CE394M: Tresca and Mohr-Coulomb

Krishna Kumar

University of Texas at Austin

krishnak@utexas.edu

April 20, 2019

Overview

Stress invariants

- The magnitudes of the component of the stress vector depend on the chosen direction of the coordinate axes (in 3D: 6 variables).
- Principal stresses always act on the same planes and have the same magnitude (invariant to the coordinate axes), but still need to define the corresponding orientations (in 3D: 6 variables).
- For isotropic materials, it is very convenient to work with alternative invariant quantities which are combinations of principal stresses.

Stress invariants

- Mean effective stress $p = \frac{1}{3}(\sigma_I + \sigma_I I + \sigma_I II)$
- Deviatoric stress: $J = \frac{1}{\sqrt{6}}\sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}$
- Lode's angle $\theta = \tan^{-1}\left[\frac{1}{\sqrt{3}}\left(2\frac{(\sigma_{II} - \sigma_{III})}{\sigma_I - \sigma_{III}} - 1\right)\right]$

Principal stresses can be expressed in terms of invariants:

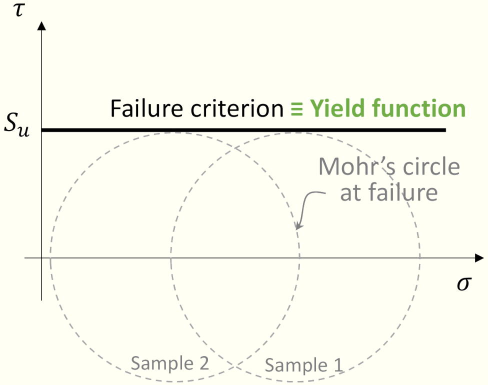
$$\begin{bmatrix}\sigma_I \\ \sigma_{II} \\ \sigma_{III}\end{bmatrix} = p\begin{bmatrix}1 \\ 1 \\ 1\end{bmatrix} + \frac{2}{\sqrt{3}}J\begin{bmatrix}\sin\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta \\ \sin\left(\theta - \frac{2\pi}{3}\right)\end{bmatrix}$$

Tresca model

Simulation of undrained behavior of saturated clay
Failure criteria:

$$\tau_f = s_u$$

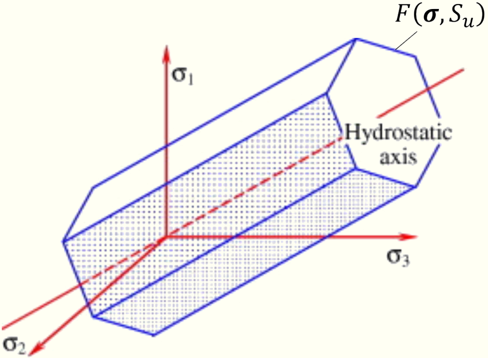
where τ_f is the shear stress at failure. s_u is the undrained strength.

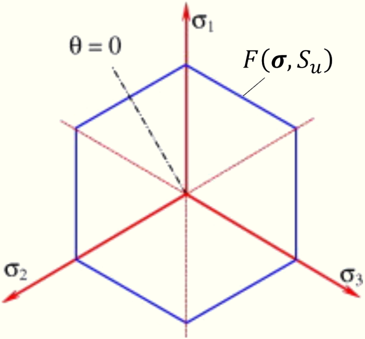


Tresca model

Yield function

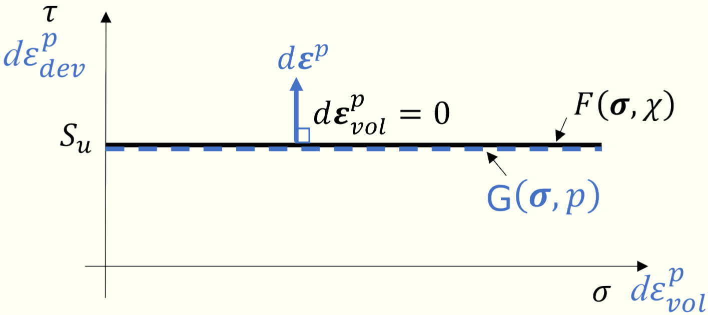
$$F(\sigma, W_p) = \sigma_I - \sigma_{III} - 2s_u = 0$$
$$= J\cos\theta - s_u = 0$$





