Lecture #7:

- **Basic Notions of Fracture Mechanics**
- **Ductile Fracture**

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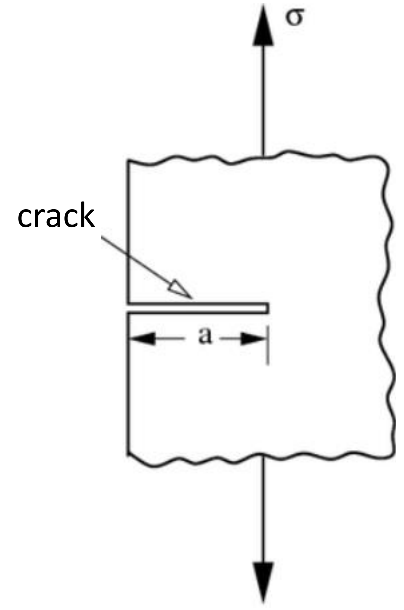
Basic Notions of Fracture Mechanics

Fracture Mechanics

Fracture mechanics is a branch of mechanics that is concerned with the study of the propagation of cracks and growth of flaws. The starting point of a fracture mechanics analysis therefore is a structure with a **pre-existing crack or flaw**. Central questions in fracture mechanics are for example:

- Under which mechanical loads does a pre-existing crack propagate?
- What is the maximum size of a crack that can be tolerated in a structure that is subject to a known mechanical load such that the crack does not propagate?

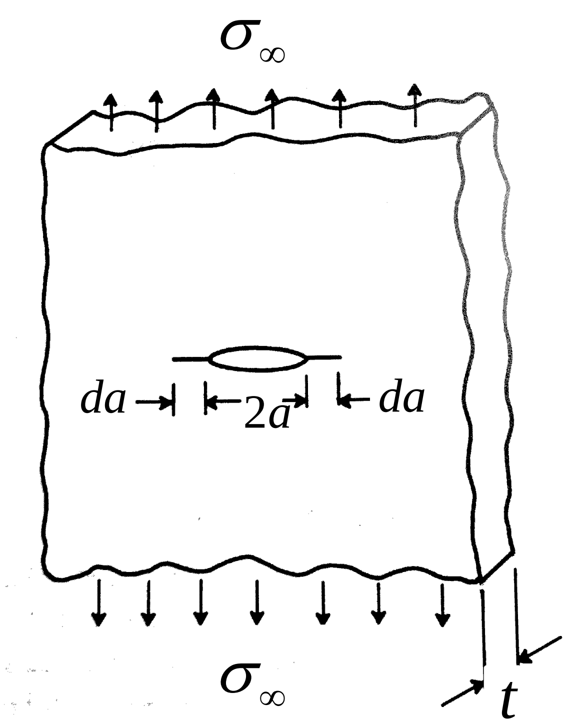


Griffith theory – an energy approach

Griffith‘s (1921) made an attempt to come up with a fracture criterion for brittle solids by writing down the energy balance during the growth of a crack.

Consider an **isotropic linear elastic** plate subject to uniaxial tension and let W_0 denote the elastic strain energy stored in that system. According to Inglis (1913) analysis, the introduction of a through thickness crack of length $2a$ reduces the elastic strain energy (energy release) by

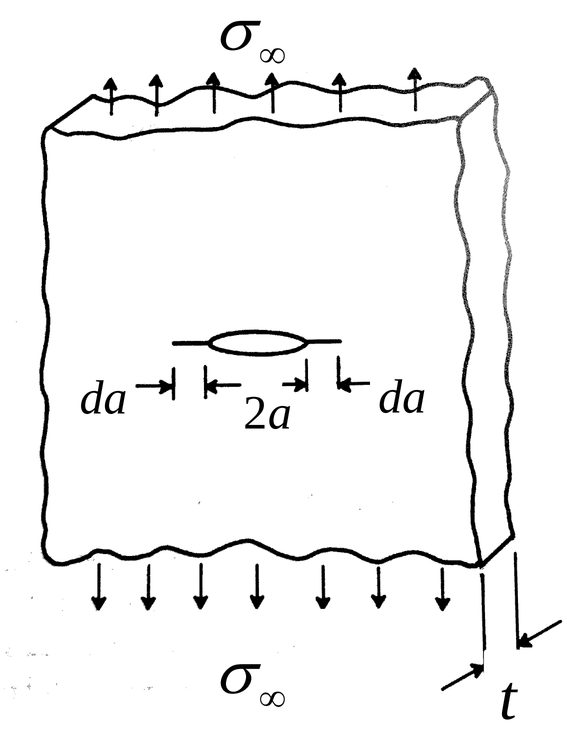
$$\frac{\pi a^2 t \sigma_\infty^2}{E}$$



Griffith theory

Introducing the **free surface energy** per unit area, γ_s, the total energy of the system then reads

$$U_{tot} = U_0 - a^2 \frac{\pi t \sigma_\infty^2}{E} + 4 a t \gamma_s$$

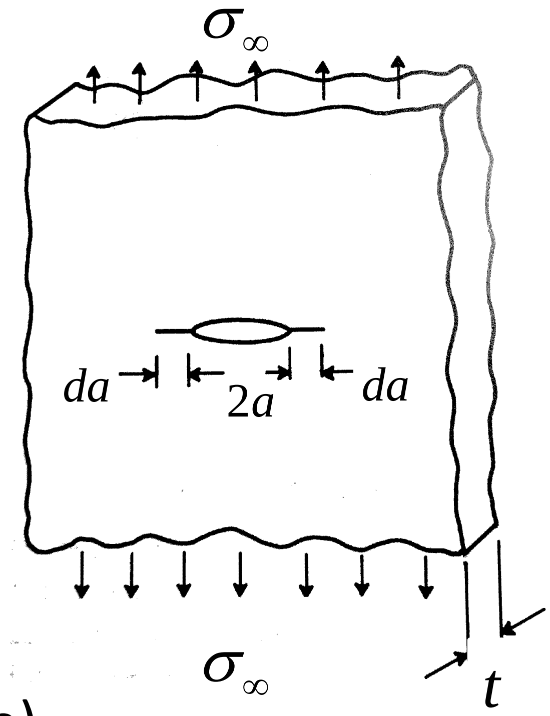


After differentiating with respect to a, we obtain the rate of change of the total energy as a function of the crack length and the applied stress

$$\frac{dU_{tot}}{da} = -2a \frac{\pi t \sigma_\infty^2}{E} + 4 t \gamma_s$$

Griffith theory

$$\frac{dU_{tot}}{da} = -2a\frac{\pi t\sigma_\infty^2}{E} + 4t\gamma_s$$



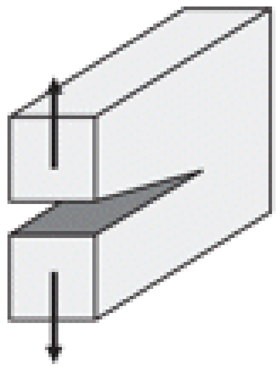
Note that for small flaws and for low applied stresses, the surface energy term dominates, i.e. the total energy of the system would increase as the crack advances. However, when the **critical condition** $dU_{tot}/da=0$ is met, the change in total energy becomes negative (energy release).

According to Griffith, this condition defines the onset of unstable crack growth. The **critical far field stress for fracture initiation** therefore reads

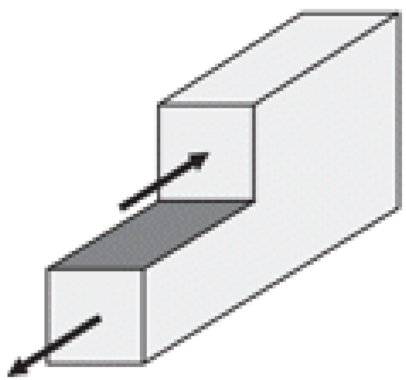
$$-2a\frac{\pi t\sigma_\infty^2}{E} + 4t\gamma_s = 0 \quad \Rightarrow \quad \sigma_\infty^c = \sqrt{\frac{2\gamma_s E}{\pi a}}$$

Linear Elastic Fracture Mechanics (LEFM)

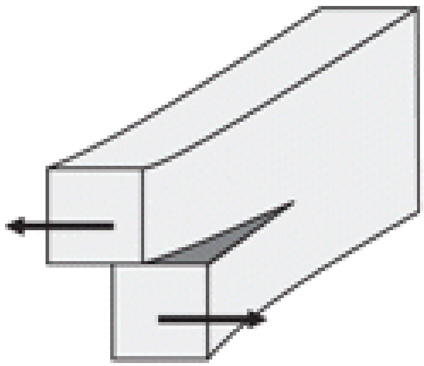
The stress fields in the vicinity of cracks in elastic media can be calculated using linear elastic stress analysis. In particular, the analytical solutions have been developed for three **basic modes of fracture**:



Mode I
"opening mode"



Mode II
"in-plane shear mode"



Mode III
"out-of-plane shear mode"

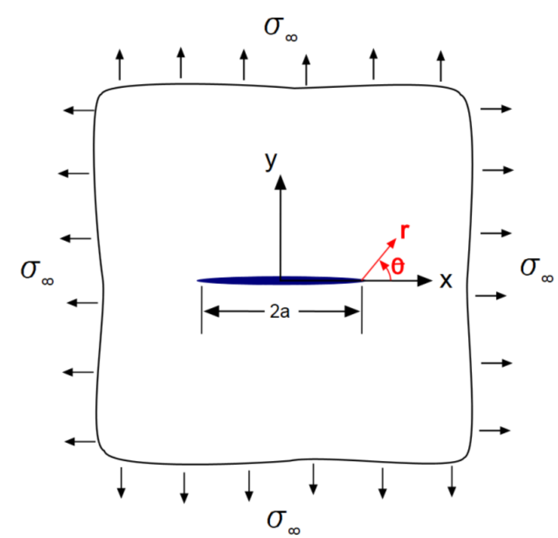
Linear elastic fracture mechanics

The leading terms of analytical solutions for the crack tip stress fields are typically expressed through the product of a radial and circumferential term. For example, the stress field for Mode I plane stress loading reads

$$\sigma_x = \frac{\sigma_\infty \sqrt{\pi a}}{\sqrt{2\pi r}} \cos\tfrac{\theta}{2}\left(1 - \sin\tfrac{\theta}{2}\sin\tfrac{3\theta}{2}\right)$$

$$\sigma_y = \frac{\sigma_\infty \sqrt{\pi a}}{\sqrt{2\pi r}} \cos\tfrac{\theta}{2}\left(1 + \sin\tfrac{\theta}{2}\sin\tfrac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{\sigma_\infty \sqrt{\pi a}}{\sqrt{2\pi r}} \cos\tfrac{\theta}{2}\sin\tfrac{\theta}{2}\cos\tfrac{3\theta}{2}$$



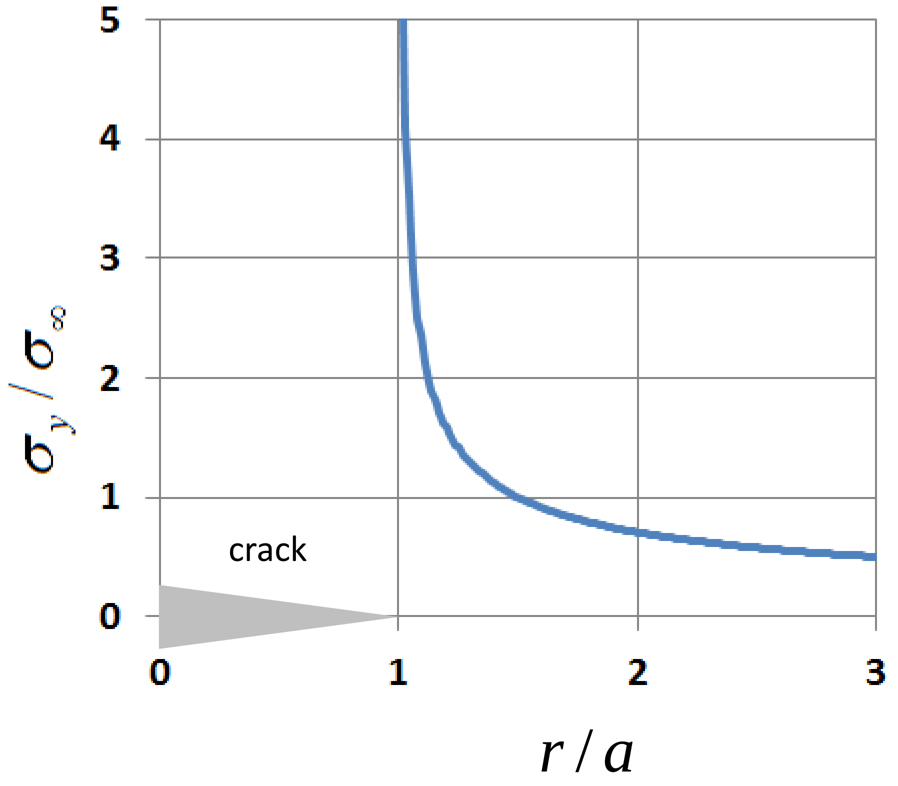
http://www.fracturemechanics.org

Linear elastic fracture mechanics

$$\sigma_y = \frac{\sigma_\infty \sqrt{\pi a}}{\sqrt{2\pi r}} \cos\tfrac{\theta}{2}\left(1+\sin\tfrac{\theta}{2}\sin\tfrac{3\theta}{2}\right)$$

Note that the governing terms all exhibit a singularity at the crack tip,

$$\lim_{r\to 0}\left|\sigma_{ij}\right| = \infty$$



Stress Intensity Factor

The limit

$$K_I := \lim_{r\to 0}\left\{\sqrt{2\pi r}\,\sigma_y\Big|_{\theta=0}\right\}$$

is called **stress intensity factor**. It characterizes the magnitude of the stresses at the crack tip. An analog factor can be defined for Mode II and Mode III fracture. In the above example, we have

$$\sigma_y[\theta=0]=\frac{\sigma_\infty\sqrt{\pi a}}{\sqrt{2\pi r}} \qquad \text{and thus} \qquad K_I=\sigma_\infty\sqrt{\pi a}$$

The expressions of the stress intensity factor for different crack shapes and loading conditions can be found in many textbooks.

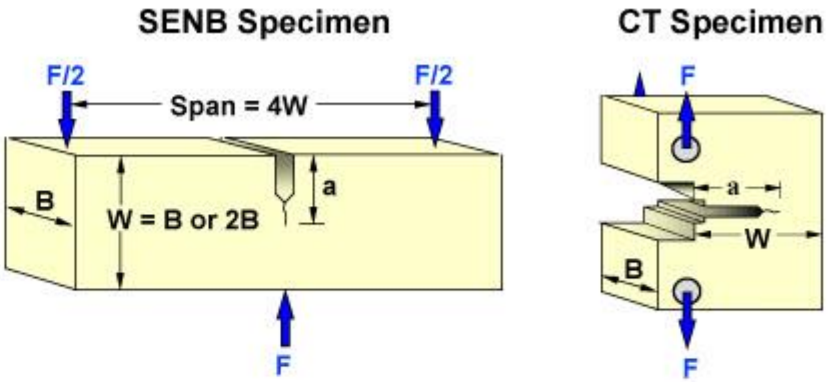
Fracture toughness

In linear elastic fracture mechanics, it is assumed that a crack propagates if the stress intensity factor reaches a critical value,

$$K_I \geq K_c$$

The critical value K_C for Mode I fracture under plane strain conditions is called **fracture toughness**. Its units are $MPa\sqrt{m}$.

It is considered as a material property which is measured using Single Edge Notch Bend (SENB) or Compact Tension (CT) specimens, see ASTM E-399 standard.

