16.06 11/03/17

Prof. J. P. How Due: 11/10/17

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16.06 Homework Assignment # 9

Format: Please staple your solutions and submit it at the beginning of the session on Friday.

Turn in any MATLAB plots, code, or Simulink systems generated as part of this Problem Set.

1. Consider the system:

$$G(s) = \frac{s+z}{s^2+2s+10}$$

a) If $z = 2$, is the point in the s-plane associated with $\zeta = 0.625$ and $\omega_n = 4$ on the root locus? Show enough work to justify your answer.
b) What value of z would be necessary to ensure that the point in the s-plane associated with $\zeta = 0.707$ and $\omega_n = 6$ is on the root locus?
c) Find the gain that produces the closed-loop poles at the desired location in b).

2. Consider the plant:

$$G(s) = \frac{29}{s^2+10s+29}$$

Overall goal: Design a Phase-Lead Compensator

(a) The percent overshoot (P.O.) should not exceed 5% and ω_n = 8.5 rad/s for the dominant quadratic mode. Where should the closed loop poles be located?

(b) Using root locus techniques, work through, by hand, one iteration in the design of a phase-lead compensator to satisfy the above performance characteristics.

(c) Sketch the pole-zero plot of the closed-loop compensated system. Give the exact location of all closed-loop poles and zeros.

(d) Use MATLAB to plot the step response of the compensated system. Does the compensated system meet the design specifications? If not, explain why.

(e) Conduct one more iteration of your compensator design. For this second iteration you can use MATLAB (e.g., the SISOtool). You do not need to show detailed workings by hand, but please explain the steps you went through. Give your final compensator design and show that it satisfies the design requirements.

3. Please consider the compensated plant transfer function found in Problem 2 above. This time, please design a phase lag compensator.

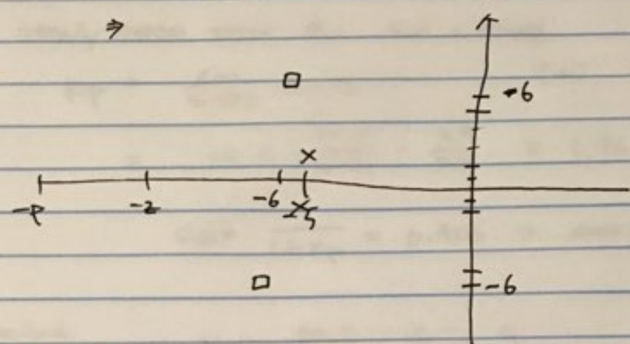
   (a) Your design specifications are now: P.O = 5%, ω_n = 8.5 rad/s for the dominant quadratic mode, and e_{ss} < 10% for a step response. Where should the closed loop poles be located?

   (b) Using root locus techniques, work through one iteration in the design of the phase-lag compensator to satisfy the design requirements (P.O., frequency, and steady state error).

   (c) Sketch the pole-zero plot of the closed-loop compensated system. Give the exact location of all closed-loop poles and zeros.

   (d) Use MATLAB to plot the step response of the compensated system. Does the compensated system meet the design specifications? If not, explain why, and describe (but do not carry out) the steps you would perform in the next iteration of your phase lag compensator.

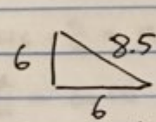
2. (a) $G(s) = \dfrac{29}{(s+5+\cancel{4}2i)(s+5-2i)}$



$M_P < 0.5$

$\zeta > 0.7$

$\omega_n = 8.5$ rads



the poles need to be at $-6 \pm 6i$

(b) Phase condition.

$$180^\circ = \tan^{-1}\left(\frac{6}{z-6}\right) - \tan^{-1}\left(\frac{6}{p-6}\right) - 90^\circ - \tan^{-1}\left(\frac{1}{4}\right) - 90^\circ - \tan^{-1}\left(\frac{1}{8}\right)$$

$$21.16^\circ = \tan^{-1}\left(\frac{6}{z-6}\right) - \tan^{-1}\left(\frac{6}{p-6}\right)$$

