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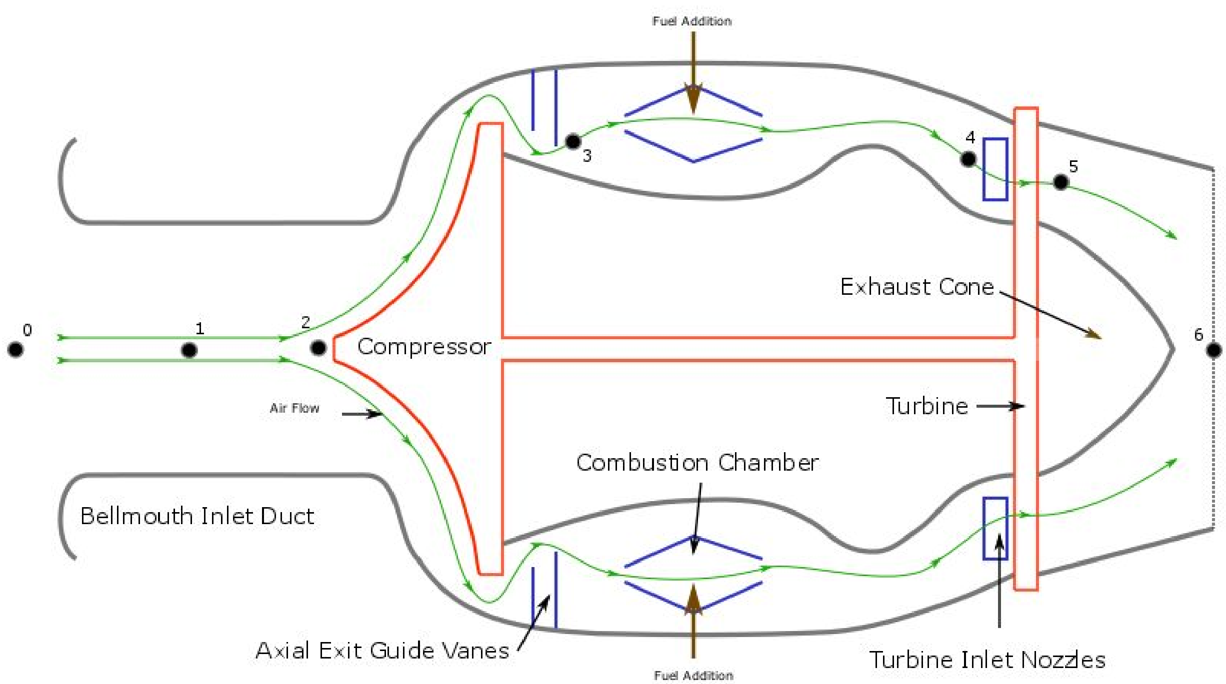
Jet Joe Lab 5 Write-up

Thermodynamic Model and Assumptions

For our thermodynamic model, we implemented similar assumptions to those that we've been utilizing all semester. The simplified engine diagram and h-s diagram of our model are shown below in Figures 1 and 2.

Figure 1

Simplified Engine Diagram



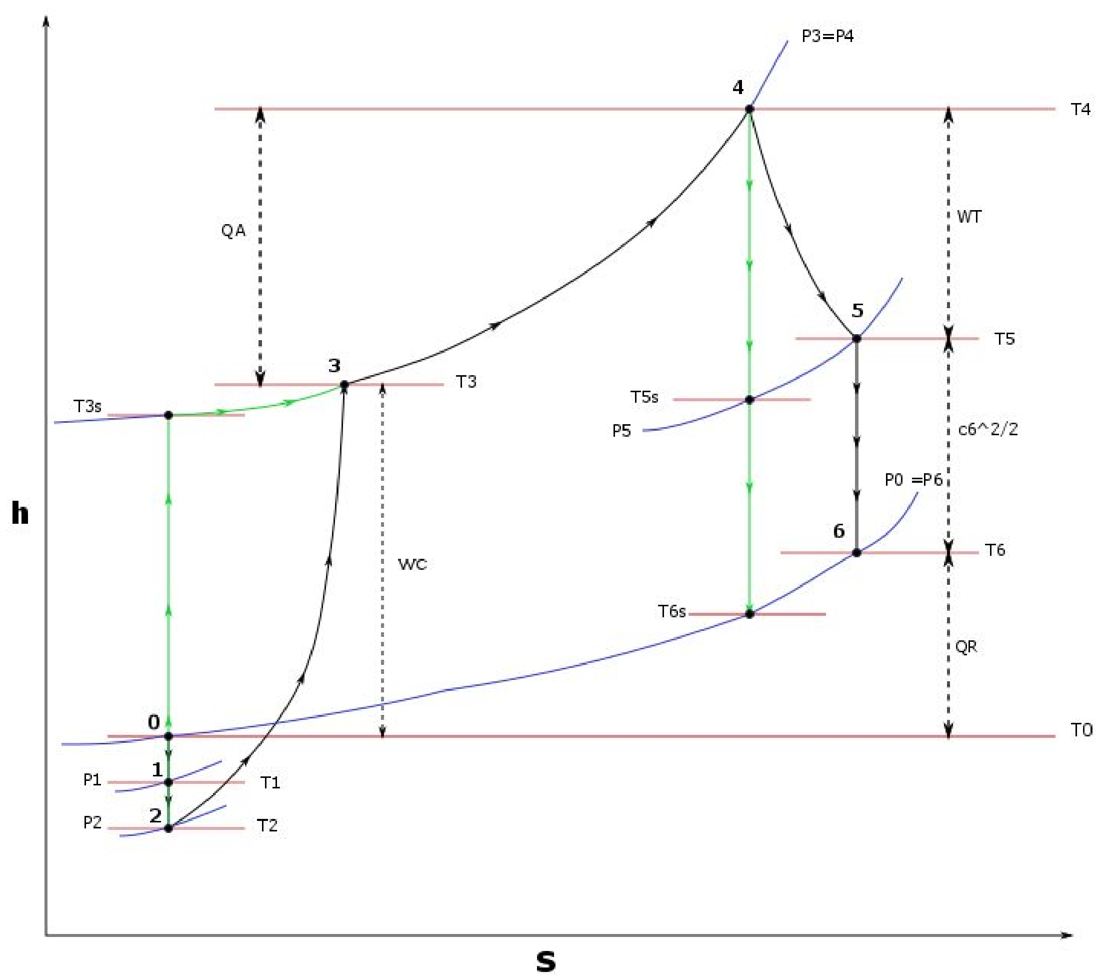
Assumptions:

