To David S. Moore,
with enduring affection, admiration, and thanks:

Thank you, David, for all that your leadership has done for our profession, and thank you also for all that your friendship, support, and guidance have done for each of us personally.

CHAPTER 0

What Is a Statistical Model?

The unifying theme of this book is the use of models in statistical data analysis. Statistical models are useful for answering all kinds of questions. For example:

- Can we use the number of miles that a used car has been driven to predict the price that is being asked for the car? How much less can we expect to pay for each additional 1000 miles that the car has been driven? Would it be better to base our price predictions on the age of the car in years, rather than its mileage? Is it helpful to consider both age and mileage, or do we learn roughly as much about price by considering only one of these? Would the impact of mileage on the predicted price be different for a Honda as opposed to a Porsche?
- Do babies begin to walk at an earlier age if they engage in a regimen of special exercises? Or does any kind of exercise suffice? Or does exercise have no connection to when a baby begins to walk?
- If we find a footprint and a handprint at the scene of a crime, are they helpful for predicting the height of the person who left them? How about for predicting whether the person is male or female?
- Can we distinguish among different species of hawks based solely on the lengths of their tails?
- Do students with a higher grade point average really have a better chance of being accepted to medical school? How much better? How well can we predict whether or not an applicant is accepted based on his or her GPA? Is there a difference between male and female students' chances for admission? If so, does one sex retain its advantage even after GPA is accounted for?
- Can a handheld device that sends a magnetic pulse into the head reduce pain for migraine sufferers?
- When people serve ice cream to themselves, do they take more if they are using a bigger bowl? What if they are using a bigger spoon?
- Which is more strongly related to the average score for professional golfers: driving distance, driving accuracy, putting performance, or iron play? Are all of these useful for predicting a

golfer's average score? Which are most useful? How much of the variability in golfers' scores can be explained by knowing all of these other values?

These questions reveal several purposes of statistical modeling:

a. **Making predictions.** Examples include predicting the price of a car based on its age, mileage, and model; predicting the length of a hawk's tail based on its species; predicting the probability of acceptance to medical school based on grade point average.

b. **Understanding relationships.** For example, after taking mileage into account, how is the age of a car related to its price? How does the relationship between foot length and height differ between men and women? How are the various measures of a golfer's performance related to each other and to the golfer's scoring average?

c. **Assessing differences.** For example, is the difference in ages of first walking different enough between an exercise group and a control group to conclude that exercise really does affect age of first walking? Is the rate of headache relief for migraine sufferers who experience a magnetic pulse sufficiently higher than those in the control group to advocate for the magnetic pulse as an effective treatment?

As with all models, statistical models are simplifications of reality. George Box, a renowned statistician, famously said that "all statistical models are wrong, but some are useful." Statistical models are not deterministic, meaning that their predictions are not expected to be perfectly accurate. For example, we do not expect to predict the exact price of a used car based on its mileage. Even if we were to record every imaginable characteristic of the car and include them all in the model, we would still not be able to predict its price exactly. And we certainly do not expect to predict the exact moment that a baby first walks based on the kind of exercise he or she engaged in. Statistical models merely aim to explain as much of the variability as possible in whatever phenomenon is being modeled. In fact, because human beings are notoriously variable and unpredictable, social scientists who develop statistical models are often delighted if the model explains even a small part of the variability.

A distinguishing feature of statistical models is that we pay close attention to possible simplifications and imperfections, seeking to quantify how much the model explains and how much it does not. So, while we do not expect our model's predictions to be exactly correct, we are able to state how confident we are that our predictions fall within a certain range of the truth. And while we do not expect to determine the exact relationship between two variables, we can quantify how far off our model is likely to be. And while we do not expect to assess exactly how much two groups may differ, we can draw conclusions about how likely they are to differ and by what magnitude.

More formally, a statistical model can be written as

$$DATA = MODEL + ERROR$$

or as

$$Y = f(X) + \epsilon$$

The Y here represents the variable being modeled, X is the variable used to do the modeling, and f is a function.[1] We start in Chapter 1 with just one quantitative, explanatory variable X and with a linear function f. Then we will consider more complicated functions for f, often by transforming Y or X or both. Later, we will consider multiple explanatory variables, which can be either quantitative or categorical. In these initial models we assume that the response variable Y is quantitative. Eventually, we will allow the response variable Y to be categorical.

The ϵ term in the model above is called the "error," meaning the part of the response variable Y that remains unexplained after considering the predictor X. Our models will sometimes stipulate a probability distribution for this ϵ term, often a normal distribution. An important aspect of our modeling process will be checking whether the stipulated probability distribution for the error term seems reasonable, based on the data, and making appropriate adjustments to the model if it does not.

0.1 Fundamental Terminology

Before you begin to study statistical modeling, you will find it very helpful to review and practice applying some fundamental terminology.

The **observational units** in a study are the people, objects, or cases on which data are recorded. The **variables** are the characteristics that are measured or recorded about each observational unit.

Example 0.1: ***Car prices***

In the study about predicting the price of a used car, the observational units are the cars. The variables are the car's price, mileage, age (in years), and manufacturer (Porsche or Honda). ◇

Example 0.2: ***Walking babies***

In the study about babies walking, the observational units are the babies. The variables are whether or not the baby was put on an exercise regimen and the age at which the baby first walked. ◇

[1] The term "model" is used to refer to the entire equation or just the structural part that we have denoted by $f(X)$.

	C1-T	C2	C3	C4	C5	C6
	City	NumMDs	RateMDs	NumHospitals	NumBeds	RateBeds
1	Holland-Grand Haven, MI	349	140	3	316	127
2	Louisville, KY-IN	4042	340	18	3909	328
3	Battle Creek, MI	256	184	3	517	372
4	Madison, WI	2679	510	7	1467	279
5	Fort Smith, AR-OK	502	179	8	975	348
6	Sarasota-Bradenton-Venice, FL	2352	371	7	1899	299
7	Anderson, IN	200	153	2	231	176
8	Honolulu, HI	3478	389	13	2160	242
9	Asheville, NC	1489	389	5	1213	317
10	Winston-Salem, NC	2018	462	6	1901	435

Figure 0.1: *Health facilities in U.S. metropolitan areas*

Example 0.3: ***Metropolitan health care***

You may find it helpful to envision the data in a spreadsheet format. The row labels are cities, which are observational units, and the columns correspond to the variables. For example, Figure 0.1 shows part of a Minitab worksheet with data compiled by the U.S. Census Bureau on health-care facilities in metropolitan areas. The observational units are the metropolitan areas and the variables count the number of doctors, hospitals, and beds in each city as well as rates (number of doctors or beds per 100,000 residents). The full dataset for 83 metropolitan areas is in the file **MetroHealth83**. ◇

Variables can be classified into two types: quantitative and categorical. A **quantitative** variable records numbers about the observational units. It must be sensible to perform ordinary arithmetic operations on these numbers, so zip codes and jersey numbers are not quantitative variables. A **categorical** variable records a category designation about the observational units. If there are only two possible categories, the variable is also said to be **binary**.

Example 0.1 (continued): The price, mileage, and age of a car are all *quantitative* variables. The model of the car is a *categorical* variable. ◇

Example 0.2 (continued): Whether or not a baby was assigned to a special exercise regimen is a *categorical* variable. The age at which the baby first walked is a *quantitative* variable. ◇

Example 0.4: ***Medical school admission***

Whether or not an applicant is accepted for medical school is a *binary* variable, as is the gender of the applicant. The applicant's undergraduate grade point average is a *quantitative* variable. ◇

Another important consideration is the role played by each variable in the study. The variable that measures the outcome of interest is called the **response** variable. The variables whose relationship to the response is being studied are called **explanatory** variables. (When the primary goal of the model is to make predictions, the explanatory variables are also called **predictor** variables.)

Example 0.1 (continued): The price of the car is the *response* variable. The mileage, age, and model of the car are all *explanatory* variables. ◇

Example 0.2 (continued): The age at which the baby first walked is the *response* variable. Whether or not a baby was assigned to a special exercise regimen is an *explanatory* variable. ◇

Example 0.4 (continued): Whether or not an applicant is accepted for medical school is the *response* variable. The applicant's undergraduate grade point average and sex are *explanatory* variables. ◇

One reason that these classifications are important is that the choice of the appropriate analysis procedure depends on the type of variables in the study and their roles. Regression analysis (covered in Chapters 1–4) is appropriate when the response variable is quantitative and the explanatory variables are also quantitative. In Chapter 3, you will also learn how to incorporate binary explanatory variables into a regression analysis. Analysis of variance (ANOVA, considered in Chapters 5–8) is appropriate when the response variable is quantitative, but the explanatory variables are categorical. When the response variable is categorical, logistic regression (considered in Chapters 9–11) can be used with either quantitative or categorical explanatory variables. These various scenarios are displayed in Table 0.1.

Keep in mind that variables are not always clear-cut to measure or even classify. For example, measuring headache relief is not a straightforward proposition and could be done with a quantitative measurement (intensity of pain on a 0–10 scale), a categorical scale (much relief, some relief, no relief), or as a binary categorical variable (relief or not).

We collect data and fit models in order to understand **populations**, such as all students who are

Response	Predictor/explanatory	Procedure	Chapter
Quantitative	Single quantitative	Simple linear regression	1, 2
Quantitative	Single categorical	One-way analysis of variance	5
Categorical	Single quantitative	Simple logistic regression	9
Categorical	Single binary	2×2 table	11
Quantitative	Multiple quantitative	Multiple linear regression	3, 4
Quantitative	Multiple categorical	Multiway analysis of variance	6, 7
Categorical	Multiple quantitative	Multiple logistic regression	10, 11
Categorical	Multiple categories	$2 \times k$ table	11

Table 0.1: *Classifying general types of models*

applying to medical school, and **parameters**, such as the acceptance rate of all students with a grade point average of 3.5. The collected data are a **sample** and a characteristic of a sample, such as the percentage of students with grade point averages of 3.5 who were admitted to medical school, out of those who applied, is a **statistic**. Thus, sample statistics are used to estimate population parameters.

Another crucial distinction is whether a research study is a controlled experiment or an observational study. In a **controlled experiment**, the researcher manipulates the explanatory variable by assigning the explanatory group or value to the observational units. (These observational units may be called **experimental units** or **subjects** in an experiment.) In an **observational study**, the researchers do not assign the explanatory variable but rather passively observe and record its information. This distinction is important because the type of study determines the scope of conclusion that can be drawn. Controlled experiments allow for drawing *cause-and-effect* conclusions. Observational studies, on the other hand, only allow for concluding that variables are *associated*. Ideally, an observational study will anticipate alternative explanations for an association and include the additional relevant variables in the model. These additional explanatory variables are then called **covariates**.

Example 0.5: ***Handwriting and SAT essay scores***

An article about handwriting appeared in the October 11, 2006, issue of *The Washington Post*. The article mentioned that among students who took the essay portion of the SAT exam in 2005–2006, those who wrote in cursive style scored significantly higher on the essay, on average, than students who used printed block letters. This is an example of an observational study since there was no controlled assignment of the type of writing for each essay. While it shows an association between handwriting and essay scores, we can't tell whether better writers tend to choose to write in cursive or if graders tend to score cursive essays more generously and printed ones more harshly. We might also suspect that students with higher GPAs are more likely to use cursive writing. To examine this carefully, we could fit a model with GPA as a covariate.

The article also mentioned a different study in which the identical essay was shown to many graders, but some graders were shown a cursive version of the essay and the other graders were shown a version with printed block letters. Again, the average score assigned to the essay with the cursive style was significantly higher than the average score assigned to the essay with the printed block letters. This second study involved an experiment since the binary explanatory factor of interest (cursive versus block letters) was controlled by the researchers. In that case, we can infer that using cursive writing produces better essay scores, on average, than printing block letters. ◇

0.2 Four-Step Process

We will employ a four-step process for statistical modeling throughout this book. These steps are:

- **Choose** a form for the model. This involves identifying the response and explanatory variable(s) and their types. We usually examine graphical displays to help suggest a model that might summarize relationships between these variables.
- **Fit** that model to the data. This usually entails estimating model parameters based on the sample data. We will almost always use statistical software to do the necessary number-crunching to fit models to data.
- **Assess** how well the model describes the data. One component of this involves comparing the model to other models. Are there elements of the model that are not very helpful in explaining the relationships or do we need to consider a more complicated model? Another component of the assessment step concerns analyzing residuals, which are deviations between the actual data and the model's predictions, to assess how well the model fits the data. This process of assessing model adequacy is as much art as science.
- **Use** the model to address the question that motivated collecting the data in the first place. This might be to make predictions, or explain relationships, or assess differences, bearing in mind possible limitations on the scope of inferences that can be made. For example, if the data were collected as a random sample from a population, then inference can be extended to that population; if treatments were assigned at random to subjects, then a cause-and-effect relationship can be inferred; but if the data arose in other ways, then we have little statistical basis for drawing such conclusions.

The specific details for how to carry out these steps will differ depending on the type of analysis being performed and, to some extent, on the context of the data being analyzed. But these four steps are carried out in some form in all statistical modeling endeavors. To illustrate the process, we consider an example in the familiar setting of a two-sample t-procedure.

Example 0.6: ***Financial incentives for weight loss***

Losing weight is an important goal for many individuals. An article[2] in the *Journal of the American Medical Association* describes a study in which researchers investigated whether financial incentives would help people lose weight more successfully. Some participants in the study were randomly assigned to a treatment group that offered financial incentives for achieving weight loss goals, while others were assigned to a control group that did not use financial incentives. All participants were monitored over a four-month period and the net weight change ($Before - After$ in pounds) was recorded for each individual. Note that a positive value corresponds to a weight loss and a negative change is a weight gain. The data are given in Table 0.2 and stored in **WeightLossIncentive4**.

[2] K. Volpp, L. John, A.B. Troxel, L. Norton, J. Fassbender, and G. Lowenstein (2008), "Financial Incentive-based Approaches for Weight Loss: A Randomized Trial", *JAMA*, 300(22): 2631–2637.

Control	12.5	12.0	1.0	−5.0	3.0	−5.0	7.5	−2.5	20.0	−1.0
	2.0	4.5	−2.0	−17.0	19.0	−2.0	12.0	10.5	5.0	
Incentive	25.5	24.0	8.0	15.5	21.0	4.5	30.0	7.5	10.0	18.0
	5.0	−0.5	27.0	6.0	25.5	21.0	18.5			

Table 0.2: *Weight loss after four months (pounds)*

The response variable in this situation (weight change) is quantitative and the explanatory factor of interest (control versus incentive) is categorical and binary. The subjects were assigned to the groups at random so this is a statistical experiment. Thus, we may investigate whether there is a statistically significant difference in the distribution of weight changes due to the use of a financial incentive.

CHOOSE

When choosing a model, we generally consider the question of interest and types of variables involved, then look at graphical displays, and compute summary statistics for the data. Since the weight loss incentive study has a binary explanatory factor and quantitative response, we examine dotplots of the weight losses for each of the two groups (Figure 0.2) and find the sample mean and standard deviation for each group.

