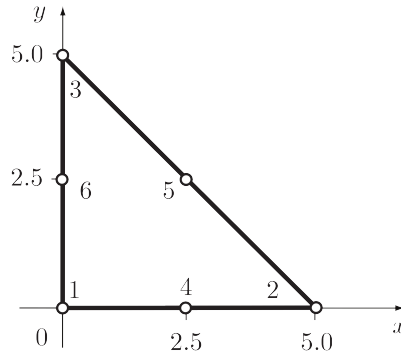


Assignment 5: FEM: Isoparametric elements, Gauss integration, solvers and errors (20 points)

Assigned: 9th March 2020

Due: 23rd March 2020

1. For the six noded-triangle shown. The nodal pwp are $T^e = [300 \ 0 \ 0 \ 0 \ 340 \ 0]^T$. Compute the PWP and its gradient at the point P with the coordinates $x = 1.5$ and $y = 2.0$.

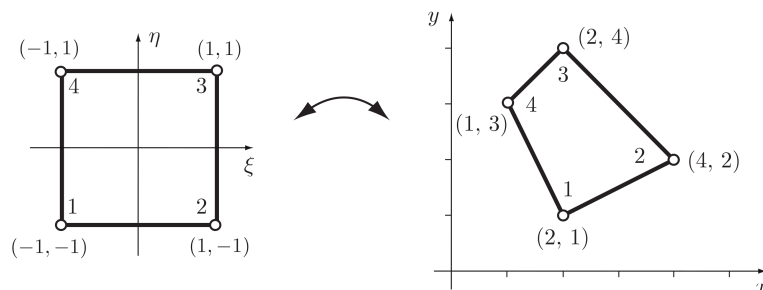


The shape functions for a 6-noded triangle element are:

$$\begin{aligned} N_1 &= 2(1 - \xi - \eta)^2 - (1 - \xi - \eta) & N_2 &= 2\xi^2 - \xi \\ N_3 &= 2\eta^2 - \eta & N_4 &= 4\xi(1 - \xi - \eta) \\ N_5 &= 4\xi\eta & N_6 &= 4\eta(1 - \xi - \eta) \end{aligned}$$

2. For the isoparametric mapping shown below

- (a) Compute the x and y coordinates of the point $\xi = 0.5, \eta = 0.5$ in the physical domain.
- (b) Compute $\frac{\partial N_1}{\partial x}$ for the same point.



3. Evaluate the following three integrals using one, two and three-point Gauss integration. Compare your results with the results of analytic integration.

(a) $\int_{-1}^{+1} (3\xi^2 + 2\xi) d\xi$

(b) $\int_{-1}^{+1} \cos \xi d\xi$

(c) $\int_0^3 (3x^2 + x) dx$

4. Consider the following quadrilateral mesh, sketch (1D is fine) the porepressure and its gradient distribution across the mesh assuming a water pressure source on the left boundary of the mesh and comment on the suitability of the mesh for finite element computation and the need for the transition element (element #2)?