- Engine casing is perfectly adiabatic
- The freestream flow is stationary (no kinetic energy at station 0)
- Engine inlet is adiabatic reversible (station 0 - 2)
- Compressor is adiabatic irreversible (station 2 - 3)

- Combustor is isobaric (station 3 - 4)
- Turbine is adiabatic irreversible (station 4 - 5)
- Exhaust is adiabatic reversible (station 5 - 6)
- Static pressure at exhaust face (station 6) is equal to freestream pressure
- Mach number through entire engine is subsonic
- Ratio of specific heats in cold section is γ_c and ratio of specific heats in hot section is γ_h
- R is constant everywhere

Figure 2

h-s Diagram with actual path (black) and ideal path (green). Isobars are marked in blue, and isotherms in red.



Our reasonings behind these assumptions and possible improvements to the model are detailed more in the Results and Model Accuracy section.

Results and Model Accuracy

Our model's predictions were within +/- 3% of the averaged measured values for all past years (including 2017) which is quite accurate (see Figure 3). The small error is due to the fact that our assumptions are not exactly what is happening in the real world.

Our results also seem reasonable when looking at our $\Delta\eta$ values (included in Table 1 below). All of our $\Delta\eta$ values are reasonably small, and are the same order of magnitude as the efficiencies themselves. None of our $\Delta\eta$ values neared one, which makes sense as this engine was far from ideally efficient. The signs of the $\Delta\eta$ values are also expected. As the engine RPM speeds up, initially, the turbine is more efficient than the compressor, causing $\Delta\eta$ to be positive. After the engine has reached more than half speed, the compressor becomes more efficient than the turbine, so the $\Delta\eta$ values will be negative. An explanation for this is that when the engine isn't producing much power, there's not much turning the turbine, so the compressor doesn't draw in very much air. The performance is in fact impeded under these conditions. Alternatively, when the engine is producing a lot of power under high RPMs, the turbine is spinning very quickly and the compressor has no trouble at all pulling in more air. The variations in efficiency between the compressor and the turbine are simply caused by slightly different ideal operating conditions. Our small $\Delta\eta$ values support this, and attest to the validity of our model.

Table 1

$\Delta\eta$ for Different Values of RPM

$\Delta\eta$	0	0.5	-0.045	-0.1	-0.1
RPM	0	43070	80110	112200	159800

As stated above, our small errors came from the fact that our assumptions weren't exactly consistent with the real world.

We assumed the freestream flow was stationary, and had no kinetic energy associated with it at the engine inlet. In reality, there was wind the day of testing, and the kinetic energy of the air going into the engine was small, but non-zero. In a real-world application, a jet is going to be moving, and the speed of the air coming into the inlet will be non-negligible. On the scale of the engine we were analyzing, the small shifts in wind speed will be nowhere near the speed of an operating aircraft, and could be neglected.

We also assumed the engine casing was perfectly adiabatic, and was not rejecting any heat to the atmosphere. In reality, while the casing is designed to be insulating, it isn't perfect. There will be heat rejected to the atmosphere through the casing, and while the effect would be small due to the large volume to surface area ratio, accounting for this would have improved the model.

Our model also assumed the engine inlet and exit were adiabatic reversible. Because we assumed the engine casing was adiabatic, we were able to model stations 0 - 2 as having zero heat added. In reality, the entire engine heats up under operating conditions, and the inlet would likely reject heat to the incoming air, and similarly, the hot exhaust gas would reject heat to the relatively cool casing. The heat transfer can be neglected because the ratio of the volume of air compared to the surface area of these inlet/exit surfaces is large, so the heat transfer is small. This is very true in the case of actual aircraft engines. On the scale of our engine, this ratio is not as large, and our model might have been improved by not assuming adiabatic. We also modeled these paths as reversible, however frictional forces within the flow and with the engine itself would produce very small irreversibilities, so small we opted to neglect them.

