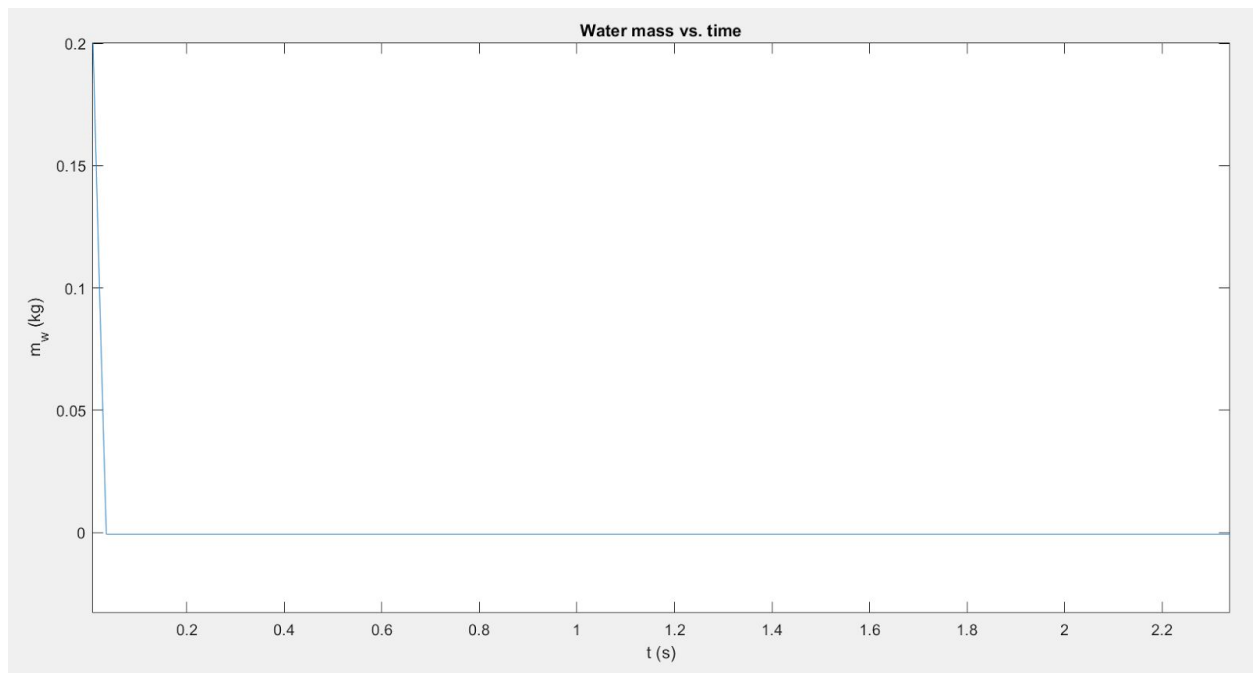
Estimation of Center of Pressure, Center of Gravity, and Stability Margin

Center of Gravity

The CG of the rocket is a time dependent equation because water is expelled from the rocket during flight. Because of this, it is impossible for us to define a singular CG.

As seen in figure 3 below, however, the mass of water is expelled so quickly (within the first 50ms) that we will consider the CG of the rocket to be the CG of the unfilled rocket for flight stability calculations. Below we will present both the CG of the unfilled rocket and the CG of the filled rocket in order to show the rocket's transition from unstable on the launch stand to stable almost immediately after takeoff

