CE394M: Stress paths and invariants

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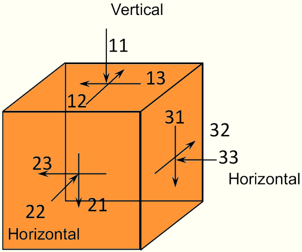
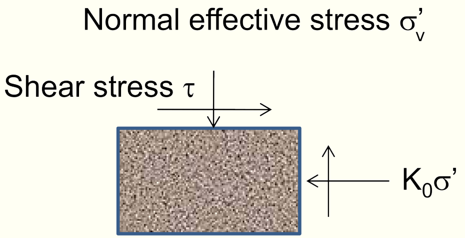
1D consolidation / simple shear

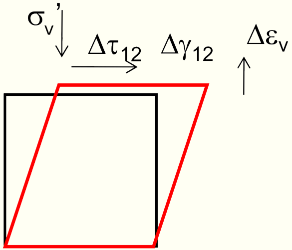


2D plane strain

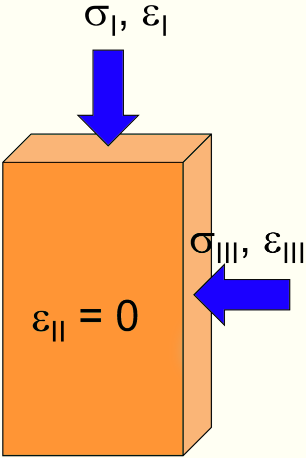
3D general (axi-symmetric as a special case)

1D simple shear





Stresses and strains: independent components



$\sigma' = \sigma - u$

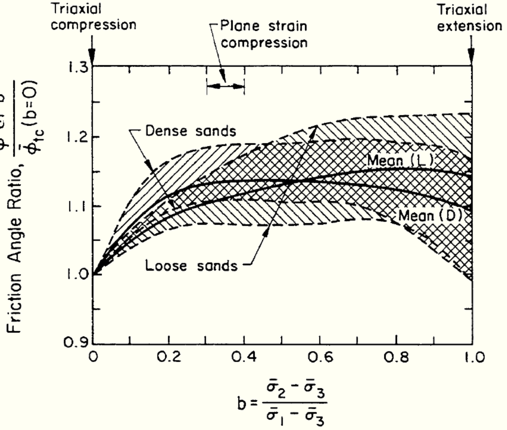
2D Mohr circle

Mohr circle to 2D stress components

Successive Mohr circles and stress path for constant σ_3 and increasing σ_1:

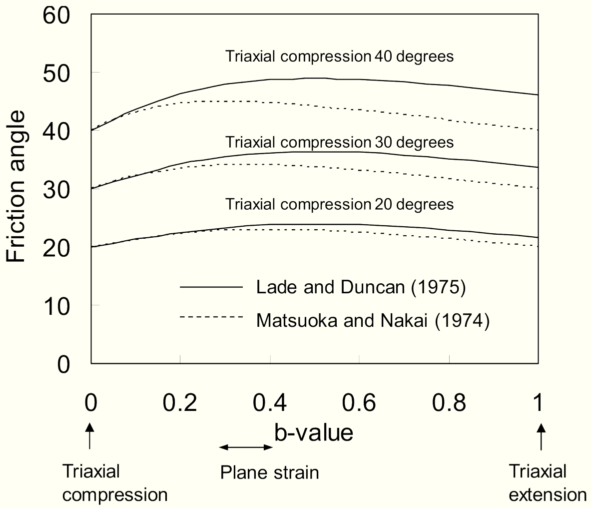
Effect of σ_{II}

Bishop (1966) defined **b-value**:



Effect of σ_{II} on friction angle (Kulhawy and Mayne, 1990)

Effect of σ_{II} on friction



Chapter 11., Mitchell and Soga, 2005

Effect of σ_{II} on friction

Various models fit the experimental data showing the intermediate stress effect. Lade and Duncan (1975)

$$\frac{I_1^3}{I_3} = const$$

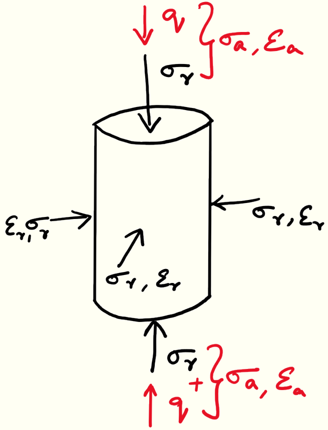
Matsuoka and Nakai (1974)

$$\frac{I_1 I_2}{I_3} = const$$

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\ I_2 &= \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \\ I_3 &= \sigma_1\sigma_2\sigma_3 \end{aligned}$$

Matsuoka and Nakai's model gives the same friction angle for compression and extension, whereas Lade and Duncan's model gives the ratio of the TXE friction angle (ϕ'_{TXE}) to the TXC friction angle (ϕ'_{TXC}) to be 1.08 at $\phi'_{TXC} = 20^o$ to 1.15 at $\phi'_{TXC} = 40^o$.