We modeled both the compressor and turbine as adiabatic (but not reversible). This assumption is not exactly real-world. In reality, the hot, compressed gas would reject heat to the compressor itself, and the gas leaving the combustor would heat up the turbine. However, the adiabatic assumption is valid because, in most engines, the volume of air is very large compared to the surface area of the compressor or turbine, so the heat flow to the blades is negligible. In a smaller engine like the one we're testing, the volume to surface area ratio is much smaller, so heat flow begins to matter. Accounting for this heat transfer would have improved our model.

We also assume the combustor is isobaric. Combustors are specifically designed to be as close to isobaric as possible, with the geometry changing to counteract pressure differences. In reality, it's not perfectly isobaric, however it is very close.

The last assumption we made is that the exit pressure was equal to the atmospheric freestream pressure. This is a valid assumption, as the flow through the entire engine is subsonic (when testing, we heard no shock waves, and we would be very impressed if such a small engine was able to produce a supersonic flow).

Our model could not be adapted to every engine. For one, we assumed the mach number was subsonic, so supersonic engines would not be able to use these same assumptions. Smaller engines than the one we tested would also not fit this model, as the effect of heat transfer is more pronounced when the volume of air to surface area ratio decreases. In fact, our engine was even a bit small for these assumptions. We also did not neglect the added mass flow of the fuel, however, in a larger engine, the mass flow fraction of the fuel is negligible compared to the mass flow of the air. Lastly, we assumed there was no kinetic energy of the gas, so any moving engines would not be able to use this model as-is.

Trends

The adiabatic compressor and turbine efficiency trends are as expected, seen in Figure 4. The adiabatic efficiency for both the turbine and the compressor are initially very low at low RPM, and then the efficiency rises and levels off. This makes sense, because when the engine is moving small amounts of air, it is unable to extract as much useful work from it, simply due to the design of the engine. In general, a smaller RPM means less mass flow of air. With less mass flow, the turbine spins slowly, so the compressor is unable to draw in much air. This accounts for the difference in performance. The definition of adiabatic efficiency is the ratio of actual work to ideal work, and the engine is farther from ideal at lower mass flows, so the percentage of actual work that can be extracted is less. The trend in adiabatic efficiency is also expressed in the graph for thrust specific fuel consumption. As the RPM increases, the ratio of $\dot{m}_f$ to thrust grows smaller, meaning the engine produces more thrust per unit of mass at higher RPMs, or, in other words, the engine is more efficient.

One anomaly we did see was the strange spike in turbine adiabatic efficiency in Figure 4, where at low RPM the efficiency is larger than the efficiency at higher RPMs. This is probably due to the fact that the data at low RPM isn't particularly indicative of the performance of the engine. If we had taken data at more intermediary RPM, the trend of the adiabatic efficiency at low RPMs would become more clear.

The trends of the compressor ratio and overall engine pressure ratio are also to be expected, seen in Figure 5. At higher RPMs, the compressor is pulling in a larger mass of air, so we'd expect the pressure to scale with mass flow, and mass flow scales with RPM. It makes sense then that we'd observe a rising trend in compressor pressure ratio as a function of RPM. Similarly, the overall engine pressure ratio makes sense, because at higher RPM, the pressure ratio of the compressor is higher, so the gas leaves the compressor at a higher exit pressure. The combustor is isobaric, and enters the turbine at the same pressure it left the compressor. Because gas enters the turbine at a higher pressure, it would make sense to leave at a higher pressure relative to inlet conditions and relative to lower RPM exit pressures.

The turbine inlet temperature as a function of RPM also produces an expected, rising curve, seen in Figure 6. Over the entirety of the curve, the turbine inlet temperature is much higher than either the compressor inlet temperature or the compressor exit temperature, even at very low RPM. This makes sense, as the air passes through the compressor and the combustor before entering the turbine. The compressor heats air through the very act of compression, and the combustor is burning fuel within the air, raising the temperature dramatically. However, the upwards curve also makes sense because as the engine reaches higher RPM, the fuel mass flow increases. This causes there to be more fuel in the combustor to ignite in the first place, and the gas will get hotter as a result. Similarly, to produce higher RPMs, the air must be relatively hotter at the turbine inlet to allow it to expand the necessary amount over the turbine.

The trends seen in Figure 7 describing the mass flow are also expected. As rotor speed increases, the mass flow of the air being pulled through the engine increases. At a higher RPM, the compressor would be expected to pull in more air, and this behavior is observed. Similarly, the faster the rotor is spinning, the more fuel is needed to power the engine, and so the higher the fuel mass flow becomes.

Calculating Turbine Efficiency

With the measurements we have taken, it is not possible to calculate the adiabatic turbine efficiency.

The equation for adiabatic efficiency is

$$\eta_t = \frac{h_1 - h_2^s - (h_2 - h_2^s)}{h_1 - h2^s}$$

Or, equivalently, in this case,

$$\eta_t = \frac{T_{t4} - T_{t5}^s - (T_{t5} - T_{t5}^s)}{T_{t4} - Tt5^s}$$

With the measurements we have taken, we only have an expression for T_{t5}^s that depends on η_t. To find T_{t5}^s, we would need to take more measurements.

The only new measurement we would need to take would be the stagnation pressure at station 5. Using the stagnation pressure at 5, we could use adiabatic reversible relationships between pressure and temperature, or

$$T_{t5}^s = T_{t4}(\frac{p_{t5}}{p_{t4}})^{\frac{\gamma_h - 1}{\gamma_h}}$$

However, this would require putting a pitot static tube directly in the turbine exhaust of an operating engine. Most pitot static tubes are not designed to withstand such high temperatures, and would fare poorly in such conditions. Because of this, while we theoretically could make another measurement in order to find the turbine adiabatic efficiency, it would be impossible without very specialized equipment.