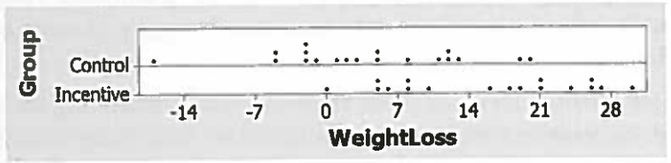


Figure 0.2: *Weight loss for* Control *versus* Incentive *groups*

```
Variable    Group       N   Mean  StDev
WeightLoss  Control    19   3.92   9.11
            Incentive  17  15.68   9.41
```

The dotplots show a pair of reasonably symmetric distributions with roughly the same variability, although the mean weight loss for the incentive group is larger than the mean for the control group. One model for these data would be for the weight losses to come from a pair of normal distributions, with different means (and perhaps different standard deviations) for the two groups. Let the parameter μ_1 denote the mean weight loss after four months without a financial incentive, and let μ_2 be the mean with the incentive. If σ_1 and σ_2 are the respective standard deviations and we let the variable Y denote the weight losses, we can summarize the model as $Y \sim N(\mu_i, \sigma_i)$,

where the subscript indicates the group membership[3] and the symbol $\sim$ signifies that the variable has a particular distribution. To see this in the $DATA = MODEL + ERROR$ format, this model could also be written as

$$Y = \mu_i + \epsilon$$

where μ_i is the population mean for the i^{th} group and $\epsilon \sim N(0, \sigma_i)$ is the random error term. Since we only have two groups, this model says that

$$\begin{aligned} Y = \ & \mu_1 + \epsilon \sim N(\mu_1, \sigma_1) && \text{for individuals in the control group} \\ Y = \ & \mu_2 + \epsilon \sim N(\mu_2, \sigma_2) && \text{for individuals in the incentive group} \end{aligned}$$

FIT

To fit this model, we need to estimate four parameters (the means and standard deviations for each of the two groups) using the data from the experiment. The observed means and standard deviations from the two samples provide obvious estimates. We let $\overline{y}_1 = 3.92$ estimate the mean weight loss for the control group and $\overline{y}_2 = 15.68$ estimate the mean for a population getting the incentive. Similarly, $s_1 = 9.11$ and $s_2 = 9.41$ estimate the respective standard deviations. The fitted model (a prediction for the typical weight loss in either group) can then be expressed as [4]

$$\hat{y} = \overline{y}_i$$

that is, that $\hat{y} = 3.92$ pounds for individuals without the incentive and $\hat{y} = 15.68$ pounds for those with the incentive.

Note that the error term does not appear in the fitted model since, when predicting a particular weight loss, we don't know whether the random error will be positive or negative. That does not mean that we expect there to be no error, just that the best guess for the *average* weight loss under either condition is the sample group mean, $\overline{y}_i$.

ASSESS

Our model indicates that departures from the mean in each group (the random errors) should follow a normal distribution with mean zero. To check this, we examine the sample *residuals* or deviations between what is predicted by the model and the actual data weight losses:

$$residual = \text{observed} - \text{predicted} = y - \hat{y}$$

For subjects in the control group, we subtract $\hat{y} = 3.92$ from each weight loss and we subtract $\hat{y} = 15.68$ for the incentive group. Dotplots of the residuals for each group are shown in Figure 0.3.

[3] For this example, an assumption that the variances are equal, $\sigma_1^2 = \sigma_2^2$, might be reasonable, but that would lead to the less familiar pooled variance version of the t-test. We explore this situation in more detail in a later chapter.

[4] We use the carat $\hat{}$ symbol above a variable name to indicate predicted value, and refer to this as y-hat.

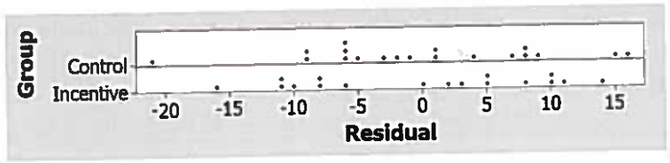
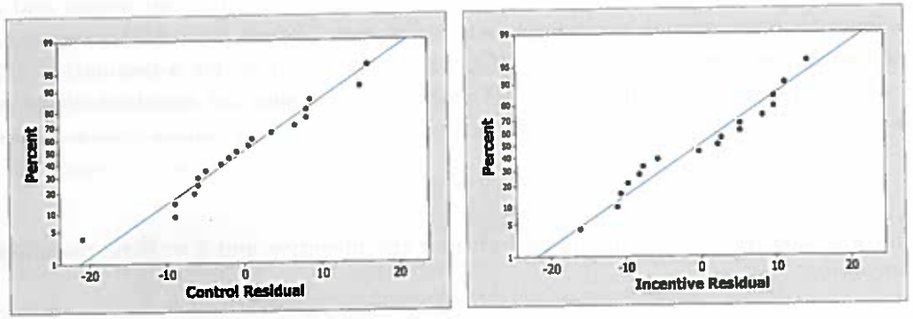


Figure 0.3: *Residuals from group weight loss means*

Note that the distributions of the residuals are the same as the original data, except that both are shifted to have a mean of zero. We don't see any significant departures from normality in the dotplots, but it's difficult to judge normality from dotplots with so few points. Normal probability plots (as shown in Figure 0.4) are a more informative technique for assessing normality. Departures from a linear trend in such plots indicate a lack of normality in the data. Normal probability plots will be examined in more detail in the next chapter.



(a) Control (b) Incentive

Figure 0.4: *Normality probability plots for residuals of weight loss*

As a second component of assessment, we consider whether an alternate (simpler) model might fit the data essentially as well as our model with different means for each group. This is analogous to testing the standard hypotheses for a two sample t-test:

$H_0 : \mu_1 = \mu_2$
$H_a : \mu_1 \neq \mu_2$

The null hypothesis (H_0) corresponds to the simpler model $Y = \mu + \epsilon$, which uses the same mean for both the control and incentive groups. The alternative (H_a) reflects the model we have considered here that allows each group to have a different mean. Would the simpler (common mean) model suffice for the weight loss data or do the two separate groups means provide a *significantly* better explanation for the data? One way to judge this is with the usual two-sample t-test (as shown in the computer output below).


```
Two-sample T for WeightLoss

Group       N   Mean  StDev  SE Mean
Control    19   3.92   9.11      2.1
Incentive  17  15.68   9.41      2.3

Difference = mu (Control) - mu (Incentive)
Estimate for difference:  -11.76
95% CI for difference:  (-18.05, -5.46)
T-Test of difference = 0 (vs not =): T-Value = -3.80  P-Value = 0.001  DF = 33
```

The extreme value for this test statistic ($t = -3.80$) and very small p-value (0.001) provide strong evidence that the means of the two groups are indeed significantly different. If the two group means were really the same (i.e., the common mean model was accurate and the financial incentives had no effect on weight loss), we would expect to see a difference as large as was observed in this experiment for only about 1 in 1000 replications of the experiment. Thus, the model with separate means for each group does a substantially better job at explaining the results of the weight loss study.

USE

Since this was a designed experiment with random allocation of the control and incentive conditions to the subjects, we can infer that the financial incentives did produce a difference in the average weight loss over the four-month period; that is, the random allocation of conditions to subjects allows us to draw a cause-and-effect relationship. A person who is on the incentive-based treatment can be expected to lose about 11.8 pounds more ($15.68 - 3.92 = 11.76$), on average, in four months, than control subjects who are not given this treatment. Note that for most individuals, approximately 12 pounds is a substantial amount of weight to lose in four months. Moreover, if we interpret the confidence interval from the Minitab output, we can be 95% confident that the incentive treatment is worth between 5.5 and 18.1 pounds of additional weight loss, on average, over four months.

Before leaving this example, we note three cautions. First, all but two of the participants in this study were adult men, so we should avoid drawing conclusions about the effect of financial incentives on weight loss in women. Second, if the participants in the study did not arise from taking a random sample, we would have difficulty justifying a statistical basis for generalizing the findings to other adults. Any such generalization must be justified on other grounds (such as a belief that most adults respond to financial incentives in similar ways). Third, the experimenters followed up with subjects to see if weight losses were maintained at a point seven months after the start of the study (and three months after any incentives expired). The results from the followup study appear in Exercise 0.14. ◇

0.3 Chapter Summary

In this chapter, we reviewed basic terminology, introduced the four-step approach to modeling that will be used throughout the text, and revisited a common two-sample inference problem.

After completing this chapter, you should be able to distinguish between a **sample** and a **population**, describe the difference between a **parameter** and a **statistic**, and identify variables as **categorical** or **quantitative**. Prediction is a major component to modeling so identifying **explanatory** (or predictor) **variables** that can be used to develop a model for the **response variable** is an important skill. Another important idea is the distinction between **observational studies** (where researchers simply observe what is happening) and **experiments** (where researchers impose "treatments").

The fundamental idea that a **statistical model** partitions data into two components, one for the model and one for error, was introduced. Even though the models will get more complex as we move through the more advanced settings, this statistical modeling idea will be a major theme throughout the text. The error term and conditions associated with this term are important features in distinguishing statistical models from mathematical models. You saw how to compute **residuals** by comparing the observed data to predictions from a model as a way to begin quantifying the errors.

The **four-step process** of choosing, fitting, assessing, and using a model is vital. Each step in the process requires careful thought and the computations will often be the easiest part of the entire process. Identifying the response and explanatory variable(s) and their types (categorical or quantitative) helps us **choose** the appropriate model(s). Statistical software will almost always be used to **fit** models and obtain estimates. Comparing models and **assessing** the adequacy of these models will require a considerable amount of practice and this is a skill that you will develop over time. Try to remember that **using** the model to make predictions, explain relationships, or assess differences is only one part of the four-step process.

0.4 Exercises

Conceptual Exercises

0.1 ***Categorical or quantitative?*** Suppose that a statistics professor records the following for each student enrolled in her class:

- Gender
- Major
- Score on first exam
- Number of quizzes taken (a measure of class attendance)
- Time spent sleeping the previous night
- Handedness (left- or right-handed)
- Political inclination (liberal, moderate, or conservative)
- Time spent on the final exam
- Score on the final exam

For the following questions, identify the response variable and the explanatory variable(s). Also classify each variable as quantitative or categorical. For categorical variables, also indicate whether the variable is binary.

a. Do the various majors differ with regard to average sleeping time?

b. Is a student's score on the first exam useful for predicting his or her score on the final exam?

c. Do male and female students differ with regard to the average time they spend on the final exam?

d. Can we tell much about a student's handedness by knowing his or her major, gender, and time spent on the final exam?

0.2 ***More categorical or quantitative?*** Refer to the data described in Exercise 0.1 that a statistics professor records for her students. For the following questions, identify the response variable and the explanatory variable(s). Also, classify each variable as quantitative or categorical. For categorical variables, also indicate whether the variable is binary.

a. Do the proportions of left-handers differ between males and females on campus?

b. Are sleeping time, exam 1 score, and number of quizzes taken useful for predicting time spent on the final exam?

c. Does knowing a student's gender help to predict his or her major?

d. Does knowing a student's political inclination and time spent sleeping help to predict his or her gender?

0.3 ***Sports projects.*** For each of the following sports-related projects, identify observational units and the response and explanatory variables when appropriate. Also, classify the variables as quantitative or categorical.

a. Interested in predicting how long it takes to play a Major League Baseball game, an individual recorded the following information for all 15 games played on August 26, 2008: time to complete the game, total number of runs scored, margin of victory, total number of pitchers used, ballpark attendance at the game, and which league (National or American) the teams were in.

b. Over the course of several years, a golfer kept track of the length of all of his putts and whether or not he made the putt. He was interested in predicting whether or not he would make a putt based on how long it was.

c. Some students recorded lots of information about all of the football games played by LaDainian Tomlinson during the 2006 season. They recorded his rushing yardage, number of rushes, rushing touchdowns, receiving yardage, number of receptions, and receiving touchdowns.

0.4 ***More sports projects.*** For each of the following sports-related projects, identify observational units and the response and explanatory variables when appropriate. Also, classify the variables as quantitative or categorical.

a. A volleyball coach wants to see if a player using a jump serve is more likely to lead to winning a point than using a standard overhand serve.

b. To investigate whether the "home-field advantage" differs across major team sports, researchers kept track of how often the home team won a game for all games played in the 2007 and 2008 seasons in Major League Baseball, National Football League, National Basketball Association, and National Hockey League.

c. A student compared men and women professional golfers on how far they drive a golf ball (on average) and the percentage of their drives that hit the fairway.

0.5 ***Scooping ice cream.*** In a study reported in the *Journal of Preventative Medicine*, 85 nutrition experts were asked to scoop themselves as much ice cream as they wanted. Some of them were randomly given a large bowl (34 ounces) as they entered the line, and the others were given a smaller bowl (17 ounces). Similarly, some were randomly given a large spoon (3 ounces) and the others were given a small spoon (2 ounces). Researchers then recorded how much ice cream each subject scooped for him- or herself. Their conjecture was that those given a larger bowl would tend to scoop more ice cream, as would those given a larger spoon.

a. Identify the observational units in this study.

b. Is this an observational study or a controlled experiment? Explain how you know.

c. Identify the response variable in this study, and classify it as quantitative or categorical.

d. Identify the explanatory variable(s) in this study, and classify it(them) as quantitative or categorical.

0.6 ***Wine model.*** In his book *SuperCrunchers: Why Thinking by Numbers Is the New Way to Be Smart*, Ian Ayres writes about Orley Ashenfelter, who has gained fame and generated considerable controversy by using statistical models to predict the quality of wine. Ashenfelter developed a model based on decades of data from France's Bordeaux region, which Ayres reports as

$$\text{WineQuality} = 12.145 + 0.00117\text{WinterRain} + 0.0614\text{AverageTemp} - 0.00386\text{HarvestRain} + \epsilon$$

where *WineQuality* is a function of the price, rainfall is measured in millimeters, and temperature is measured in degrees Celsius.

a. Identify the response variable in this model. Is it quantitative or categorical?

b. Identify the explanatory variables in this model. Are they quantitative or categorical?

c. According to this model, is higher wine quality associated with more or with less winter rainfall?

d. According to this model, is higher wine quality associated with more or with less harvest rainfall?

e. According to this model, is higher wine quality associated with more or with less average growing season temperature?

f. Are the data that Ashenfelter analyzed observational or experimental? Explain.

0.7 ***Measuring students.*** The registrar at a small liberal arts college computes descriptive summaries for all members of the entering class on a regular basis. For example, the mean and standard deviation of the high school GPAs for all entering students in a particular year were 3.16 and 0.5247, respectively. The Mathematics Department is interested in helping all students who want to take mathematics to identify the appropriate course, so they offer a placement exam. A randomly selected subset of students taking this exam during the past decade had an average score of 71.05 with a standard deviation of 8.96.

a. What is the population of interest to the registrar at this college?

b. Are the descriptive summaries computed by the registrar (3.16 and 0.5247) statistics or parameters? Explain.

c. What is the population of interest to the Mathematics Department?

d. Are the numerical summaries (71.05 and 8.96) statistics or parameters? Explain.

Guided Exercises

0.8 ***Scooping ice cream.*** Refer to Exercise 0.5 on self-serving ice cream. The following table reports the average amounts of ice cream scooped (in ounces) for the various treatments:

	17-ounce bowl	34-ounce bowl
2-ounce spoon	4.38	5.07
3-ounce spoon	5.81	6.58

a. Does it appear that the size of the bowl had an effect on amount scooped? Explain.

b. Does it appear that the size of the spoon had an effect on amount scooped? Explain.

c. Which appears to have more of an effect: size of bowl or size of spoon? Explain.

d. Does it appear that the effect of the bowl size is similar for both spoon sizes, or does it appear that the effect of the bowl size differs substantially for the two spoon sizes? Explain.

0.9 ***Diet plans.*** An article in the *Journal of the American Medical Association* (Dansinger et al., 2005) reported on a study in which 160 subjects were randomly assigned to one of four popular diet plans: Atkins, Ornish, Weight Watchers, and Zone. Among the variables measured were:

- Which diet the subject was assigned to
- Whether or not the subject completed the 12-month study
- The subject's weight loss after 2 months, 6 months, and 12 months (in kilograms, with a negative value indicating weight gain)
- The degree to which the subject adhered to the assigned diet, taken as the average of 12 monthly ratings, each on a 1–10 scale (with 1 indicating complete nonadherence and 10 indicating full adherence)

a. Classify each of these variables as quantitative or categorical.

b. The primary goal of the study was to investigate whether weight loss tends to differ significantly among the four diets. Identify the explanatory and response variables for investigating this question.

c. A secondary goal of the study was to investigate whether weight loss is affected by the adherence level. Identify the explanatory and response variables for investigating this question.

d. Is this an observational study or a controlled experiment? Explain how you know.

e. If the researchers' analysis of the data leads them to conclude that there is a significant difference in weight loss among the four diets, can they legitimately conclude that the difference is because of the diet? Explain why or why not.

f. If the researchers' analysis of the data analysis leads them to conclude that there is a significant association between weight loss and adherence level, can they legitimately conclude that a cause-and-effect association exists between them? Explain why or why not.