Figure 3: Water Mass vs. Time



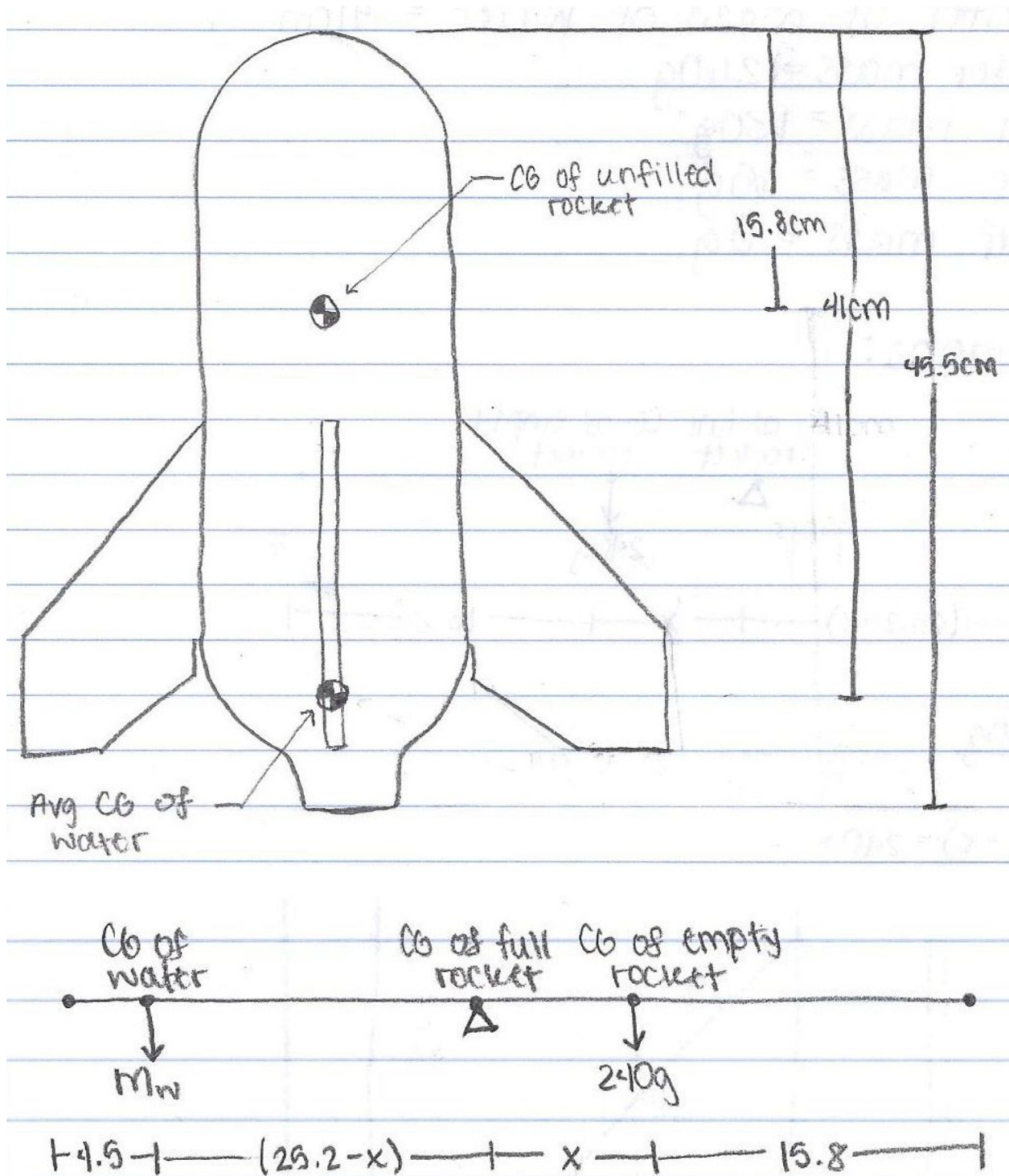
To determine the CG of the unfilled rocket (CG during flight), we used a simple balance test. We began by clamping a thin piece of wood in a vice. We then placed the rocket on top of the thin wooden piece such that the rocket acted like a teeter-totter on top. We then moved the rocket along its length until the rocket balanced on the thin piece of wood without extra support on either end of its length. The point along the rocket's length at which it balanced on the wood is the rocket's CG.

Through experimentation we determined the CG of the unfilled water rocket to be **15.8 cm** from the top of the rocket.

To determine the CG of the filled rocket (CG while sitting on launch pad), we started by estimating the CG of the initial water mass within the rocket. We then used then used moment equations along with the experimentally determined CG of the unfilled water rocket to obtain a CG estimate for the filled rocket. As shown in the figure below, we can model the water rocket as a simple lever arm. We model the CG of the unfilled rocket and the CG of the water propellant as torques on the filled rocket to calculate the new CG. We will calculate the CG of the filled rocket twice, once with our predicted water mass (260g) and once with the actual water mass we used at launch (350g).

For this calculation we estimate the CG of the water mass to be at 41cm from the top of the rocket. The calculations are as follows in figure 4:

Figure 4: Center of Gravity Calculation



Center of Gravity of filled rocket for initial water mass 260g:

$$\Sigma M = 0$$

$$260 * (25.2 - x) - 240x = 0$$

$$260 * (25.2 - x) = 240x$$

$$25.2 - x = \frac{12}{13}x$$

$$x = 13.10 \text{ cm}$$

This translates to a CG of $13.1 + 15.8 = \mathbf{28.9 \text{ cm}}$ from top of the rocket for a rocket filled with 260g of water.

