CE394M: Tresca and Mohr-Coulomb

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Overview

Stress invariants

- The magnitudes of the component of the stress vector depend on the chosen direction of the coordinate axes (in 3D: 6 variables).
- Principal stresses always act on the same planes and have the same magnitude (invariant to the coordinate axes), but still need to define the corresponding orientations (in 3D: 6 variables).
- For isotropic materials, it is very convenient to work with alternative invariant quantities which are combinations of principal stresses.

Stress invariants

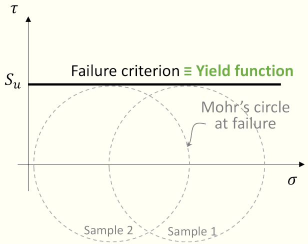
- Mean effective stress $p = \frac{1}{3}(\sigma_I + \sigma_I I + \sigma_I II)$
- Deviatoric stress: $J = \frac{1}{\sqrt{6}}\sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}$
- Lode's angle $\theta = \tan^{-1}\left[\frac{1}{\sqrt{3}}\left(2\frac{(\sigma_{II} - \sigma_{III})}{\sigma_I - \sigma_{III}} - 1\right)\right]$

Principal stresses can be expressed in terms of invariants:

$$\begin{bmatrix}\sigma_I \\ \sigma_{II} \\ \sigma_{III}\end{bmatrix} = p\begin{bmatrix}1 \\ 1 \\ 1\end{bmatrix} + \frac{2}{\sqrt{3}}J\begin{bmatrix}\sin\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta \\ \sin\left(\theta - \frac{2\pi}{3}\right)\end{bmatrix}$$

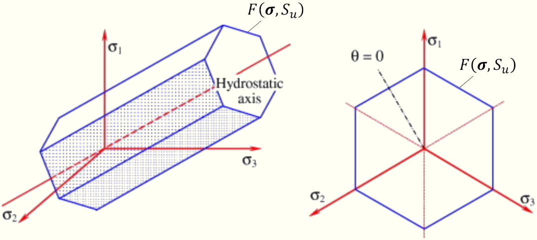
Tresca model

Simulation of undrained behavior of saturated clay
Failure criteria:

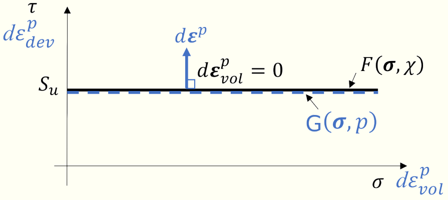


Tresca model

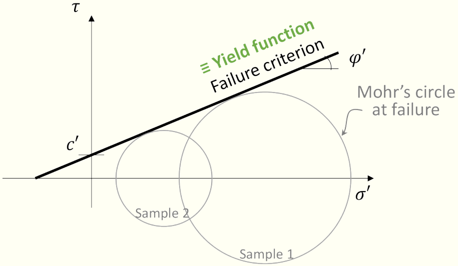
Yield function



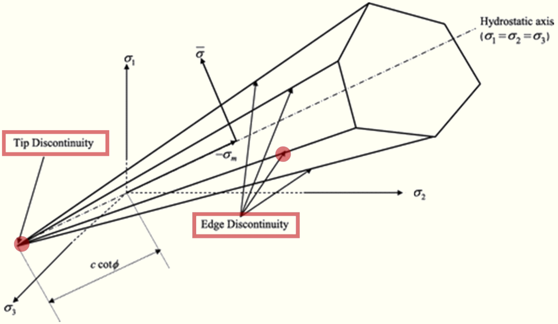
Tresca model



Mohr-Coulomb model

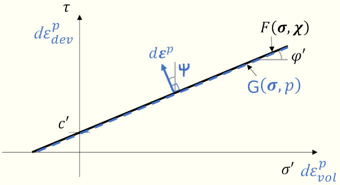


Mohr-Coulomb model



Mohr-Coulomb: flow rule?

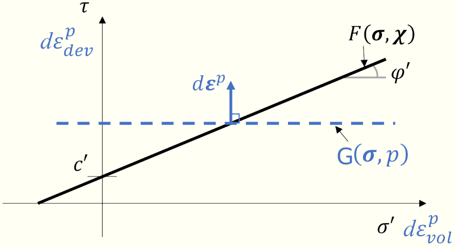
Can we consider an associative flow rule?



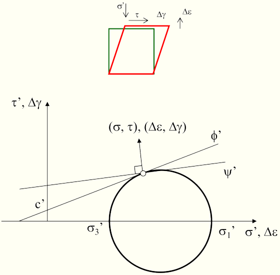
Mohr-Coulomb: Drawbacks (associative)

Solutions:

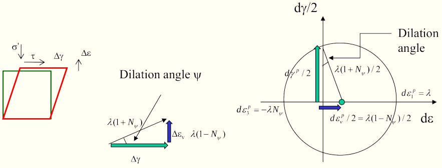
Mohr-Coulomb: non-associative model



Mohr-Coulomb: Yield and Potential fn



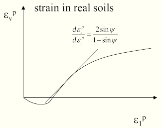
Mohr-Coulomb: plastic strains

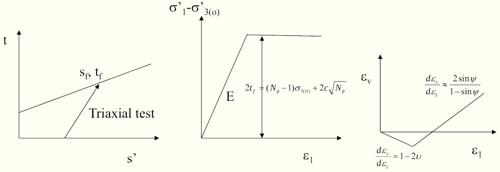


Interpretation of drained TXC

Plastic volumetric strain:

Note that ψ changes with plastic strain in real soils





Idealized M-C curves

Mohr-Coulomb: Tension cut-off