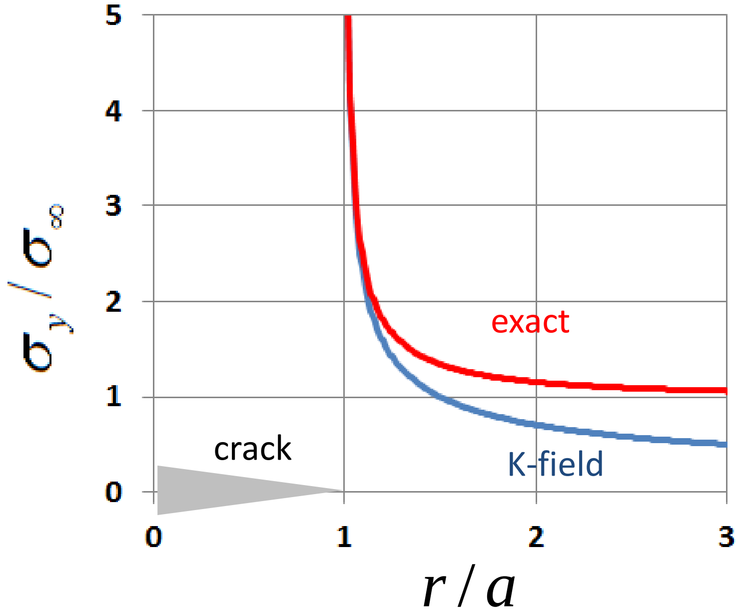


K-dominance

The stress intensity factor characterizes only the leading term of the stress field near the crack tip. The exact solution for the near-tip fields includes also non-singular higher order terms

$$\sigma_{ij}[\theta,r]=\frac{K}{\sqrt{2\pi r}}f_{ij}[\theta]+\text{higher order terms}$$

The annular region within which the singular terms (so-called **K-fields**) dominate is described by the radius r_K of the **zone of K-dominance**.



Small scale yielding condition

The K-fields can still be meaningful even if the “real” mechanical system is different from that assumed in the theoretical analysis. Examples include situations where

- the crack is not sharp;
- the material deforms plastically
- micro-cracks are present near the crack tip

The condition of applicability of linear elastic fracture mechanics is that the radius r_p of the **zone of inelastic deformation** at the crack tip must be well confined inside the region of K-dominance.

$$r_p << r_K$$

An alternative to fracture mechanics

The **classical fracture mechanics approach** is based on the assumption that failure is the outcome of the growth of a pre-existing crack (and that all materials contain flaws).

An alternative approach consists of **assuming that a solid is initially crack-free**. Phenomenological fracture criteria in terms of macroscopic stresses and strains are then often employed to predict the onset of fracture.

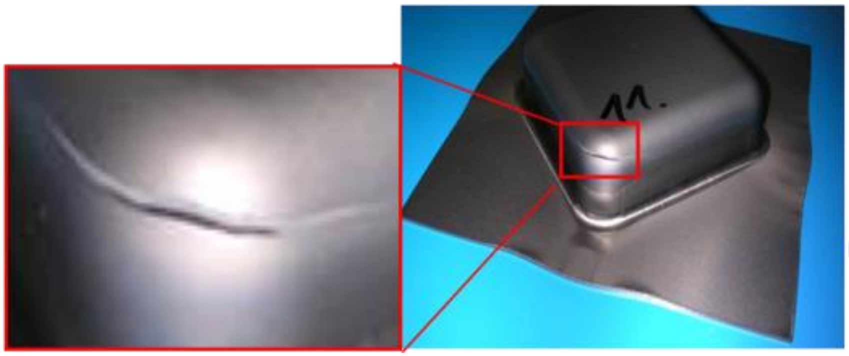
A simple example of a **phenomenological criterion for brittle solids** is to assume that fracture initiates when the maximum principal stress exceeds a critical value,

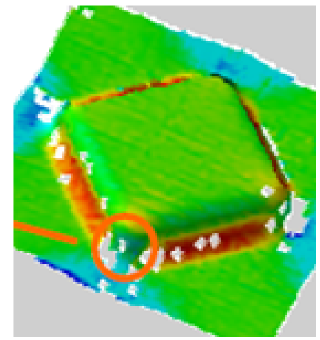
$$\sigma_I \geq \sigma_{crit}$$

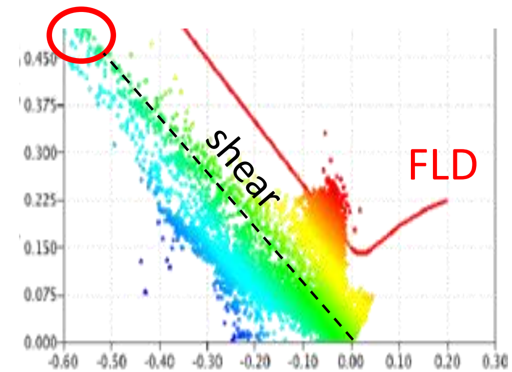
Ductile Fracture

Fracture in Automotive Applications

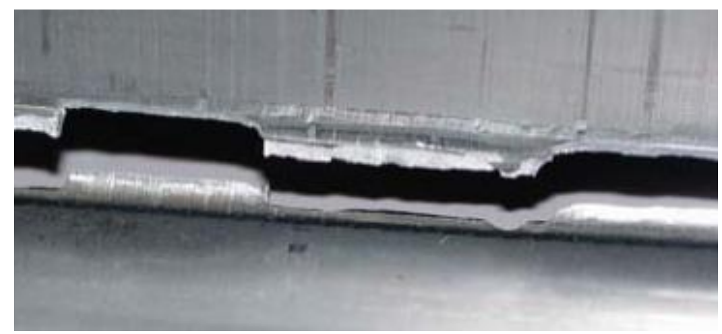
- **Shear induced fracture** *(Courtesy of ThyssenKrupp)*







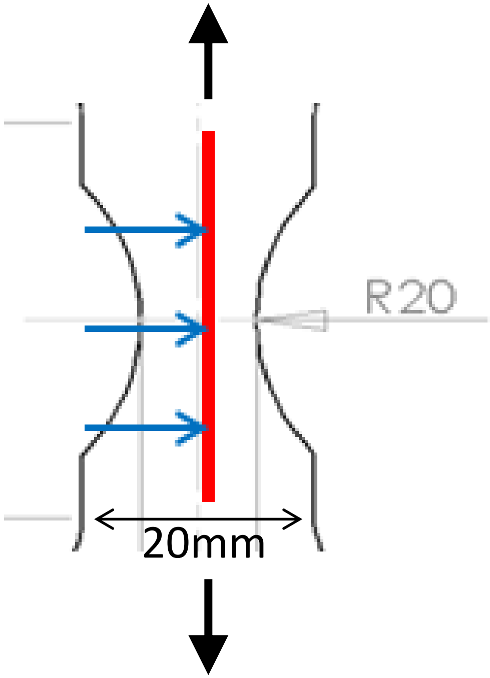
- **Bending under tension** *(Courtesy of US Steel)*

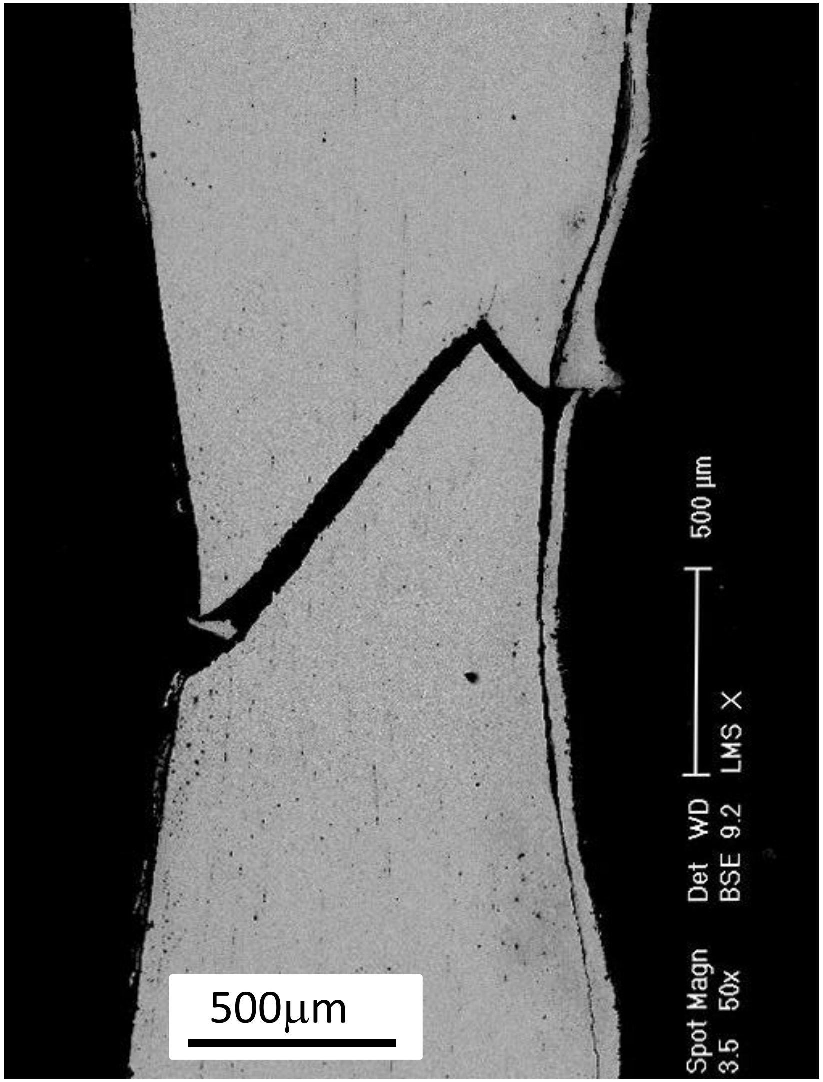


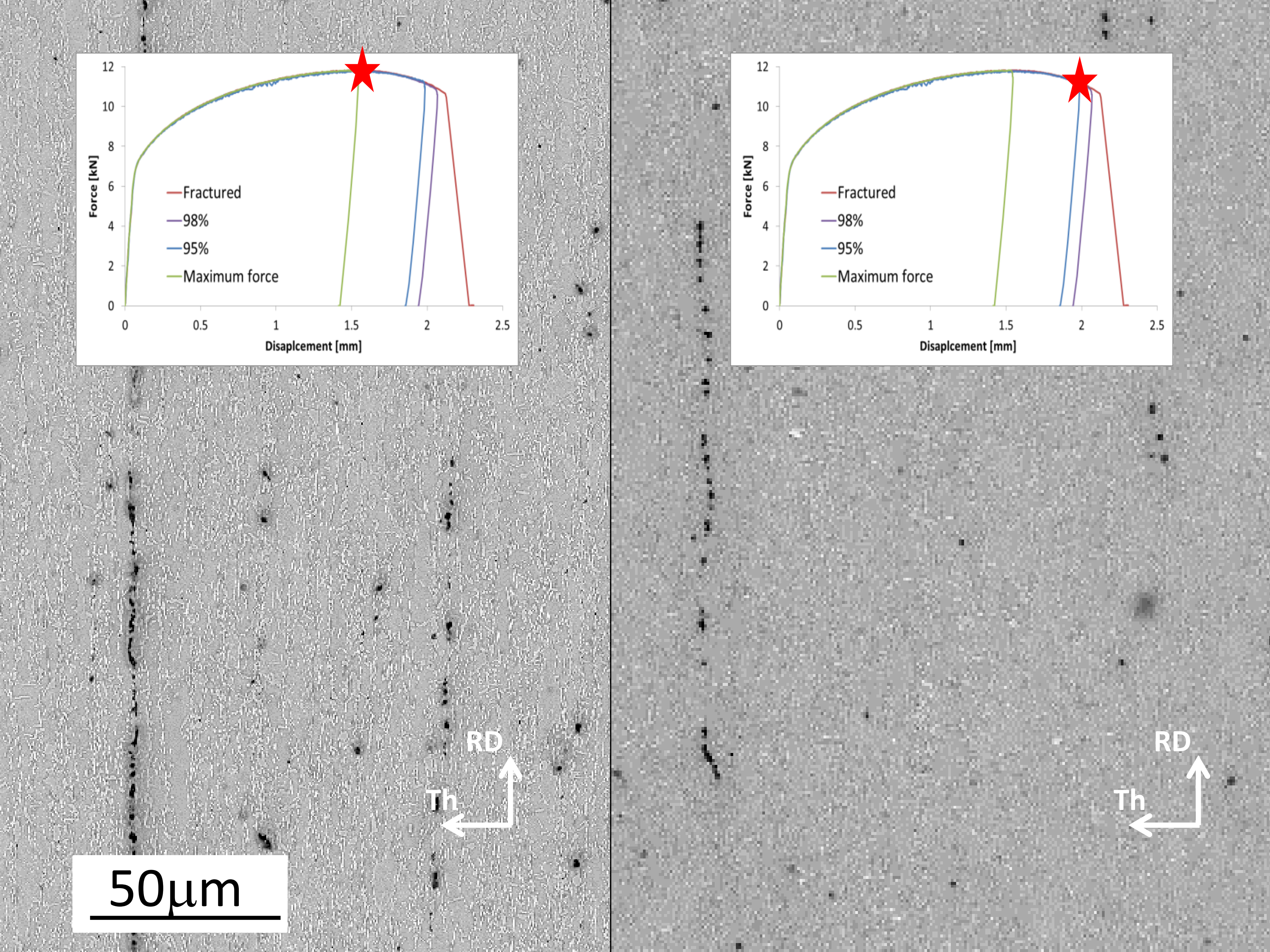
- Fractures on tight radii during stamping cannot be predicted by Forming Limit Diagram (FLD)
- Usually termed as **shear fracture**, presents little necking, shows slant fracture

Interrupted tension experiments

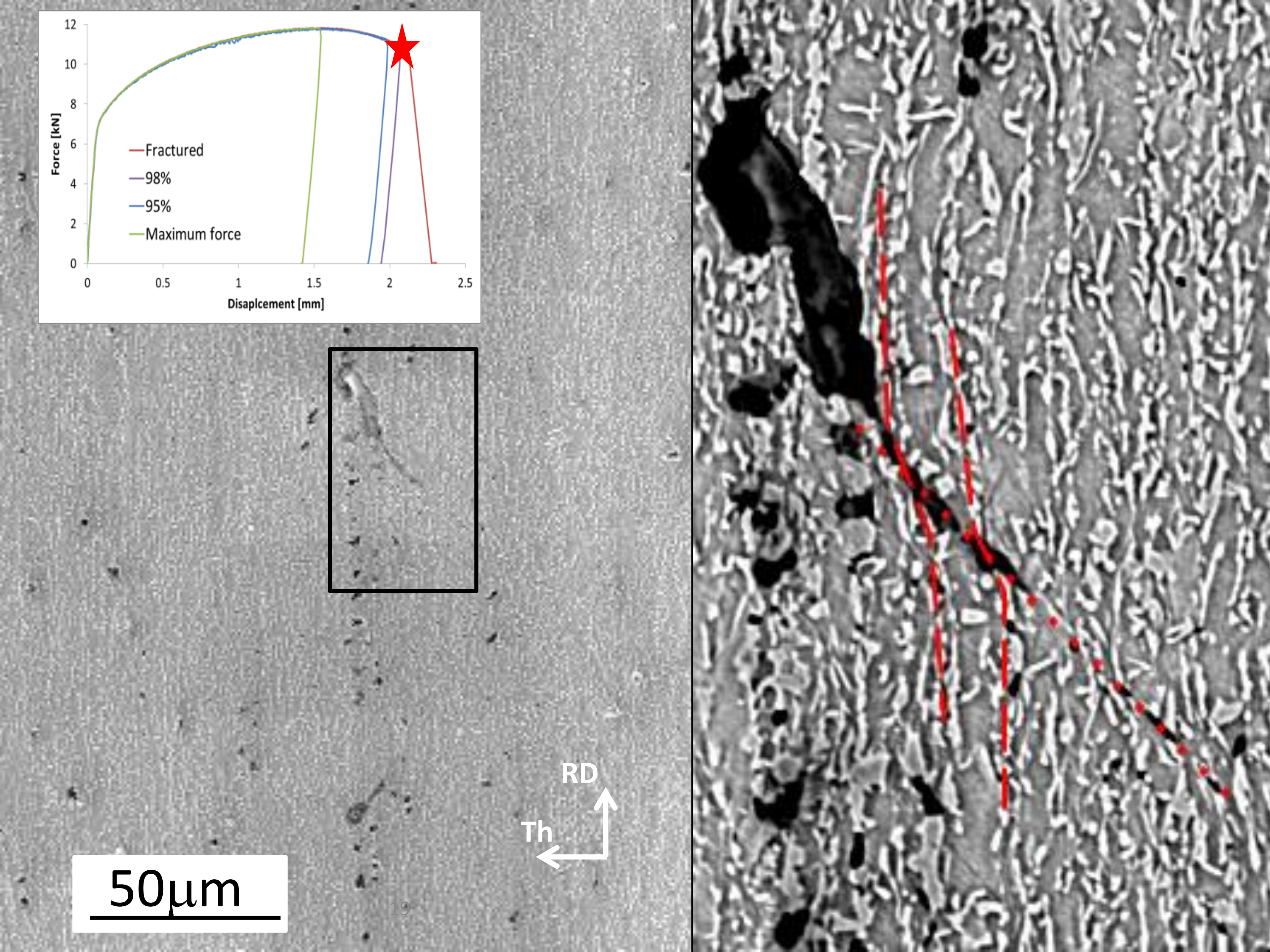
- **Flat** notched tensile specimens (1.4mm initial thickness)