Triaxial stresses and strains: independent components



Triaxial deviatoric strain

- Deviatoric / shear strain: $\varepsilon_s = 2/3(\varepsilon_a - \varepsilon_r)$
- Why 2/3?:

Triaxial stress - strain relationship

Relationship in terms of axial and radial direction:

$$\begin{bmatrix} \varepsilon_a \\ \varepsilon_r \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -2\nu \\ -\nu & (1-\nu) \end{bmatrix} \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \tag{1}$$

Relationship in terms of 'p' and 'q':

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \quad \text{invert} \ \rightarrow \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \begin{bmatrix} 1 & 2/3 \\ 1 & -1/3 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \tag{2}$$

Strain:

$$\begin{bmatrix} \varepsilon_v \\ \varepsilon_s \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2/3 & -2/3 \end{bmatrix} \begin{bmatrix} \varepsilon_a \\ \varepsilon_r \end{bmatrix} \tag{3}$$

Volume - shear coupling

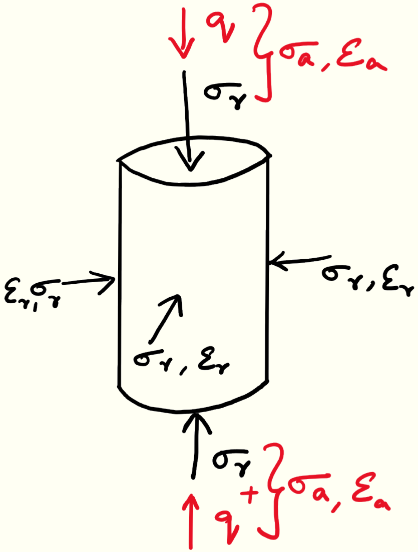
Equation (1) and eq. (2) in eq. (3) to relate $\varepsilon_v, \varepsilon_s$ to p, q:

$$\begin{bmatrix} \varepsilon_v \\ \varepsilon_s \end{bmatrix} = \begin{bmatrix} 3(1-2\nu)/E & 0 \\ 0 & 2(1+\nu)/3E \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}$$

Volume shear coupling

Usually volume change and shear deformation are decoupled (i.e. bulk modulus and shear modulus), but in soils they are coupled:

Triaxial test



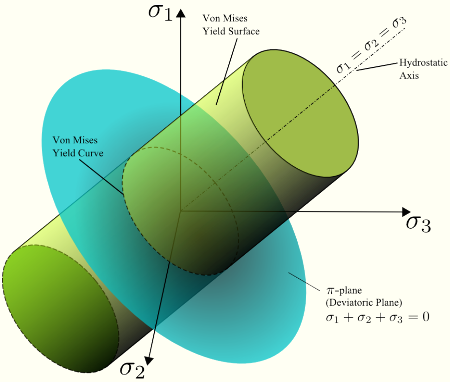
Mohr circle to 3D stress components

Successive Mohr circles and stress path for constant σ_3 and increasing σ_1:

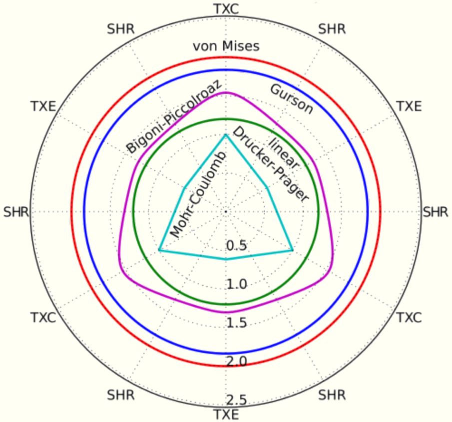
Stress paths p-q

Different stress paths for initially hydrostatic stress conditions:

Pi-plane

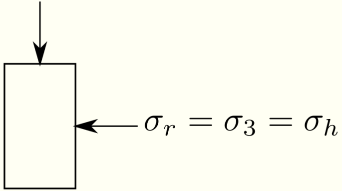


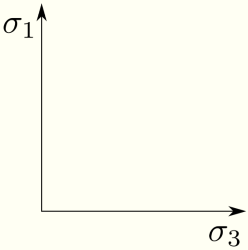
σ_1
Von Mises
Yield Surface
$\sigma_1 = \sigma_2 = \sigma_3$
Hydrostatic
Axis
Von Mises
Yield Curve
σ_3
π-plane
(Deviatoric Plane)
$\sigma_1 + \sigma_2 + \sigma_3 = 0$
σ_2