0.10 ***Predicting NFL wins.*** Consider the following model for predicting the number of games that a National Football League (NFL) team wins in a season:

$$Wins = 4.6 + 0.5PF - 0.3PA + \epsilon$$

where PF stands for average points a team scores per game over an entire season and PA stands for points allowed per game. Currently, each team plays 16 games in a season.

a. According to this model, how many more wins is a team expected to achieve in a season if they increase their scoring by an average of 3 points per game?

b. According to this model, how many more wins is a team expected to achieve in a season if they decrease their points allowed by an average of 3 points per game?

c. Based on your answers to (a) and (b), does it seem that a team should focus more on improving its offense or improving its defense?

d. The Green Bay Packers had the best regular season record in 2010, winning 15 games and losing only 1. They averaged 35.0 points scored per game, while giving up an average of 22.44 points per game against them. Find the residual for the Green Bay Packers in 2010 using this model.

0.11 ***More predicting NFL wins.*** Refer to the model in Exercise 0.10 for predicting the number of games won in a 16-game NFL season based on the average number of points scored per game (PF) and average number of points allowed per game (PA).

a. Use the model to predict the number of wins for the 2010 New England Patriots, who scored 513 points and allowed 342 points in their 16 games.

b. The Patriots actually won 13 games in 2010. Determine their residual from this model, and interpret what this means.

c. The largest positive residual value from this model for the 2010 season belongs to the Kansas City Chiefs, with a residual value of 2.11 games. The Chiefs actually won seven games. Determine this model's predicted number of wins for the Chiefs.

d. The largest negative residual value from this model for the 2010 season belongs to the Minnesota Vikings, with a residual value of −3.81 games. Interpret what this residual means.

0.12 ***Roller coasters.*** The Roller Coaster Database (rcdb.com) contains lots of information about roller coasters all over the world. The following statistical model for predicting the top speed (in miles per hour) of a coaster was based on more than 100 roller coasters in the United States and data displayed on the database in November 2003:

$$TopSpeed = 54 + 7.6TypeCode + \epsilon$$

where $TypeCode = 1$ for steel roller coasters and $TypeCode = 0$ for wooden roller coasters.

a. What top speed does this model predict for a wooden roller coaster?

b. What top speed does this model predict for a steel roller coaster?

c. Determine the difference in predicted speeds in miles per hour for the two types of coasters. Also identify where this number appears in the model equation, and explain why that makes sense.

0.13 ***More roller coasters.*** Refer to the information about roller coasters in Exercise 0.12. Some other predictor variables available at the database include: age, total length, maximum height, and maximum vertical drop. Suppose that we include all of these predictor variables in a statistical model for predicting the top speed of the coaster.

a. For each of these predictor variables, indicate whether you expect its coefficient to be positive or negative. Explain your reasoning for each variable.

b. Which of these predictor variables do you expect to be the best single variable for predicting a roller coaster's top speed? Explain why you think that.

The following statistical model was produced from these data:

$$Speed = 33.4 + 0.10Height + 0.11Drop + 0.0007Length - 0.023Age - 2.0TypeCode + \epsilon$$

c. Comment on whether the signs of the coefficients are as you expect.

d. What top speed would this model predict for a steel roller coaster that is 10 years old, with a maximum height of 150 feet, maximum vertical drop of 100 feet, and length of 4000 feet?

Open-ended Exercises

0.14 ***Incentive for weight loss.*** The study (Volpp et al., 2008) on financial incentives for weight loss in Example 0.6 on page 7 used a follow-up weight check after seven months to see whether weight losses persisted after the original four months of treatment. The results are given in Table 0.3 and in the variable $Month7Loss$ of the **WeightLossIncentive7** data file. Note that a few participants dropped out and were not re-weighed at the seven-month point. As with the earlier example, the data are the change in weight (in pounds) from the beginning of the study and positive values correspond to weight losses. Using Example 0.6 as an outline, follow the four-step process to see whether the data provide evidence that the beneficial effects of the financial incentives still apply to the weight losses at the seven-month point.

Control	−2.0	7.0	19.5	−0.5	−1.5	−10.0	0.5	5.0	8.5	$\overline{y}_1 = 4.64$
	18.0	16.0	−9.0	4.5	23.5	5.5	6.5	−9.5	1.5	$s_1 = 9.84$
Incentive	11.5	20.0	−22.0	2.0	7.5	16.5	19.0	18.0	−1.0	$\overline{y}_2 = 7.80$
	5.5	24.5	9.5	10.0	−8.5	4.5				$s_2 = 12.06$

Table 0.3: *Weight loss after seven months (pounds)*

0.15 ***Statistics students survey.*** An instructor at a small liberal arts college distributed the data collection card similar to what is shown below on the first day of class. The data for two different sections of the course are shown in the file **Day1Survey**. Note that the names have not been entered into the dataset.

```
Data Collection Card

Directions: Please answer each question and return to me.
1.  Your name (as you prefer): _____________________________________________
2.  What is your current class standing? ____________________________________
3.  Sex:  Male _____    Female _____
4.  How many miles (approximately) did you travel to get to campus? ___________
5.  Height (estimated) in inches: __________________________________________
6.  Handedness (Left, Right, Ambidextrous): ________________________________
7.  How much money, in coins (not bills), do you have with you? $_____________
8.  Estimate the length of the white string (in inches): ____________________
9.  Estimate the length of the black string (in inches): ____________________
10. How much do you expect to read this semester (in pages/week)? ____________
11. How many hours do you watch TV in a typical week? ______________________
12. What is your resting pulse? _____________________________________________
13. How many text messages have you sent and received in the last 24 hours? _____
```

The data for this survey are stored in **Day1Survey**.

a. Apply the four-step process to the survey data to address the question: "Is there evidence that the mean resting pulse rate for women is different from the mean resting pulse rate for men?"

b. Pick another question that interests you from the survey and compare the responses of men and women.

0.16 ***Statistics student survey (continued).*** Refer to the survey of statistics students described in Exercise 0.15 with data in **Day1Survey**. Use the survey data to address the question: "Do women expect to do more reading than men?"

0.17 ***Marathon training.*** Training records for a marathon runner are provided in the file **Marathon**. The *Date*, *Miles* run, *Time* (in minutes:seconds:hundredths), and running *Pace* (in minutes:seconds:hundredths per mile) are given for a five-year period from 2002 to 2006. The time and pace have been converted to decimal minutes in *TimeMin* and *PaceMin*, respectively. The brand of the running shoe is added for 2005 and 2006. Use the four-step process to investigate if a runner has a tendency to go faster on short runs (5 or less miles) than long runs. The variable *Short* in the dataset is coded with 1 for short runs and 0 for longer runs. Assume that the data for this runner can be viewed as a sample for runners of a similar age and ability level.

0.18 ***More marathon training.*** Refer to the data described in Exercise 0.17 that contains five years' worth of daily training information for a runner. One might expect that the running patterns change as the runner gets older. The file **Marathon** also contains a variable called *After2004*, which has the value 0 for any runs during the years 2002–2004 and 1 for runs during 2005 and 2006. Use the four-step process to see if there is evidence of a difference between these two time periods in the following aspects of the training runs:

a. The average running pace (*PaceMin*)

b. The average distance run per day (*Miles*)

Supplementary Exercises

0.19 ***Pythagorean theorem of baseball.*** Renowned baseball statistician Bill James devised a model for predicting a team's winning percentage. Dubbed the "Pythagorean Theorem of Baseball," this model predicts a team's winning percentage as

$$\text{Winning percentage} = \frac{(\text{runs scored})^2}{(\text{runs scored})^2 + (\text{runs against})^2} \times 100 + \epsilon$$

a. Use this model to predict the winning percentage for the New York Yankees, who scored 915 runs and allowed 753 runs in the 2009 season.

b. The New York Yankees actually won 103 games and lost 59 in the 2009 season. Determine the winning percentage, and also determine the residual from the Pythagorean model (by taking the observed winning percentage minus the predicted winning percentage).

c. Interpret what this residual value means for the 2009 Yankees. (*Hints*: Did the team do better or worse than expected, given their runs scored and runs allowed? By how much?)

d. Repeat (a–c) for the 2009 San Diego Padres, who scored 638 runs and allowed 769 runs, while winning 75 games and losing 87 games.

e. Which team (Yankees or Padres) exceeded their Pythagorean expectations by more?

Table 0.4 provides data,[5] predictions, and residuals for all 30 Major League Baseball teams in 2009.

[5] *Source*: www.baseball-reference.com.

f. Which team exceeded their Pythagorean expectations the most? Describe how this team's winning percentage compares to what is predicted by their runs scores and runs allowed.

g. Which team fell furthest below their Pythagorean expectations? Describe how this team's winning percentage compares to what is predicted by their runs scored and runs allowed.

TEAM	W	L	WinPct	RunScored	RunsAgainst	Predicted	Residual
Arizona Diamondbacks	70	92	43.21	720	782	45.88	−2.67
Atlanta Braves	86	76	53.09	735	641	56.80	−3.71
Baltimore Orioles	64	98	39.51	741	876	41.71	−2.20
Boston Red Sox	95	67	58.64	872	736	58.40	0.24
Chicago Cubs	83	78	51.55	707	672	52.54	−0.98
Chicago White Sox	79	83	48.77	724	732	49.45	−0.69
Cincinnati Reds	78	84	48.15	673	723	46.42	1.73
Cleveland Indians	65	97	40.12	773	865	44.40	−4.28
Colorado Rockies	92	70	56.79	804	715	55.84	0.95
Detroit Tigers	86	77	52.76	743	745	49.87	2.90
Florida Marlins	87	75	53.70	772	766	50.39	3.31
Houston Astros	74	88	45.68	643	770	41.08	4.59
Kansas City Royals	65	97	40.12	686	842	39.90	0.23
Los Angeles Angels	97	65	59.88	883	761	57.38	2.50
Los Angeles Dodgers	95	67	58.64	780	611	61.97	−3.33
Milwaukee Brewers	80	82	49.38	785	818	47.94	1.44
Minnesota Twins	87	76	53.37	817	765	53.28	0.09
New York Mets	70	92	43.21	671	757	44.00	−0.79
New York Yankees	103	59	63.58	915	753		
Oakland Athletics	75	87	46.30	759	761	49.87	−3.57
Philadelphia Phillies	93	69	57.41	820	709	57.22	0.19
Pittsburgh Pirates	62	99	38.51	636	768	40.68	−2.17
San Diego Padres	75	87	46.30	638	769		
San Francisco Giants	88	74	54.32	657	611	53.62	0.70
Seattle Mariners	85	77	52.47	640	692	46.10	6.37
St. Louis Cardinals	91	71	56.17	730	640	56.54	−0.37
Tampa Bay Rays	84	78	51.85	803	754	53.14	−1.29
Texas Rangers	87	75	53.70	784	740	52.88	0.82
Toronto Blue Jays	75	87	46.30	798	771	51.72	−5.42
Washington Nationals	59	103	36.42	710	874	39.76	−3.34

Table 0.4: *Winning percentage and Pythagorean predictions for baseball teams in 2009*

Unit A: Linear Regression

Response: Quantitative
Predictor(s): Quantitative

Chapter 1: Inference for Simple Linear Regression
Identify and fit a linear model for a quantitative response based on a quantitative predictor. Check the conditions for a simple linear model and use transformations when they are not met. Detect outliers and influential points.

Chapter 2: Inference for Simple Linear Regression
Test hypotheses and construct confidence intervals for the slope of a simple linear model. Partition variability to create an ANOVA table and determine the proportion of variability explained by the model. Construct intervals for predictions made with the simple linear model.

Chapter 3: Multiple Regression
Extend the ideas of the previous two chapters to consider regression models with two or more predictors. Use a multiple regression model to compare two regression lines. Create and assess models using functions of predictors, interactions, and polynomials. Recognize issues of multicolinearity with correlated predictors. Test a subset of predictors with a nested F-test.

Chapter 4: Topics in Regression
Construct and interpret an added variable plot. Consider techniques for choosing predictors to include in a model. Identify unusual and influential points. Incorporate categorical predictors using indicator variables. Use computer simulation techniques (bootstrap and randomization) to do inference for regression parameters.

CHAPTER 1

Simple Linear Regression

How is the price of a used car related to the number of miles it's been driven? Is the number of doctors in a city related to the number of hospitals? How can we predict the price of a textbook from the number of pages?

In this chapter, we consider a single quantitative predictor X for a quantitative response variable Y. A common model to summarize the relationship between two quantitative variables is the *simple linear regression model*. We assume that you have encountered simple linear regression as part of an introductory statistics course. Therefore, we review the structure of this model, the estimation and interpretation of its parameters, the assessment of its fit, and its use in predicting values for the response. Our goal is to introduce and illustrate many of the ideas and techniques of statistical model building that will be used throughout this book in a somewhat familiar setting. In addition to recognizing when a linear model may be appropriate, we also consider methods for dealing with relationships between two quantitative variables that are not linear.

1.1 The Simple Linear Regression Model

Example 1.1: ***Porsche prices***

Suppose that we are interested in purchasing a Porsche sports car. If we can't afford the high sticker price of a new Porsche, we might be interested in finding a used one. How much should we expect to pay? Obviously, the price might depend on many factors including the age, condition, and special features on the car. For this example, we will focus on the relationship between $X = Mileage$ of a used Porsche and $Y = Price$. We used an Internet sales site[1] to collect data for a sample of 30 used Porsches, with price (in thousands of dollars) and mileage (in thousands of miles), as shown in Table 1.1. The data are also stored in the file named **PorschePrice**. We are interested in predicting the price of a used Porsche based on its mileage, so the explanatory variable is *Mileage*, the response is *Price*, and both variables are quantitative.

[1] *Source:* Autotrader.com, Spring 2007.

Price ($1000s)	Mileage (thousands)
69.4	21.5
56.9	43.0
49.9	19.9
47.4	36.0
42.9	44.0
36.9	49.8
83.0	1.3
72.9	0.7
69.9	13.4
67.9	9.7
66.5	15.3
64.9	9.5
58.9	19.1
57.9	12.9
54.9	33.9
54.7	26.0
53.7	20.4
51.9	27.5
51.9	51.7
49.9	32.4
44.9	44.1
44.8	49.8
39.9	35.0
39.7	20.5
34.9	62.0
33.9	50.4
23.9	89.6
22.9	83.4
16.0	86.0
52.9	37.4

Table 1.1: Price *and* Mileage *for used Porsches*

Choosing a Simple Linear Model

Recall that data can be represented by a **model** plus an **error** term:

$$Data = Model + Error$$

When the data involve a quantitative response variable Y and we have a single quantitative predictor X, the model becomes

$$\begin{aligned} Y &= f(X) + \epsilon \\ &= \mu_Y + \epsilon \end{aligned}$$

where $f(X)$ is a function that gives the mean value of Y, μ_Y, at any value of X and ϵ represents the error (deviation) from that mean.[2]

We generally use graphs to help visualize the nature of the relationship between the response and potential predictor variables. Scatterplots are the major tool for helping us choose a model when both the response and predictor are quantitative variables. If the scatterplot shows a consistent linear trend, then we use in our model a mean that follows a straight-line relationship with the predictor. This gives a **simple linear regression** model where the function, $f(X)$, is a linear function of X. If we let β_0 and β_1 represent the intercept and slope, respectively, of that line, we have

$$\mu_Y = f(X) = \beta_0 + \beta_1 X$$

and

$$Y = \beta_0 + \beta_1 X + \epsilon$$

Example 1.2: ***Porsche prices (continued)***

CHOOSE

A scatterplot of price versus mileage for the sample of used Porsches is shown in Figure 1.1. The plot indicates a negative association between these two variables. It is generally understood that cars with lots of miles cost less, on average, than cars with only limited miles and the scatterplot supports this understanding. Since the rate of decrease in the scatterplot is relatively constant as the mileage increases, a linear model might provide a good summary of the relationship between the average prices and mileages of used Porsches for sale on this Internet site. In symbols, we express the mean price as a linear function of mileage:

$$\mu_{Price} = \beta_0 + \beta_1 \cdot Mileage$$

[2] More formal notation for the mean value of Y at a given value of X is $\mu_{Y|X}$. To minimize distractions in most formulas, we will use just μ_Y when the role of the predictor is clear.