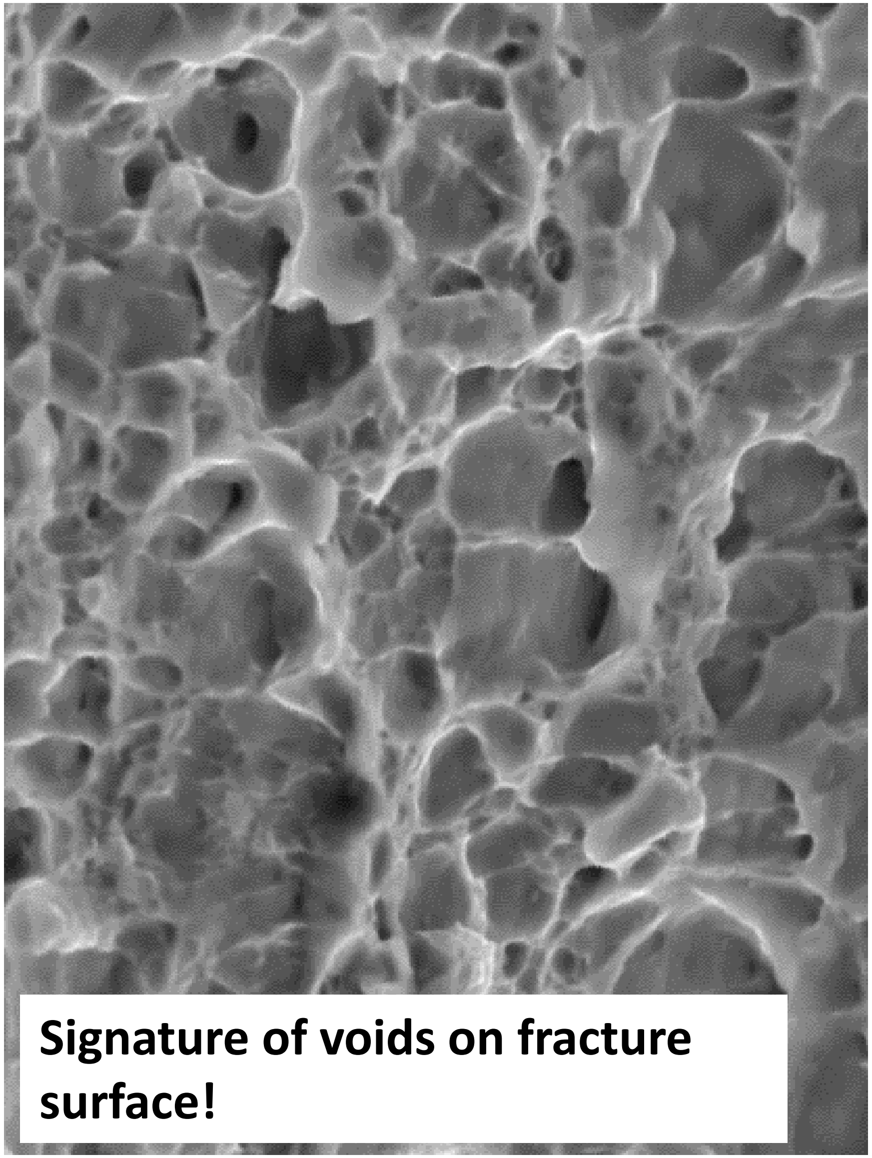


12
10
8
6
4
2
0
Force [kN]
Fractured
98%
95%
Maximum force
0
0.5
1
1.5
2
2.5
Disaplcement [mm]
RD
Th
50μm
12
10
8
6
4
2
0
Force [kN]
Fractured
98%
95%
Maximum force
0
0.5
1
1.5
2
2.5
Disaplcement [mm]
RD
Th

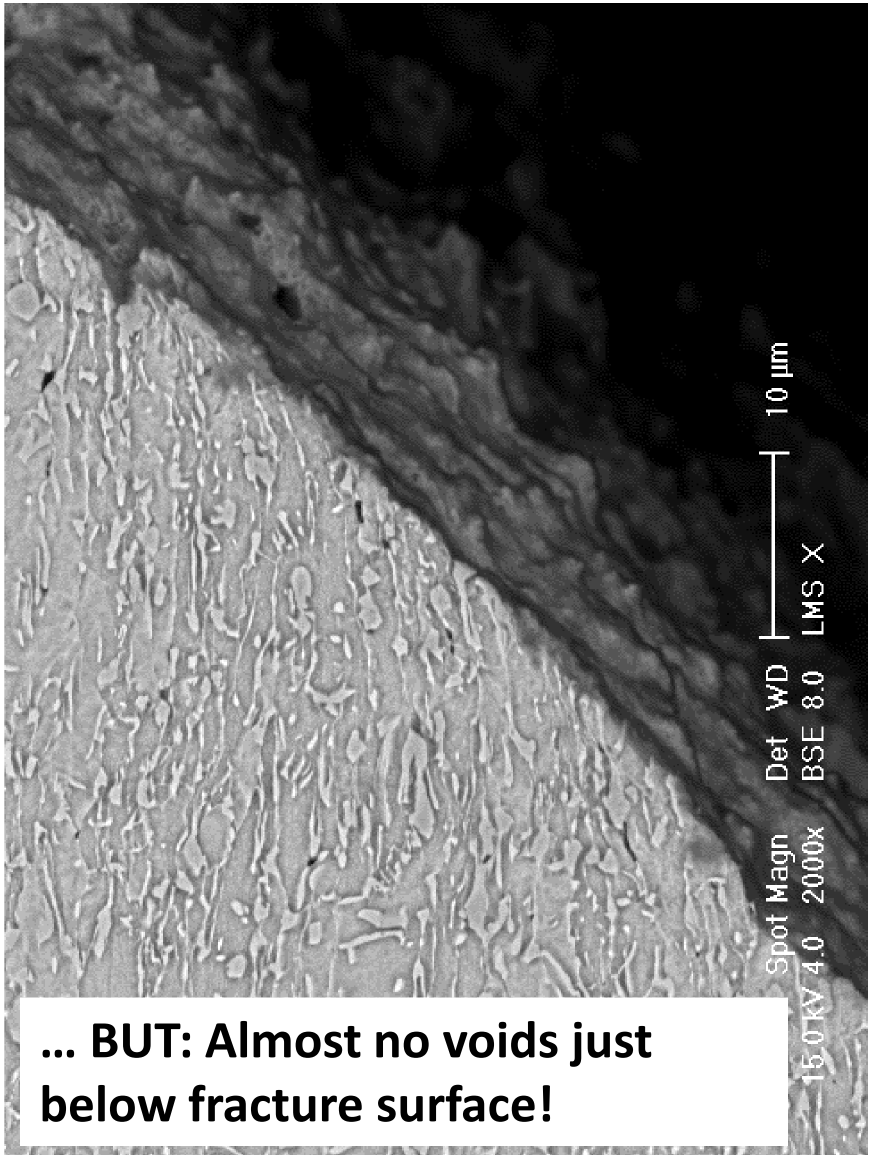


Force [kN]
Disaplcement [mm]
0
2
4
6
8
10
12
0.5
1
1.5
2
2.5
Fractured
98%
95%
Maximum force
RD
Th
50μm

Surface versus Cross-section View

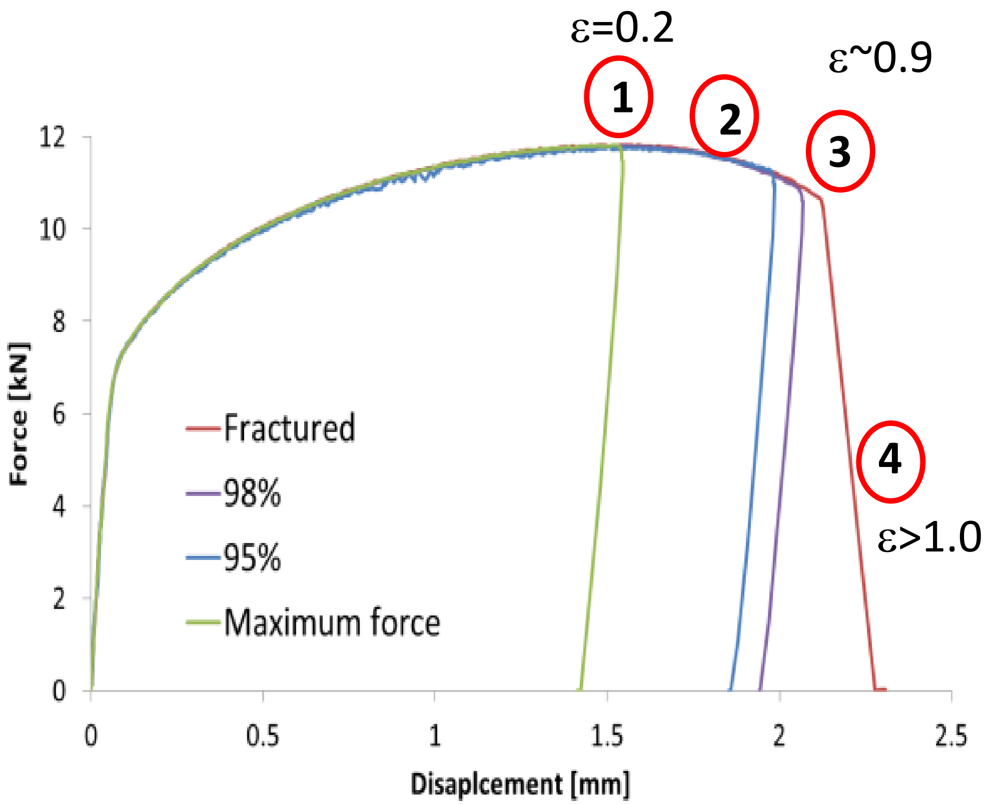


Signature of voids on fracture surface!



… BUT: Almost no voids just below fracture surface!

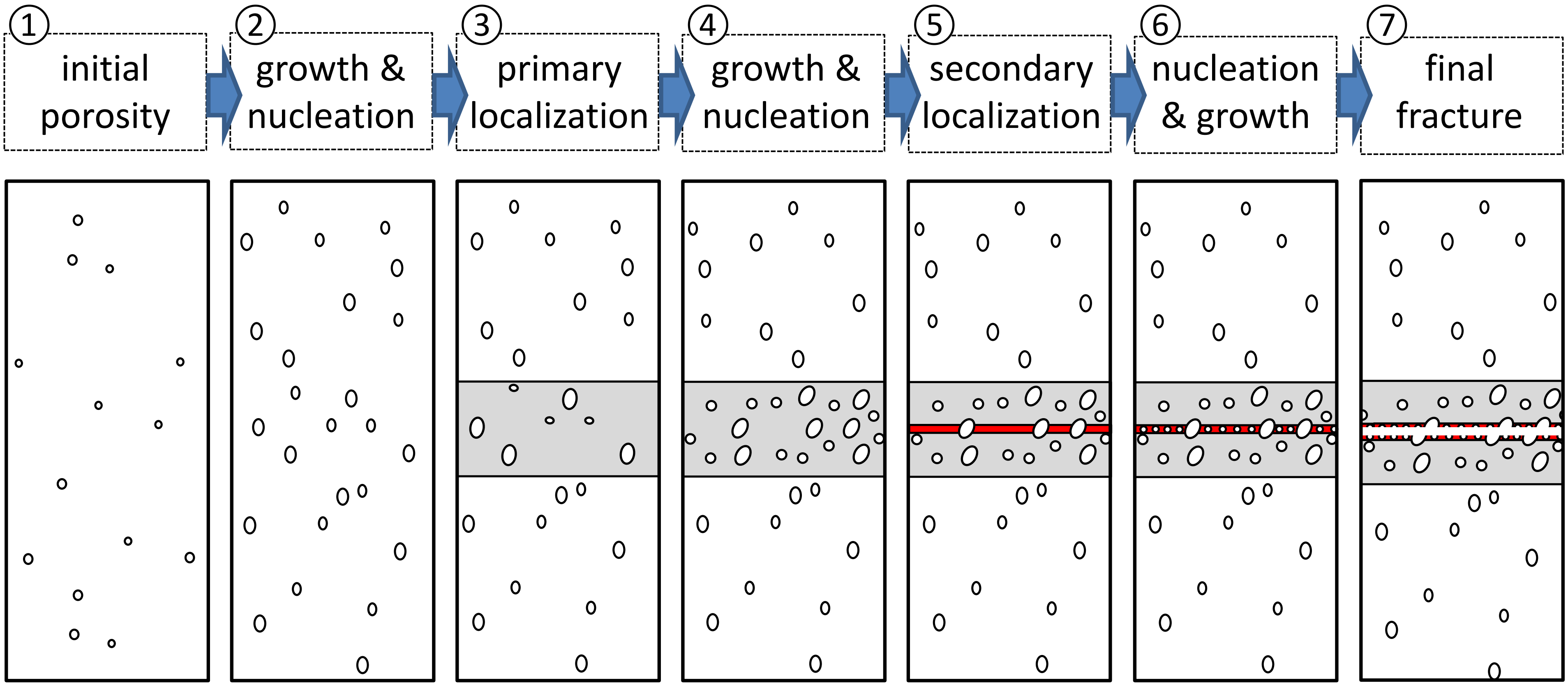
Failure Mechanism Summary



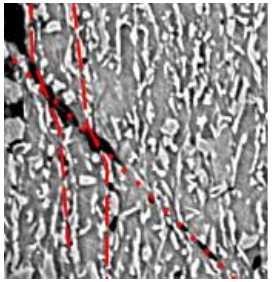
1. Onset of necking
2. Void volume fraction increases (more nucleation)
3. Shear localization
4. Void sheet failure

