16.06 9/29/17
Prof. J. P. How Due: 10/6/17
T.A. P. Singh

16.06 Homework Assignment #4

Format: Please staple your solutions and submit it at the beginning of the session on Friday.

1. Consider a signal output from a sensor "sensor_output.mat" (This data file is uploaded on stellar with HW4).
    a) Find a first order low-pass filter with a cutoff frequency of 10 rad/sec.
    b) Find a first order high-pass filter with a cutoff frequency of 1 rad/sec.
    c) Filter the sensor output through: i) low-pass filter only, ii) high-pass filter only, iii) low-pass AND high-pass filter. Plot the outputs in MATLAB and comment on the differences from the original sensor output signal.

2. Consider a system with the transfer function:

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2+bs+k} \qquad \text{(m = 2 N, b = 1 N-s/m, k = 0.5 N/M)}$$

    a) Using Matlab, plot the <u>open loop response</u> of this system for a step input of $f(t) = 1$ N occurring at 2 seconds with a duration of 30 sec.

    b) Using the plot, calculate the time response parameters below (i~vi), using equations provided.
        i. overshoot percentage (OS)
        ii. damping ratio (ζ)
        iii. peak time (T_p)
        iv. natural frequency (ω_n)
        v. steady state time (T_s)
        vi. rise time (T_r)

$$\text{OS} = \frac{A_{max}-A_{final}}{A_{final}} \qquad \zeta = \sqrt{\frac{(\ln OS)^2}{\pi^2+(\ln OS)^2}} \qquad T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$T_s = \frac{4}{\zeta\omega_n} \qquad T_r = \frac{1+1.1\zeta+1.4\zeta^2}{\omega_n}$$

3. For each of the three systems below:

   i. $G(s) = \frac{8(s-12)}{s^2+4s+4}$, $R(s) = 1$

   ii. $G(s) = \frac{s^2-9}{(s^2+7s+12)}$, $R(s) = \frac{1}{s}$

   iii. $G(s) = \frac{\frac{s}{6}-\frac{1}{9}}{(s+3)(s^2+9s+18)}$, $r(t) = e^{-t}$

   a) Plot the poles and zeros of $Y(s) = G(s)R(s)$.

   b) Determine the response of y(t) using the graphical method of residues.

4. a) For the system G(s):

$$G(s) = \frac{100}{(s^2 + 3s + 10)(s + 10)}$$

   estimate the following for a step input:

   i) Maximum Overshoot M_p

   ii) Peak time t_p

   iii) Settling time t_s

   iv) Rise time t_r

   v) Final value $y(t_f)$

   b) Given the parameters estimated in 4(a), sketch what you would expect the step response of G(s) to look like (I do not expect that you would perform an inverse Laplace transform to compute this) – in particular show that the plot agrees with your response estimates.