Lecture 3

Zed

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1 General Formulation of Explicit 1-Step Method

We now write 1-step methods in a general form,

$$y_{n+1} = y_n + h\Phi(t_n, y_n, h) \quad (\dagger)$$

Def. **Truncation Error**: We insert true values of the solution into the formulation above, and define the truncation error:

$$T_n = \frac{y(t_{n+1}) - y(t_n)}{h} - \Phi(t_n, y(t_n), h)$$

Def. **Consistency**: The numerical method $(\dagger)$ is *consistent* with $y' = f(t, y)$ if the truncation error goes to 0 as $h \to 0$, i.e. $\lim_{h\to 0} T_n = 0$.

Cor. Let $\Phi(t_n, y(t_n), \cdot)$ continuous on h, then the definition of consistency is equivalent to

$$\lim_{h\to 0} \frac{y(t_n + h) - y(t_n)}{h} = \lim_{h\to 0} \Phi(t_n, y(t_n), h)$$

i.e. $y'(t_n) = \Phi(t_n, y(t_n), 0)$. Hence we have consistency $\iff$

$$y'(t_n) = \Phi(t_n, y(t_n), 0) \iff f(t_n, y(t_n)) = \Phi(t_n, y(t_n), 0)$$