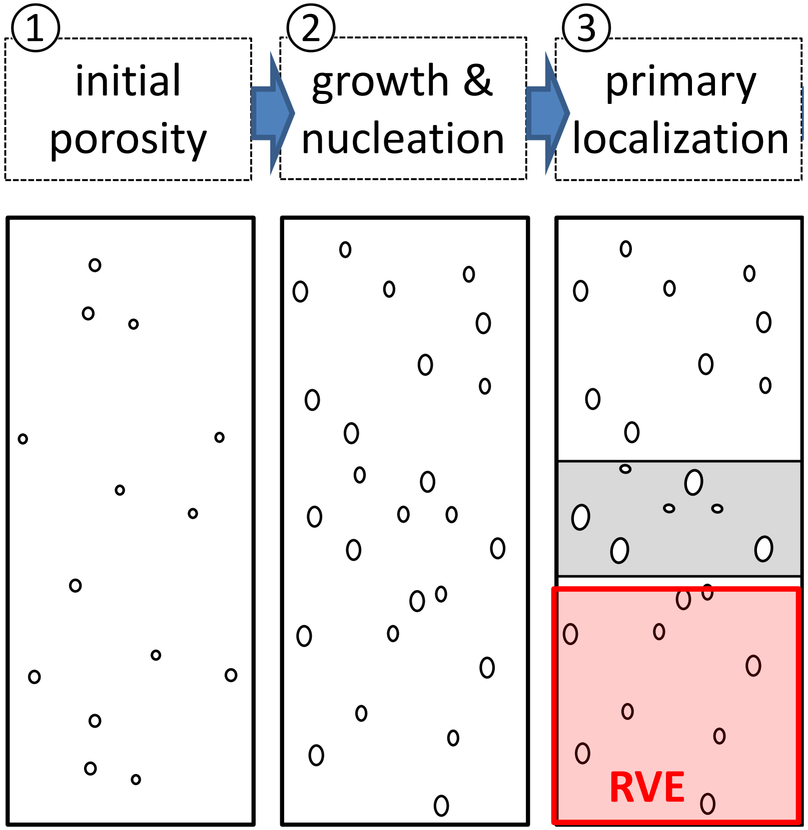
Define "strain to fracture" as strain at the onset of shear localization

A Think Model of the Ductile Fracture Process



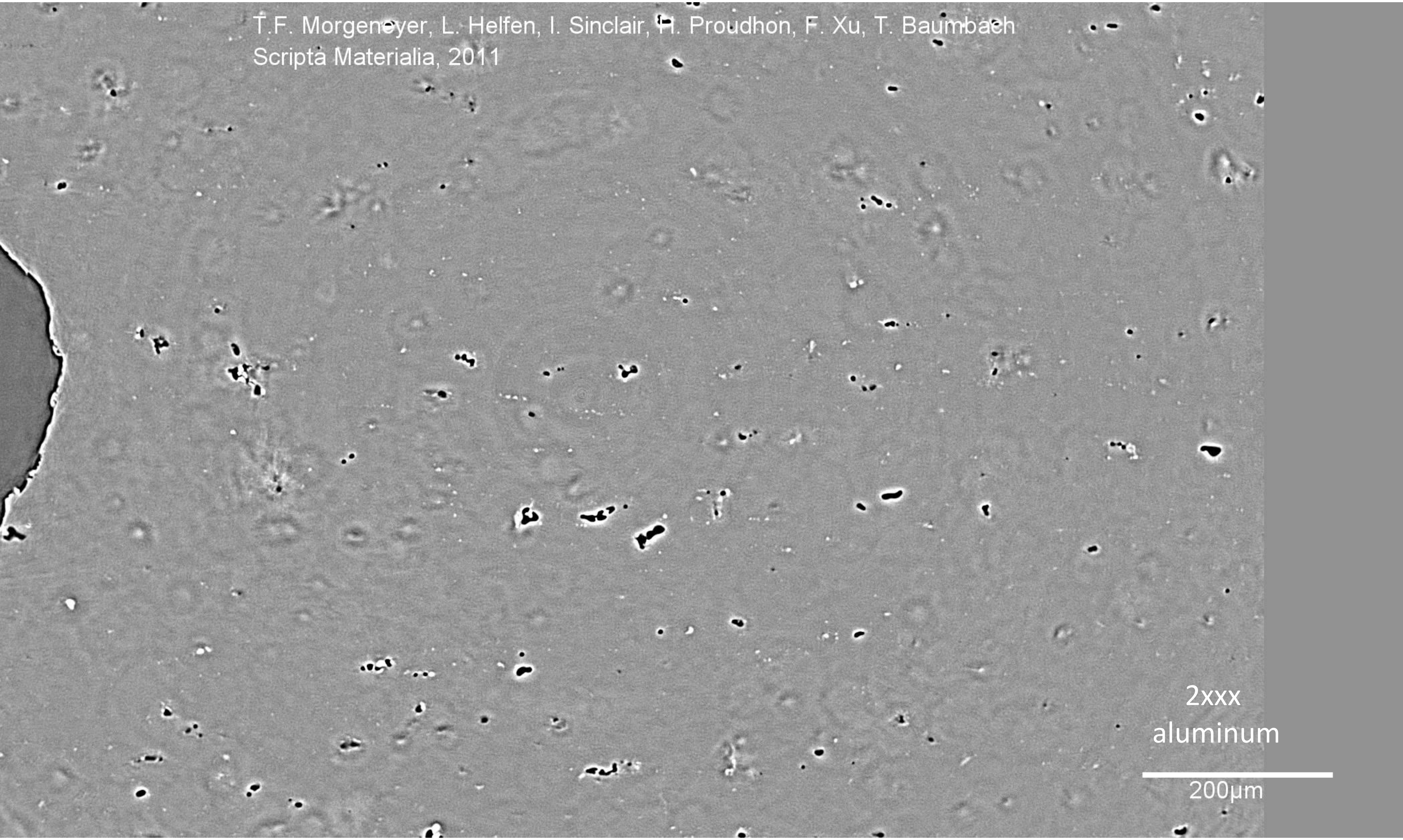
Definition of "strain to fracture"



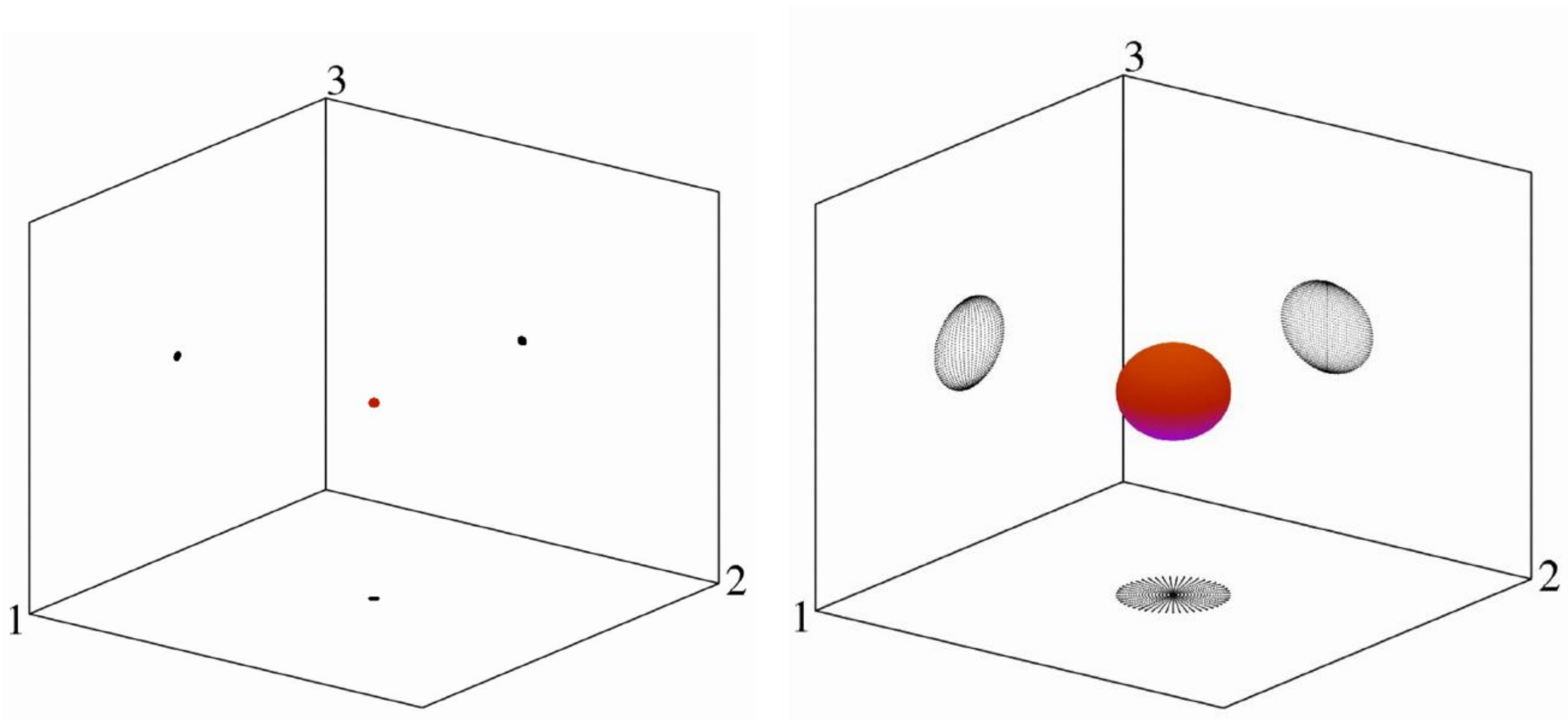


Strain to fracture =
macroscopic equiv. plastic strain at instant of first localization

Tomographic observations

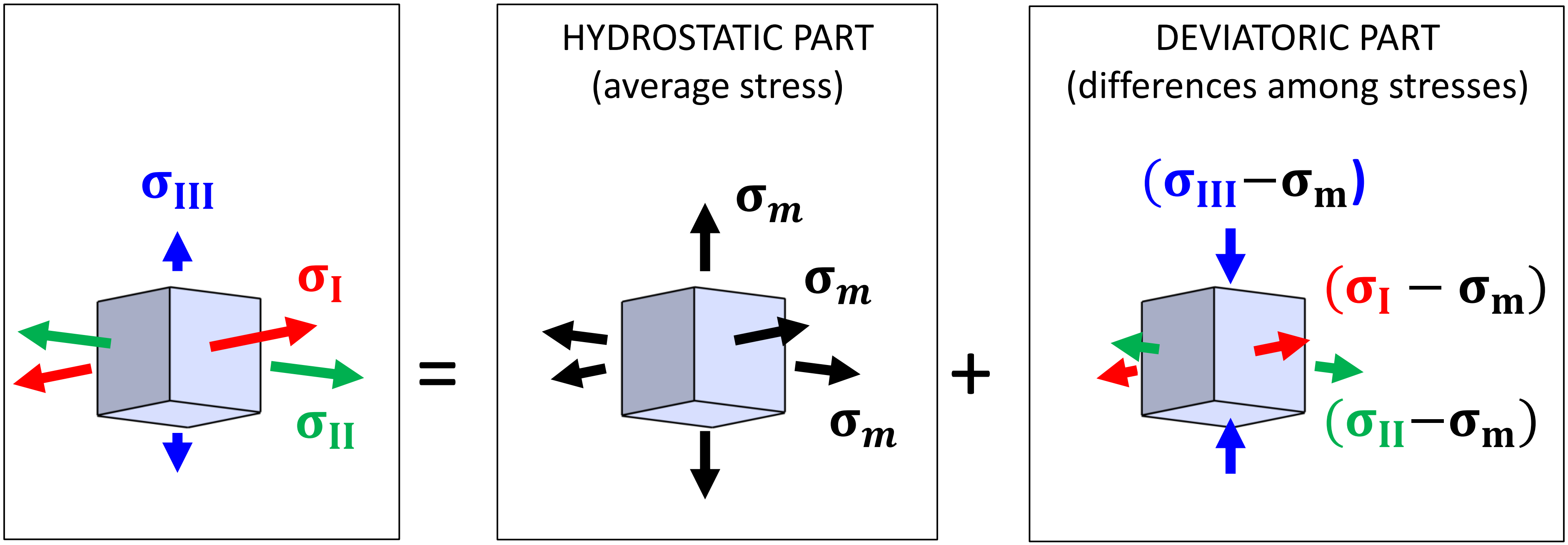


Void evolution in a plastic solid



…. void evolution depends on stress state

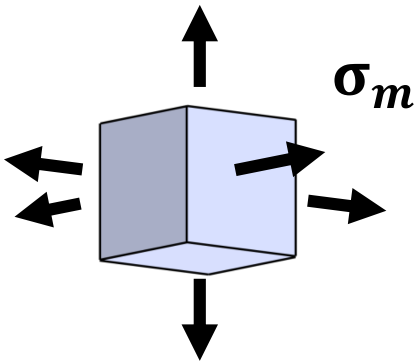
Decomposition of the Stress Tensor



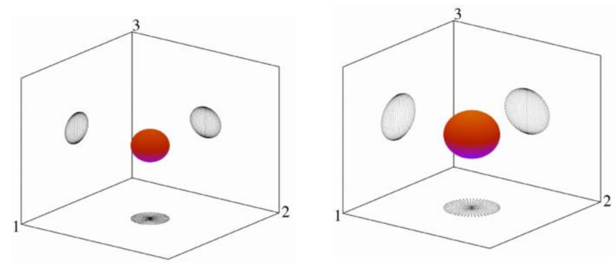
$$\sigma_m = \frac{\sigma_I + \sigma_{II} + \sigma_{III}}{3}$$

Effect of Stress State on Void Evolution

HYDROSTATIC PART



… controls void growth

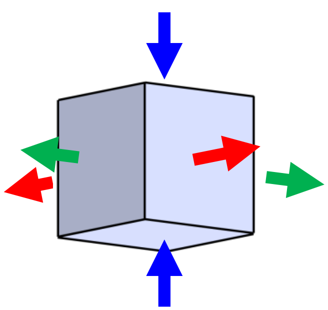


… is characterized by:

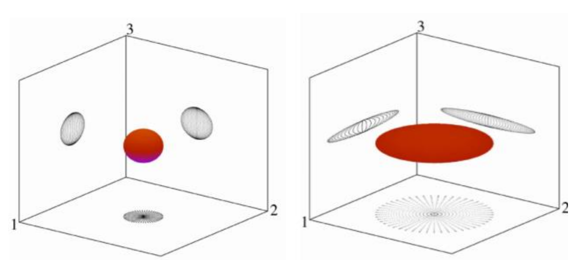
STRESS TRIAXIALITY

$$\eta = \frac{\sigma_m}{\bar{\sigma}}$$

DEVIATORIC PART



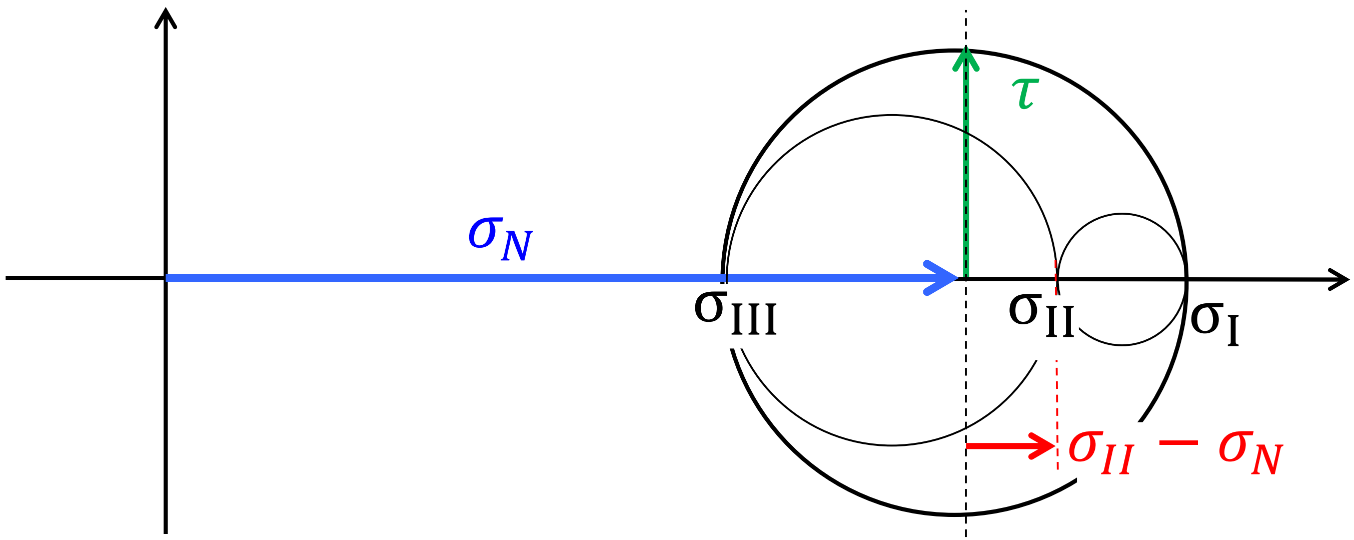
… controls shape change



… is characterized by:

LODE PARAMETER

Definition of Lode Parameter



- Maximum shear stress (radius of biggest circle):

$$\tau = \frac{\sigma_I - \sigma_{III}}{2}$$

- Normal stress on plane of max. shear (center of biggest circle)

$$\sigma_N = \frac{\sigma_I + \sigma_{III}}{2}$$

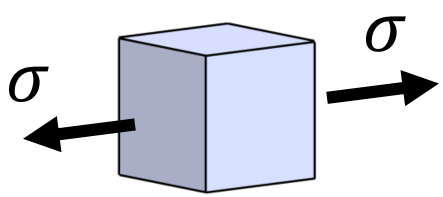
- Position of the intermediate principal stress:

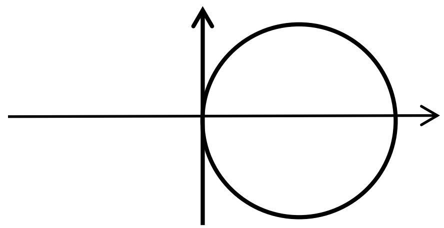
$$\boxed{L = \frac{\sigma_{II} - \sigma_N}{\tau}}$$

LODE PARAMETER

Which states have the same Lode parameter?

- Uniaxial tension

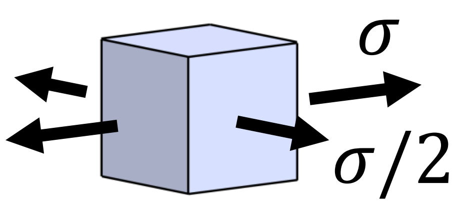


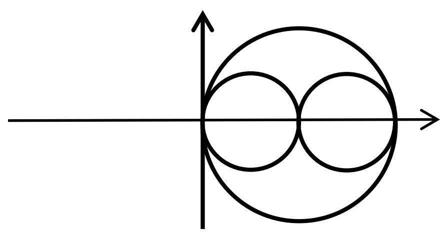


$$\left.\begin{array}{l}\sigma_m = \sigma/3 \\ \bar{\sigma} = \sigma\end{array}\right\} \Rightarrow \eta = 1/3$$

$$\sigma_{II} = \sigma_{III} = 0 \quad \Rightarrow L = -1$$

- Plane strain tension

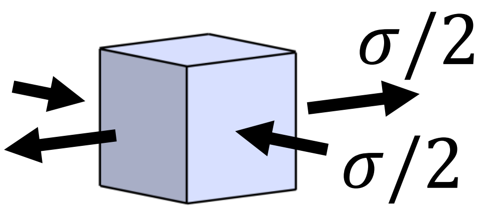


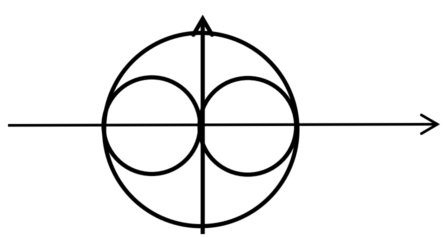


$$\left.\begin{array}{l}\sigma_m = \sigma/2 \\ \bar{\sigma} = \sqrt{3}/2\sigma\end{array}\right\} \Rightarrow \eta = 1/\sqrt{3}$$

$$\boxed{\sigma_{II} = (\sigma_I + \sigma_{III})/2 \Rightarrow L = 0}$$

- Pure shear





$$\left.\begin{array}{l}\sigma_m = 0 \\ \bar{\sigma} = \sqrt{3}/2\sigma\end{array}\right\} \Rightarrow \eta = 0$$

$$\boxed{\sigma_{II} = (\sigma_I + \sigma_{III})/2 \Rightarrow L = 0}$$

Lode angle parameter

- Stress triaxiality:

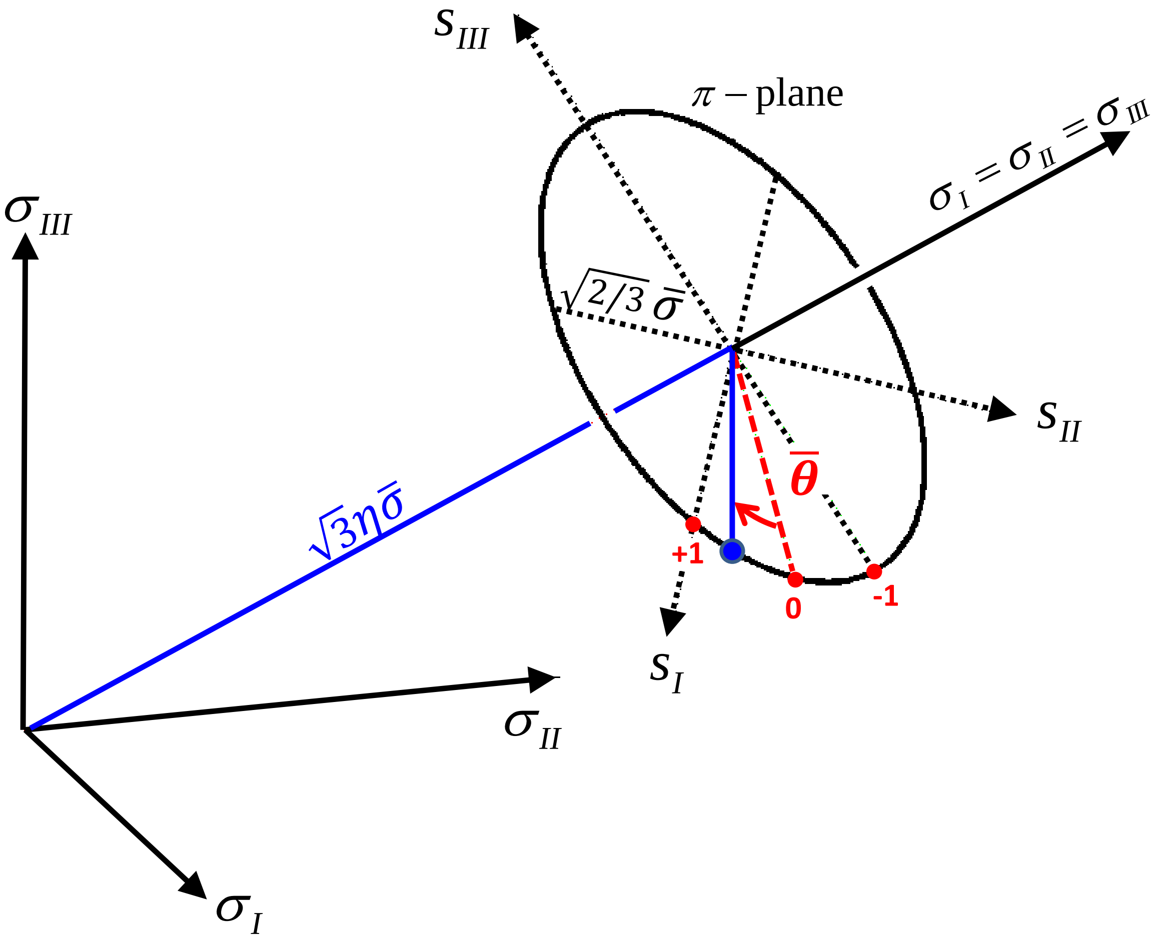
$$\eta = \frac{\sigma_m}{\bar{\sigma}}$$

- Normalized third stress invariant

$$\xi = \frac{27}{2}\frac{J_3}{\bar{\sigma}^3}$$

- Lode angle parameter

$$\bar{\theta} = 1 - \frac{2}{\pi}\arccos(\xi)$$



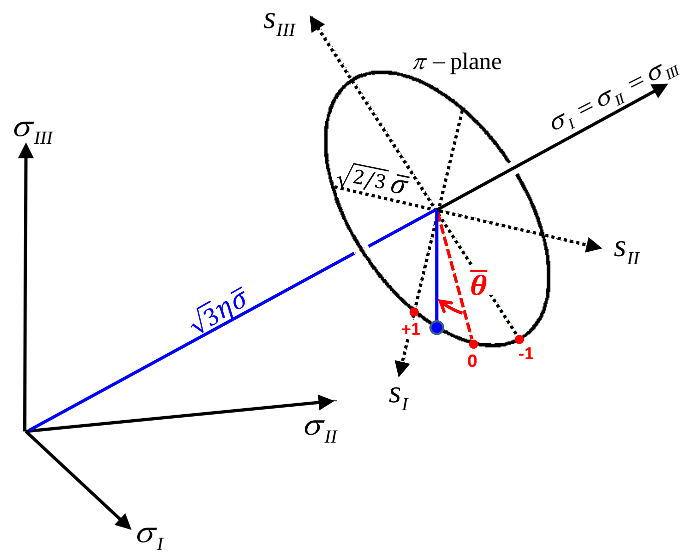
Lode angle parameter

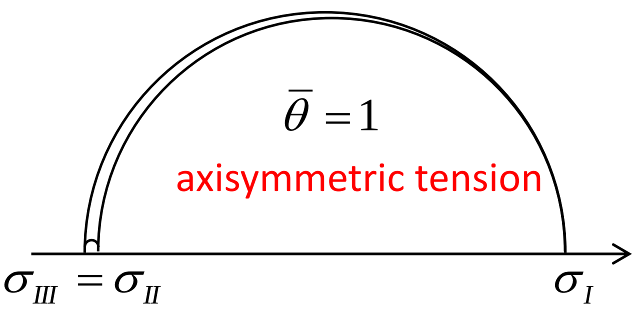
- Lode parameter (Lode, 1926)

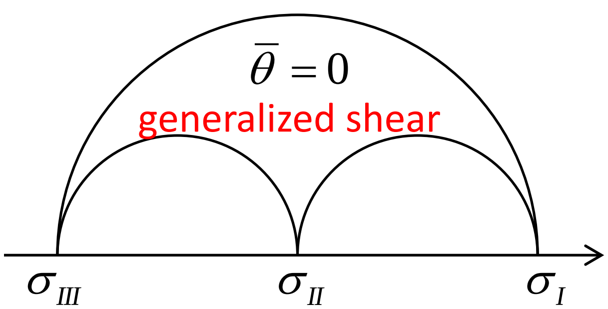
$$L = \frac{2\sigma_{II} - \sigma_I - \sigma_{III}}{\sigma_I - \sigma_{III}}$$

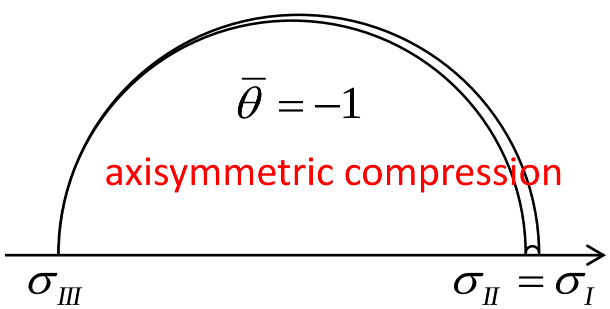
- Lode angle parameter

$$\bar{\theta} = 1 - \frac{2}{\pi}\arccos(\xi) \cong -L$$









Plane stress states

For isotropic materials, the **stress tensor** is fully characterized by three stress tensor invariants,

$$\{I_1, J_2, J_3\} \quad \text{or} \quad \{\sigma_I, \sigma_{II}, \sigma_{III}\}$$

while the **stress state** is characterized by the **two** dimensionless ratios of the invariants, e.g.

$$\{I_1/\sqrt{J_2},\ J_3/J_2^{3/2}\} \quad \text{or} \quad \{\eta,\ \bar{\theta}\} \quad \text{or} \quad \{\sigma_{II}/\sigma_I,\ \sigma_{III}/\sigma_I\}$$

with

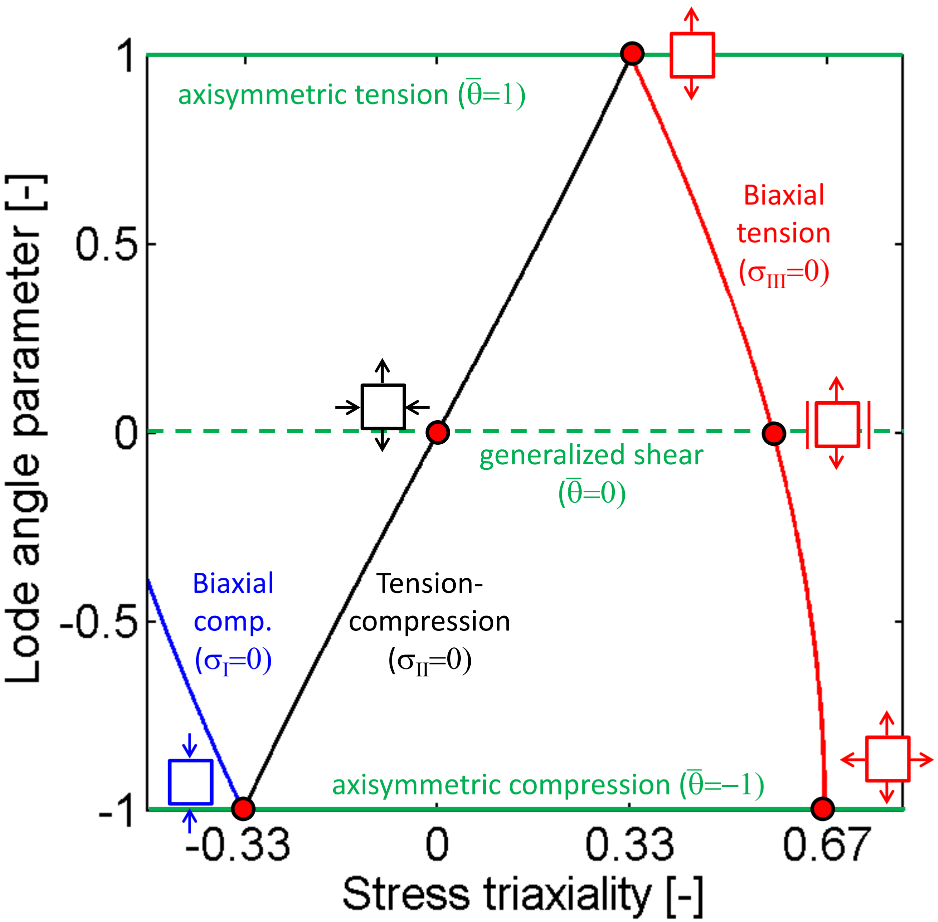
$$\eta = \frac{I_1}{3\sqrt{3J_2}} \quad \text{and} \quad \bar{\theta} = 1 - \frac{2}{\pi}\arccos\left[\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right]$$

Plane stress states

Under plane stress conditions, one principal stress is zero. The stress state may thus be characterized by the ratio of the two non-zero principal stresses.

