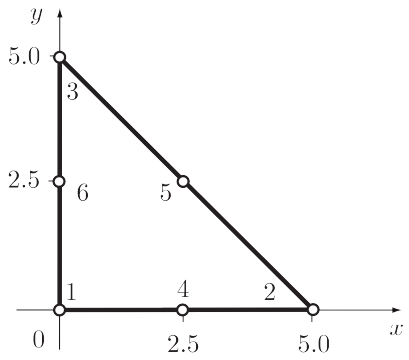
Assignment 5: FEM: Isoparametric elements and Gauss integration (10 points)

Assigned: 9th March 2020

Due: 23rd March 2020

1. For the six noded-triangle shown. The nodal pwp are $P^e = \begin{bmatrix}300 & 0 & 0 & 0 & 340 & 0\end{bmatrix}^T$. Compute the PWP and its gradient at the point P with the coordinates $x = 1.5$ and $y = 2.0$.



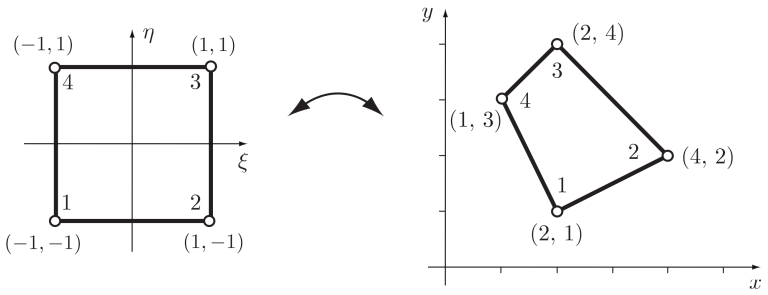
The shape functions for a 6-noded triangle element are:

$$N_1 = 2(1-\xi-\eta)^2 - (1-\xi-\eta) \qquad N_2 = 2\xi^2 - \xi$$
$$N_3 = 2\eta^2 - \eta \qquad N_4 = 4\xi(1-\xi-\eta)$$
$$N_5 = 4\xi\eta \qquad N_6 = 4\eta(1-\xi-\eta)$$

2. For the isoparametric mapping shown below

   (a) Compute the x and y coordinates of the point $\xi = 0.5$, $\eta = 0.5$ in the physical domain.

   (b) Compute $\frac{\partial N_1}{\partial x}$ for the same point.



3. Evaluate the following three integrals using one, two and three-point Gauss integration. Compare your results with the results of analytic integration.

(a) $\int_{-1}^{+1}(3\xi^2+2\xi)d\xi$

(b) $\int_{-1}^{+1}\cos\xi d\xi$

(c) $\int_{0}^{3}(3x^2+x)dx$

4. Consider the following quadrilateral mesh, sketch (1D is fine) the porepressure and its gradient distribution across the mesh assuming a water pressure source on the left boundary of the mesh and comment on the suitability of the mesh for finite element computation and the need for the transition element (element #2)?