Appendix

Figure 3

Model vs. Measured Specific Thrust and Model vs. Measured Thrust Specific Fuel Consumption

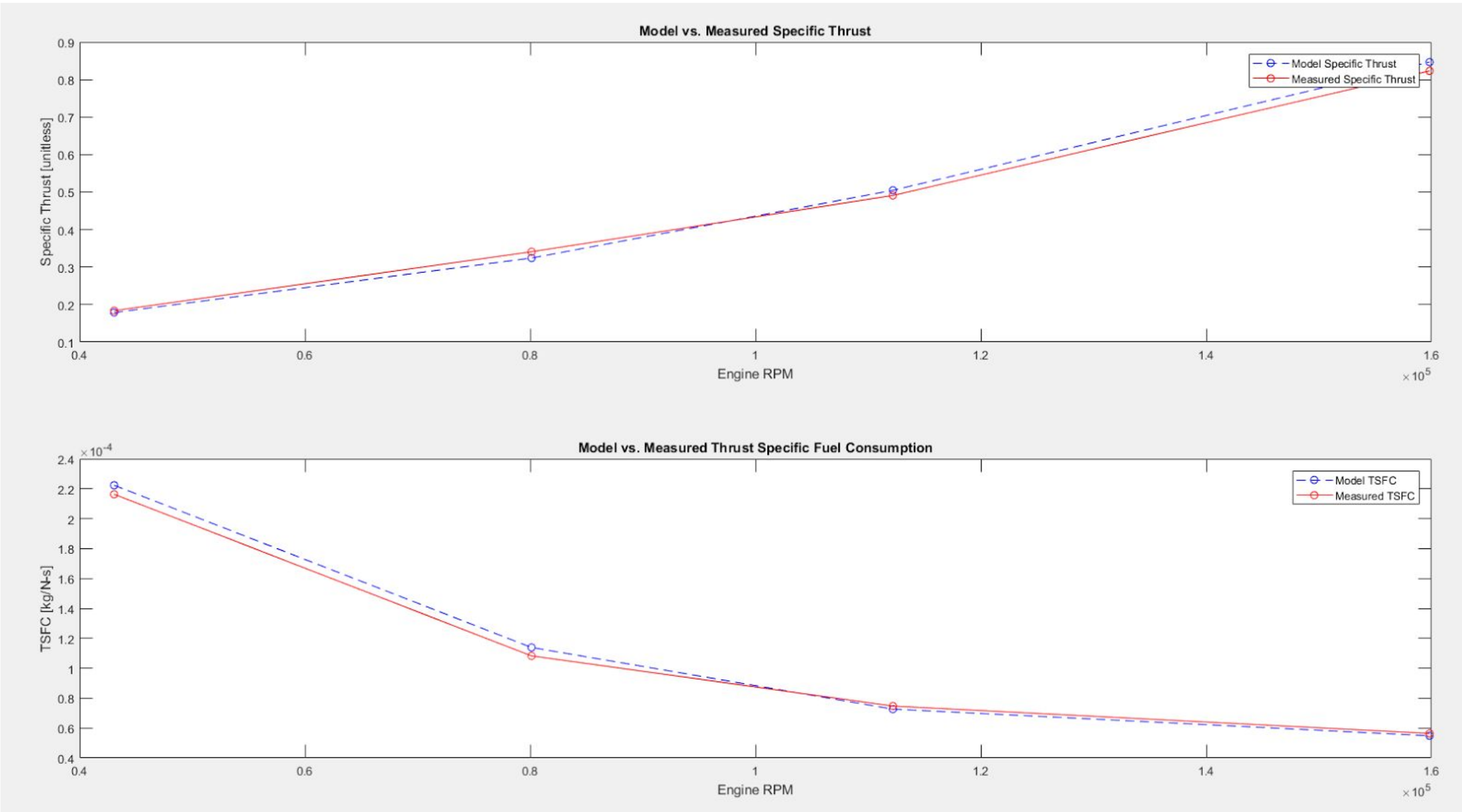


Figure 4

Compressor and Turbine Adiabatic Efficiency and Thermal Efficiency

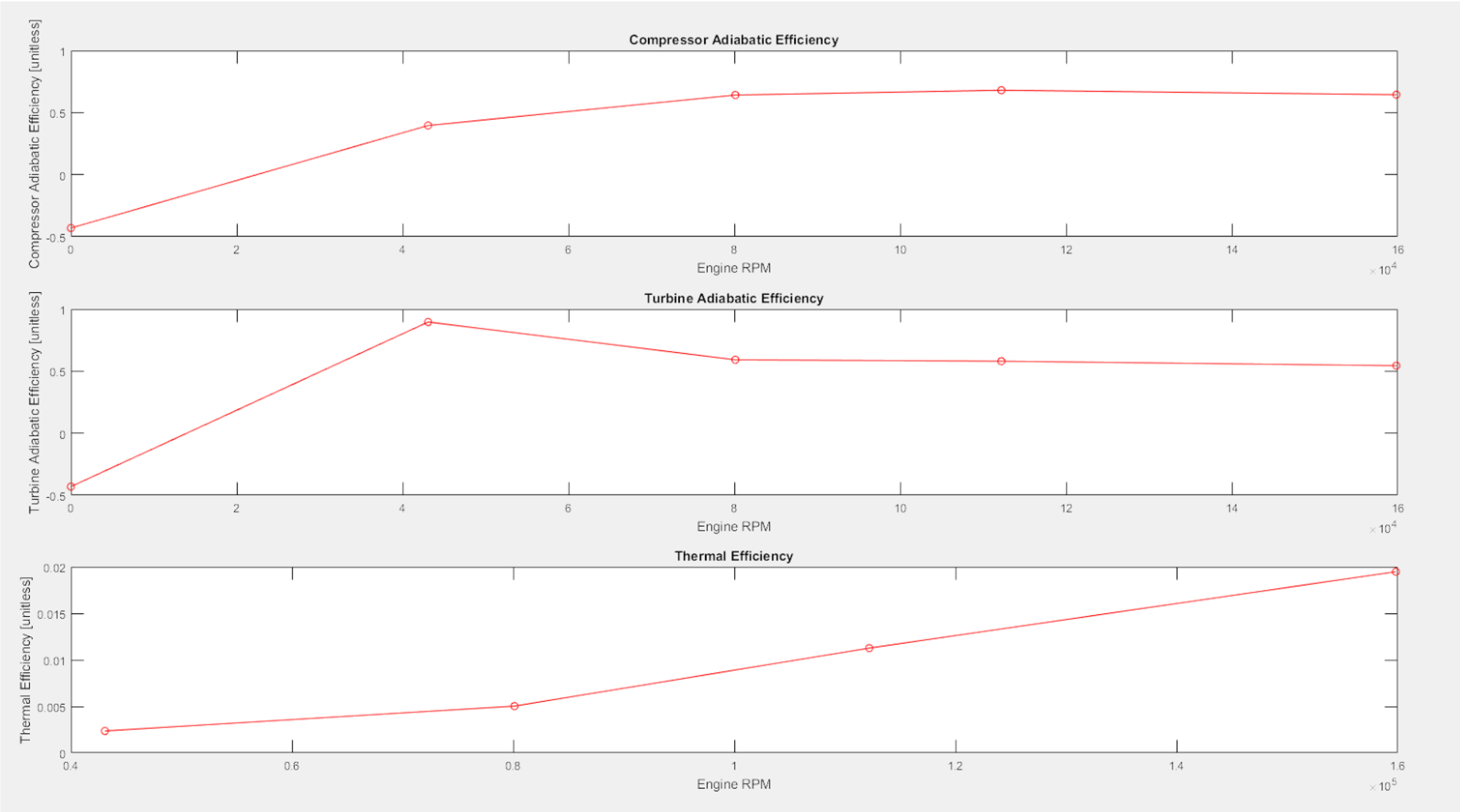


Figure 5

Compressor Pressure Ratio and Overall Engine Pressure Ratio

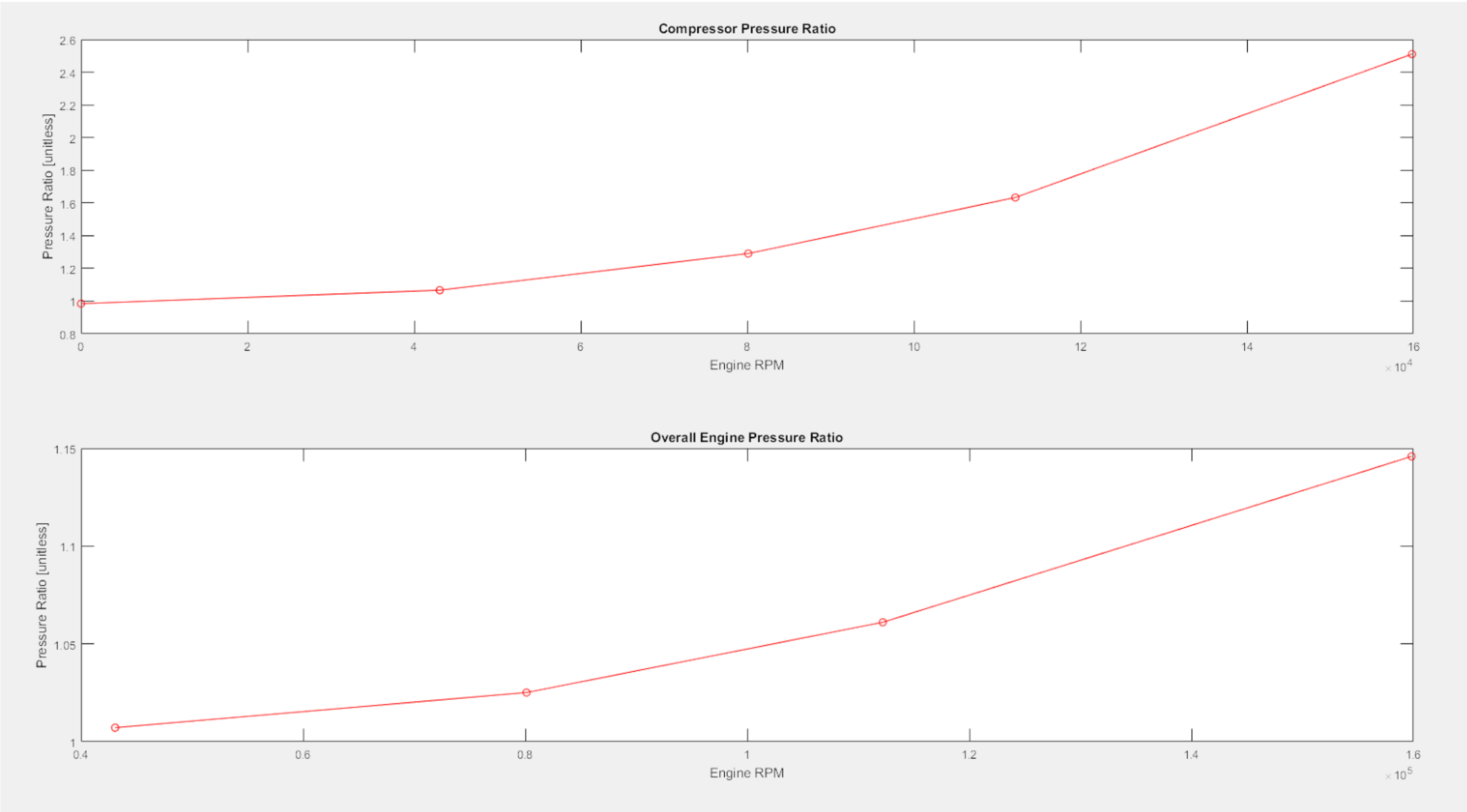

Compressor Pressure Ratio
Pressure Ratio [unitless]
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
2
4
6
8
10
12
14
16
Engine RPM
× 10^4
Overall Engine Pressure Ratio
Pressure Ratio [unitless]
1.15
1.1
1.05
1
0.4
0.6
0.8
1
1.2
1.4
1.6
Engine RPM
× 10^5

Figure 6

Compressor Exit to Inlet Temperature Ratio and Turbine Inlet to Compressor Inlet Temperature Ratio

