Project 2: Implemntation of Modified Cam Clay in Python
Assigned: 24th April 2019
Due: 10th May 2019

Explain in detail the formulation of the Modified Cam-Clay model and derive the D-matrix of the model. Simulate two undrained triaxial tests using the following Cam-Clay model parameters:

$$
\begin{aligned}
M &= 0.95 \\
N &= 2.7 \\
\lambda &= 0.16 \\
\kappa &= 0.06 \\
\nu &= 0.2
\end{aligned}
$$

1. Isotopically consolidated undrained compression test of normally consolidated clay. Normally consolidated to 100 kPa and then sheared in undrained conditions. For the undrained shear part of the test:
   (a) Determine the initial void ratio before shearing.
   (b) Plot deviatoric stress q versus axial strain ε_a
   (c) Plot the stress path in $q - p'$ plane and the state path in $e - \ln p'$ plane
   (d) Plot excess pore pressure u versus axial strain
2. Isotopically consolidated undrained compression test of overconsolidated clay. Normally consolidated to 450 kPa isotopically, unloaded isotropically to 100 kPa and then sheared in undrained conditions. For the undrained shear part of the test:
   (a) Determine the initial void ratio before shearing.
   (b) Plot deviatoric stress q versus axial strain ε_a
   (c) Plot the stress path in $q - p'$ plane and the state path in $e - \ln p'$ plane
   (d) Plot excess pore pressure u versus axial strain

   Note:
   (a) Using zero volumetic strain condition (undrained), $\varepsilon_r = -\varepsilon_a/2$
   (b) Compute $\sigma_1', \sigma_3'(= \sigma_2')$ using the D-matrix and then compute p' and q
   (c) For the overconsolidated condition, use the elasto-plastic D matrix after the stress state reaches the yield surface. Use the elastic D matrix before yielding.
3. Discuss how to simulate an isotropically consolidated 'drained' triaxial compression test.

Elasto-plastic relation:

$$d\sigma' = \left[D^e - \frac{D^e \left(\frac{\partial G}{\partial \sigma'}\right) \left(\frac{\partial F}{\partial \sigma'}\right)^T D^e}{-\left(\frac{\partial F}{\partial W_p}\right) \left(\frac{\partial W_p}{\partial \varepsilon^p}\right)^T \left(\frac{\partial G}{\partial \sigma'}\right) + \left(\frac{\partial F}{\partial \sigma'}\right)^T D^e \left(\frac{\partial G}{\partial \sigma'}\right)} \right] d\varepsilon$$

Yield function of modified Cam-Clay:

$$F = \frac{q^2}{M^2} - p'p_c + p^2 = 0$$

$$\frac{\partial F}{\partial \sigma'} = \frac{\partial F}{\partial p'}\frac{\partial p'}{\partial \sigma'} + \frac{\partial F}{\partial q}\frac{\partial q}{\partial \sigma'}$$

$$\frac{\partial F}{\partial p'} = 2p - p_c$$

$$\frac{\partial F}{\partial q} = 2q/M^2$$

$$\frac{\partial F}{\partial \sigma'} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial q}{\partial \sigma'} = (3/2q) \begin{bmatrix} \sigma_x x - p \\ \sigma_y y - p \\ \sigma_z z - p \\ 2\sigma_{xy} \\ 2\sigma_{yz} \\ 2\sigma_{zx} \end{bmatrix}$$

Calculate $(\partial F/\partial p_c)(dp_c/d\varepsilon_v^p)(\partial F/\partial p)$ by differentiating the relevant terms.

$$\frac{\partial p_c}{\partial d\varepsilon_v^p} = \frac{v p_c}{(\lambda - \kappa)}$$