Center of Gravity of filled rocket for initial water mass 350g:

$$\Sigma M = 0$$

$$350 * (25.2 - x) - 240x = 0$$

$$350 * (25.2 - x) = 240x$$

$$25.2 - x = \frac{24}{35}x$$

$$x = 14.95 \text{ cm}$$

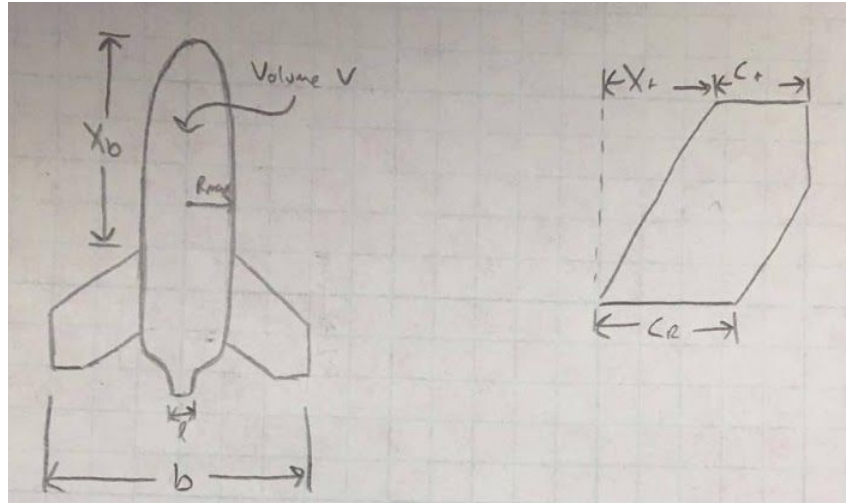
This translates to a CG of $14.95 + 15.8 = \mathbf{30.75 \text{ cm}}$ from top of the rocket for a rocket filled with 350g of water.

Center of Pressure

Determining the center of pressure was more challenging than initially expected. The notes given in class explained how to find the center of pressure using an X_f based on a design that utilized trapezoidal fins. Using OpenRocket, our team found it beneficial to use fins that were a five-sided polygon shape rather than a trapezoidal shape, which made it impossible for our team to use the given methods for determining X_f . We instead used the Barrowman Equations to find X_{cp} , and then from this calculated the actual center of pressure using the equations provided in class.

All equations use variables defined in the following figure 5.

Figure 5: Variable Definitions for Center of Pressure Calculations



$$X_{cp} = \frac{IA_{max} + X_f A_{eff} - V}{A_{max} + A_{eff}}$$

$$A_{max} = R_{max}^2 \Pi$$

$$R_{max} = 5.08cm$$

$$A_{max} = 5.08^2 \Pi = 81.07cm^2$$

$$X_f = \frac{X_t(C_r + 2C_t)}{3(C_r + C_t)} + \frac{1(C_r^2 + C_t^2 + C_r C_t)}{6(C_r + C_t)} + X_b$$

$$X_t = 10.5cm$$

$$C_r = 14cm$$

$$C_t = 9.5cm$$

$$X_b = 25.5cm$$

$$X_f = \frac{10.5(14 + 19)}{3(14 + 9.5)} + \frac{1(14^2 + 9.5^2 + (14)(9.5))}{6(14 + 9.5)} + 25.5 = 33.388cm$$

$$A_{eff} = \Pi\left(\frac{b}{2}\right)^2\left(1 - \left(\frac{R_{max}}{b}\right)^2\right)^2$$

$$b = 30.56cm$$

$$R_{max} = 5.08cm$$

$$A_{eff} = \Pi\left(\frac{30.56}{2}\right)^2\left(1 - \left(\frac{5.08}{30.56}\right)^2\right)^2 = 693.517$$

$$l = 1.25cm$$

$$V \sim 3250cm^3$$

$$X_{cp} = \frac{(1.25)(81.07) + (33.388)(693.5170) - 3250}{81.07 + 693.517} = 25.829cm$$

We found that the center of pressure was located **25.829 cm** from the top of the rocket. This was consistent with the prediction we found in OpenRocket.