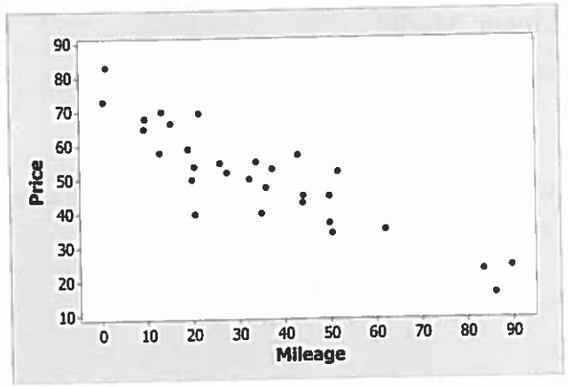


Figure 1.1: *Scatterplot of Porsche* Price *versus* Mileage

Thus, the model for actual used Porsche prices would be

$$Price = \beta_0 + \beta_1 \cdot Mileage + \epsilon$$

This model indicates that Porsche prices should be scattered around a straight line with deviations from the line determined by the random error component, ϵ. We now turn to the question of how to choose the slope and intercept for the line that best summarizes this relationship. ◇

Fitting a Simple Linear Model

We want the best possible estimates of β_0 and β_1. Thus, we use least squares regression to fit the model to the data. This chooses coefficient estimates to minimize the sum of the squared errors and leads to the best set of predictions when we use our model to predict the data. In practice, we rely on computer technology to compute the least squares estimates for the parameters. The fitted model is represented by

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

In general, we use Greek letters (β_0, β_1, etc.) to denote parameters and hats ($\hat{\beta}_0, \hat{\beta}_1$, etc.) are added to denote estimated (fitted) values of these parameters.

A key tool for fitting a model is to compare the values it predicts for the individual data cases[3] to the actual values of the response variable in the dataset. The discrepancy in predicting each response is measured by the **residual**:

$$residual = \text{observed } y - \text{predicted } y = y - \hat{y}$$

[3]We generally use a lowercase y when referring to the value of a variable for an individual case and an uppercase Y for the variable itself.

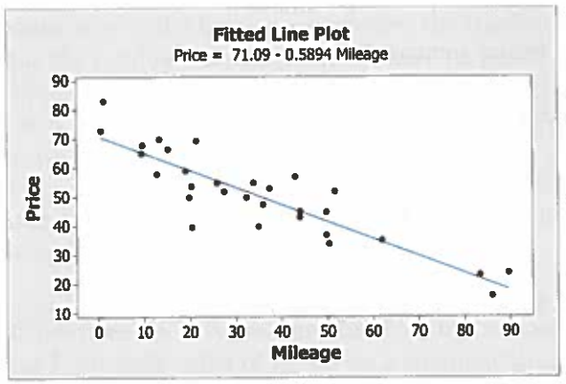


Figure 1.2: *Linear regression to predict Porsche* Price *based on* Mileage

The **sum of squared residuals** provides a measure of how well the line predicts the actual responses for a sample. We often denote this quantity as **SSE** for the sum of the squared errors. Statistical software calculates the fitted values of the slope and intercept so as to minimize this sum of squared residuals; hence, we call this the **least squares line**.

Example 1.3: ***Porsche prices (continued)***

FIT

For the i^{th} car in the dataset, with mileage x_i, the model is

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

The parameters, β_0 and β_1 in the model, represent the true, population-wide intercept and slope for all Porsches for sale. The corresponding statistics, $\hat{\beta}_0$ and $\hat{\beta}_1$, are estimates derived from this particular sample of 30 Porsches. (These estimates are determined from statistical software, for example, in the Minitab fitted line plot shown in Figure 1.2 or the output shown in Figure 1.3.)

The least squares line is

$$\widehat{Price} = 71.09 - 0.5894 \cdot Mileage$$

Thus, for every additional 1000 miles on a used Porsche, the predicted price goes down by about \$589. Also, if a (used!) Porche had zero miles on it, we would predict the price to be \$71,090. In many cases, the the intercept lies far from the data used to fit the model and has no practical interpretation.

```
Regression Analysis: Price versus Mileage

The regression equation is Price = 71.1 - 0.589 Mileage

Predictor       Coef  SE Coef       T      P
Constant      71.090    2.370   30.00  0.000
Mileage     -0.58940  0.05665  -10.40  0.000

S = 7.17029   R-Sq = 79.5%   R-Sq(adj) = 78.7%

Analysis of Variance
Source           DF      SS      MS       F      P
Regression        1  5565.7  5565.7  108.25  0.000
Residual Error   28  1439.6    51.4
Total            29  7005.2
```

Figure 1.3: *Computer output for regression of Porsche* Price *on* Mileage

Note that car #1 in Table 1.1 had a mileage level of 21.5 (21,500 miles) and a price of 69.4 ($69,400), whereas the fitted line predicts a price of

$$\widehat{Price} = 71.09 - 0.5894 \cdot Mileage = 58.4$$

The residual here is $Price - \widehat{Price} = 69.4 - 58.4 = 11.0$.

If we do a similar calculation for each of the 30 cars, square each of the resulting residuals, and sum the squares, we get 1439.6. If you were to choose any other straight line to make predictions for these Porsche prices based on the mileages, you could never obtain an SSE less than 1439.6. ◇

1.2 Conditions for a Simple Linear Model

We know that our model won't fit the data perfectly. The discrepancies that result from fitting the model represent what the model did not capture in each case. We want to check whether our model is reasonable and captures the main features of the dataset. Are we justified in using our model? Do the assumptions of the model appear to be reasonable? How much can we trust predictions that come from the model? Do we need to adjust or expand the model to better explain features of the data or could it be simplified without much loss of predictive power?

In specifying any model, certain conditions must be satisfied for the model to make sense. We often make assumptions about the nature of the relationship between variables and the distribution of the

errors. A key part of assessing any model is to check whether the conditions are reasonable for the data at hand. We hope that the residuals are small and contain no pattern that could be exploited to better explain the response variable. If our assessment shows a problem, then the model should be refined. Typically, we will rely heavily on graphs of residuals to assess the appropriateness of the model. In this section, we discuss the conditions that are commonly placed on a simple linear model. The conditions we describe here for the simple linear regression model are typical of those that will be used throughout this book. In the following section, we explore ways to use graphs to help us assess whether the conditions hold for a particular set of data.

Linearity – The overall relationship between the variables has a linear pattern. The average values of the response Y for each value of X fall on a common straight line.

The other conditions deal with the distribution of the errors.

Zero Mean – The error distribution is centered at zero. This means that the points are scattered at random above and below the line. (*Note:* By using least squares regression, we force the residual mean to be zero. Other techniques would not necessarily satisfy this condition.)

Constant Variance – The variability in the errors is the same for all values of the predictor variable. This means that the spread of points around the line remains fairly constant.

Independence – The errors are assumed to be independent from one another. Thus, one point falling above or below the line has no influence on the location of another point.

When we are interested in using the model to make formal inferences (conducting hypothesis tests or providing confidence intervals), additional assumptions are needed.

Random – The data are obtained using a random process. Most commonly, this arises either from random sampling from a population of interest or from the use of randomization in a statistical experiment.

Normality – In order to use standard distributions for confidence intervals and hypothesis tests, we often need to assume that the random errors follow a normal distribution.

We can summarize these conditions for a simple linear model using the following notation.

Simple Linear Regression Model

For a quantitative response variable Y and a single quantitative explanatory variable X, the **simple linear regression model** is

$$Y = \beta_0 + \beta_1 X + \epsilon$$

where ϵ follows a normal distribution, that is, $\epsilon \sim N(0, \sigma_\epsilon)$, and the errors are independent from one another.

Estimating the Standard Deviation of the Error Term

The simple linear regression model has three unknown parameters: the slope, β_1; the intercept, β_0; and the standard deviation, σ_ϵ, of the errors around the line. We have already seen that software will find the least squares estimates of the slope and intercept. Now we must consider how to estimate σ_ϵ, the standard deviation of the distribution of errors. Since the residuals estimate how much Y varies about the regression line, the sum of the squared residuals (SSE) is used to compute the estimate, $\widehat{\sigma_\epsilon}$. The value of $\widehat{\sigma_\epsilon}$ is referred to as the **regression standard error** and is interpreted as the size of a "typical" error.

Regression Standard Error

For a simple linear regression model, the estimated standard deviation of the error term based on the least squares fit to a sample of n observations is

$$\widehat{\sigma_\epsilon} = \sqrt{\frac{\sum (y - \hat{y})^2}{n-2}} = \sqrt{\frac{SSE}{n-2}}$$

The predicted values and resulting residuals are based on a sample slope and intercept that are calculated from the data. Therefore, we have $n - 2$ **degrees of freedom** for estimating the regression standard error.[4] In general, we lose an additional degree of freedom in the denominator for each new beta parameter that is estimated in the prediction equation.

Example 1.4: ***Porsche prices (continued)***

The sum of squared residuals for the Porsche data is shown in Figure 1.3 as 1439.6 (see the SS column of the Residual Error line of the Analysis of Variance table in the Minitab output). Thus, the regression standard error is

$$\widehat{\sigma_\epsilon} = \sqrt{\frac{1439.6}{30-2}} = 7.17$$

Using mileage to predict the price of a used Porsche, the typical error will be around \$7170. So we have some feel for how far individual cases might spread above or below the regression line. Note that this value is labeled S in the Minitab output of Figure 1.3. ◇

[4] If a scatterplot only has 2 points, then it's easy to fit a straight line with residuals of zero, but we have no way of estimating the variability in the distribution of the error term. This corresponds to having zero degrees of freedom.

1.3 Assessing Conditions

A variety of plots are used to assess the conditions of the simple linear model. Scatterplots, histograms, and dotplots will be helpful to begin the assessment process. However, plots of residuals versus fitted values and normal plots will provide more detailed information, and these visual displays will be used throughout the text.

Residuals versus Fits Plots

A scatterplot with the fitted line provides one visual method of checking linearity. Points will be randomly scattered above and below the line when the linear model is appropriate. Clear patterns, for example clusters of points above and below the line in a systematic fashion, indicate that the linear model is not appropriate.

A more informative way of looking at how the points vary about the regression line is a scatterplot of the residuals versus the fitted values for the prediction equation. This plot reorients the axes so that the regression line is represented as a horizontal line through zero. Positive residuals represent points that are above the regression line. The residuals versus fits plot allows us to focus on the estimated errors and look for any clear patterns without the added complexity of a sloped line.

The residual versus fits plot is especially useful for assessing the linearity and constant variance conditions of a simple linear model. The ideal pattern will be random variation above and below zero in a band of relatively constant width. Figure 1.4 shows a typical residual versus fits plot when these two conditions are satisfied.

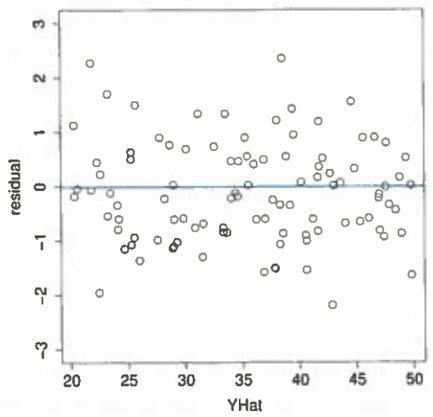
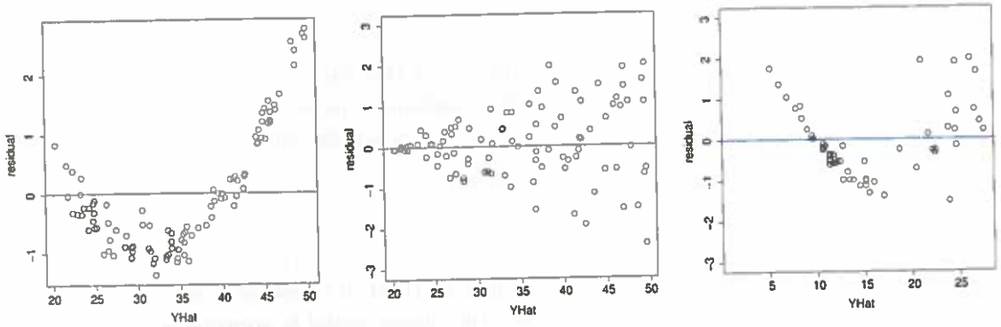


Figure 1.4: *Residuals versus fitted values plot when linearity and constant variance conditions hold*



(a) Nonlinear (b) Nonconstant variance (c) Both nonlinear and nonconstant variance

Figure 1.5: *Residuals versus fitted values plots illustrating problems with conditions*

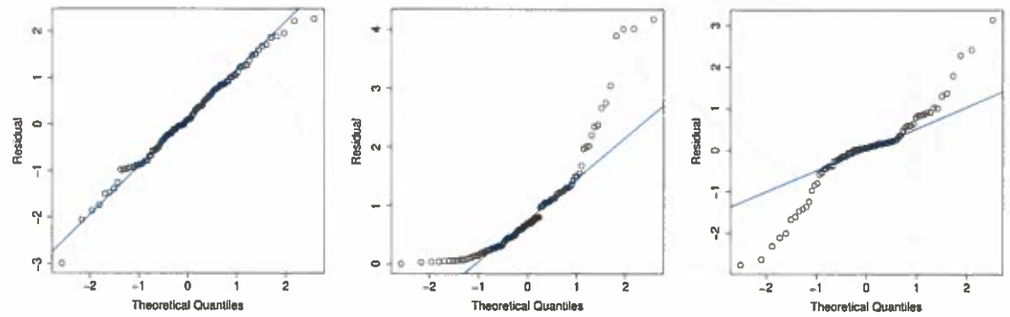
Figure 1.5 shows some examples of residual versus fits plots that exhibit some typical patterns indicating a problem with linearity, constant variance, or both conditions.

Figure 1.5(a) illustrates a curved pattern demonstrating a lack of linearity in the relationship. The residuals are mostly positive at either extreme of the graph and negative in the middle, indicating more of a curved relationship. Despite this pattern, the vertical width of the band of residuals is relatively constant across the graph, showing that the constant variance condition is probably reasonable for this model.

Figure 1.5(b) shows a common violation of the equal variance assumption. In many cases, as the predicted response gets larger, its variability also increases, producing a fan shape as in this plot. Note that a linearity assumption might still be valid in this case since the residuals are still equally dispersed above and below the zero line as we move across the graph.

Figure 1.5(c) indicates problems with both the linearity and constant variance conditions. We see a lack of linearity due to the curved pattern in the plot and, again, variance in the residuals that increases as the fitted values increase.

In practice, the assessment of a residual versus fits plot may not lead to as obvious a conclusion as in these examples. Remember that no model is "perfect" and we should not expect to always obtain the ideal plot. A certain amount of variation is natural, even for sample data that are generated from a model that meets all of the conditions. The goal is to recognize when departures from the model conditions are sufficiently evident in the data to suggest that an alternative model might be preferred or we should use some caution when drawing conclusions from the model.



(a) Normal residuals (b) Skewed right residuals (c) Long-tailed residuals

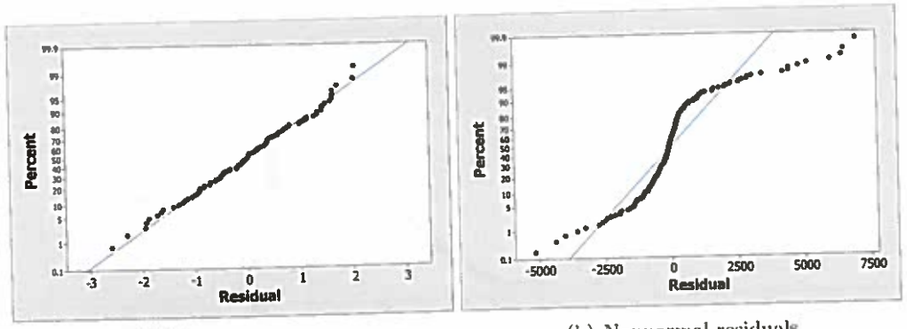
Figure 1.6: *Examples of normal quantile plots*

Normal Plots

Data from a normal distribution should exhibit a "bell-shaped" curve when plotted as a histogram or dotplot. However, we often need a fairly large sample to see this shape accurately, and even then it may be difficult to assess whether the symmetry and curvature of the tails are consistent with a true normal curve. As an alternative, a **normal plot** shows a different view of the data where an ideal pattern for a normal sample is a straight line. Although a number of variations exist, there are generally two common methods for constructing a normal plot.