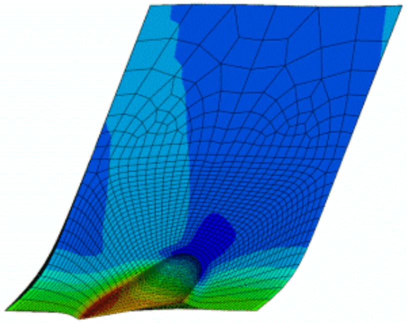
As a result, the stress triaxiality and the Lode angle parameter are no longer independent for plane stress, i.e. we have a functional relationship

$$\bar{\theta} = \bar{\theta}[\eta]$$

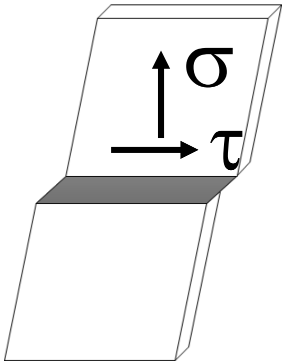


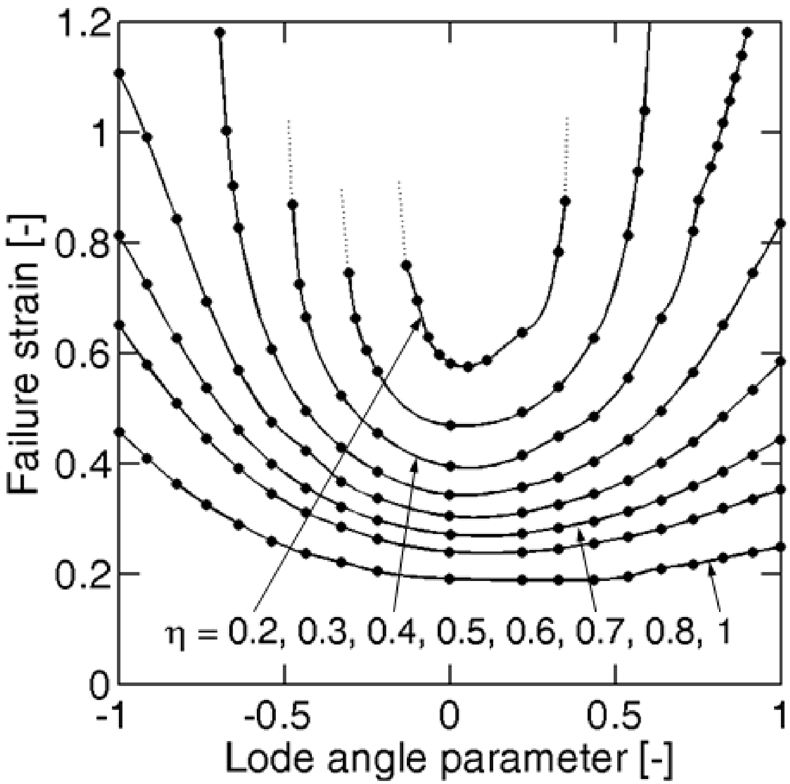
Results from Localization Analysis

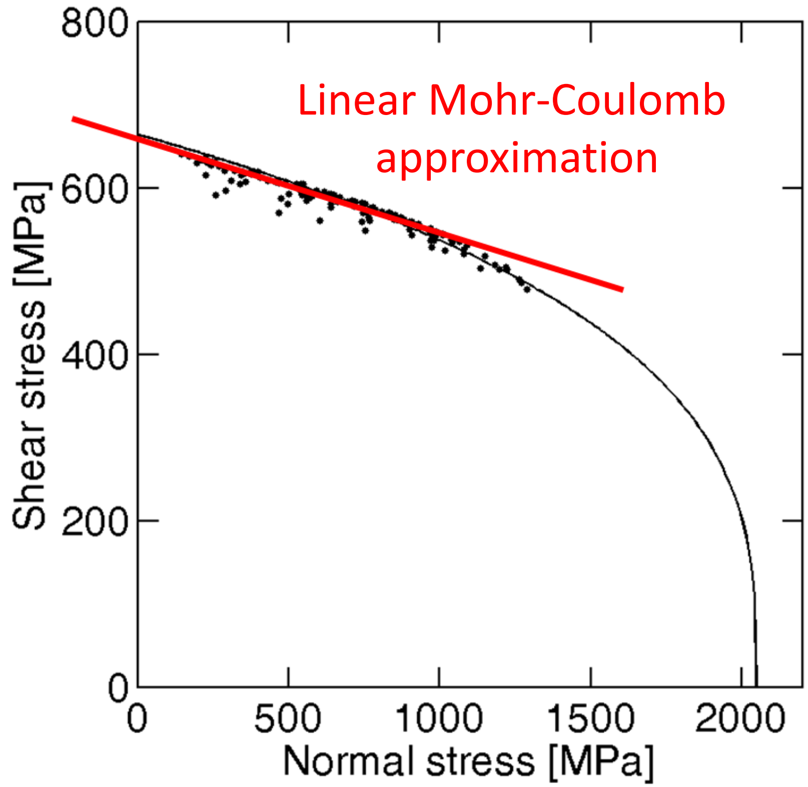
Unit Cell with Central Void



Stresses on Plane of Localization







Hosford-Coulomb Ductile Fracture Model

Principal stress space $\{\sigma_I, \sigma_{II}, \sigma_{III}\}$

Haigh-Westergaard space $\{\eta, \bar{\theta}, \bar{\sigma}\}$

Mixed strain-stress space $\{\eta, \bar{\theta}, \bar{\varepsilon}_p\}$

Hosford- Coulomb

$$\bar{\sigma}_{Hf} + c(\sigma_I + \sigma_{III}) = b$$

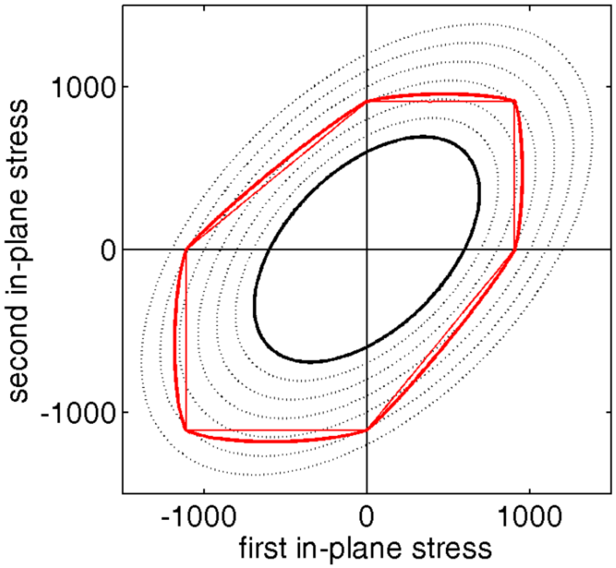
Coordinate transformation

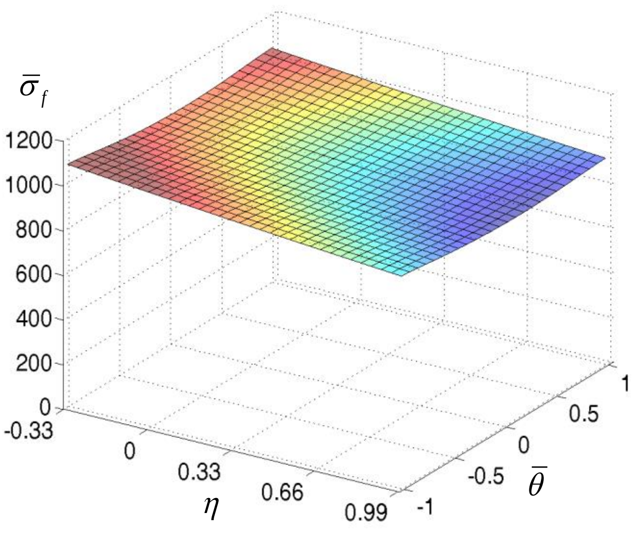
$$\bar{\sigma}_f = \bar{\sigma}_f[\bar{\theta}, \eta]$$

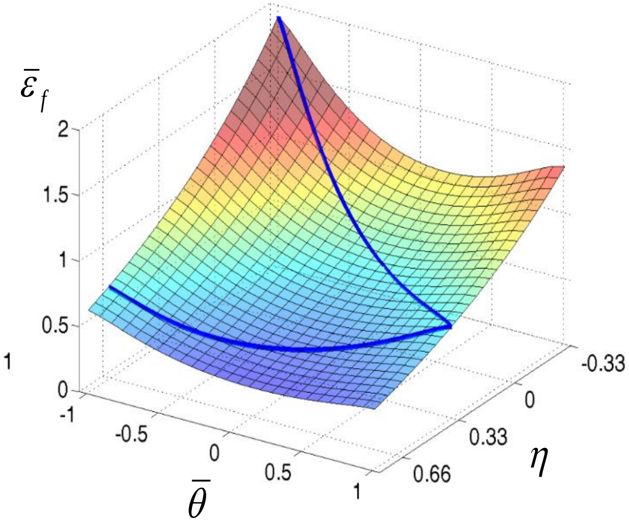
Isotropic hardening law

$$\bar{\sigma} = k[\bar{\varepsilon}_p]$$

$$\bar{\varepsilon}_f = k^{-1}\left[\bar{\sigma}_f[\eta, \bar{\theta}]\right]$$







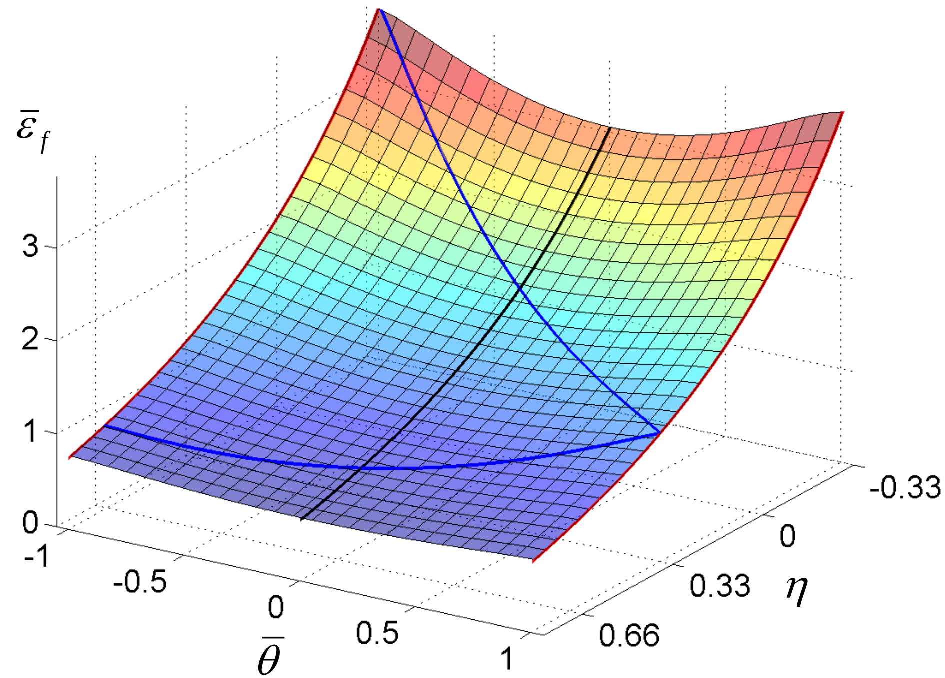
Hosford-Coulomb Ductile Fracture Model

- **General form**

von Mises equivalent plastic strain to fracture

$$\bar{\varepsilon}_f = \bar{\varepsilon}_f[\eta, \bar{\theta}, a, b, c]$$

Stress triaxiality (η) — Lode angle parameter ($\bar{\theta}$) — 3 material parameters (a, b, c)



- **Detailed expressions**

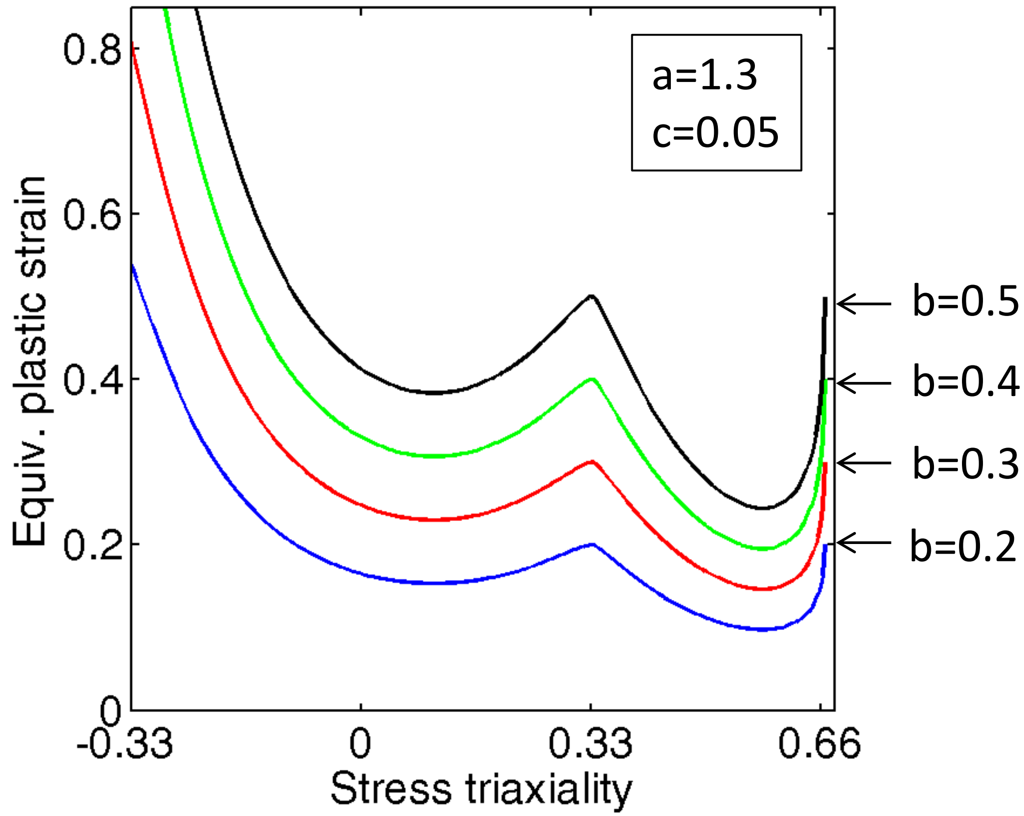
$$\bar{\varepsilon}_f = b\left(\frac{1+c}{g_{HC}[\eta,\bar{\theta}]}\right)^{\frac{1}{n}}$$

$$g_{HC} = \left(\tfrac{1}{2}|f_I - f_{II}|^a + \tfrac{1}{2}|f_{II} - f_{III}|^a + \tfrac{1}{2}|f_I - f_{III}|^a\right)^{\frac{1}{a}} + c\left(2\eta + f_I + f_{III}\right)$$

$$f_I[\bar{\theta}] = \frac{2}{3}\cos\left[\frac{\pi}{6}(1-\bar{\theta})\right] \qquad f_{II}[\bar{\theta}] = \frac{2}{3}\cos\left[\frac{\pi}{6}(3+\bar{\theta})\right] \qquad f_3[\bar{\theta}] = -\frac{2}{3}\cos\left[\frac{\pi}{6}(1+\bar{\theta})\right]$$