Tresca model

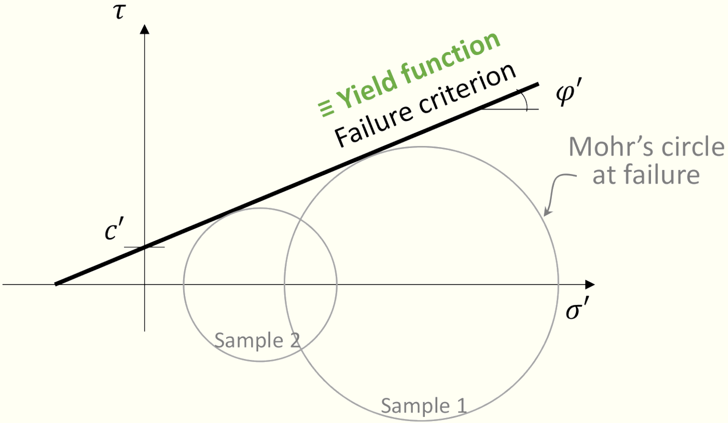
1. The model is intended to predict the undrained behavior of saturated clay, hence it should predict zero volumetric strains.
2. This fact constrains the plastic potential, and an associated flow rule should be considered.
3. In general, this is a perfectly plastic model (no hard/soft law)
4. Parameters required (3): s_u, E_u, ν_u.



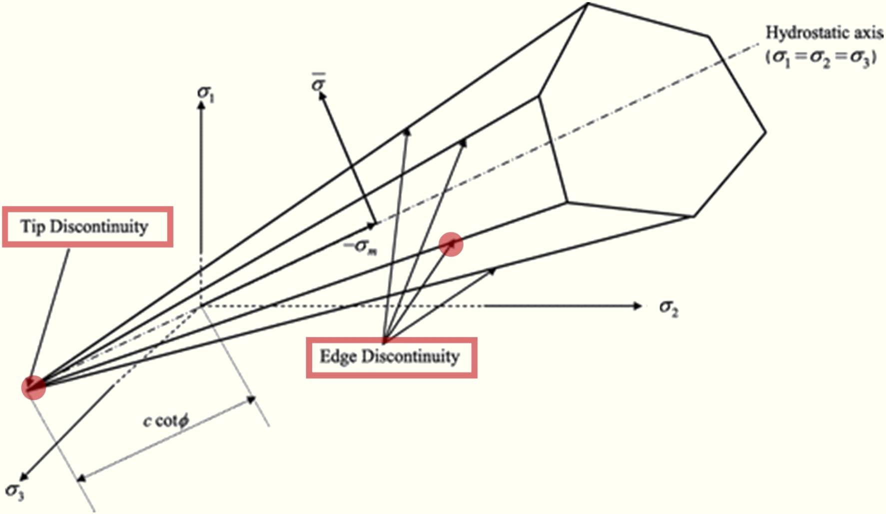
Mohr-Coulomb model

Failure criteria: $\tau_f = c' = \sigma'_f \tan \phi'$.

$$F(\sigma', c', \phi') = \frac{1}{2}(\sigma'_I - \sigma'_{III}) - \frac{1}{2}(\sigma'_I + \sigma'_{III}) \sin \phi' - c' \cos \phi' = 0.$$

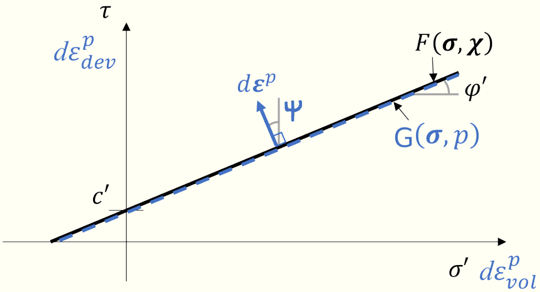


Mohr-Coulomb model



Mohr-Coulomb: flow rule?

Can we consider an associative flow rule? Failure criteria: $F(\sigma', c', \phi') = G(\sigma', c', \phi')$



ε^p is inclined at an angle of ϕ', and indicates negative (tensile) plastic strains. This results in a dilatant volumetric plastic strain ($d\varepsilon^p_{vol}$), in which the angle of dilation ψ is equal to the effective friction angle (ϕ').

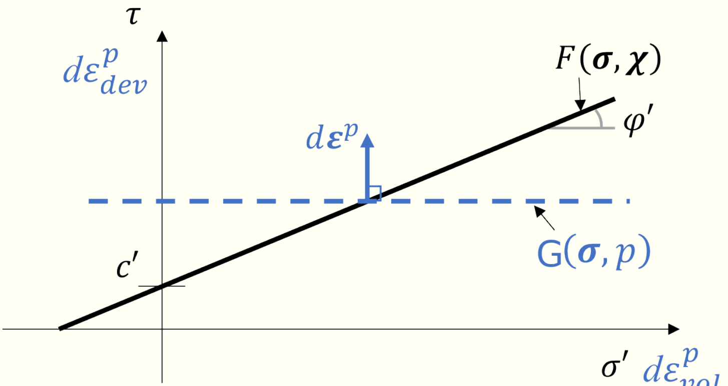
1. The magnitude of the plastic volumetric strains ($d\varepsilon$) is much larger than that observed in real soils.
2. Once the soil yields it will dilate for ever! (note: real soil, which may dilate initially on meeting the failure surface, will often reach a constant volume condition at large strains).