The first, called a **normal quantile plot**, is a scatterplot of the ordered observed data versus values (the theoretical quantiles) that we would expect to see from a "perfect" normal sample of the same size. If the ordered residuals are increasing at the rate we would expect to see for a normal sample, the resulting scatterplot is a straight line. If the distribution of the residuals is skewed in one direction or has tails that are overly long due to some extreme outliers at both ends of the distribution, the normal quantile plot will bend away from a straight line. Figure 1.6 shows several examples of normal quantile plots. The first (Figure 1.6(a)) was generated from residuals where the data were generated from a linear model with normal errors and the other two from models with nonnormal errors.

The second common method of producing a normal plot is to use a **normal probability plot**, such as those shown in Figure 1.7. Here, the ordered sample data are plotted on the horizontal axis while the vertical axis is transformed to reflect the rate that normal probabilities grow. As with a normal quantile plot, the values increase as we move from left to right across the graph, but the revised scale produces a straight line when the values increase at the rate we would expect for a sample from a normal distribution. Thus, the interpretation is the same. A linear pattern (as in Figure 1.7(a)) indicates good agreement with normality and curvature, or bending away from a straight line (as in Figure 1.7(b)), shows departures from normality.



(a) Normal residuals (b) Nonnormal residuals

Figure 1.7: *Examples of normal probability plots*

Since both normal plot forms have a similar interpretation, we will use them interchangeably. The choice we make for a specific problem often depends on the options that are most readily available in the statistical software we are using.

Example 1.5: ***Porsche prices (continued)***

ASSESS

We illustrate these ideas by checking the conditions for the model to predict Porsche prices based on mileage.

Linearity: Figure 1.2 shows that the linearity condition is reasonable as the scatterplot shows a consistent decline in prices with mileage and no obvious curvature. A plot of the residuals versus fitted values is shown in Figure 1.8. The horizontal band of points scattered randomly above and below the zero line illustrates that a linear model is appropriate for describing the relationship between price and mileage.

Zero mean: We used least squares regression, which forces the sample mean of the residuals to be zero when estimating the intercept β_0. Also note that the residuals are scattered on either side of zero in the residual plot of Figure 1.8 and a histogram of the residuals, Figure 1.9, is centered at zero.

Constant variance: The fitted line plot in Figure 1.2 shows the data spread in roughly equal width bands on either side of the least squares line. Looking left to right in the plot of residuals versus fitted values in Figure 1.8 reinforces this finding as we see a fairly constant spread of the residuals above and below zero (where zero corresponds to actual prices that fall on the least squares regression line). This supports the constant variance condition.

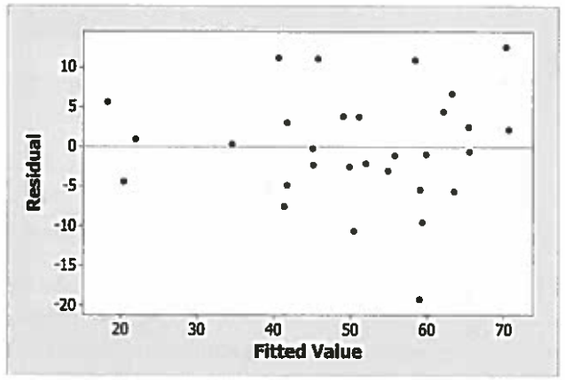


Figure 1.8: *Plot of Porsche residuals versus fitted values*

Independence and Random: We cannot tell from examining the data whether these conditions are satisfied. However, the context of the situation and the way the data were collected make these reasonable assumptions. There is no reason to think that one seller changing the asking price for a used car would necessarily influence the asking price of another seller. We were also told that these data were randomly selected from the Porsches for sale on the Autotrader.com website. So, at the least, we can treat it as a random sample from the population of all Porsches on that site at the particular time the sample was collected. We might want to be cautious about extending the findings to cars from a different site, an actual used car lot, or a later point in time.

Normality: In assessing normality, we can refer to the histogram of residuals in Figure 1.9, where a reasonably bell-shaped pattern is displayed. However, a histogram based on this small sample

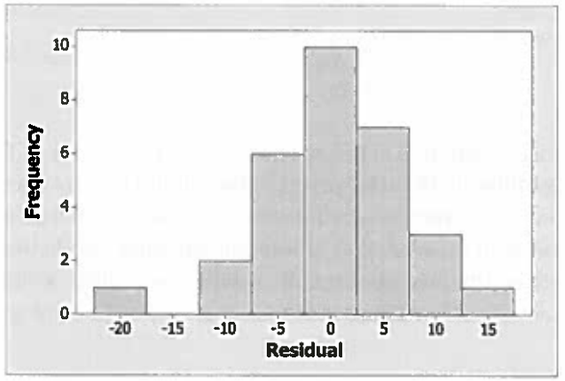


Figure 1.9: *Histogram of Porsche residuals*

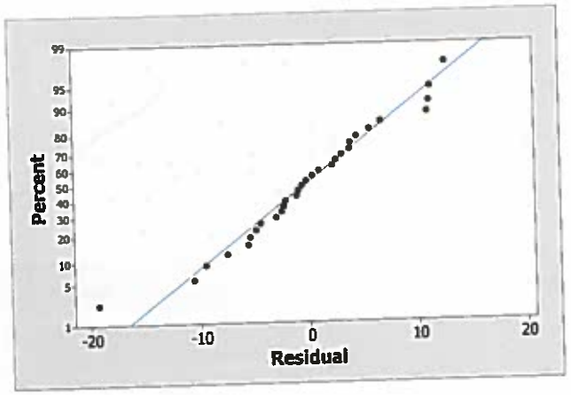


Figure 1.10: *Normal probability plot of residuals for Porsche data*

may not be particularly informative and can change considerably depending on the bins used to determine the bars. A more reliable plot for assessing normality is the normal probability plot of the residuals shown in Figure 1.10. This graph shows a consistent linear trend that supports the normality condition. We might have a small concern about the single point in the lower left corner of the plot, but we are looking more at the overall pattern when assessing normality.

USE

After we have decided on a reasonable model, we interpret the implications for the question of interest. For example, suppose we find a used Porsche for sale with 50,000 miles and we believe that it is from the same population from which our sample of 30 used Porsches was drawn. What should we expect to pay for this car? Would it be an especially good deal if the owner was asking $38,000?

Based on our model, we would expect to pay

$$\widehat{Price} = 71.09 - 0.5894 \cdot 50 = 41.62$$

or $41,620. The asking price of $38,000 is below the expected price of $41,6200, but is this difference large relative to the variability in Porsche prices? We might like to know if this is a really good deal or perhaps such a low price that we should be concerned about the condition of the car. This question will be addressed in a Section 2.4, where we consider prediction intervals. For now, we can observe that the car's residual is about half of what we called a "typical error" ($\widehat{\sigma}_\epsilon$ = $7.17 thousand) below the expected price. Thus, it is low, but not unusually so. ◇

1.4 Transformations

If one or more of the conditions for a simple linear regression model are not satisfied, then we can consider transformations on one or both of the variables. In this section, we provide two examples where this is the case.

Example 1.6: ***Doctors and hospitals in metropolitan areas***

We expect the number of doctors in a city to be related to the number of hospitals, reflecting both the size of the city and the general level of medical care. Finding the number of hospitals in a given city is relatively easy, but counting the number of doctors is a more challenging task. Fortunately, the U.S. Census Bureau regularly collects such data for many metropolitan areas in the United States. The data in Table 1.2 show values for these two variables (and the *City* names) from the first few cases in the data file **MetroHealth83**, which has a sample of 83 metropolitan areas[5] that have at least two community hospitals.

City	NumMDs	NumHospitals
Holland-Grand Haven, MI	349	3
Louisville, KY-IN	4042	18
Battle Creek, MI	256	3
Madison, WI	2679	7
Fort Smith, AR-OK	502	8
Sarasota-Bradenton-Venice, FL	2352	7
Anderson, IN	200	2
Honolulu, HI	3478	13
Asheville, NC	1489	5
Winston-Salem, NC	2018	6
⋮	⋮	⋮

Table 1.2: *Number of MDs and community hospitals for sample of $n = 83$ metropolitan areas*

CHOOSE

As usual, we start the process of finding a model to predict the number of MDs ($NumMDs$) from the number of hospitals ($NumHospitals$) by examining a scatterplot of the two variables as seen in Figure 1.11. As expected, this shows an increasing trend with cities having more hospitals also tending to have more doctors, suggesting that a linear model might be appropriate.

[5] *Source:* U.S. Census Bureau, *2006 State and Metropolitan Area Data Book*, Table B-6.

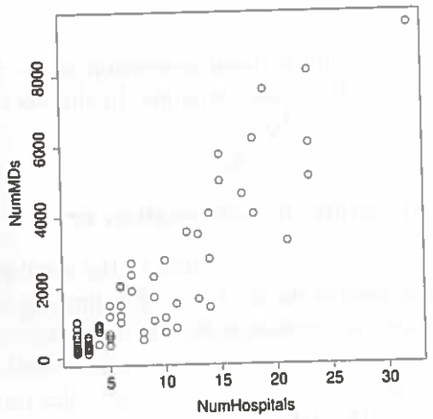


Figure 1.11: *Scatterplot for* Doctors *versus* Hospitals

FIT

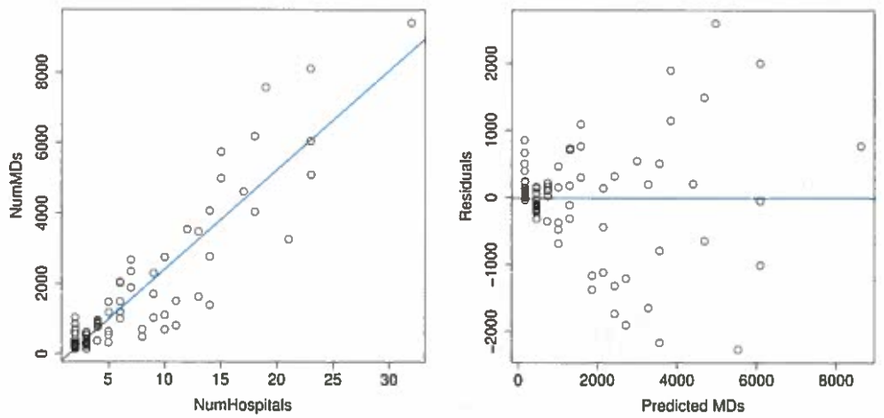
Fitting a least squares line in R produces estimates for the slope and intercept as shown below, giving the prediction equation $\widehat{NumMDs} = -385.1 + 282.0 \cdot NumHospitals$. Figure 1.12(a) shows the scatterplot with this regression line as a summary of the relationship.

```
Call:
lm(formula = NumMDs ~ NumHospitals)

Coefficients:
 (Intercept)  NumHospitals
      -385.1         282.0
```

ASSESS

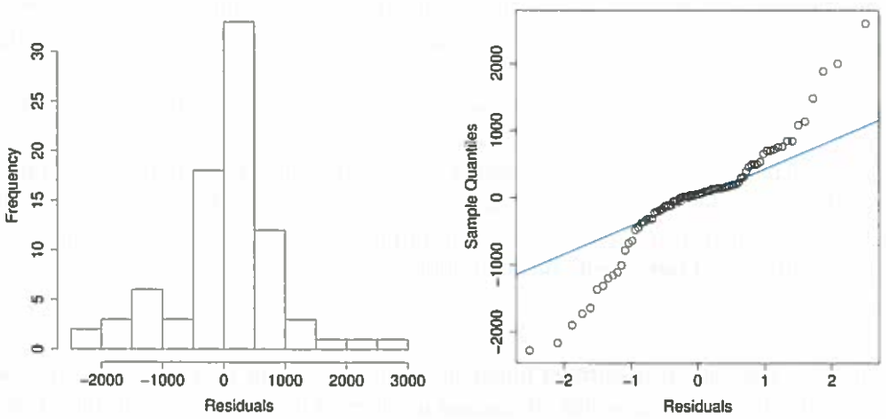
The line does a fairly good job of following the increasing trend in the relationship between number of doctors and number of hospitals. However, a closer look at plots of the residuals shows some considerable departures from our standard regression assumptions. For example, the plot of residuals versus fitted values in Figure 1.12(b) shows a fan shape, with the variability in the residuals tending to increase as the fitted values get larger. This often occurs with count data like the number of MDs and number of hospitals where variability increases as the counts grow larger. We can also observe this effect in a scatterplot with the regression line, Figure 1.12(a).



(a) Least squares line (b) Residuals versus fits

Figure 1.12: *Regression for number of doctors based on number of hospitals*

We also see from a histogram of the residuals, Figure 1.13(a), and normal quantile plot, Figure 1.13(b), that an assumption of normality would not be reasonable for the residuals in this model. Although the histogram is relatively unimodal and symmetric, the peak is quite narrow with very long "tails." This departure from normality is seen more clearly in the normal quantile plot that has significant curvature away from a straight line at both ends.



(a) Histogram of residuals (b) Normal quantile plot

Figure 1.13: *Normality plots for residuals of* Doctors *versus* Hospitals

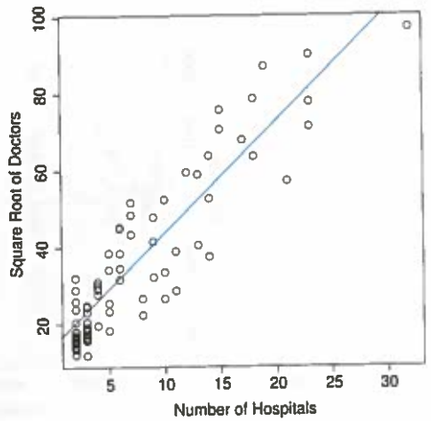


Figure 1.14: *Least squares line for* Sqrt(Doctors) *versus* Hospitals

CHOOSE (again)

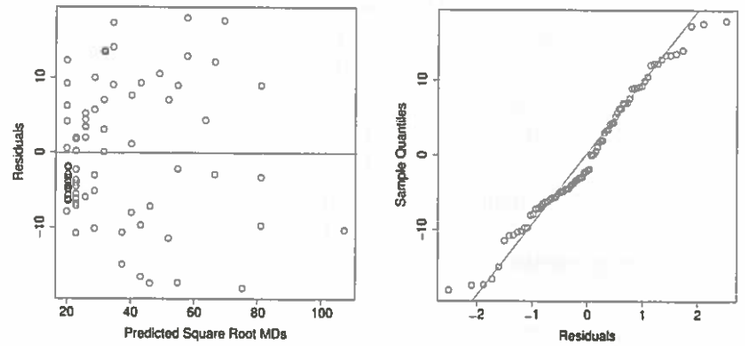
To stabilize the variance in a response (Y) across different values of the predictor (X) we often try transformations on either Y or X. Typical options include raising a variable to a power (such as $\sqrt{Y}$, X^2, or $1/X$) or taking a logarithm (e.g., using $\log(Y)$ as the response). For count data, such as the number of doctors or hospitals where the variability increases along with the magnitudes of the variables, a square root transformation is often helpful. Figure 1.14 shows the least square line fit to the transformed data to predict the square root of the number of doctors based on the number of hospitals. The prediction equation is now $\widehat{\sqrt{NumMDs}} = 14.033 + 2.915 \cdot NumHospitals$.

When the equal variance assumption holds, we should see roughly parallel bands of data spread along the line. Although there might still be slightly less variability for the smallest numbers of hospitals, the situation is much better than for the data on the original scale. The residuals versus fitted values plot for the transformed data in Figure 1.15(a) and normal quantile plot of the residuals in Figure 1.15(b) also show considerable improvement at meeting the constant variance and normality conditions of our simple linear model.