Hosford-Coulomb Ductile Fracture Model

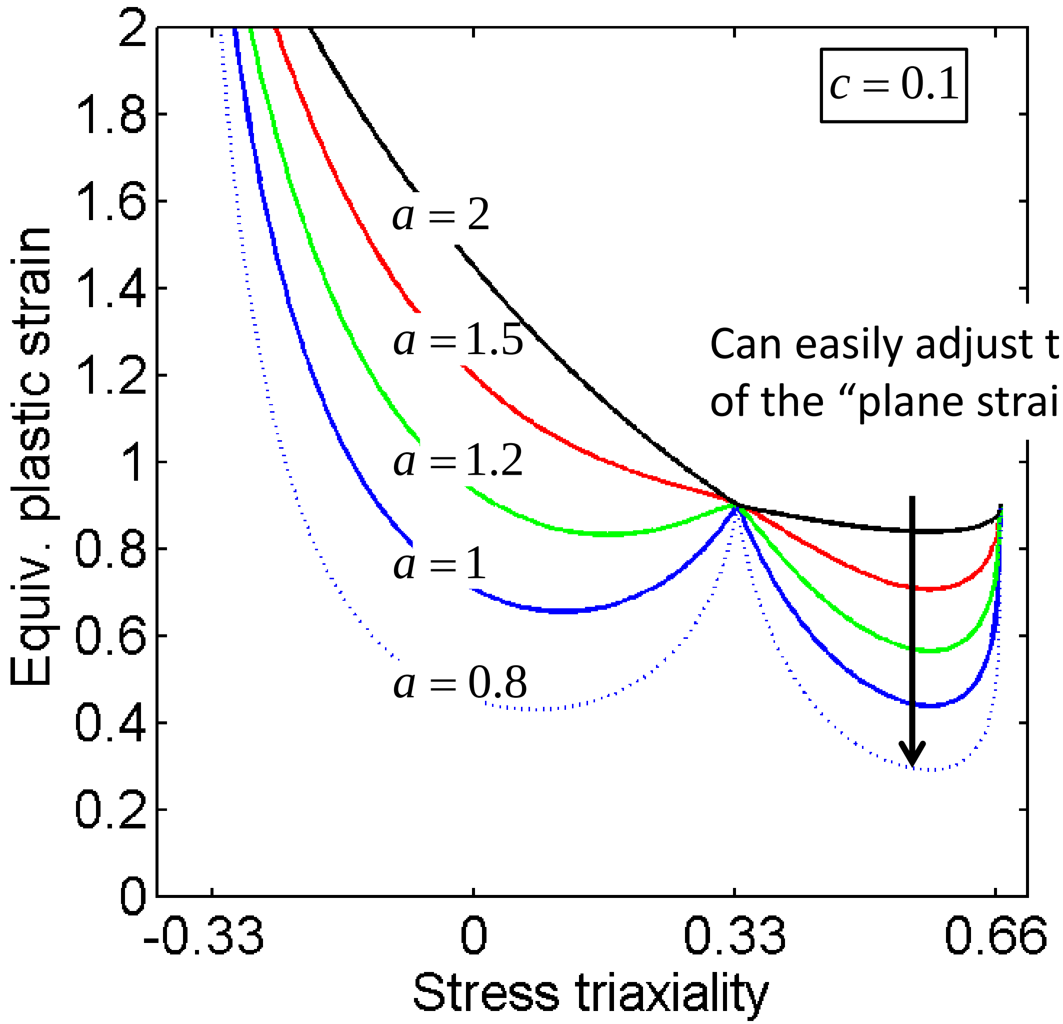
- Influence of parameter b



b = strain to fracture for uniaxial tension (or equi-biaxial tension)

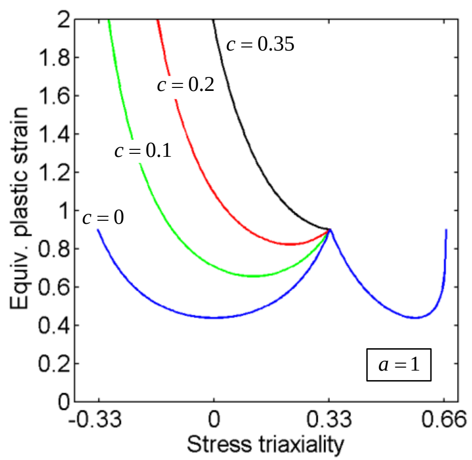
Hosford-Coulomb Ductile Fracture Model

- Influence of parameter a



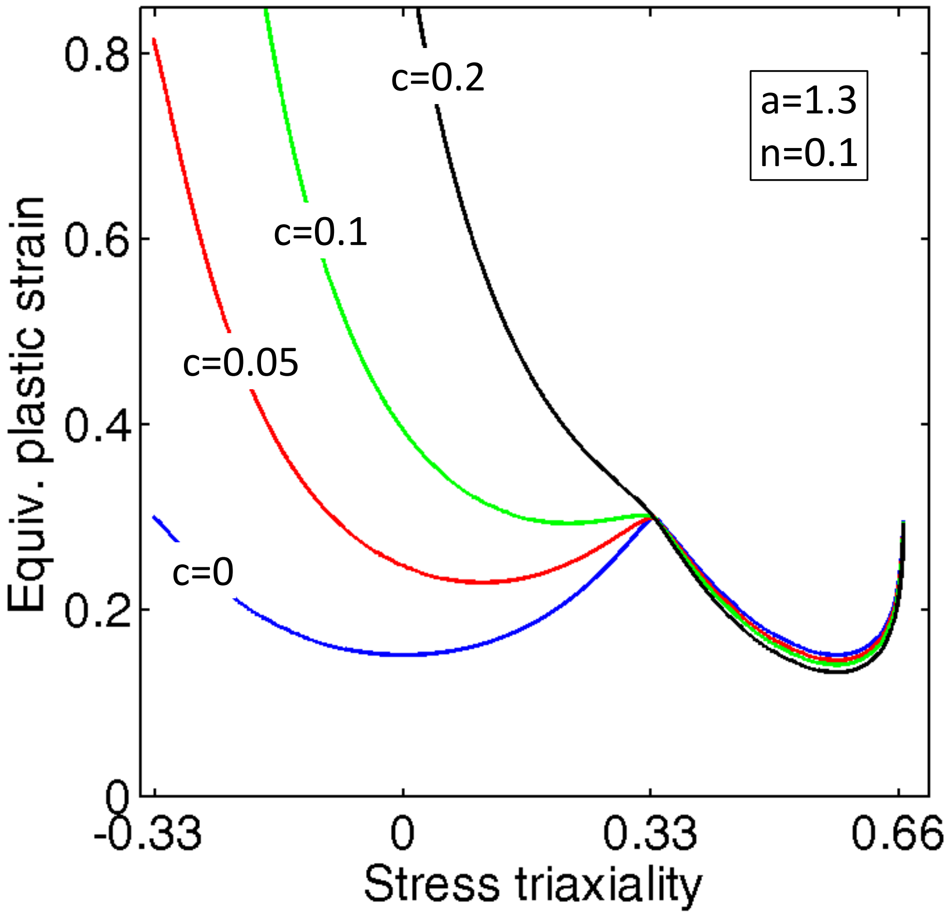
Can easily adjust the depth of the "plane strain valley"

Compare: Mohr-Coulomb



Hosford-Coulomb Ductile Fracture Model

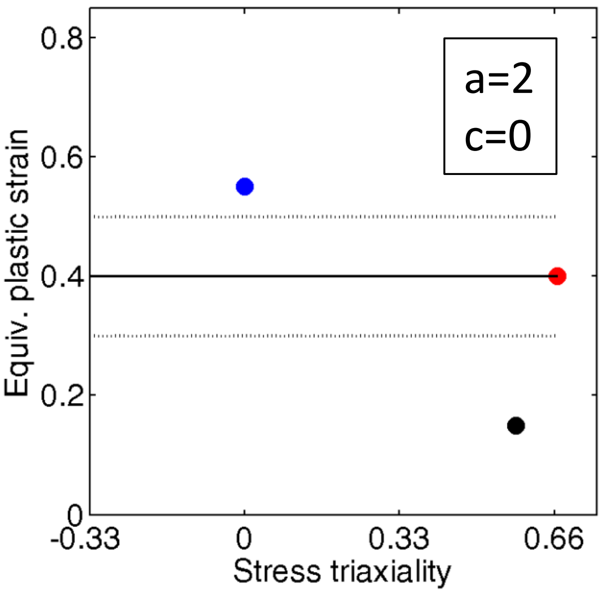
- Influence of parameter c



Hosford-Coulomb Ductile Fracture Model

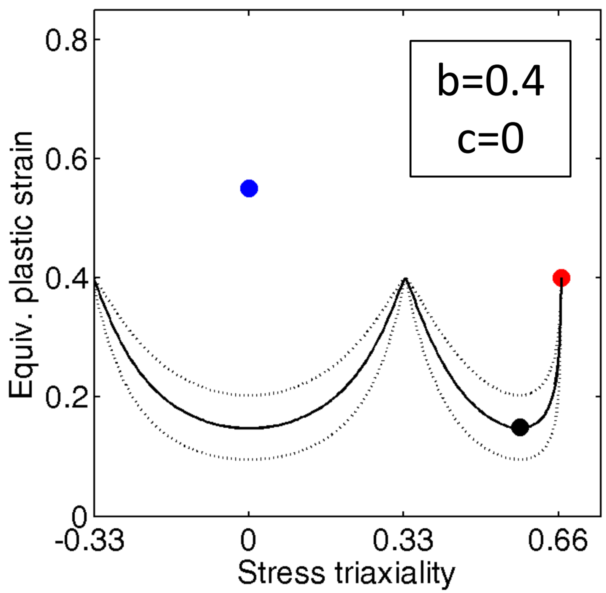
Rapid calibration guideline

Step I: Identify b



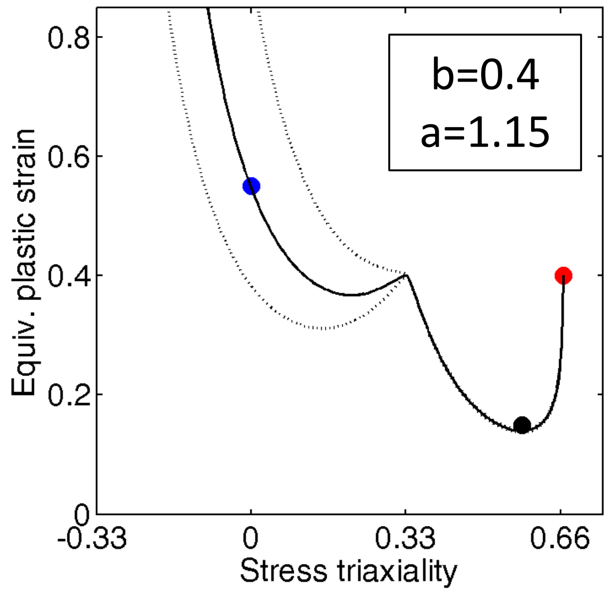
b=0.4

Step II: Identify a



a=1.15

Step III: Identify c



c=0.14

Hosford-Coulomb Ductile Fracture Model

Excel program for calibration

*INPUT

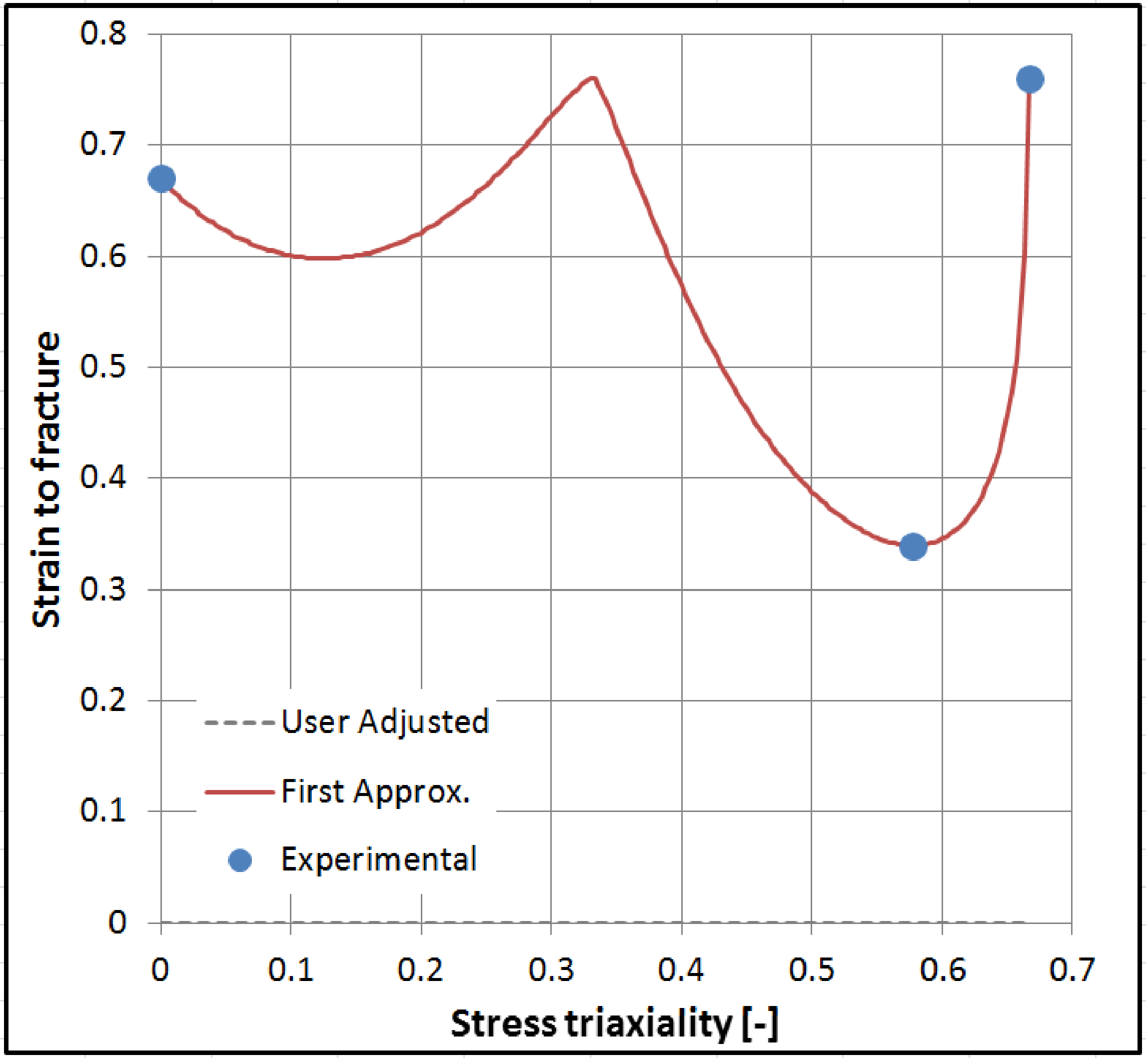
loading	EPS_F
equi-biaxial tension	0.76
plane strain tension	0.34
pure shear	0.67