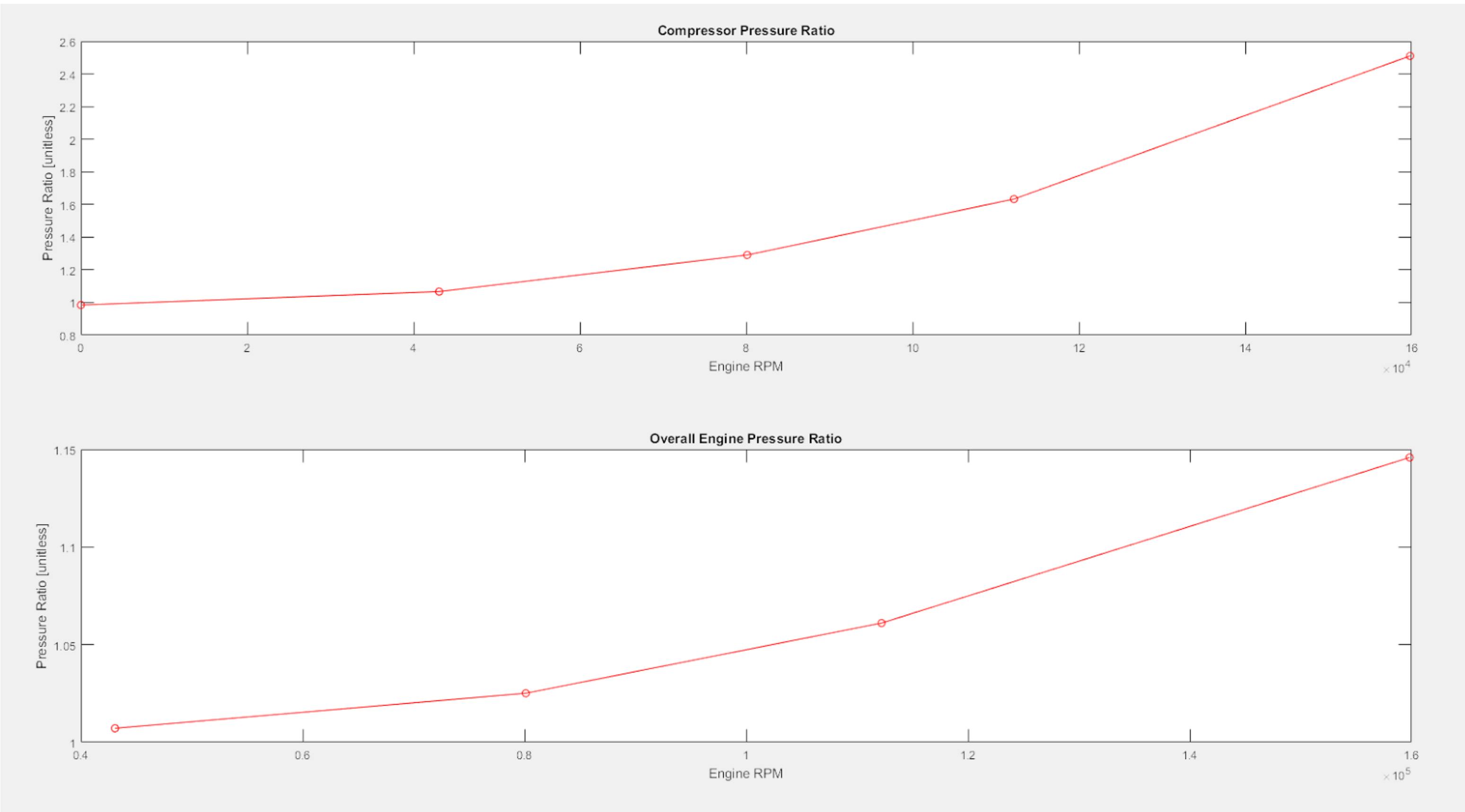
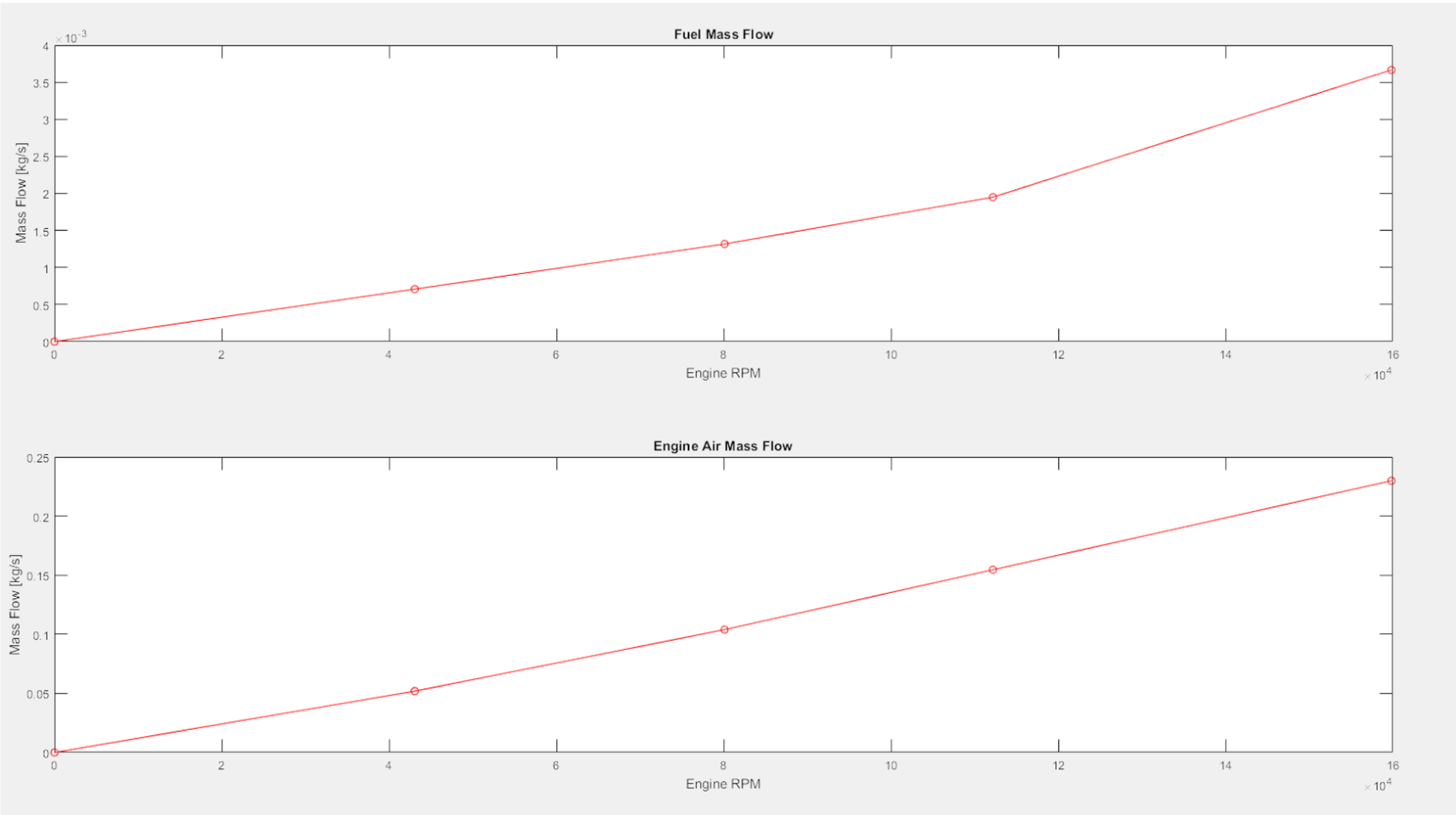


Figure 7

Fuel Mass Flow and Engine Air Mass Flow



Part 4

Given $p_0,\ T_0,\ p_1,\ A_1$:

a. Stagnation temperature and stagnation pressure at station 1

At Station 0, the air is assumed stationary, so the stagnation quantities are equal to the static quantities. We modeled stations 0 - 1 is adiabatic reversible, so the stagnation pressure and temperature at station 1 are equal to the stagnation pressure and temperature at station 0.

$$p_{t1} = p_{t0} = p_0 \text{ and } T_{t1} = T_{t0} = T_0$$

b. Station 1 Mach number

We know the pressure at station 0 and the pressure at station 1, so using the static jump relations and solving for mach number, we obtain,

$$M_1 = \sqrt{\frac{\frac{p_{t1}}{p_1}^{\frac{\gamma_c-1}{\gamma_c}} - 1}{\frac{\gamma_c-1}{2}}}$$

c. Corrected mass flow)