USE

We must remember that our transformed linear model is predicting $\sqrt{MDs}$, so we must square its predicted values to obtain estimates for the actual number of doctors. For example, if we consider the case from the data of Louisville, Kentucky, which has 18 community hospitals, the transformed model would predict

$$\widehat{\sqrt{NumMDs}} = 14.033 + 2.915 \cdot 18 = 66.50$$



(a) Residuals versus fits (b) Normal plot of residuals

Figure 1.15: *Plots of residuals for* Sqrt(Doctors) *versus* Hospitals

so the predicted number of doctors is $66.50^2 = 4422.3$, while Louisville actually had 4042 doctors at the time of this sample. Figure 1.16 shows the scatterplot with the predicted number of doctors after transforming the linear model for the square roots of the number of doctors back to the original scale so that

$$\widehat{NumMDs} = (14.033 + 2.915 \cdot NumHospitals)^2$$

Note that we could use this model to make predictions for other cities, but in doing so, we should feel comfortable only to the extent that we believe the sample to be representative of the larger population of cities with at least two community hospitals. ◇

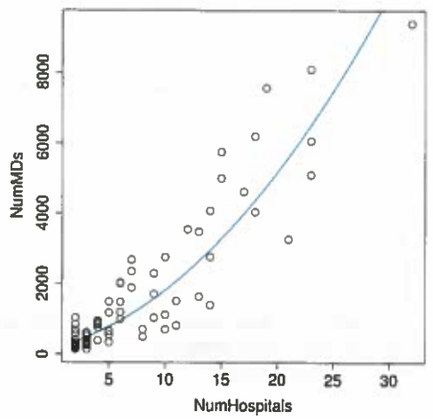


Figure 1.16: *Predicted* NumMDs *from the linear model for* Sqrt(NumMDs)

Island	Area (km^2)	Mammal Species
Borneo	743244	129
Sumatra	473607	126
Java	125628	78
Bangka	11964	38
Bunguran	1594	24
Banggi	450	18
Jemaja	194	15
Karimata Besar	130	19
Tioman	114	23
Siantan	113	16
Sirhassan	46	16
Redang	25	8
Penebangan	13	13
Perhentian Besar	8	6

Table 1.3: Species *and* Area *for Southeast Asian islands*

Example 1.7: ***Species by area***

The data in Table 1.3 (and the file **SpeciesArea**) show the number of mammal species and the area for 13 islands[6] in Southeast Asia. Biologists have speculated that the number of species is related to the size of an island and would like to be able to predict the number of species given the size of an island.

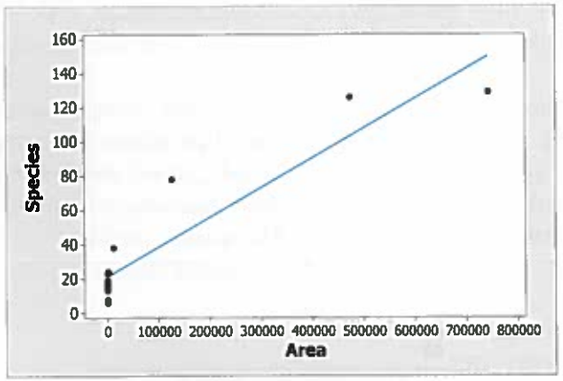
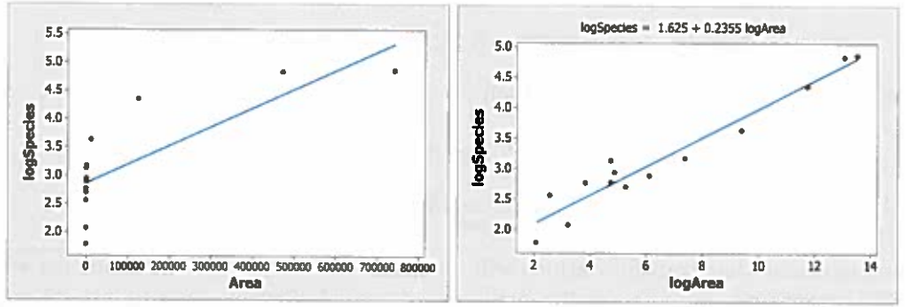


Figure 1.17: *Number of* Mammal Species *versus* Area *for S.E. Asian islands*

[6]Source: Lawrence R. Heaney, (1984), Mammalian Species Richness on Islands on the Sunda Shelf, Southeast Asia, *Oecologia* vol. 61, no. 1, pages 11–17.

Figure 1.17 shows a scatterplot with least squares line added. Clearly, the line does not provide a good summary of this relationship because it doesn't reflect the curved pattern shown in the plot.

In a case like this, where we see strong curvature and extreme values in a scatterplot, a logarithm transformation of either the response variable, the predictor, or possibly both, is often helpful. Applying a log transformation[7] to the species variable results in the scatterplot of Figure 1.18(a).



(a) Log *Species* versus *Area* (b) Log *Species* versus log *Area*

Figure 1.18: *Log transformations of* Species *versus* Area *for S.E. Asian islands*

Clearly, this transformation has failed to produce a linear relationship. However, if we also take a log transformation of the area, we obtain the plot illustrated in Figure 1.18(b), which does show a linear pattern. Figure 1.19 shows a residual plot from this regression, which does not indicate any striking patterns.

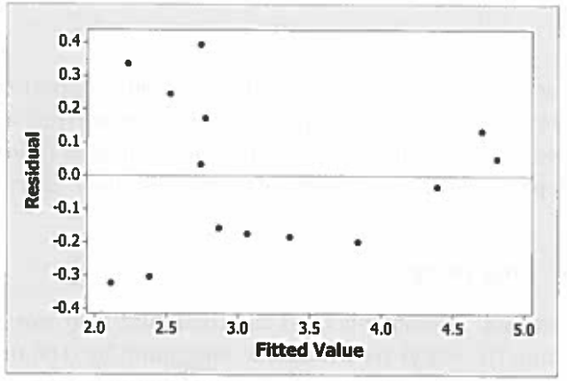


Figure 1.19: *Residual plot after log transform of response and predictor*

[7]In this text we use log to denote the natural logarithm.

Based on the fitted model, we can predict $\log(Species)$ based on $\log(Area)$ for an island with the model

$$\widehat{\log(Species)} = 1.625 + 0.2355 \cdot \log(Area)$$

Suppose we wanted to use the model to find the predicted value for Java, which has an area of 125,628 square kilometers. We substitute 125,628 into the equation as the *Area* and compute an estimate for the log of the number of species:

$$\widehat{\log(Species)} = 1.625 + 0.2355 \cdot \log(125,628) = 4.390$$

Our estimate for the number of species is then

$$e^{4.390} = 80.6 \text{ species}$$

The actual number of mammal species found on Java for this study was 78. ◇

There is no guarantee that transformations will eliminate or even reduce the problems with departures from the conditions for a simple linear regression model. Finding an appropriate transformation is as much an art as a science.

1.5 Outliers and Influential Points

Sometimes, a data point just doesn't fit within a linear trend that is evident in the other points. This can be because the point doesn't fit with the other points in a scatterplot vertically—it is an *outlier*—or a point may differ from the others horizontally and vertically so that it is an *influential* point. In this section, we examine methods for identifying outliers and influential points using graphs and summary statistics.

Outliers

We classify a data point as an *outlier* if it stands out away from the pattern of the rest of the data and is not well described by the model. In the simple linear model setting, an outlier is a point where the magnitude of the residual is unusually large. How large must a residual be for a point to be called an outlier? That depends on the variability of all the residuals, as we see in the next example.

Example 1.8: *Olympic long jump*

During the 1968 Olympics, Bob Beamon shocked the track and field world by jumping 8.9 meters (29 feet 2.5 inches), breaking the world record for the long jump by 0.65 meters (more than 2 feet). Figure 1.20 shows the winning men's Olympic long jump distance (labeled as *Gold*) versus *Year*, together with the least squares regression line, for the $n = 26$ Olympics held during the period 1900–2008. The data are stored in **LongJumpOlympics**.

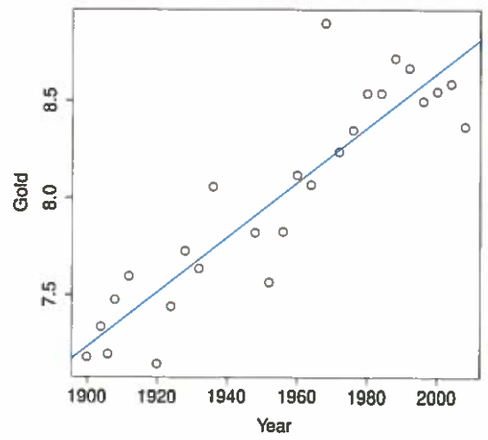


Figure 1.20: *Gold-medal-winning distances (m) for the men's Olympic long jump, 1900–2008*

The 1968 point clearly stands above the others and is far removed from the regression line. Because this point does not fit the general pattern in the scatterplot, it is an outlier. The unusual nature of this point is perhaps even more evident in Figure 1.21, a residual plot for the fitted least squares model.

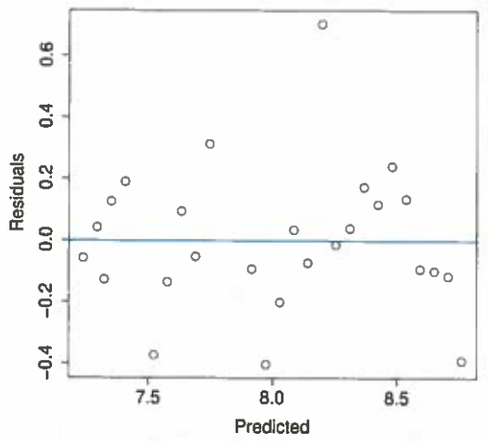


Figure 1.21: *Residual plot for long jump model*

The fitted regression model is

$$\widehat{Gold} = -19.48 + 0.014066 \cdot Year$$

Thus, the predicted 1968 winning long jump is $\widehat{Gold} = -19.48 + 0.014066 \cdot 1968 = 8.20$ meters. The 1968 residual is $8.90 - 8.20 = 0.70$ meters.

Even when we know the context of the problem, it can be difficult to judge whether a residual of 0.70 m is unusually large. One method to help decide when a residual is extreme is to put the residuals on a standard scale. For example, since the estimated standard deviation of the regression error, $\hat{\sigma}_\epsilon$, reflects the size of a "typical" error, we could standardize each residual using

$$\frac{y - \hat{y}}{\hat{\sigma}_\epsilon}$$

In practice, most statistical packages make some modifications to this formula when computing a **standardized** residual to account for how unusual the predicted value is for a particular case. Since an extreme outlier might have a significant effect on the estimation of σ_ϵ, another common adjustment is to estimate the standard deviation of the regression error using a model that is fit after omitting the point in question. Such residuals are often called **studentized**[8] (or **deleted-t**) residuals.

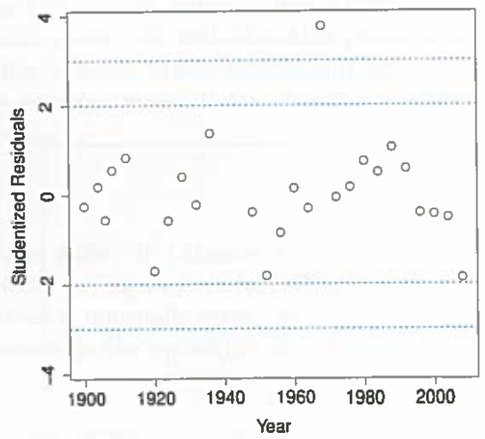


Figure 1.22: *Studentized residuals for the long jump model*

If the conditions of a simple linear model hold, approximately 95% of the residuals should be within 2 standard deviations of the residual mean of zero, so we would expect most standardized

[8]You may recall the t-distribution, which is sometimes called Student's t, from an introductory statistics course.

or studentized residuals to be less than 2 in absolute value. We may be slightly suspicious about points where the magnitude of the standardized or studentized residual is greater than 2 and even more wary about points beyond ±3. For example, the standardized residual for Bob Beamon's 1968 jump is 3.03, indicating this point is an outlier. Figure 1.22 shows the studentized residuals for the long jump data plotted against the predicted values. The studentized residual for the 1968 jump is 3.77, while none of the other studentized residuals are beyond ±2, clearly pointing out the exceptional nature of that performance. ◇

Influential Points

When we fit a regression model and make a prediction, we combine information across several observations, or cases, to arrive at the prediction, or fitted value, for a particular case. For example, we use the mileages and prices of many cars to arrive at a predicted price for a particular car. In doing this, we give equal weight to all of the cases in the dataset; that is, every case contributes equally to the creation of the fitted regression model and to the subsequent predictions that are based on that model.

Usually, this is a sensible and useful thing to do. Sometimes, however, this approach can be problematic, especially when the data contain one or more extreme cases that might have a significant impact on the coefficient estimates in the model.

Example 1.9: *Butterfly ballot*

The race for the presidency of the United States in the fall of 2000 was very close, with the electoral votes from Florida determining the outcome. Nationally, George W. Bush received 47.9% of the popular vote, Al Gore received 48.4%, and the rest of the popular vote was split among several other candidates. In the disputed final tally in Florida, Bush won by just 537 votes over Gore (48.847% to 48.838%) out of almost 6 million votes cast. About 2.3% of the votes cast in Florida were awarded to other candidates. One of those other candidates was Pat Buchanan, who did much better in Palm Beach County than he did anywhere else. Palm Beach County used a unique "butterfly ballot" that had candidate names on either side of the page with "chads" to be punched in the middle. This nonstandard ballot seemed to confuse some voters, who punched votes for Buchanan that may have been intended for a different candidate. Figure 1.23 shows the number of votes that Buchanan received plotted against the number of votes that Bush received for each county, together with the fitted regression line ($\widehat{Buchanan} = 45.3 + 0.0049 * Bush$). The data are stored in **PalmBeach**.

The data point near the top of the scatterplot is Palm Beach County, where Buchanan picked up over 3000 votes. Figure 1.24 is a plot of the residuals versus fitted values for this model; clearly, Palm Beach County stands out from the rest of the data. Using this model, Minitab computes the standardized residual for Palm Beach to be 7.65 and the studentized residual to be 24.08! No

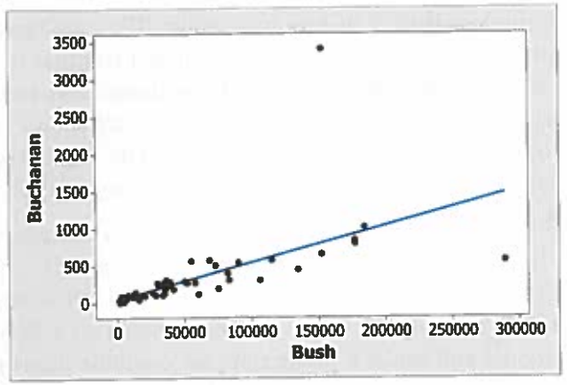


Figure 1.23: *2000 presidential election totals in Florida counties*

question that this point should be considered an outlier. Also, the data point at the far right on the plots (Dade County) has a large negative residual of −907.5, which gives a standardized residual of −3.06; certainly something to consider as an outlier, although not as dramatic as Palm Beach.

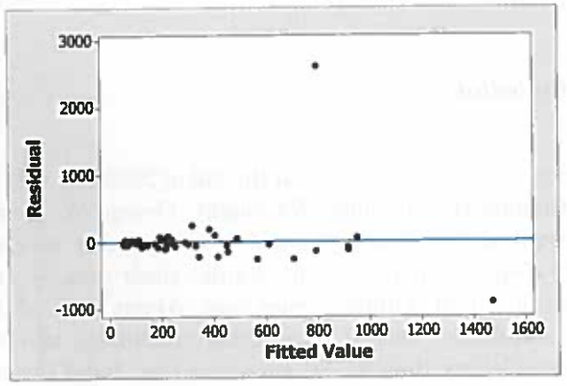


Figure 1.24: *Residual plot for the butterfly ballot data*

Other than recognizing that the model does a poor job of predicting Palm Beach County (and to a lesser extent Dade County), should we worry about the effect that such extreme values have on the rest of the predictions given by the model? Would removing Palm Beach County from the dataset produce much change in the regression equation? Portions of the Minitab output for fitting the simple linear model with and without the Palm Beach County data point are shown below. Figure 1.25 shows both regression lines, with the steeper slope (0.0049) occurring when Palm Beach County is included and the shallower slope (0.0035) when that point is omitted. Notice that the effect of the extreme value for Palm Beach is to "pull" the regression line in its direction.