*OUTPUT

EMC Parameters	
a	1.257
b	0.76
c	0.07

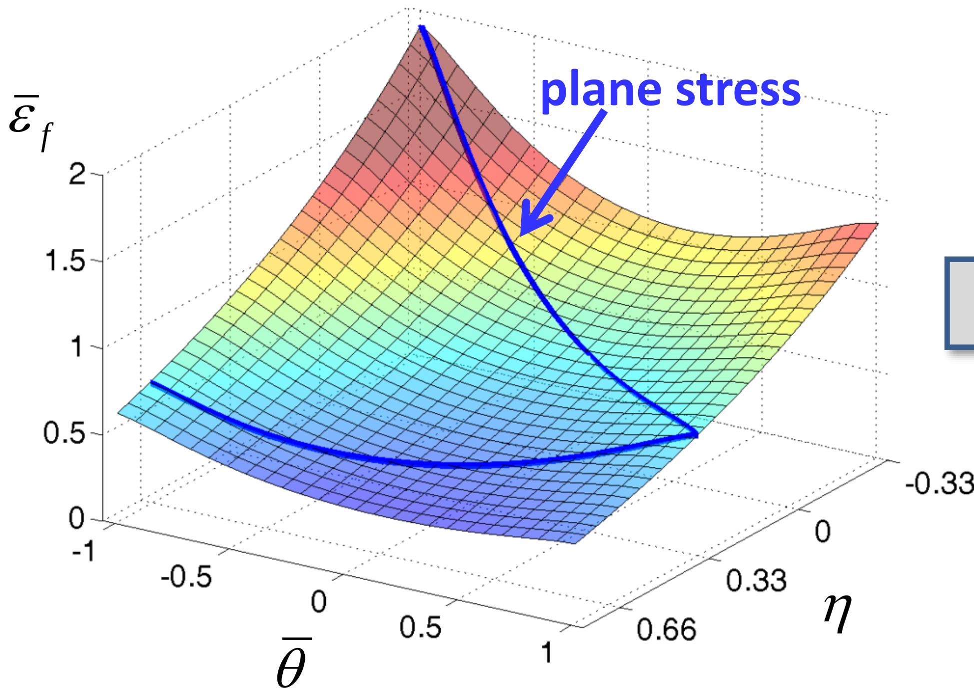
*USER ADJUSTED

EMC Parameters	
a	2
b	0
c	0

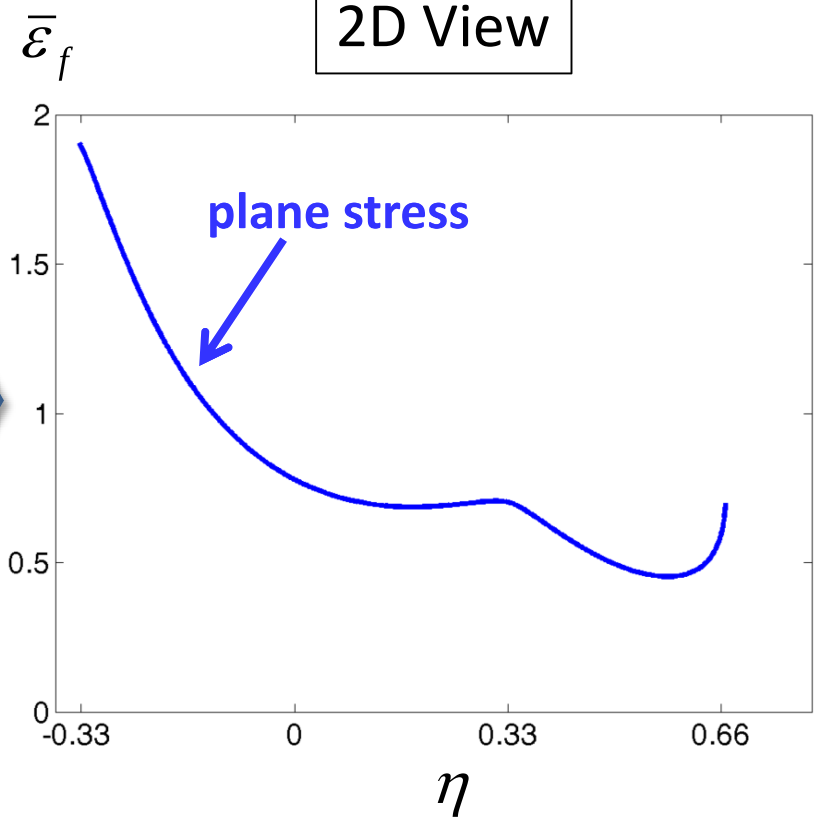


Hosford-Coulomb Ductile Fracture Model

3D View



2D View



"heart" of the model:

$$\bar{\varepsilon}_f = \bar{\varepsilon}_f[\eta, \bar{\theta}]$$

Damage Accumulation

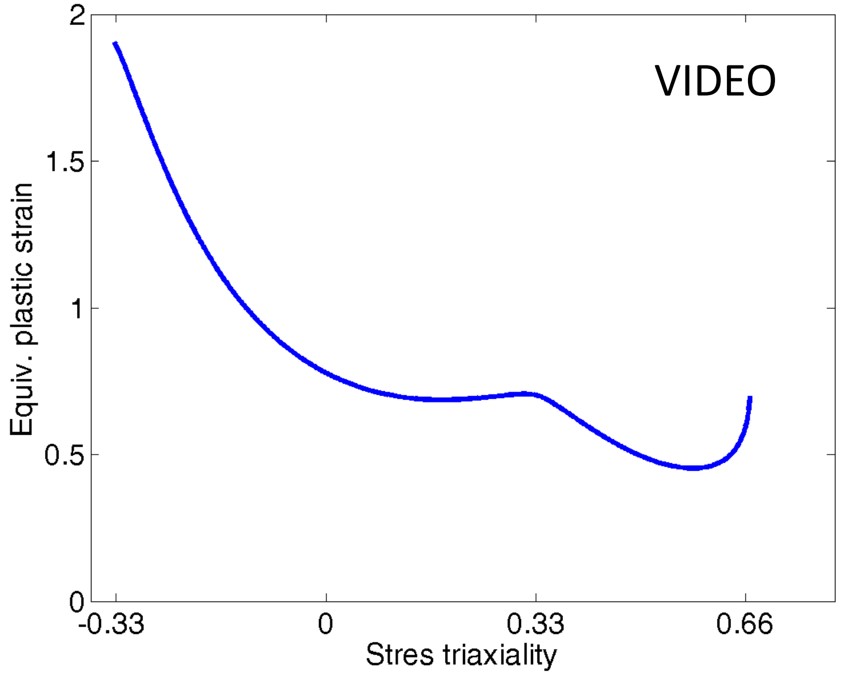
Define "damage indicator"

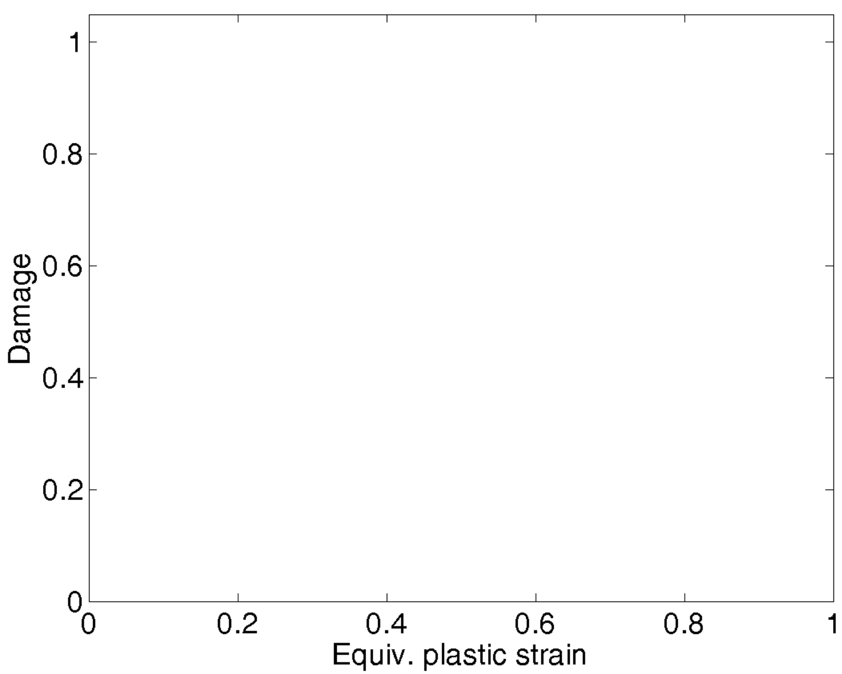
$$\bar{\varepsilon}_f = \bar{\varepsilon}_f[\eta, \bar{\theta}] \quad \Rightarrow \quad D = \int \frac{d\bar{\varepsilon}_p}{\bar{\varepsilon}_f[\eta, \bar{\theta}]}$$

$D = 0$ (initial)

$D = 1$ (fracture)

- Example: uniaxial tension





Damage Accumulation

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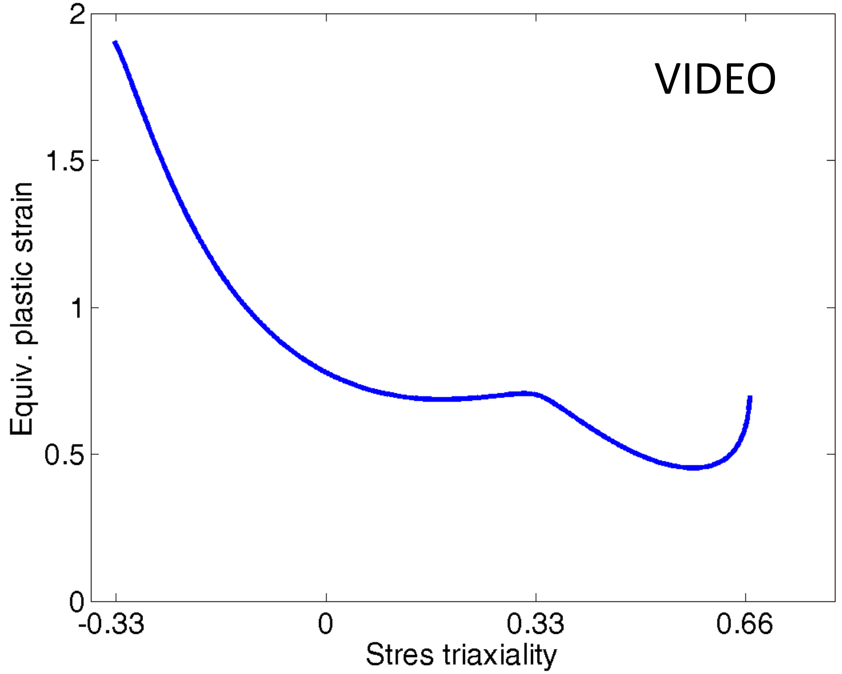
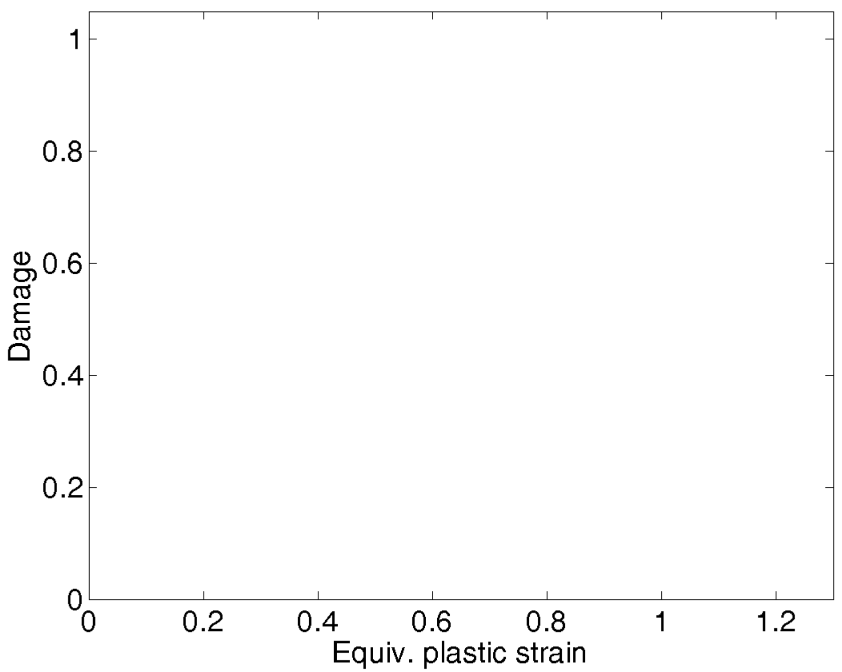
$D = 1$ (fracture)

- **Example: uniaxial compression followed by tension**





Damage Accumulation

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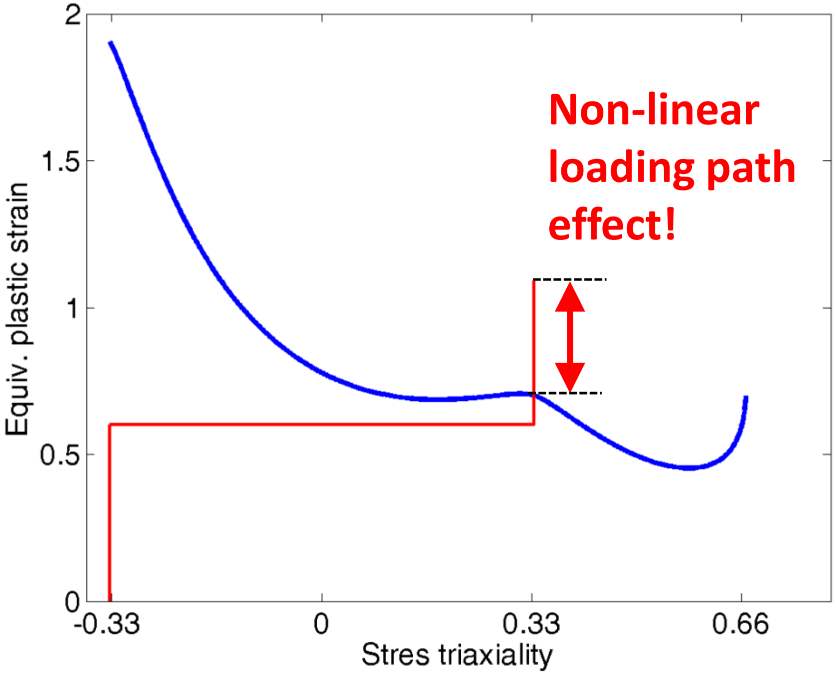
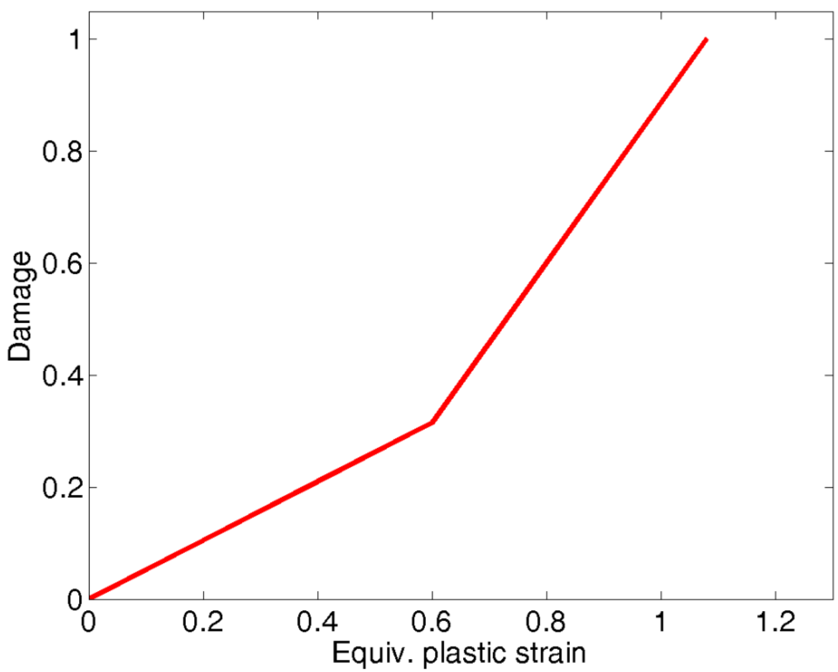
$D = 1$ (fracture)

- **Example: uniaxial compression followed by tension**





Reading Materials for Lecture #7

- S. Suresh, “Fatigue of Materials”, Cambridge University Press, 1998.
- D. Mohr and S.J. Marcadet, http://www.sciencedirect.com/science/article/pii/S0020768315000700