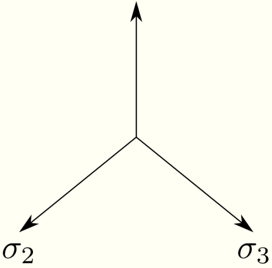


TXC
SHR
SHR
von Mises
Gurson
Bigoni-Piccolroaz
TXE
TXE
linear
Drucker-Prager
Mohr-Coulomb
SHR
SHR
0.5
1.0
TXC
TXC
1.5
2.0
SHR
SHR
2.5
TXE

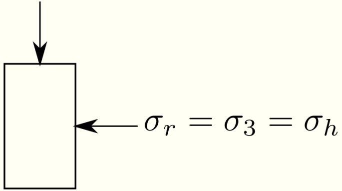
π plane: Triaxial compression

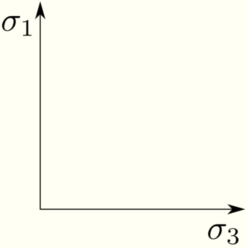


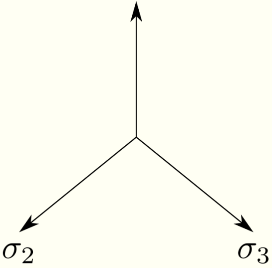




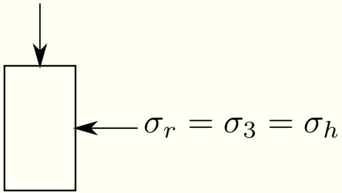
π plane: Triaxial extension

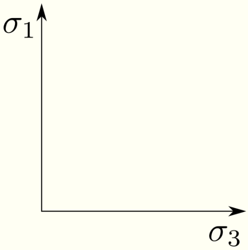


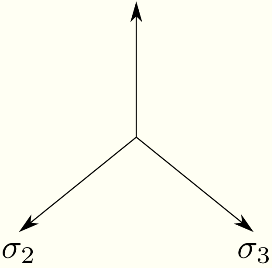




π plane: Random stress paths







Stress and strain invariants in 3D

6 stresses and strains

- Mean pressure:
- Deviator stress: $q = \sqrt{3/2}\sqrt{s_{11}^2 + s_{22}^2 + s_{33}^2 + 2\tau_{12}^2 + 2\tau_{23}^2 + 2\tau_{31}^2}$
- where $s_{11} = \sigma'_{11} - p', s_{22} = \sigma'_{22} - p', s_{33} = \sigma'_{33} - p'$
- Volumetric strain:
- Deviatoric strain:
  $\varepsilon_s = \sqrt{2/3}\sqrt{e_{11}^2 + e_{22}^2 + e_{33}^2 + \gamma_{12}^2/2 + \gamma_{23}^2/2 + \gamma_{31}^2/2}$
- where $e_{11} = \varepsilon_{11} - \varepsilon_v/3, e_{22} = \varepsilon_{22} - \varepsilon_v/3, e_{33} = \varepsilon_{33} - \varepsilon_v/3$

Stress and strain invariants in 3D

3 Principal stresses and strains

- Mean pressure:
- Deviator stress: $q = \sqrt{1/2}\sqrt{(\sigma'_I - \sigma'_{II})^2 + (\sigma'_{II} - \sigma'_{III})^2 + (\sigma'_{III} - \sigma'_I)^2}$
- Volumetric strain:
- Deviatoric strain: $\varepsilon_s = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon'_I - \varepsilon'_{II})^2 + (\varepsilon'_{II} - \varepsilon'_{III})^2 + (\varepsilon'_{III} - \varepsilon'_I)^2}$

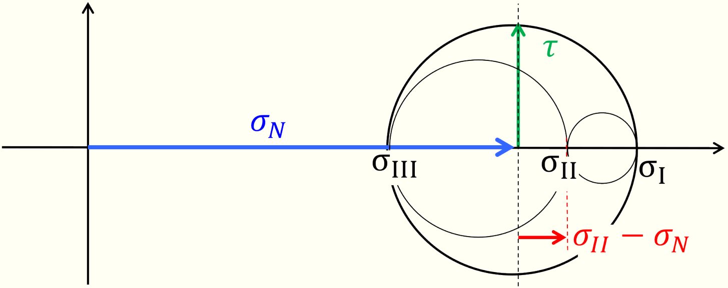
In triaxial condition (principal stresses/strains)