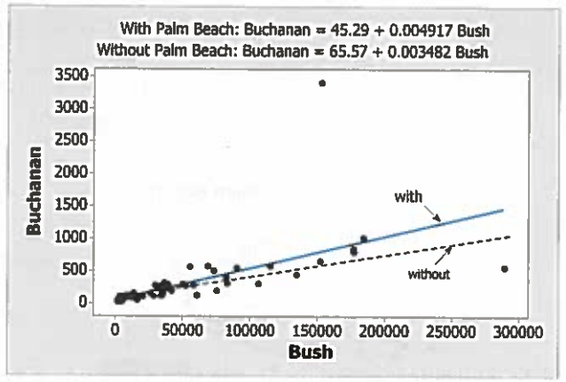


Figure 1.25: *Regression lines with and without Palm Beach*

Regression output with Palm Beach County:

```
The regression equation is
Buchanan = 45.3 + 0.00492 Bush

Predictor       Coef    SE Coef      T      P
Constant       45.29      54.48   0.83  0.409
Bush      0.0049168  0.0007644   6.43  0.000

S = 353.922   R-Sq = 38.9%   R-Sq(adj) = 38.0%
```

Regression output without Palm Beach County:

```
The regression equation is
Buchanan = 65.6 + 0.00348 Bush

Predictor       Coef    SE Coef      T      P
Constant       65.57      17.33   3.78  0.000
Bush      0.0034819  0.0002501  13.92  0.000

S = 112.453   R-Sq = 75.2%   R-Sq(adj) = 74.8%
```

◇

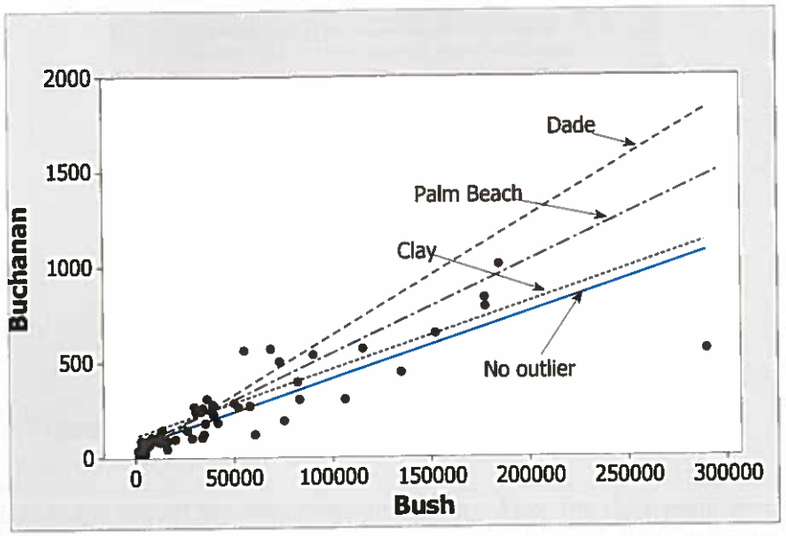


Figure 1.26: *Regression lines with an outlier of 3407 "moved" to different counties*

The amount of **influence** that a single point has on a regression fit depends on how well it aligns with the pattern of the rest of the points and on its value for the predictor variable. Figure 1.26 shows the regression lines we would have obtained if the extreme value (3407 Buchanan votes) had occurred in Dade County (with 289,456 Bush votes), Palm Beach County (152,846 Bush votes), Clay County (41,745 Bush votes), or not occurred at all. Note that the more extreme values for the predictor (large Bush counts in Dade or Palm Beach) produced a bigger effect on the slope of the regression line than when the outlier was placed in a more "average" Bush county such as Clay.

Generally, points farther from the mean value of the predictor ($\overline{x}$) have greater potential to influence the slope of a fitted regression line. This concept is known as the **leverage** of a point. Points with high leverage have a greater capacity to pull the regression line in their direction than do low leverage points near the predictor mean. Although in the case of a single predictor, we could measure leverage as just the distance from the mean, we introduce a somewhat more complicated statistic in Section 4.3 that can be applied to more complicated regression settings.

1.6 Chapter Summary

In this chapter, we considered a **simple linear regression model** for predicting a single quantitative response variable Y from a single quantitative predictor X:

$$Y = \beta_0 + \beta_1 \cdot X + \epsilon$$

You should be able to use statistical software to estimate and interpret the slope and intercept for this model to produce a least squares regression line:

$$\hat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot X$$

The coefficient estimates, $\widehat{\beta}_0$ and $\widehat{\beta_1}$, are obtained using a method of least squares that selects them to provide the smallest possible sum of squared residuals (SSE). A **residual** is the observed response (y) minus the predicted response ($\hat{y}$), or the vertical distance from a point to the line. You should be able to interpret, in context, the intercept and slope, as well as residuals.

The **scatterplot**, which shows the relationship between two quantitative variables, is an important visual tool to help choose a model. We look for the direction of association (positive, negative, or a random scatter), the strength of the relationship, and the degree of linearity. Assessing the fit of the model is a very important part of the modeling process. The conditions to check when using a simple linear regression model include **linearity**, **zero mean** (of the residuals), **constant variance** (about the regression line), **independence**, **random selection** (or random assignment), and **normality** (of the residuals). We can summarize several of these conditions by specifying the distribution of the error term, $\epsilon \sim N(0, \sigma_\epsilon)$. In addition to a scatterplot with the least squares line, various residual plots, such as a **histogram of the residuals** or a **residuals versus fits plot**, are extremely helpful in checking these conditions. Once we are satisfied with the fit of the model, we use the estimated model to make inferences or predictions.

Special plots, known as a **normal quantile plot** or **normal probability plot**, are useful in assessing the normality condition. These two plots are constructed using slightly different methods, but the interpretation is the same. A linear trend indicates that normality is reasonable, and departures from linearity indicate trouble with this condition. Be careful not to confuse linearity in normal plots with the condition of a linear relationship between the predictor and response variable.

You should be able to estimate the **standard deviation of the error** term, σ_ϵ, the third parameter in the simple linear regression model. The estimate is based on the sum of squared errors (SSE) and the associated degrees of freedom ($n - 2$):

$$\widehat{\sigma_\epsilon} = \sqrt{\frac{SSE}{n-2}}$$

and is the typical error, often referred to as the **regression standard error**.

If the conditions are not satisfied, then **transformations** on the predictor, response, or both variables should be considered. Typical transformations include the square root, reciprocal, logarithm, and raising the variable(s) to another power. Identifying a useful transformation for a particular dataset (if one exists at all) is as much art as science. Trial and error is often a good approach.

You should be able to identify obvious **outliers** and **influential points**. Outliers are points that are unusually far away from the overall pattern shown by the other data. Influential points exert considerable impact on the estimated regression line. We will look at more detailed methods for identifying outliers and influential points in Section 4.3. For now, you should only worry about recognizing very extreme cases and be aware that they can affect the fitted model and analysis. One common guideline is to tag all observations with **standardized** or **studentized residuals** smaller than -2 or larger than 2 as possible outliers. To see if a point is influential, fit the model with and without that point to see if the coefficients change very much. In general, points far from the average value of the predictor variable have greater potential to influence the regression line.

1.7 Exercises

Conceptual Exercises

1.1 ***Equation of a line.*** Consider the fitted regression equation $\hat{Y} = 100 + 15 \cdot X$. Which of the following is *false*?

a. The sample slope is 15.

b. The predicted value of Y when $X = 0$ is 100.

c. The predicted value of Y when $X = 2$ is 110.

d. Larger values of X are associated with larger values of Y.

1.2–1.5 ***Sparrows.*** Priscilla Erickson from Kenyon College collected data on a stratified random sample of 116 Savannah sparrows at Kent Island. The weight (in grams) and wing length (in mm) were obtained for birds from nests that were reduced, controlled, or enlarged. The data[9] are in the file **Sparrows**. Use the computer output below in Exercises 1.2–1.5.

```
The regression equation is  Weight = 1.37 + 0.467 WingLength

Predictor       Coef  SE Coef      T      P
Constant      1.3655   0.9573   1.43  0.156
WingLength    0.4674  0.03472  13.46  0.000

S = 1.39959   R-Sq = 61.4%   R-Sq(adj) = 61.1%

Analysis of Variance
Source           DF      SS      MS      F      P
Regression        1  355.05  355.05  181.25  0.000
Residual Error  114  223.31    1.96
Total           115  578.36
```

1.2 ***Sparrows slope.*** Based on the regression output, what is the slope of the least squares regression line for predicting sparrow weight from wing length?

1.3 ***Sparrows intercept.*** Based on the regression output, what is the intercept of the least squares regression line for predicting sparrow weight from wing length?

1.4 ***Sparrows regression standard error.*** Based on the regression output, what is the size of the typical error when predicting weight from wing length?

[9] We thank Priscilla Erickson and Professor Robert Mauck from the Department of Biology at Kenyon College for allowing us to use these data.

1.5 ***Sparrow degrees of freedom.*** What are the degrees of freedom associated with the standard regression error when predicting weight from wing length for these sparrows?

1.6 ***Computing a residual.*** Consider the fitted regression equation $\hat{Y} = 25 + 7 \cdot X$. If $x_1 = 10$ and $y_1 = 100$, what is the residual for the first data point?

1.7 ***Residual plots to check conditions.*** For which of the following conditions for inference in regression does a residual plot *not* aid in assessing whether the condition is satisfied?

a. Linearity

b. Constant variance

c. Independence

d. Zero mean

Guided Exercises

1.8 ***Breakfast cereal.*** The number of calories and number of grams of sugar per serving were measured for 36 breakfast cereals. The data are in the file **Cereal**. We are interested in trying to predict the calories using the sugar content.

a. Make a scatterplot and comment on what you see.

b. Find the least squares regression line for predicting calories based on sugar content.

c. Interpret the value (not just the sign) of the slope of the fitted model in the context of this setting.

1.9 ***More breakfast cereal.*** Refer to the data on breakfast cereals in **Cereal** that is described in Exercise 1.8. The number of calories and number of grams of sugar per serving were measured for 36 breakfast cereals. The data are in the file **Cereal**. We are interested in trying to predict the calories using the sugar content.

a. How many calories would the fitted model predict for a cereal that has 10 grams of sugar?

b. Cheerios has 110 calories but just 1 gram of sugar. Find the residual for this data point.

c. Does the linear regression model appear to be a good summary of the relationship between calories and sugar content of breakfast cereals?

1.10 ***Sparrow residuals.*** Exercises 1.2–1.5 deal with a model for the weight (in grams) of sparrows using the wing length as a predictor and the data in **Sparrows**. Construct and interpret the following plots for the residuals of this model. In each case, discuss what the plot tells you about potential problems (if any) with the regression conditions.

a. Histogram of the residuals.

b. Normal probability plot of the residuals.

c. Scatterplot that includes the least squares line. Are there any obvious outliers or influential points in this plot?

1.11 ***Capacitor voltage.*** A capacitor was charged with a 9-volt battery and then a voltmeter recorded the voltage as the capacitor was discharged. Measurements were taken every 0.02 seconds. The data are in the file **Volts**.

a. Make a scatterplot with *Voltage* on the vertical axis versus *Time* on the horizontal axis. Comment on the pattern.

b. Transform *Voltage* using a log transformation and then plot log(*Voltage*) versus *Time*. Comment on the pattern.

c. Regress log(*Voltage*) on *Time* and write down the prediction equation.

d. Make a plot of residuals versus fitted values from the regression from part (c). Comment on the pattern.

1.12–1.15 ***Caterpillars.*** Student and faculty researchers at Kenyon College conducted numerous experiments with *Manduca Sexta* caterpillars to study biological growth.[10] A subset of the measurements from some of the experiments is in the file **Caterpillars**. Exercises 1.12–1.15 deal with some relationships in these data. The variables in the dataset include:

Variable	Description
Instar	A number from 1 (smallest) to 5 (largest) indicating stage of the caterpillar's life
ActiveFeeding	An indicator (Y or N) of whether or not the animal is actively feeding
Fgp	An indicator (Y or N) of whether or not the animal is in a free growth period
Mgp	An indicator (Y or N) of whether or not the animal is in a maximum growth period
Mass	Body mass of the animal in grams
Intake	Food intake in grams/day
WetFrass	Amount of frass (solid waste) produced by the animal in grams/day
DryFrass	Amount of frass, after drying, produced by the animal in grams/day
Cassim	CO_2 assimilation (ingestion–excretion)
Nfrass	Nitrogen in frass
Nassim	Nitrogen assimilation (ingestion–excretion)

Log (base 10) transformations are also provided as *LogMass*, *LogIntake*, *LogWetFrass*, *LogDryFrass*, *LogCassim*, *LogNfrass*, and *LogNassim*.

[10] We thank Professors Harry Itagaki, Drew Kerkhoff, Chris Gillen, Judy Holdener, and their students for sharing this data from research supported by NSF InSTaRs grant No. 0827208.

1.12 ***Caterpillar waste versus mass.*** We might expect that the amount of waste a caterpillar produces per day (*WetFrass*) is related to its size (*Mass*). Use the data in **Caterpillars** to examine this relationship as outlined below.

a. Produce a scatterplot for predicting *WetFrass* based on *Mass*. Comment on any patterns.

b. Produce a similar plot using the log (base 10) transformed variables, *LogWetFrass* versus *LogMass*. Again, comment on any patterns.

c. Would you prefer the plot in part (a) or part (b) to predict the amount of wet frass produced for caterpillars? Fit a linear regression model for the plot you chose and write down the prediction equation.

d. Add a plotting symbol for the grouping variable *Instar* to the scatterplot that you chose in (c). Does the linear trend appear consistent for all five stages of a caterpillar's life? (*Note:* We are not ready to fit more complicated models yet, but we will return to this experiment in Chapter 3.)

e. Repeat part (d) using plotting symbols (or colors) for the groups defined by the free growth period variable *Fgp*. Does the linear trend appear to be better when the caterpillars are in a free growth period? (Again, we are not ready to fit more complicated models, but we are looking at the plot for linear trend in the two groups.)

1.13 ***Caterpillar nitrogen assimilation versus mass.*** The *Nassim* variable in the **Caterpillars** dataset measures nitrogen assimilation, which might be associated with the size of the caterpillars as measured with *Mass*. Use the data to examine this relationship as outlined below.

a. Produce a scatterplot for predicting nitrogen assimilation (*Nassim*) based on *Mass*. Comment on any patterns.

b. Produce a similar plot using the log (base 10) transformed variables, *LogNassim* versus *LogMass*. Again, comment on any patterns.

c. Would you prefer the plot in part (a) or part (b) to predict the nitrogen assimilation of caterpillars with a linear model? Fit a linear regression model for the plot you chose and write down the prediction equation.

d. Add a plotting symbol for the grouping variable *Instar* to the scatterplot that you chose in (c). Does the linear trend appear consistent for all five stages of a caterpillar's life? (*Note:* We are not ready to fit more complicated models yet, but we will return to this experiment in Chapter 3.)

e. Repeat part (d) using plotting symbols (or colors) for the groups defined by the free growth period variable *Fgp*. Does the linear trend appear to be better when the caterpillars are in a free growth period? (Again, we are not ready to fit more complicated models, but we are looking at the plot for linear trend in the two groups.)

1.14 ***Caterpillar body mass and food intake.*** We might expect that larger caterpillars would consume more food. Use the data in **Caterpillars** to look at using food intake to predict *Mass* as outlined below.

a. Plot body mass (*Mass*) as the response variable versus food intake (*Intake*) as the explanatory variable. Comment on the pattern.

b. Plot the log (base 10) transformed variables, *LogMass* versus *LogIntake*. Again, comment on any patterns.

c. Do you think the linear model should be used to model either of the relationships in part (a) or (b)? Explain.