$$0.3871 = \frac{6(p-z)}{zp-6p-6z+36+36} = \frac{6(9z)}{10z^2-66z+72}$$

$p = 10z$

$z = 20.2$

$p = 202$

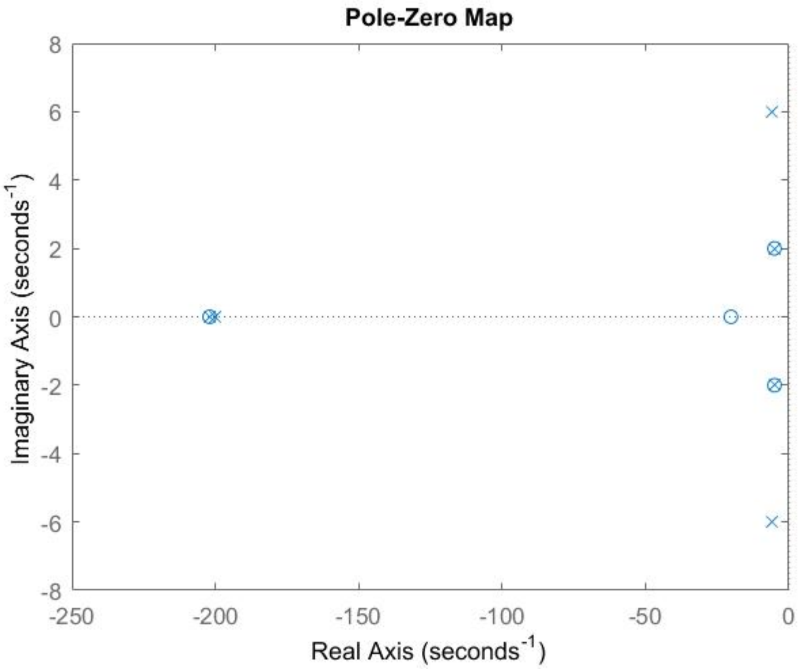
$$\boxed{G_{lead} = \overset{14.6}{K} \cdot \frac{(s+20.2)}{(s+202)}}$$

$$|K| = \frac{|-6-6i+202| \cdot |-6-6i+5-2i| \cdot |-6-6i+5+2i|}{|-6-6i+20.2|}$$

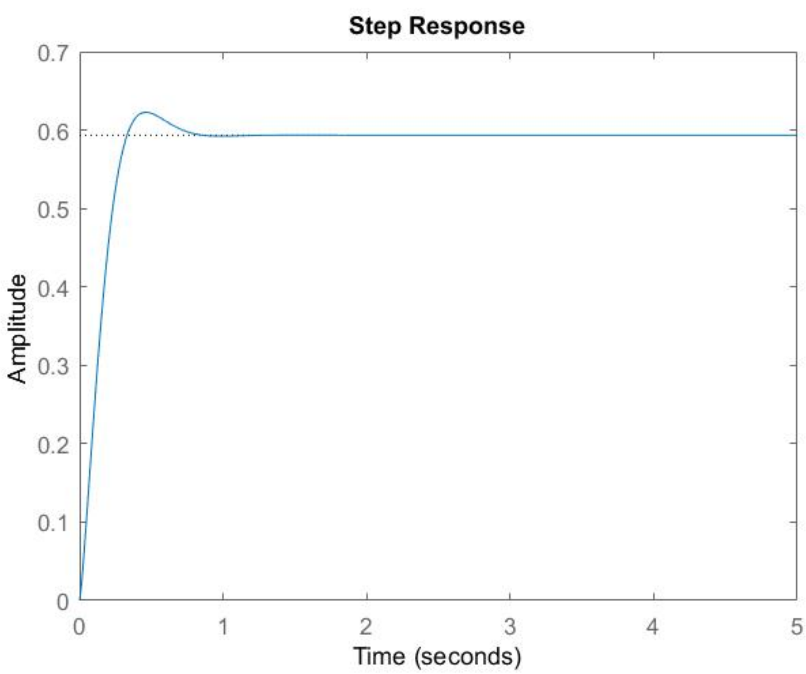
$K = 422.8 / 29 = \underline{14.6}$

(c) $\boxed{\begin{array}{l}\text{closed loop pole locations} = -6 \pm 6i,\ -200 \\ \text{" " zero locations} = -20.2\end{array}}$

C)



D,E)



It is clear that the transient requirement was met, however there exist steady state error.

3. $G(s) = 14.6 \cdot \frac{(s+20.2)}{(s+202)} \cdot \frac{29}{s^2+10s+29}$

(a) steady state error e_{ss} for a step.