Stability Margin

The stability margin is the subtraction between the center of gravity and center of pressure $Stability\ Margin = X_{cg} - X_{cp}$. In order for the bottle rocket to be stable, the center of gravity must be above the center of pressure; therefore, a stable bottle rocket will have a negative stability margin. While the center of pressure should stay constant throughout the entirety of the rocket's flight, the center of mass of the rocket will vary as water is expended. Our calculations demonstrate that the rocket will be unstable with all of the water mass, but will become very stable as water is expended. With the limitation in materials and other size constraints, we found it difficult to design a rocket that would be stable with the entire water mass in the rocket. Our resulting design was the most stable design we could create. While our calculations show the rocket to be slightly unstable at takeoff, we found through simulations and during the actual launch that the rate of water expenditure was high enough that the rocket was stable during the entirety of its flight.

Using the values obtained above for the center of gravity during flight (without water) and the center of pressure, we obtain a stability margin of $15.8\text{ cm} - 25.829\text{ cm} = \mathbf{-10.029\text{ cm}}$. As described above, the rocket will be stable during flight because it has a negative stability margin.

Estimation of Mass of Water

We used the provided trajectory model for water mass and angle calculations.

Our Inputs:

- Initial Pressure Difference = 2.75×10^6 Pa
 - The team used the maximum possible pressure difference (40 psi) for launch. The conversion ratio is 1 psi = 6895 Pa, so 40 psi = 2.75×10^6 Pa.
- Rocket Mass Including Payload = 240g
 - This value was reached by weighing the completed rocket on a digital scale. The actual measured value was about 285g. We rounded to 300g for calculations to account for any added tape or glue needed to attach the fins or to repair the rocket.
- Drag Coefficient = 0.38
 - This value was reached experimentally. The rocket was attached to a load cell in the wind tunnel; then using the provided conversion equation, we determined the CD of the rocket.
- Bottle Exhaust Area = $3.5 \times 10^{-4} \text{ m}^2$
 - This value was reached by carefully measuring the diameter of the bottle opening. The diameter we measured was 21.3mm. We found the exhaust area using the equation $A = \pi * r^2$.
- Volume of Bottle = $2 \times 10^{-3} \text{ m}^3$
 - It's marketed as a 2L bottle. We trust Pepsi. 1L = $1 \times 10^{-3} \text{ m}^3$, so 2L = $2 \times 10^{-3} \text{ m}^3$.

Necessary Water Mass for 50m Flight = 260g

We concluded through the provided model with the above input parameters that the minimum water mass we would need to reach 50m was 260g. The corresponding angle to be used with this water mass was 41deg.

Actual Performance vs Estimates

In order to reach a target 50 meters away, our team estimated that it would take 260 grams of water. For a stable flight we estimated we would need four excessively large fins and extra weight in the nose cone to create a stable water rocket. Because of water leakage during the first launch, our rocket lost most of its "fuel", causing it to land 44 meters away from the target (only 6 meters from the launch pad). During the second launch, after some physical modification to the nozzle of the bottle to fix the water leakage problem, there was little to no lost water. Unfortunately, two of the four fins fell

off mid-flight. Even so, the rocket was able to land 3.8 meters away from the 50-meter target. During the third launch, we experienced similar problems to our second launch, and all four fins fell off the rocket immediately after launch. This made the rocket (expectedly) very unstable, landing 31.4 meters away from the target.

Out of the three launches, the second launch performed most accurately based on our estimates.

Challenges we faced during the actual launch included water leakage and the loss of fins during flight. Because the nozzle of the rocket was too large, most of the water leaked out during the first launch, so the rocket was not able to travel very far. Our team fixed this problem by taping the insides of the nozzle between flights to make the exit area smaller. This improved the seal between the nozzle and the launch tube and prevented water from leaking out. In order to compensate for possible water leakage, we also overestimated the amount of water we initially calculated after the first launch, using 350 grams of water during the actual launch instead of the estimated 260 grams. Another problem we encountered was that the fins did not handle stress very well; this was especially apparent in launch 3. We had glued the fins such that the grain of the wood was parallel to the direction of flight rather than perpendicular to it, so the slightest amount of stress was able to break the fins.

Our team designed the fins such that the rocket would be stable when filled with water. As mentioned above, this caused our fins to be oversized. After actually launching the rocket, we realized that most of the water was expelled in less than half a second. Therefore, after recalculating the stability of the rocket without water, such excessively large fins were not actually needed, and smaller fins would have sufficed. Furthermore, if we had taken into account water leakage and the force necessary to get off the launchpad itself, our water estimate would have been more realistic.