1.15 ***Caterpillar body mass and food intake—again.*** This exercise is similar to Exercise 1.14 except that we will reverse the rolls of predictor and response, using caterpillar size (*Mass*) to predict food intake (*Intake*) for the data in **Caterpillars**.

a. Plot *Intake* as the response variable versus *Mass* as the explanatory variable. Comment on the pattern.

b. Plot the log (base 10) transformed variables, *LogIntake* versus *LogMass*. Again, comment on any patterns.

c. Would you prefer the plot in part (a) or (b) to fit with a linear model? Fit a linear regression model for the plot you chose and write down the prediction equation.

d. Add plotting symbols (or colors) for the grouping variable *Instar* to the scatterplot that you chose in (c). Is a linear model more appropriate for this relationship during some of the stages of caterpillar development?

1.16 ***U.S. stamp prices.*** Historical prices[11] of U.S. stamps for mailing a letter weighing less than 1 ounce are provided in the file **USStamps**.

a. Plot *Price* (in cents) versus *Year* and comment on any patterns.

b. Regular increases in the postal rates started in 1958. Remove the first four observations from the dataset and fit a regression line for predicting *Price* from *Year*. What is the equation of the regression line?

c. Plot the regression line along with the prices from 1958 to 2012. Does the regression line appear to provide a good fit?

d. Analyze appropriate residual plots for the linear model relating stamp price and year. Are the conditions for the regression model met?

[11] The data were obtained from Wikipedia, the URL is http://en.wikipedia.org/wiki/History_of_United_States_postage_rates.

e. Identify any unusual residuals.

1.17 ***Enrollment in mathematics courses.*** Total enrollments in mathematics courses at a small liberal arts college[12] were obtained for each semester from Fall 2001 to Spring 2012. The academic year at this school consists of two semesters, with enrollment counts for *Fall* and *Spring* each year as shown in Table 1.4. The variable *AYear* indicates the year at the beginning of the academic year. The data are also provided in the file **MathEnrollment**.

AYear	Fall	Spring
2001	259	246
2002	301	206
2003	343	288
2004	307	215
2005	286	230
2006	273	247
2007	248	308
2008	292	271
2009	250	285
2010	278	286
2011	303	254

Table 1.4: *Math enrollments*

a. Plot the mathematics enrollment for each semester against time. Is the trend over time roughly the same for both semesters? Explain.

b. A faculty member in the Mathematics Department believes that the fall enrollment provides a very good predictor of the spring enrollment. Do you agree?

c. After examining a scatterplot with the least squares regression line for predicting spring enrollment from fall enrollment, two faculty members begin a debate about the possible influence of a particular point. Identify the point that the faculty members are concerned about.

d. Fit the least squares line for predicting math enrollment in the spring from math enrollment in the fall, with and without the point you identified in part (c). Would you tag this point as influential? Explain.

[12]The data were obtained from http://Registrar.Kenyon.edu on June 1, 2012.

1.18 ***Pines.*** The dataset **Pines** contains data from an experiment conducted by the Department of Biology at Kenyon College at a site near the campus in Gambier, Ohio.[13] In April 1990, student and faculty volunteers planted 1000 white pine (*Pinus strobes*) seedlings at the Brown Family Environmental Center. These seedlings were planted in two grids, distinguished by 10- and 15-foot spacings between the seedlings. Several variables, described below, were measured and recorded for each seedling over time.

Variable	Description
Row	Row number in pine plantation
Col	Column number in pine plantation
*Hgt*90	Tree height at time of planting (cm)
*Hgt*96	Tree height in September 1996 (cm)
*Diam*96	Tree trunk diameter in September 1996 (cm)
*Grow*96	Leader growth during 1996 (cm)
*Hgt*97	Tree height in September 1997 (cm)
*Diam*97	Tree trunk diameter in September 1997 (cm)
*Spread*97	Widest lateral spread in September 1997 (cm)
*Needles*97	Needle length in September 1997 (mm)
*Deer*95	Type of deer damage in September 1995: 0 = none, 1 = browsed
*Deer*97	Type of deer damage in September 1997: 0 = none, 1 = browsed
*Cover*95	Thorny cover in September 1995: 0 = none; 1 = some; 2 = moderate; 3 = lots
Fert	Indicator for fertilizer: 0 = no, 1 = yes
Spacing	Distance (in feet) between trees (10 or 15)

a. Construct a scatterplot to examine the relationship between the initial height in 1990 and the height in 1996. Comment on any relationship seen.

b. Fit a least squares line for predicting the height in 1996 from the initial height in 1990.

c. Are you satisfied with the fit of this simple linear model? Explain.

1.19 ***Pines: 1997 versus 1990.*** Refer to the **Pines** data described in Exercise 1.18. Examine the relationship between the initial seedling height and the height of the tree in 1997.

a. Construct a scatterplot to examine the relationship between the initial height in 1990 and the height in 1997. Comment on any relationship seen.

b. Fit a least squares line for predicting the height in 1997 from the initial height in 1990.

c. Are you satisfied with the fit of this simple linear model? Explain.

1.20 ***Pines: 1997 versus 1996.*** Refer to the **Pines** data described in Exercise 1.18. Consider fitting a line for predicting height in 1997 from height in 1996.

[13]Thanks to the Kenyon College Department of Biology for sharing these data.

a. Before doing any calculations, do you think that the height in 1996 will be a better predictor than the initial seedling height in 1990? Explain.

b. Fit a least squares line for predicting height in 1997 from height in 1996.

c. Does this simple linear regression model provide a good fit? Explain.

1.21 ***Caterpillar CO_2 assimilation and food intake.*** Refer to the data in **Caterpillars** that is described on page 57 for Exercises 1.12–1.15. Consider a linear model to predict CO_2 assimilation (*Cassim*) using food intake (*Intake*) for the caterpillars.

a. Plot *Cassim* versus *Intake* and comment on the pattern.

b. Find the least squares regression line for predicting CO_2 assimilation from food intake.

c. Are the conditions for inference met? Comment on the appropriate residual plots.

1.22 ***Caterpillar nitrogen assimilation and wet frass.*** Repeat the analysis described in Exercise 1.21 for a model to predict nitrogen assimilation (*Nassim*) based on the amount of solid waste (*WetFrass*) in the **Caterpillars** data.

1.23 ***Fluorescence experiment.*** Suzanne Rohrback used a novel approach in a series of experiments to examine calcium-binding proteins. The data from one experiment[14] are provided in **Fluorescence**. The variable *Calcium* is the log of the free calcium concentration and *ProteinProp* is the proportion of protein bound to calcium.

a. Find the regression line for predicting the proportion of protein bound to calcium from the transformed free calcium concentration.

b. What is the regression standard error?

c. Plot the regression line and all of the points on a scatterplot. Does the regression line appear to provide a good fit?

d. Analyze appropriate residual plots. Are the conditions for the regression model met?

1.24 ***Goldenrod galls.*** Biology students collected measurements on goldenrod galls at the Brown Family Environmental Center.[15] The file **Goldenrod** contains the gall diameter (in mm), stem diameter (in mm), wall thickness (in mm), and codes for the fate of the gall in 2003 and 2004.

a. Are stem diameter and gall diameter positively associated in 2003?

[14] Thanks to Suzanne Rohrback for providing these data from her honors experiments at Kenyon College.
[15] Thanks to the Kenyon College Department of Biology for sharing these data.

b. Plot wall thickness against stem diameter and gall diameter on two separate scatterplots for the 2003 data. Based on the scatterplots, which variable has a stronger linear association with wall thickness? Explain.

c. Fit a least squares regression line for predicting wall thickness from the variable with the strongest linear relationship in part (b).

d. Find the fitted value and residual for the first observation using the fitted model in (c).

e. What is the value of a typical residual for predicting wall thickness based on your linear model in part (c)?

1.25 ***More goldenrod galls.*** Refer to the data on goldenrod galls described in Exercise 1.24. Repeat the analysis in that exercise for the measurements made in 2004 instead of 2003. The value of *Wall04* is missing for the first observation, so use the second case for part (e).

Open-ended Exercises

1.26 ***Textbook prices.*** Two undergraduate students at Cal Poly took a random sample[16] of 30 textbooks from the campus bookstore in the fall of 2006. They recorded the price and number of pages in each book, in order to investigate the question of whether the number of pages can be used to predict price. Their data are stored in the file **TextPrices** and appear in Table 1.5.

Pages	Price	Pages	Price	Pages	Price
600	95.00	150	16.95	696	130.50
91	19.95	140	9.95	294	7.00
200	51.50	194	5.95	526	41.25
400	128.50	425	58.75	1060	169.75
521	96.00	51	6.50	502	71.25
315	48.50	930	70.75	590	82.25
800	146.75	57	4.25	336	12.95
800	92.00	900	115.25	816	127.00
600	19.50	746	158.00	356	41.50
488	85.50	104	6.50	248	31.00

Table 1.5: *Pages and price for textbooks*

a. Produce the relevant scatterplot to investigate the students' question. Comment on what the scatterplot reveals about the question.

b. Determine the equation of the regression line for predicting price from number of pages.

[16] *Source:* Cal Poly Student project.

c. Produce and examine relevant residual plots, and comment on what they reveal about whether the conditions for inference are met with these data.

1.27 ***Baseball game times.*** What factors can help to predict how long a Major League Baseball game will last? The data in Table 1.6 were collected at www.baseball-reference.com for the 15 games played on August 26, 2008, and stored in the file named **BaseballTimes**. The *Time* is recorded in minutes. *Runs* and *Pitchers* are totals for both teams combined. *Margin* is the difference between the winner's and loser's scores.

Game	League	Runs	Margin	Pitchers	Attendance	Time
CLE-DET	AL	14	6	6	38774	168
CHI-BAL	AL	11	5	5	15398	164
BOS-NYY	AL	10	4	11	55058	202
TOR-TAM	AL	8	4	10	13478	172
TEX-KC	AL	3	1	4	17004	151
OAK-LAA	AL	6	4	4	37431	133
MIN-SEA	AL	5	1	5	26292	151
CHI-PIT	NL	23	5	14	17929	239
LAD-WAS	NL	3	1	6	26110	156
FLA-ATL	NL	19	1	12	17539	211
CIN-HOU	NL	3	1	4	30395	147
MIL-STL	NL	12	12	9	41121	185
ARI-SD	NL	11	7	10	32104	164
COL-SF	NL	9	5	7	32695	180
NYM-PHI	NL	15	1	16	45204	317

Table 1.6: *Major League Baseball game times*

a. First, analyze the distribution of the response variable (*Time* in minutes) alone. Use a graphical display (dotplot, histogram, boxplot) as well as descriptive statistics. Describe the distribution. Also, identify the outlier (which game is it?) and suggest a possible explanation for it.

b. Examine scatterplots to investigate which of the quantitative predictor variables appears to be the best single predictor of time. Comment on what the scatterplots reveal.

c. Choose the one predictor variable that you consider to be the best predictor of time. Determine the regression equation for predicting time based on that predictor. Also, interpret the slope coefficient of this equation.

d. Analyze appropriate residual plots and comment on what they reveal about whether the conditions for inference appear to be met here.

1.28 ***Baseball game times (continued).*** Refer to the previous Exercise 1.27 on the playing time of baseball games.

a. Which game has the largest residual (in absolute value) for the model that you selected? Is this the same game that you identified as an outlier based on your analysis of the time variable alone?

b. Repeat the entire analysis from the previous exercise, with the outlier omitted.

c. Comment on the extent to which omitting the outlier changed the analysis and your conclusions.

1.29 ***Retirement SRA.*** A faculty member opened a supplemental retirement account in 1997 to investment money for retirement. Annual contributions were adjusted downward during sabbatical years in order to maintain a steady family income. The annual contributions are provided in the file **Retirement**.

a. Fit a simple linear regression model for predicting the amount of the annual contribution (SRA) using $Year$. Identify the two sabbatical years that have unusually low SRA residuals and compute the residual for each of those cases. Are the residuals for the two sabbatical years outliers? Provide graphical and numerical evidence to support your conclusion.

b. Sabbaticals occur infrequently and are typically viewed by faculty to be different from other years. Remove the two sabbatical years from the dataset and refit a linear model for predicting the amount of the annual contribution (SRA) using $Year$. Does this model provide a better fit for the annual contributions? Make appropriate graphical and numerical comparisons for the two models.

1.30 ***Metabolic rate of caterpillars.*** Marisa Stearns collected and analyzed body size and metabolic rates for Manduca Sexta caterpillars.[17] The data are in the file **MetabolicRate** and the variables are:

Variable	Description
Computer	Number of the computer used to obtain metabolic rate
BodySize	Size of the animal in grams
CO2	Carbon dioxide concentration in parts per million
Instar	Number from 1 (smallest) to 5 (largest) indicating stage of the caterpillar's life
Mrate	Metabolic rate

The dataset also has variables *LogBodySize* and *LogMrate* containing the logs (base 10) of the size and metabolic rate variables. The researchers would like to build a linear model to predict metabolic rate (either *Mrate* directly or on a log scale with *LogMrate*) using a measure of body size for the caterpillars (either *BodySize* directly or on a log scale with *LogBodySize*).

[17] We thank Professor Itagaki and his research students for sharing these data.

a. Which variables should they use as the response and predictor for the model? Support your choice with appropriate plots.

b. What metabolic rate does your fitted model from (a) predict for a caterpillar that has a body size of 1 gram?

1.31 ***More metabolic rate of caterpillars.*** Refer to Exercise 1.30 that considers linear models for predicting metabolic rates (either *Mrate* or *LogMrate*) for caterpillars using a measure of body size (either *BodySize* or *LogBodySize*) for the data in **MetabolicRate**. Produce a scatterplot for the model you chose in Exercise 1.30 and add a plotting symbol for the grouping variable *Instar* to show the different stages of development. Does the linear trend appear to be consistent across all five stages of a caterpillar's life? (*Note:* We are not ready to fit more complicated models yet, but we will return to this experiment in Chapter 3.)

Supplemental Exercises

1.32 ***Zero mean.*** One of the neat consequences of the least squares line is that the sample means $(\overline{x}, \overline{y})$ always lie on the line so that $\overline{y} = \hat{\beta}_0 + \hat{\beta}_1 \overline{x}$. From this, we can get an easy way to calculate the intercept if we know the two means and the slope:

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \cdot \overline{x}$$

We could use this formula to calculate an intercept for *any* slope, not just the one obtained by least squares estimation. See what happens if you try this. Pick any dataset with two quantitative variables, find the mean for both variables, and assign one to be the predictor and the other the response. Pick any slope you like and use the formula above to compute an intercept for your line. Find predicted values and then residuals for each of the data points using this line as a prediction equation. Compute the sample mean of your residuals. What do you notice?

CHAPTER 2

Inference for Simple Linear Regression

Recall that in Example 1.1 we considered a simple linear regression model to predict the price of used Porches based on mileage. How can we evaluate the effectiveness of this model? Are prices significantly related to mileage? How much of the variability in Porsche prices can we explain by knowing their mileages? If we are interested in a used Porsche with about 50,000 miles, how accurately can we predict its price?

In this chapter, we consider various aspects of inference based on a simple linear regression model. By inference, we mean methods such as confidence intervals and hypothesis tests that allow us to answer questions of interest about the population based on the data in our sample. Recall that the simple linear model is

$$Y = \beta_0 + \beta_1 X + \epsilon$$

where $\epsilon \sim N(0, \sigma_\epsilon)$. Many of the inference methods, such as those introduced in Section 2.1, deal with the slope parameter β_1. Note that if $\beta_1 = 0$ in the model, there is no linear relationship between the predictor and the response. In Section 2.2, we examine how the variability in the response can be partitioned into one part for the linear relationship and another part for variability due to the random error. This partitioning can be used to assess how well the model explains variability in the response. The correlation coefficient r is another way to measure the strength of linear association between two quantitative variables. In Section 2.3, we connect this idea of correlation to the assessment of a simple linear model. Finally, in Section 2.4, we consider two forms of intervals that are important for quantifying the accuracy of predictions based on a regression model.

2.1 Inference for Regression Slope

We saw in Example 1.3 on page 29 that the fitted regression model for the Porsche cars dataset is

$$\widehat{Price} = 71.1 - 0.589 \cdot Mileage$$