$K_p = \lim_{s \to 0} G_c G$ (a) same as 2. $\boxed{-6 \pm 6i}$

$= 14.6 \cdot \frac{20.2}{202} \cdot \frac{29}{29} = 1.46$

$e_{ss} = \frac{1}{1+K_p} = 0.406 > 0.1$

therefore $= 14.6 \cdot \frac{20.2}{202} \cdot \frac{a}{b} = 9$

$\frac{1}{1+K_p} = 0.1$

$1.46 \cdot \frac{a}{b} = 9$

$1 = 0.1 + 0.1 K_p$

$\frac{a}{b} = 6.16$

$\frac{0.9}{0.1} = K_p = 9$

$\omega_n = 8.5$

$\frac{\omega_n}{10} = a = 0.85$

(b) $b = 0.138$

$G_c = 14.6 \cdot \frac{(s+20.2)}{(s+202)} \cdot \frac{(s+0.85)}{(s+0.138)}$

(c) Closed-loope pole locations $= -5.7 \pm 5.9i, -0.59, 200$

zero locations $= -0.85, -20.2$

(d) the steady state error to a step goes to .9

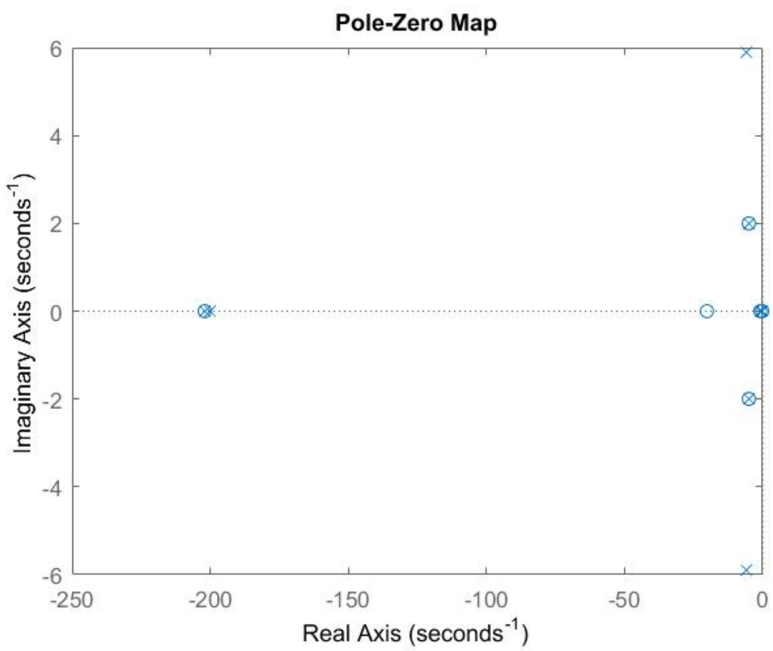
which is $e_{ss} \simeq 10\%$

but the settling time is now large

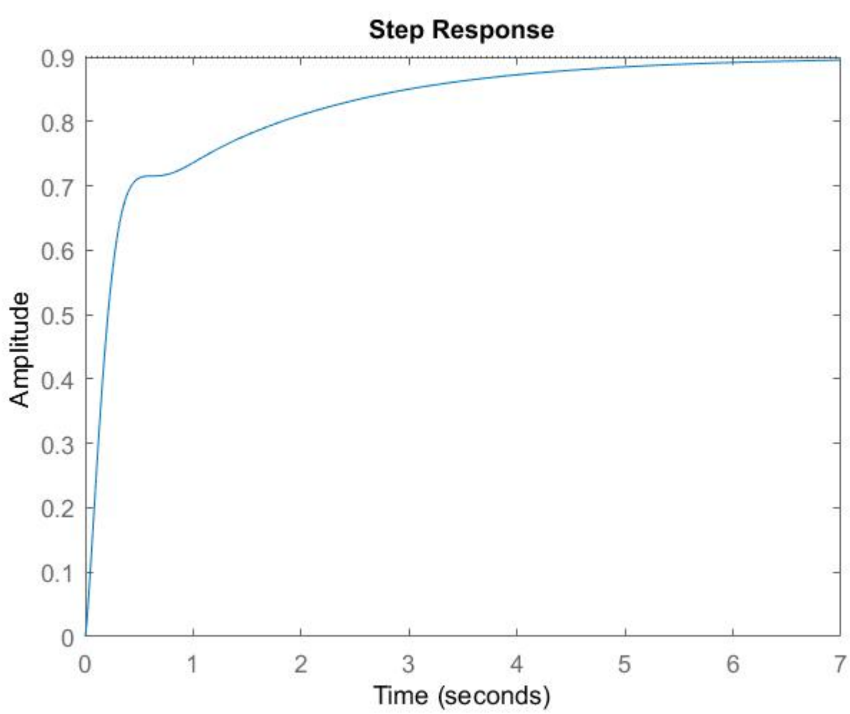
need to make in response faster.

Maybe need to increase the gain

C)



D)



The steady-state error reaches 10%. However there is slow settling time. We can increase the gain to meet the requirement, or can move the desire pole location little more towards LHP.