The formula for uncorrected mass flow is,

$$\dot{m} = \frac{p_1 A_1 u_1}{RT_1}$$

Then, correcting it as described in the lab, the mass flow becomes the following,

$$\dot{m}_{corrected} = \frac{p_1 A_1 u_1}{RT_1} \cdot \frac{\sqrt{T_0}}{A_1 p_0}\sqrt{\frac{R}{\gamma_c}}$$

Simplifying and substituting, the corrected mass flow becomes the following, which is only dependent on mach number.

$$\dot{m}_{corrected} = M_1 \cdot (1 + \frac{\gamma_c - 1}{\gamma_c} M_1^2)^{\frac{1}{2}\frac{1+\gamma_c}{1-\gamma_c}}$$

Part 5

Given $p_0,\ T_0,\ p_{t3},\ T_{t3},\ T_{t5}, f = \frac{\dot{m}_f}{\dot{m}},\ \gamma_c,\ \gamma_h$

a. Compressor stagnation pressure ratio and compressor adiabatic efficiency

The compressor inlet pressure is p_{t2} and the exit pressure is p_{t3}. We're given p_{t3}, and knowing that $p_{t2} = p_0$, we can solve the ratio of exit pressure to inlet pressure, and we obtain the following,

$$\pi_c = \frac{p_{t3}}{p_0}$$

The definition of the adiabatic compressor efficiency is,

$$\eta_c = \frac{h_3^s - h_2}{h_3 - h_2}$$

Or, in this case, equivalently,

$$\eta_c = \frac{T_{t3}^s - T_{t2}}{T_{t3} - T_{t2}}$$

The ideal stagnation temperature at station three can be found using adiabatic reversible pressure - temperature relations.

$$T_{t3}^s = T_0 \left(\frac{p_{t3}}{p_{t2}}\right)^{\frac{\gamma_c - 1}{\gamma_c}}$$

Plugging in the compressor stagnation pressure ratio and ideal stagnation temperature at station 3 into the definition of the compressor adiabatic efficiency, we found,

$$\eta_c = \frac{T_0 (\pi_c)^{\frac{\gamma_c - 1}{\gamma}} - T_0}{T_{t3} - T_0}$$

b. Station 4 stagnation temperature

The work of the compressor is,

$$w_c = h_3 - h_2$$

And the work of the turbine is (accounting for mass flow of the fuel),

$$w_t = (1 + f)(h_4 - h_5)$$

Equating the work of the turbine and the work of the compressor, and recognizing that change in enthalpy is equal to,

$$\Delta h = c_p \Delta T$$

We solve for the stagnation temperature at station 4 as below,

$$T_{t4} = T_{t5} + \frac{c_{pc}(T_3 - T_2)}{c_{ph}(1+f)}$$

c. Static pressure at station 6

Under the assumption that the engine has a subsonic mach number at the exhaust, the exit pressure will be equal to the back pressure, or,

$$p_6 = p_0 .$$

Part 6

**Given $\eta_t = \eta_c + \Delta\eta$*

a. Isentropic turbine exit stagnation temperature at station 5

The isentropic turbine exit temperature is the ideal turbine exit temperature, or the temperature the air would be if the path was adiabatic reversible. The definition for η_t is,

$$\eta_t = \frac{h_{t4}^s - h_{t5}}{h_{t4} - h_{t5}^s}$$

Assuming we have η_t, and using the fact that,

$$\Delta h = c_p \Delta T$$

We can solve for the ideal turbine exit stagnation temperature as below.

$$T_{t5}^s = \frac{T_{t5} - T_{t4}}{\eta_T} + T_{t4}$$

b. Static temperature at station 6

The ideal static pressure at station 5 is the same as the actual static pressure. Knowing this and using the adiabatic reversible relationship between pressure and temperature to find p_{t5}. We utilize the fact that the ideal pressure at station 5 is equal to the actual pressure at station 5, and that the pressure at station 4 is equal to the pressure at station three.

$$\frac{p_{t5}^s}{p_{t4}} = \left(\frac{T_{t5}^s}{T_{t4}}\right)^{\frac{\gamma_h}{\gamma_h - 1}}$$

Then, because the path between station 5 - 6 is adiabatic reversible, we again use the adiabatic relationship between pressure and temperature to find T_6.

$$T_6 = T_{t5}(\frac{p_6}{p_{t5}})^{\frac{\gamma_h - 1}{\gamma_h}}$$

c. Station 6 Mach number and and jet exhaust velocity

We again used static jump relations, this time across station 5 -6 and solve for mach number, obtaining,

$$M_6 = \sqrt{\frac{(\frac{p_{5t}}{p_0})^{\frac{\gamma_h - 1}{\gamma_h}} - 1}{\frac{\gamma_h - 1}{2}}}$$

Then, to find the jet exhaust velocity, we used the definition of mach number, velocity, and speed of sound, and rearranged to solve for velocity.

$$c_6 = M_6\sqrt{\gamma_h R T_6}$$

d. Specific thrust

To find the specific thrust using the definition given in the lab, we used the definition of the speed of sound, and plugged into the provided equation.

$$\frac{F}{\dot{m}a_0} = \frac{F}{\sqrt{\gamma_h R T_0}}$$

e. Thermal Efficiency

Using the definition of thermal efficiency, and simplifying with known quantities, we found

$$\eta_t = 1 - \frac{T_{t5} - T_0}{T_{t4} - T_{t3}}$$