The missing stress invariant

- **General stresses:**

- **Principal stresses:**

The Lode parameter

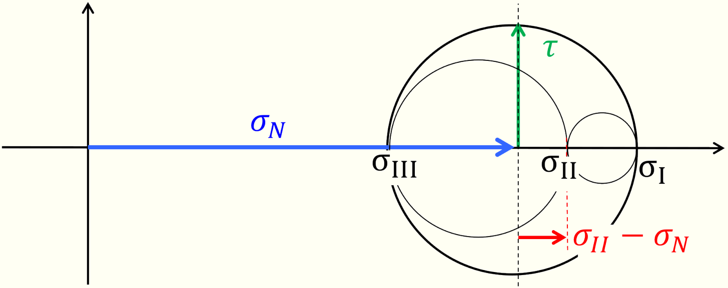


Maximum shear stress:

Normal stress:

Position of intermediate principal stress:

The Lode parameter



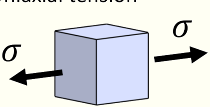
Maximum shear stress:

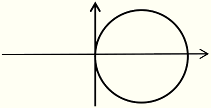
Normal stress:

Position of intermediate principal stress:

Which states have the same Lode parameter?

- Uniaxial tension

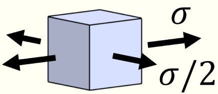


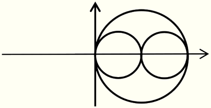


$$\left.\begin{aligned}\sigma_m &= \sigma/3\\ \bar{\sigma} &= \sigma\end{aligned}\right\}$$

$$\sigma_{II} = \sigma_{III} = 0 \quad \Rightarrow L = -1$$

- Plane strain tension

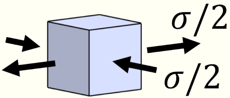


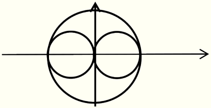


$$\left.\begin{aligned}\sigma_m &= \sigma/2\\ \bar{\sigma} &= \sqrt{3}/2\sigma\end{aligned}\right\}$$

$$\boxed{\sigma_{II} = (\sigma_I + \sigma_{III})/2 \Rightarrow L = 0}$$

- Pure shear





$$\left.\begin{aligned}\sigma_m &= 0\\ \bar{\sigma} &= \sqrt{3}/2\sigma\end{aligned}\right\}$$

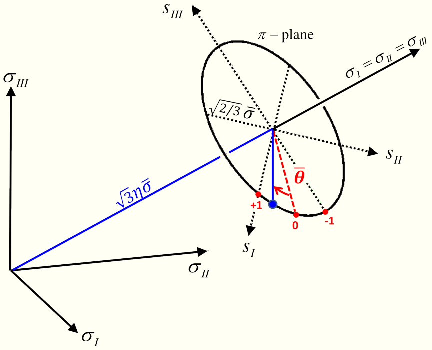
$$\boxed{\sigma_{II} = (\sigma_I + \sigma_{III})/2 \Rightarrow L = 0}$$

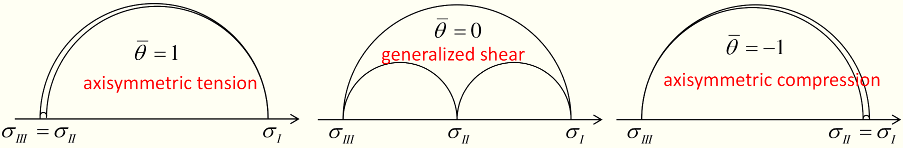
Lode angle parameter

Lode angle parameter (Lode, 1926)

$$L = \frac{2\sigma_{II} - \sigma_I - \sigma_{III}}{\sigma_I - \sigma_{III}}$$

Lode angle $\bar{\theta} \approx -L$

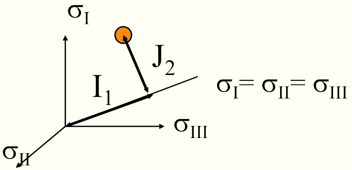




Stress invariant

Principal stresses (3 components) $\Leftrightarrow$ Invariants (3 components)

- $I_1 =$
- $s_{ij} =$
- $J_2 =$
- $J_2 =$
- $q =$



Lode angle:

- $\sin 3\theta = \frac{-3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}}$.
- $\theta = \tan^{-1}\left[\frac{1}{\sqrt{3}}\left(2\frac{(\sigma_2 - \sigma_3)}{(\sigma_1 - \sigma_3)} - 1\right)\right]$
-
-