Significant Contributions and Time Estimations

Our team made sure through every phase to divide up the work as evenly as possible, and made sure not to give any one team member more or less work than other

members of the group. Every part of the lab was a group effort. Below, table 1 describes the estimated time spent on each task.

Table 1: Estimated Time of Each Task per TEam Member

Task	Team Member Contributors	Hours Spent per Team Member	Total Man Hours Spent
Concieve/Planning Phase			
Baseline Analysis	Haley	1	1
Sensitivity Analysis	Sabina	1	2
	Jacob	1	
	Haley	1	3
Lab Planning Phase	Sabina	1	
	Jacob	1	
	Haley	2	6
Lab Report 2 Writing	Sabina	2	
	Jacob	2	
Design, Implement, Operate Phase			
Conceptual Design/Brainstorming	Haley	1	
	Sabina	1	
	Jacob	1	3
Rocket Drawing	Haley	2	2
Design/Manufacture Nose Cone	Sabina	1	
	Jacob	1	2
Design/Manufacture Fins	Sabina	1	
	Jacob	1	2
Assemble Rocket	Haley	1	
	Sabina	1	
	Jacob	1	3
Test Rocket in Wind Tunnel/Coefficient of Drag Determination	Haley	1	
	Sabina	1	
	Jacob	1	3
Determine Variable Parameters (mass, center of mass, center of pressure, water mass, launch angle)	Haley	1	
	Jacob	1	2
Final Report Phase			
Write Final Report	Haley	3	
	Sabina	3	
	Jacob	3	9

Total Man Hours for Concieve/Planning Phase: 12

Total Man Hours for Design, Implement, Operate Phase: 17

Total Man Hours for Final Report Phase: 9

The total time spent on any given activity did not exceed the lab restrictions (for Phase One 4hrs, for Phase Two 6hrs, for Phase Three 3hrs).

Comparison to Lab 2 Predictions

Lab 3 included creating a stable rocket design, manufacturing and assembling fins and a nose cone, calculating the center of mass and center of pressure and determining stability, testing the rocket in the wind tunnel and calculating the drag coefficient, assembling the rocket, launch day, and writing this final report. All of these activities were planned for in Lab 2, though some were not explicitly mentioned. For example, rather than explicitly stating “calculate center of mass and center of pressure” in the Lab 2 plan, the stability calculations were instead broadly mentioned as “Estimate and Determine Variable Parameters”.

The stability calculations took longer than originally planned due to our team’s fin design. In order most easily a stable design through OpenRocket simulations, our fin was designed as a five-sided polygon. We then had difficulty when we tried to hand-calculate the center of pressure, as the equations given in class assumed a trapezoidal fin design.

Making the actual nose cone and fins also took longer than expected because our team needed to determine ways of keeping the modelling foam and payload (bagel) inside the nose cone while making the payload easy to remove if necessary.

We assumed that our rocket would need to include a fairing. This was not the case, and we were able to use the time budgeted to the design and manufacture of a fairing on other parts of the lab.

Many of the roles that we initially assigned to each individual in Lab 2 were traded or shared during the actual design and implementation of the rocket in Lab 3. For instance, though Haley and Jacob were initially responsible for manufacturing the fins and nosecone while Sabina was responsible for determining the variable parameters, it was ultimately Jacob and Sabina who made the fins and nose cone while Haley did the stability calculations. This was due to conflicting schedules and unpredicted time constraints of all team members. Another reason for changes in roles and time underestimations was that our team only had three people on it rather than the usual four. Because of this, it was hard to accomplish everything while ensuring no team

member worked longer than the maximum 6 hours specified in Lab 2 for the Design, Implement and Operate Phase.

The list of significant contributions provided in this report is an accurate representation of each team member's role in the Water Rocket project.



Do you have any suggestions for how we can improve the Water Rocket Project for future generations of Unified students?

It was very difficult to keep to the maximum allowed time for each section. We were very worried something would go wrong, and we would not have the time to fix it. It was a very real possibility that we would not have a rocket to launch, and being unable to spend extra time on the project was very stressful. We understand why the time limit was imposed, but think that it's a little unrealistic.

This lab came in the middle of a very busy time of the semester. We pulled it off, but scheduling with our team was difficult. It would have been much easier to do well if we did not also have PSETs from Unified, especially the week of the rocket launch.

We had a team of three. Most teams had teams of four. We feel that the time limit should be adjusted for teams with less people, as we had far fewer man hours overall.

A rubric for Lab 3 would have also been appreciated.