Solutions:

1. Adopt a non-associated flow rule with a more realistic angle of dilation ψ
2. Allow angle of dilation vary with plastic strain (hardening/softening law) $\psi = f(\varepsilon)$

Mohr-Coulomb: non-associative model

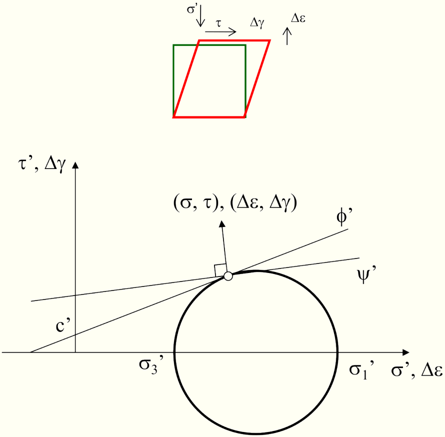
Consider a non-associated model with $\psi = 0$. In which case $d\varepsilon^p_{vol} = 0$.
Parameters (5): c', ϕ', ψ, E, ν.



Mohr-Coulomb: Yield and Potential fn

Yield fn: $F = \sigma_1' - \sigma_3' N_\phi + 2c\sqrt{N_\phi} = 0.$, where $N_\phi = \frac{1+\sin\phi'}{1-\sin\phi'}$

Potential fn: $G = \sigma_1' - \sigma_3' N_\psi + constant = 0.$, where $N_\psi = \frac{1+\sin\psi'}{1-\sin\psi'}$

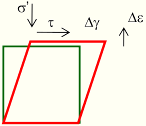


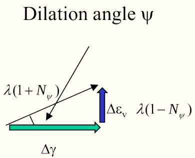
Mohr-Coulomb: plastic strains

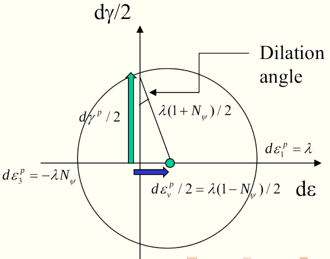
$$d\varepsilon_1^p = \lambda \frac{\partial G}{\partial \sigma_1} = \lambda \cdot 1 = \lambda$$

$$d\varepsilon_2^p = \lambda \frac{\partial G}{\partial \sigma_2} = \lambda \cdot 0 = 0 \quad \text{Zero strain increment in 2-dir}$$

$$d\varepsilon_3^p = \lambda \frac{\partial G}{\partial \sigma_3} = \lambda \cdot (-N_\psi) = -\lambda N_\psi$$





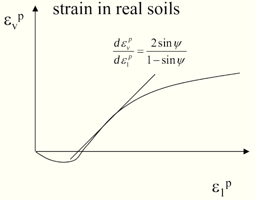


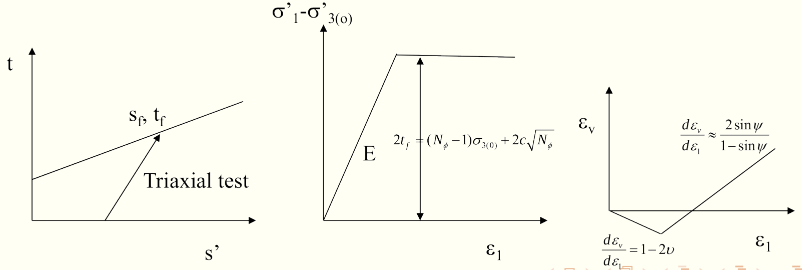
Interpretation of drained TXC

Plastic volumetric strain:

$$d\varepsilon_v^p = d\varepsilon_1^p + d\varepsilon_2^p + d\varepsilon_3^p = \lambda(1 - N_\psi)$$
$$= \lambda\frac{2\sin\psi}{1-\sin\psi}$$
$$\frac{d\varepsilon_v^p}{d\varepsilon_1^p} = \frac{\frac{2\sin\psi}{1-\sin\psi}}{\lambda} = \frac{2\sin\psi}{1-\sin\psi}$$

Note that ψ changes with plastic strain in real soils





Mohr-Coulomb: Tension cut-off

1. For $c = 0$, the standard Mohr-Coulomb criterion allows for tension. Allowable stress increase with cohesion.
2. In reality, soil can sustain none or only very small tensile stresses.
3. This behavior is usually included by specifying a tension cut-off, which is equivalent to include an additional yield surface: $F(\sigma, \sigma_t) = \sigma_i - \sigma_t = 0$.